مجلة البحوث المالية والتجارية
المجلد (22) – العدد الثالث – يوليو 2021

Simple and Instantaneous Causality for Multivariate Auto-Regressive Models: An Application On Some Monetary Variables of the Egyptian Economy

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رابط المجلة: https://jsst.journals.ekb.eg/

ABSTRACT

Granger`s (1969) concept and definitions of causality, feedback, and instantaneous causality and Akaike final prediction error criterion and extended by Chan (1982) to fit a multivariate autoregressive model are used. The objective of the paper is to distinguish between simple, feedback and instantaneous causality. The notion of feedback between endogenous and exogenous variables in the bivariate AR model and its extension to the tri-variate AR models are presented. As an application to these causality notions, the paper aimed to reach the optimal lag structure in forecasting some monetary variables in the Egyptian economy from Q12005 to Q42018. Variables selected were "current deposits of local currency", "loan totals" and "quasi-money". The three variables were correlated, and each variable was used as an endogenous function of itself lagged and the other two variables as exogeneous. The study also aimed to test if prediction is improved if current values and previous values are used in the prediction equation. The simple causal model showed that a) "current deposits" is best predicted lagged 7, lag one for "Total Loans", and current value of " Quasi-Money"; b) "Total Loans" is best predicted using " Current deposit`s" current value, "Total Loans" lagged 6„ and " Quasi-Money" lagged 1; and c) "current Quasi-Money" is best predicted from the current value of " Current Deposits" and "Total Loans", and from " Quasi-Money" lagged 4. The instantaneous causal model showed that : a) "current deposits" is best predicted lagged 7, current values for "Total Loans", and current value of " Quasi-Money"; b) "Total Loans" is best predicted using " Current deposit`s" current value, "Total Loans" lagged 6„ and current value of " Quasi-Money"; and c) "current Quasi-Money" is best predicted from the " Current Deposits" lagged 2, and "Total Loans", lagged 1 and from " Quasi-Money" lagged 4.. The analysis showed that a one-way simple causal model exists from "loans total" to "current deposits of local currency", and from "quasi-money" to "loans total". Instantaneous causality and feedback occur between the three variables.

Keywords

Final Prediction Error (FPE); Autoregressive Modelling; Granger Causality, Lag Structure; Auto-Regressive Distributive Lag; ARDL; AIC criterion, Akaike criterion; Full model; Reduced Model; simple causal model; Dynamic model. Bivariate feedback, tri-variate feedback.

ملخص

اتبع هذا البحث طريقة تتابعية بهدف الوصول الي النوذج المبطأ وفترات التأخير المثلي . اعتمد البحث علي تعريف Granger للسببة البسيطة والتبادلية والفورية وعلي معيار خطأ التنبؤ النهائي الذي قدمه Akaike في عام 1969 وقام Chan et al في عام 1982 بتطويره ليشمل المتغيرات المتعددة. يهدف البحث التفرقة بين الأنواع المختلفة للسببية للمتغيرات الخارجة Exogeneous والمتغيرات الداخلة Endogenous في المتغيرات الثنائية وإمتدادها الي المتغيرات متعددة المتغيرات . تم التطبيق علي متغيرات نقدية من الإقتصاد المصري في السنوات من 2005 الي 2018 ، وشملت المتغيرات "الودائع الحالية للعملة المحلية " ، "جملة القروض" و " أشباه النقود". واتضح وجود ارتباطات بسيطة للثلاث متغيرات. استخدم كل متغير كمتغير داخل مبطأ والمتغيرات الآخرين كمتغيرين خارجيين . اهتمت الدراسة بمعرفة وجود أو عدم وجود علاقات تبادلية و فورية بين الثلاث متغيرات ، وهل يتحسن التقدير إذا تم استخدام فترات مبطأه أم لا ؟ وتوصلت الدراسة الي انه في العلاقات السببية البسيطة يمكن التنبؤ بحجم الودائع بسبع فترات تأخير في ذات المتغير وفترة تأخير واحدة في إجمالي القروض والقيمة الحالية لأشباه النقود . يمكن التنبؤ بقيمةإجمالي القروض الحالية من القيمة الحالية للودائع و ستة فترات تأخير في متغير " اجمالي القروض" وفترة تأخير واحدة في متغير " أشباه النقود ، ويمكن التنبؤ بمتغير " أشباه النقود" من القيم الحالية لمتغيري " حجم الودائع " و " إجمالي القروض " واربعة فترات إبطاء في متغير " أشباه النقود" . أما في العلاقات السببية الفورية ، يمكن التنبؤ بحجم الودائع بسبع فترات تأخير في ذات المتغير القيمة الحالية في متغيري " إجمالي القروض" و " أشباه النقود " ، ويمكن التنبؤ بقيمة "إجمالي القروض "الحالية من القيمة الحالية "حجم الودائع و ستة فترات تأخير في متغير " اجمالي القروض" ، وأيضا يمكن التنبؤ بمتغير " أشباه النقود" من قيم " حجم الودائع " بفترتين إبطاء و فترة إبطاء واحدة لمتغير " إجمالي القروض " واربعة فترات إبطاء في متغير " أشباه النقود" . توصلت الدراسة الي وجود علاقة سببية بسيطة في إتجاه واجد من إجمالي القروض الي حجم الودائع بالعملة المحلية ومن أشباه النقود الي إجمالي القروض، وايضا وجود علاقة سببية تبادلية فورية بين الثلاث متغيرات.

1. Introduction

Analysis of economic data has been approached with two different philosophies: that is of time series analysis and that is of classical econometrics. If the endogenous and exogenous variables are distinguishable, and if prior information on them is available then the economy could be represented by some dynamic simultaneous equations model. But if theories are inexact, the time series techniques have the advantage of avoiding spurious and false restrictions. However, it is common practice in time series to develop models having long lag structure or models with lags chosen arbitrarily. The most common type of structured infinite *distributed lag* model is the geometric lag, also known as the *Koyck* lag (Koyck,1954; **Gasparrini**,2014; and Lutkepohl,1980) . The distributed lag model is a dynamic model in which the regressor x on y occurs over time, and could be given (Romer, 2012) as:

$$y_t = \alpha + \beta(L)x_i + \mu_t = \alpha + + \sum_{s=0}^{\omega} \beta_s x_{t-s} + \mu_t \qquad (1-1)$$

Where μ_t is a stationary error term. Model (**1-1**) is estimated without having a firm idea about the optimal lag structure or the possibility of any feedback between the variables. When too lengthy lags are chosen, the prediction will suffer from shortage of degrees of freedom, multicollinearity, biased or at least inefficient estimates and from specification error (Fey and Jain; 1982). Several studies have discussed several alternative to determine lag structures; Sims (1977) argues the advantage of treating all variables as endogenous and estimated an unconstrained vector autoregressive model (VAR) model in the first stage and then formulated a hypothesis testing procedure in a second stage. Fey and Jain (1982) suggested the estimation of a sequence of AR models, beginning with one lag and continuing with a higher orders of AR process until the likelihood function for a given sample attains a maximum, however the resulting model was over-specified in the order of AR process. Auto Regressive Distributed Lag (ARDL) models were used in fields other than economics; Nothdurft and Engel, (2020) have used it to evaluate the effects of species mixing on productivity and climate-related resistance via tree-ring width measurements from sample cores; Hierarchical Distributed-Lag Models have been used by Baek et al (2020) to exploring varying geographic scale and magnitude in associations between the built environment and health; Geda and Kwong (2021) have used Bayesian inference approach to parameter estimation of distributed lag models for forecasting used product returns for remanufacturing. Heaton and Peng (2012) treat the maximum lag as an additional parameter and estimate it by sampling from its posterior distribution. Belloumi (2013) applied The ARDL model to study the relationship between trade, FDI and economic growth in Tunisia. Also ARDL models have been used in hydraulic data (Rushworth et al, 2013), in attributable risk (Gasparrini et al 2014, Aboubakri et al (2019) and in climate sensitivity (2020).

Akaike (1976) introduces the AIC criterion, which enables to compare increasing model parameters, and the optimal lag is chosen as the order of the model having the minimum value of AIC. This criterion minimizes the sum of squares of error after correction for the degrees of freedom. Hsiao (1979) used this criterion in autoregressive modeling of Canadian money and income data, variables used were M1, M2 and GNP. Fey and Jain (1982) apply this AIC criterion to univariate models of money supply (M1), income (GNP) and prices. Also, Fey and Jain (1982) applied this criterion to two bivariate models, one model for money and income, and the other for income and prices; Hsiao (1981) had applied Granger causality and AIC criterion to bivariate seasonally adjusted quarterly stock and nominal GNP from 1947 to 1977 using M1 and M2 as alternative measure of stock variables.

The purpose of this paper is to analyze and determine the lag structures in forecasting models. Specifically, to determine the optimal lag structure in forecasting current deposits of local currency, loans total (guaranteed and not guaranteed) and quasi-money when using the three above variables as endogenous function of the same variable lagged, and the other two variables as endogenous. The paper also aims to discover the existence or non-existence of causality and feedback among the variables of each data set and whether there is instantaneous causality (Granger, 1965), that is if a model that uses current, past and future values of x and current and past values of y to predict y has smaller forecast error than a model than only uses current and past values of x and current and past values of y.

Section 1, of this paper gives Granger`s causality and Granger`s definitions of causality, feedback and instantaneous causality, and the Akaike (1969) Final prediction error (FPE) criterion. Section 2 gives the strategy of fitting a multivariate AR model; Section 3 is the data analysis segment, that shows how to interpret results when fitting the multi-variate model to time series of variables selected for the analysis; and section 4 gives conclusions and suggestions for future work in multivariate fitting of AR models.

2. Granger Causality and definitions

In recent years Granger causality has received considerable attention and use in many areas of research. Granger causality is only relevant to time series variables. Given two time series variables: { X_t , Y_t } ; the variable $X"_t$ is "Granger cause Y_t" if Y_t can be better predicted using the historic of both X_t and Y_t than it by using Y_t alone. In such a system, each variable depends on all other variables. Granger (1969) gave a definition of causality that does not depend on economic laws (Caines and Chan, 1975; Pierce and Haugh, 1977; Sims, 1972). Causality is defined by Granger (1969) as "incremental predictability", that examines whether the forecasts of the future values of Y can be improved if the current and lagged values of X are taken into consideration.

Granger considered a stationary stochastic process and let: x_t $\{x_s : S < T\}$ i.e., represents the set of past values and $x_t{}^*$ $\{x_s : S \leq T\}$ i.e., contains the set of past and present values, and similarly y_t and y_t*.

Let $\sigma^2(y_t|Y)$is the mean squares of the error of predicting y_t given all information in the population of interest; Granger gave the following definitions:

Definition 1:

if $\sigma^2(y_t|Y,X) < \sigma^2(y_t|Y)$ then we say t is causing Y denoted as $X \Rightarrow Y$

Definition 2:

if $\sigma^2(y_t|Y,X) < \sigma^2(y_t|Y)$ and $\sigma^2(x_t|Y,X) < \sigma^2(x_t|X)$ then we say feedback is occurring, denoted $X \Leftrightarrow Y$

Definition 3

if $\sigma^2(y_t|Y,X^*) < \sigma^2(y_t|Y)$ where $x_t{}^*\{x_s : S \leq T\}$ we say instantaneous causality exists; meaning that current values of the endogeneous variable Y is better predicted when lagged endogenous variable, present and past exogeneous variable are included in the forecasting model.

3. Strategy for fitting a Multivariate Autoregressive Model

We assume a vector stationary time series Y consists of three components (X, Y, Z). Under general conditions a regular full rank stationary process X, Y, Z posses an autoregressive (AR) model as follows (Masani, 1966):

$$\begin{bmatrix} y_t \\ x_t \\ z_t \end{bmatrix} = \begin{bmatrix} \psi_{11}(L) & \psi_{12}(L) & \psi_{13}(L) \\ \psi_{21}(L) & \psi_{22}(L) & \psi_{23}(L) \\ \psi_{31}(L) & \psi_{32}(L) & \psi_{33}(L) \end{bmatrix} \begin{bmatrix} y_t \\ x_t \\ z_t \end{bmatrix} + \begin{bmatrix} \mu_t \\ \upsilon_t \\ \omega_t \end{bmatrix} \qquad (2-1)$$

Where:

(L) is the lag operator and $L_{y_t} = y_{t-1}$

$\psi_{ij}(L) = \sum_{i=1}^{M} \psi_{ij,L}$

$\mu_t, \upsilon_t, \omega_t$ *are white noise innovations with constant variance* Ω.

Least square can be applied to each equation of model (2-1), the estimated will be consistent, unbiased and asymptotically normally distributed; however, the model is extremely sensitive to the order of the chosen lags (Hsiao, 1979, a, b). To determine the lags (m) in ψ_{ij} Akaike (1969) gave FPE criterion to each equation. In the bivariate case as:

$$FPE_{y_t} = E(y_t - \hat{y}_t)^2 = SSE \qquad (2-2)$$

And $\hat{y}_t$ is the predicted value of y_t. In the tri-variate case, Chan et al (1982) gave a generalization of Akaike FPE criterion as follows:

$$FPE_{y_t}(m,n,r) = \frac{T+m+n+r-1}{T-m-n-r-1} \times \frac{SSE}{T} \qquad (2-3)$$

Where,

$$\hat{y}_t = A + \hat{\psi}_{11}{}^{m}(L)y_t + \hat{\psi}_{12}{}^{m}(L)x_t + \hat{\psi}_{13}{}^{m}(L)z_t \qquad (2-4)$$

and, m,n,r *denote the order of the lags in* $\psi_{11}(L)$, ψ_{12} and ψ_{13}

$\hat{\psi}_{11}{}^{m}$, $\hat{\psi}_{12}{}^{m}$, $\hat{\psi}_{13}{}^{m}$ *and* A are *the coefficients* when we treat the observations from (-m+1) to (0) as fixed, i.e., $\{t = -m + 1, \ldots..0,1,2 \ldots.T\}, m, n, r, \leq M$.

When $r = 0$ in Equation (2-3) the FPE criterion balances the risk of choosing a lower order and the risk of selecting a higher order when using the specifications that give the smallest FPE (Hsiao (1978). Shibata (1976) and Bhansali (1996) have derived the asymptotic distribution and reached that the probability of choosing too low order using (2-3) approaches zero very quickly as the sample size increases; and the probability of selecting too high an order does not approach zero, but it vanishes out quickly. Bhansali (1988) and Bhansali and Downham (1977) showed that the probability of selecting too low an order is not significant in finite samples, but the cost of over-fitting is less than the cost of under-fitting. Gweke and

Mess (1979), Quandt and Trussell (1979) agree that the criterion has good properties as a fundamental criterion to select the order of an AR process.

$In\ model\ (2-1)$ If we let every variable to influence every other variable, with the same lag length, then the number of parameters grows very large and exhausts the degrees of freedom. So, in a tri-variate case, when the order of lags = M, there will be $(M+1)^3$combinations of $\psi_{11}(L),\ \psi_{12}(L),\ \psi_{13}(L)$ for y_t alone. To reduce computational burden to less than 3M, Hsioa (1979, a) made use of Granger causality`s definitions of causality and feedback, suggested the following sequential procedure for fitting a multivariate AR process (in this section, variable is denoted * if variable is not in the equation). For the y equation, proceed as follows:

1. Consider that y is the only output of the system, determine the one-dimensional AR process for y, i.e., endogenous variable y as a function of lagged exogeneous variable y. Using FPE criterion (Equation 2-2), determine the order of the AR process that gives the smallest FPE, say that order is s.
2. Introduce the first manipulated variable, say X, now compute the FPE to determine the order of the lag of ψ_{12}, say m, taking into consideration the order of the lag operator of y, obtained in (1) above.
3. Now, compare the FPE criterion in (1) and (2) above, i.e., compare: $FPE_y(m,*,*)\ with\ FPE_y(m,n,*)$ as follows:
   a) If $FPE_y(m,*,*) < FPE_y(m,n,*)$
   $then\ X \nRightarrow y\ and\ thus\ \ \psi_{12}(L) = 0\ in\ Model\ (3-1)$
   b) If $FPE_y(m,*,*) > FPE_y(m,n,*)$
   $then\ X \Rightarrow y\ and\ thus\ \ \psi_{12}(L) = n\ im\ Model\ (3-1)$
4. Introduce Z as the second manipulated variable, use FPE criterion (3-3) to determine the order of ψ_{13}, say, r.
   Then compare: $FPE_y(m,n,*)\ \ with\ \ FPE_y(m,n,r)$ as follows:
   c) If $FPE_y(m,n,*) < FPE_y(m,n,r)$
   $then\ Z \nRightarrow y\ and\ thus\ \ \psi_{13}(L) = 0\ in\ Model\ (2-1)$
   d) If $FPE_y(m,n,*) > FPE_y(sm,n,r)$
   $then\ Z \Rightarrow y\ and\ thus\ \ \psi_{13}(L) = r\ in\ Model\ (2-1)$
5. Identify the equation $Y\ lagged\ m, X\ lagged\ n\ and\ Z\ laged\ r$
6. Repeat steps 1 to 5 for the X variables (and for the Z variable) treating the other two as manipulated variables.

4. Data Analysis

In this section, the above strategy is applied to a two data sets, where possible causation and feedback exist among the variables within each data set. The analysis aims to answer the following: a) what is the optimal lag structure for the AR process? b) do exogeneous variables cause simply or instantaneously the endogenous variable? C) is there feedback between the variables? And d) what is the best-chosen AR model for each variable within a set?

Variables include: Current Deposits of local currency (D), Loans Total (L), and quasi-money (Q). Data were obtained as quarterly data from 2001 I to 2018 IV (Central Bank of Egypt, Yearbooks 2001 to 2018). However, data from the years 2011 to 2014 were excluded from the analysis (Due to the economic instability during that period). Thus, data used for the analysis were for 56 quarters. Results were compiled from the output of R package, which was used for the analysis.

Data showed a seasonal pattern (not shown), and the seasonal variations showed increase over time. To free data from trend and seasonality variations, a logarithmic transformation was performed to the time series, and then the first difference of the logarithmic values is taken (Bowerman and Occonnell;1987, p.82).

3.1 Testing for simple Causality

The FPE of treating each variable as a one-dimensional AR is presented in Table 1, for a maximum lag being set at 10. The smallest FPE for "Current Deposits (D)", "Loans (L)" and "Quasi-Money (Q)" are 7, 6, and 4 respectively.

Table 1: FPE for a one-dimensional AR Process

Order of Lags	$D \times 10^{-3}$	$L \times 10^{-3}$	$Q \times 10^{-3}$
1	8.2659	8.6086	3.6437
2	8.4746	5.2033	3.5264
3	5.7870	5.2623	3.5744
4	6.0481	4.4710	3.3029*
5	5.7976	5.5433	3.5062
6	6.0552	4.1435*	3.5376
7	5.7140*	4.1924	3.4961
8	6.1538	4.3330	3.7324
9	6.4783	4.4082	3.9148
10	6.6525	4.6161	4.1989

$R^2(L)$	$R^2(7) = .7024$	$R^2(6) =..4923$	$R^2(4) = .4482$
Standard error	.0049	.0036	.0030

The intercorrelation of the three variables were:$r_{D,L} = .20$ $r_{D,Q} = -.13$ $r_{q,L} = .33$. Thus, the first manipulated variable is chosen to be the one with the strongest correlation coefficient. The FPE of the controlled variables are obtained by varying the order of lags from 1 to 10, orders that gives the smallest FPE are given in Table 2.

Table 2: The optimum lag of first manipulated variable

Controlled variable	Ist Manipulated variable	$FPE \times 10^{-3}$	R^2	T
D(7)	Q (1)	5.9413	.7037	48
L(6)	Q (1)	4.0723	.5233	49
Q(4)	L(1)	3.3394	.4643	51

Lags are denoted as: D for "current deposits", L for "Loans Total" and Q for "quasi-money) and variable not in the equation is denoted as "*". From Table 2, it is found that, adding " quasi-Money" (lag 1) to current deposits(Lag7) did not reduce the FPE, since $FPE(7,*,*) = 5.714$ $from\ Table\ 1\ and\ FPE\ (7,*\ ,1) = 5.9413\ from\ Table\ 2, and\ thus,\ FPE(7,*,*) < FPE\ (7,*\ ,1)$. Using Hsiao (1979a) along with Granger causality definition, we conclude that " Quasi-Money does not cause " Current Deposits", i.e. $Q \nRightarrow D$, and ψ_{13} in model (3-1) could be assumed to equal zero.

Also, since $FPE\ (*, 6\ ,1)\ from\ Table\ (2) is\ more\ than\ FPE\ (*\ , 6\ ,*)\ (Table\ 1)$,, then it is concluded that ., $L \Rightarrow Q$ and $\psi_{23} = 0$; same for the quasi money variable where $FPE\ (*\ ,1\ ,4)\ from\ Table\ (2) is\ more\ than\ FPE\ (*,*\ ,4)\ (Table\ 1)$, then it is concluded that , $Q \Rightarrow L$ and $\psi_{32} = 0$, and feedback exists between " Loans Totals" and " Quasi-money.

The FPE(s) of the controlled variable are then obtained, holding the autoregressive operator of the controlled variable and the first manipulated variable to the order of lags chosen in previous steps. The order that gives the smallest FPE of the three- dimensional model is presented in Table 3.

Table 3: Optimal lags of fitting the three-dimensional AR process.

Controlled variable	1st Manipulated variable	2nd manipulated variable	$FPE \times 10^{-3}$	R^2
D(7)	Q (1)	L(1)	5.8508	.7211
L(6)	Q (1)	D (1)	4.1854	.5307
Q(4)	L (1)	D (1)	3.1747	.4652

From Table 3, it is found that $FPE\ (7, 1\ ,1)$ is less than $FPE\ (7,*\ ,1)$ from Table 2, which means that "Loans total" causes " current deposits", , i.e., $L \Rightarrow D$, i.e., $\psi_{12} = 0$. Also, $FPE\ (1, 6\ ,1)$ is $> FPE(*\ ,6,1)$ which means that $D \nRightarrow L$. Also, $FPE\ (*, 1\ ,4)$ is $> FPE(1,1,4)$ which means that $D \nRightarrow Q$.

Similarly, comparing the FPE`s from Tables 1,2 and 3. For the other two controlled variables, it is found that:

$Quasi\ Money \Rightarrow Loan\ Totals,\qquad Current\ Deposits \nRightarrow Loan\ Totals$

$Loan\ Totals \nRightarrow Quasi\ Money\qquad Current\ Deposits \nRightarrow Quasi\ Money$

And thus: $\psi_{23} \neq 0\ ,\ \psi_{21} = \psi_{31} = \psi_{32} = 0$

But, since $L \Rightarrow D$ but $D \nRightarrow L$ then no feedback occurs between those two variables. Also, no feedback exists between "Current Deposits" and "Quasi-Money".

Comparing models using the R^2 values from Tables (1), (2), and (3) using the equation (*Draper and Smith ,1998):*

$$F_{(k_2-k_1),(T-k_2-1)} = \frac{\frac{R^2{}_{full} - R^2{}_{reduced}}{(k_2-k_1)}}{(1-R^2{}_{full})/(T-k_2-1} \qquad (3-1)$$

Where $R^2{}_{full}$ is the R^2 value from the full model and $R^2{}_{reduced}$ is the R^2 value from the reduced model; T, k_2 *and* k_1 are the number of observations, number of parameters in the full model and the number of parameters in the reduced model, respectively. Thus, we are comparing the following models:

Model 1: $D_t = a + \sum_{j=1}^{7} b_j \times D_{t-j}$ $\qquad R^2 = .7024$

Model 2 : $D_t = a + \sum_{j=1}^{7} b_j \times D_{t-j} + c \times Q_{t-1}$ $\qquad R^2 = .7037$

Model 3 : $D_t = a + \sum_{j=1}^{7} b_j \times D_{t-j} + c \times Q_{t-1} + d_t \times L_{t-1}$ $\qquad R^2 = .7211$

Model 4: $L_t = a + \sum_{j=1}^{6} d_j \times L_{t-j}$ $\qquad R^2 = .4923$

Model 5: $L_t = a + \sum_{j=1}^{6} d_j \times L_{t-j} + c \times Q_{t-1}$ $R^2 = .5233$

Model 6: $L_t = a + \sum_{j=1}^{6} d_j \times L_{t-j} + c \times Q_{t-1} + b \times CD_{t-1}$ $R^2 = .5307$

Model 7: $Q_t = a + \sum_{i=1}^{4} c_j \times Q_{t-j}$ $R^2 = .4482$

Model 8: $Q_t = a + \sum_{j=1}^{4} c_{j\times} Q_{t-j} + d \times L_{t-1}$ $R^2 =$ .4643

Model 9: $Q_t = a + \sum_{j=1}^{4} c_j \times Q_{t-j} + d \times L_{t-1} + b \times CD_{t-1}$ $R^2 = .4652$

And the resulted F-ratios using $(4-1)$ are as follows:

Model (1)	vs	Model (2)	F = .1625
Model (2)	vs	Model (3)	F = .2.32
Model (4)	vs	Model (5)	F = 2.40
Model (5)	vs	Model (6)	F = .616
Model (7)	vs	Model (8)	F = 1.27
Model (8)	vs	Model (9)	F = .0723

None of the obtained F-ratios is significant. Thus, if we had to use the F-ratios alone, we would have chosen model (1), (4) and (7) to represent a simple equation for each variable. However, since Hsiao (1979a) and Granger (1969) definitions are adopted in this paper, and following Masani`s (1966) definition , the following system of equations is identified:

$$\begin{bmatrix}(1-L)Log\,D\\(1-L)Log\,L\\(1-L)Log\,Q\end{bmatrix} = \begin{bmatrix}\psi^7{}_{11}(L) & \psi^1{}_{12}(L) & 0\\ 0 & \psi^6{}_{22}(L) & \psi^1{}_{23}(L)\\ 0 & 0 & \psi^4{}_{33}(L)\end{bmatrix}\begin{bmatrix}(1-L)Log\,D\\(1-L)Log\,L\\(1-L)Log\,Q\end{bmatrix} + \begin{bmatrix}\mu_t\\ \upsilon_t\\ \omega_t\end{bmatrix} \qquad (3-2)$$

Where: $\psi_{ij}{}^{k}$ *is the variable lagged k and* μ_t,υ_t *and* ω_t are residuals error for each time series.

Full, information estimates and their corresponding standard error (given in parenthesis below each estimate) for each of the single equation of the first difference of log (D), Log (L), and log(Q) are presented below(* denotes significance coefficients ($P_{value} < .05$)):

$$\hat{D}_t = -.006 - \underset{(1.66)}{.8662}\, D^*_{t-1} - \underset{(.2036)}{.9182}\, D^*_{t-2} - \underset{(.2292)}{1.0592}\, D^*_{t-3} - \underset{(.2640)}{.6573}\, D^*_{t-4}$$

$$-\underset{(.2283)}{.5296}\, D^*_{t-5} - \underset{(.2014)}{.2477}\, D^*_{t-6}) - \underset{(.1498)}{.2562}\, D^*_{t-7} + \underset{(.1787)}{.2716}\, L^*_{t-1}$$

$R^2 = .7211$ SE $.0048$ $T = 48$

$$\hat{L}_t = -.007 - \underset{(.1500)}{.8133}\, L^*_{t-1} - \underset{(.1843)}{.8311}\, L^*_{t-2} - \underset{(.2019)}{.6078}\, L^*_{t-3} - \underset{(.1988)}{.3340}\, L_{t-4} \ \underset{(.1559)}{-.2073} L_{t-5}$$

$$-\underset{(.1179)}{.1203}\, L_{t-6} - \underset{(.1455)}{.2241} Q_{t-1}$$

$R^2 = .5301 \quad SE \ .0035 \quad T = 49$

$$\hat{Q}_t = -.001 - \underset{(.1419)}{.8188}\, Q^*_{t-1} - \underset{(.1725)}{.4570}\, Q^*_{t-2} - \underset{(.1711)}{.3987}\, Q^*_{t-3} - \underset{(.1403)}{.3214}\, Q^*_{t-4}$$

$R^2 = .4652 \quad SE = .003 \quad T = 51$

3.2 Testing for Instantaneous Causality

Granger (1969) gave the definition of instantaneous causality as given in definition (3) above. In addition, Granger gave a representation for the bivariate case, which is extended and simplified to fit the three- variable case (D, L, Q) as follows:

$$d_t = \sum_{j=1}^{m} \alpha_j\, d_{t-j} + \sum_{j=1}^{n} \beta_j\, l_{t-j+1} + \sum_{j=1}^{r} \gamma_j\, q_{t-j+1} + \varepsilon_d$$

$$l_t = \sum_{j=1}^{m} \alpha_j\, d_{t-j+1} + \sum_{j=1}^{n} \beta_j\, l_{t-j} + \sum_{j=1}^{r} \gamma_j\, q_{t-j+1} + \varepsilon_l$$

$$q_t = \sum_{j=1}^{m} \alpha_j\, d_{t-j+1} + \sum_{j=1}^{n} \beta_j\, lx_{t-j+1} + \sum_{j=1}^{r} \gamma_j\, q_{t-j} + \varepsilon_q$$

Rerunning the same data to test for instaneous causality and feedback, Table 4 gives the FPE values for the optimal lags for the controlled variable and for the first manipulated variable (a two- dimensional AR Model).Table 5, gives The FPE values for the three-dimensional AR Model.

Table 4: Two-Dimensional Instantaneous causality AR Model the FPE for D, L and Q

Optimal Lags		**FPE * 10^{-3}**
Controlled Variable	1st Manipulated variable	
D (7)	Q (1)	5.5474
L (6)	Q (1)	3.8491
Q (4)	L (1)	3.0252

Comparing FPE's of Tables (1), (4) and (5) we find that:

for the Current deposits variable:

$$\text{FPE}\,(7,*,*) > FPE\,(7,1,*) thus \qquad L \Rightarrow D, i.e.\ \psi12 \neq 0,$$

$$FPE\,(7,1,*) > FPE\,(7,1,1) thus \quad Q \Rightarrow D, i.e.\ \psi13 \neq 0.$$

$For\ the\ Total\ Loans\ \ variable$:

$$Q \Rightarrow L\ and\ \ D \Rightarrow L, i.e.\ \psi21 \neq 0\ and\ \psi23 \neq 0$$

$For\ the\ Quasi - Money\ \ variable$:

$$L \Rightarrow Q\ and\ \ D \Rightarrow Q.$$

Table 5: Three-Dimensional Instantaneous causality AR Model the FPE for D, L and Q

Optimal Lags			**FPE * 10^{-3}**
Controlled Variable	**1st Manipulated variable**	**2nd Manipulated variable**	
D (7)	Q (1)	L (1)	5.1582
L (6)	Q (1)	D (1)	3.7239
Q (4)	L (1)	D (2)	2.999

Thus, instantaneous causality and feedback occur between the three variables. This is evident, also, when comparing the MSE resulted from the simple causal model (Table 3) and the instantaneous causal model (Table 5) as given in Table 6, for each of the three variables, where "0" means " Current values at time t", MSE for the instaneous causal model is less than that of the simple causal model (definition (3) above).

Table 6: Mean Squared Error for the Simple and Instaneous causal models.

Dep. Variable	Simple causal					Instaneous causal				
	m	n	r	R^2	MSE	m	n	r	R^2	MSE
Current Deposits	7	1	0	.7211	.0048	7	0	0	.7541	.00425
Total Loans	0	6	1	.5301	.0036	0	6	0	.5825	.0032
Quasi-Money	0	0	4	.4652	.0030	2	1	4	.5564	.0026

Table 6, shows that, the R^2 values are larger and MSE is smaller for the instantaneous model than that of the simple causal model. The previous conclusion agrees with Granger's view (1969, P.427):

> "whether or not a model involves some group of economic variables can be a simple causal model depends on what one considers to be the speed with which information flows through the economy and on the sampling period of the data used. It might be true that when quarterly data are used, for example, a simple causal model is not sufficient to explain the relationships between the variables".

The resulting full information instantaneous causal model equations (in first difference of the logs) and their corresponding standard error is given below each coefficient, and * denotes significance coefficients ($P_{value} <$ .05).

$$\hat{D}_t = -.0062 - \underset{(.1422)}{.8930} D^*_{t-1} - \underset{(.1901)}{.8935} D^*_{t-2} - \underset{(.2198)}{1.0548} D^*_{t-3} - \underset{(.2447)}{.7536} D^*_{t-4} - \underset{(.2147)}{.6269} D^*_{t-5}$$

$$-\underset{(.1858)}{.3159} D^*_{t-6} - \underset{(.1408)}{.2788} D^*_{t-7} + \underset{(.1381)}{.3549} L^*_t - \underset{(.1569)}{.3374} Q^*_t$$

$$R^2 = .7541 \qquad SE \ .0043 \qquad T = 48$$

$$\hat{L}_t = -.0061 + \underset{(.0472)}{.8133} D^*_t - \underset{(.1481)}{.7140} L^*_{t-1} - \underset{(.1754)}{.7628} L^*_{t-2} - \underset{(.1921)}{.5724} L^*_{t-3} - \underset{(.1900)}{.3496} L_{t-4} - \underset{(.1481)}{.2681} L_{t-5}$$

$$-\underset{(.1032)}{.1599} L_{t-6} + \underset{(.1270)}{.3302} Q_t$$

$$R^2 = .5825 \qquad SE = .0043 \qquad T = 49$$

$$\hat{Q}_t = -.0001 - \underset{(.1419)}{.1712} D^*_t - \underset{(.1725)}{.1263} D^*_{t-1} + \underset{(.1711)}{.2780} L^*_t - \underset{(.1403)}{.7181} Q^*_{t-1} - \underset{(.1613)}{.4596} Q^*_{t-2}$$

$$-\underset{(.1602)}{.4165} Q^*_{t-3} - \underset{(.1322)}{.3361} Q^*_{t-4}$$

$$R^2 = .5564 \qquad SE = .0026 \qquad T = 51$$

4. Conclusions

In this paper, we have tried to fit a multivariate autoregressive process that could be used as an initial step for model identification. The standard technique is to let every variable in the equation with the same length. The investigation of causality and feedback cut down on running an algorithm several times. Thus, if we know that the process involves a one-way causality or if the variables are unrelated, we could fit a univariate ARIMA model (Box and Jenkins, 1970). The multivariate approach is applied to a time series that consists of 56 quarters of some monetary variables of the Egyptian economy; these variables were Current deposits in local currency, Loan Total (guaranteed and unguaranteed), and Quasi-money. It is found that:

a) Current deposits of local currency is better predicted using it previous seven quarters volume, and "Loan Totals" lagged only one quarter. However, prediction is improved when current volume of: Loans Totals" and current quasi-money are added to the : current deposits of local currency" lagged 7 quarters.
b) Loans total are affected significantly by its previous 6 quarters volume and by "Quasi-money" volume of one previous quarter. However, prediction is improved when "current deposits" and "current "quasi-money" are included in the equation. "Loans Total ", i.e., no past values of these two exogeneous variables.
c) Quasi-money is uni-dimensional, prediction of current values is good from its four previous quarters values. However, prediction is improved when current and past values of "current deposits" and "Loans Total" are included in the prediction equation.
d) Instantaneous causality and instantaneous feedback occur among all three variables. A one-way simple causal relation exists from "Loans Total" to "current deposits of local currency" and from "quasi-money to "Loans total".

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