FIELDS AND CURRENTS OF A SPHERICAL REFLECTOR ILLUMINATED BY AN ELECTRIC DIPOLE

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ABSTRACT

A technique is presented for computing the induced currents and secondary fields produced when a spherical reflector is illuminated by an electric dipole. An integral equation for the induced current is formulated. The current is obtained in a series form. Each term of the series satifies the boundary and edge conditions on the reflector. Expressions for the secondary fields in the far-zone are also given, some numerical results are then presented.

I. INTRODUCTION

The problem of diffraction of electromagnetic waves incident on open metallic surfaces has been the subject of exhaustive research. Most of these studies concentrated on the parabolic reflector because of its wide range use in microwave applications, which is mainly due to its being an ideal collimating device. Some studies have also been carried out on the spherical reflector which, although being inferior to the parabolic reflector as a collimating device, is superior to the latter when scanning ability is considered.

In the majority of these studies the geometrical optics approximation method [5,6] was employed where approximate current distributions on the reflector's surface were used. These distributions do not satisfy boundary conditions on the edge of the reflector which consequently gave rise to suspicion in the accuracy of the obtained results. This suspicion arose mainly because the application of Maxwell's equations and its boundary conditions is insufficient to solve the diffraction problem at the edge of the reflector. This was remedied by Meixner whose method included the boundary conditions at the edge of the reflector [8] .

In this contribution, the actual current distribution on spherical reflector illuminated by an electric dipole is found in a convergent series form. The secondary field due to this current is also found using a method proposed by YA.Fel'D [1-3] .

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The results obtained from this research are discussed in the conclusion section of this contribution.

II. FORMULATION OF THE PROBLEM

Consider an open infinitely-thin ideally-conducting surface S, which is a part of a sphere of radius a, shown in Fig. (1). Let $\bar{E}$ and $\bar{H}$ be the electric and magnetic field components of a wave incident on S. This wave will induce an electric current on S with a surface density $\bar{K}$. A secondary electromagnetic field will be excited. This field satisfies the homogeneous Maxwell's equations outside S and the edge conditions (Meixner condition) on the contour $\mathcal{L}$. The total field $\bar{E}$ comprising the primary and secondary fields satisfies the radiation condition at infinity as well as the boundary conditions:

$$\bar{E}_t = \bar{E}^o_t + \bar{E}^s_t = 0 \quad \text{on} \quad S \tag{1}$$

The problem of determining the secondary electromagnetic field is known to have a unique solution. As a first step toward this end, let us convert S to a closed, sufficiently smooth surface S_o by means of a geometrical surface Σ. For simplicity, Σ is chosen such that $S_o = S + \Sigma$ is a sphere of radius a.

Let $\bar{K}$ be the unknown surface current we are looking for, and $\bar{E}(\bar{K};S)$ be the secondary field induced on S by the current $\bar{K}$. Also, let $\bar{K}_n$ be one of a family of auxiliary currents on S_o and $\bar{E}(\bar{K}_n;S_o)$ be the field on S_o due to the current. Then by Lorentz reciprocity theorem:

$$\int_S \bar{K}\ \bar{E}(\bar{K}_n;S_o)\, ds = \int_{S_o} \bar{K}_n\ \bar{E}(\bar{K};S)\, ds \tag{2}$$

The following notation will be made:

$$\bar{I} = \begin{cases} \bar{K} & \text{on} \quad S \\ \frac{1}{\rho_o}\bar{E}_t(\bar{K};S) & \text{on} \quad \Sigma \end{cases} \tag{3}$$

$$\bar{F}_n = \begin{cases} \bar{E}_t(\bar{K}_n\ ;\ S_o) & \text{on} \quad S \\ -\rho_o\bar{K}_n & \text{on} \quad \Sigma \end{cases} \tag{4}$$

where, ρ_o is the wave impedance in free space. Substituting equations (3) and (4) into equation (2), we get:

$$\int_{S_o} \bar{I}\ \bar{F}_n\, ds = -\int_S \bar{K}_n\ \bar{E}^o\, ds \tag{5}$$

The set of auxiliary currents $(\bar{K}_n)$ may be chosen arbitrarily. It is convenient, however, to take the natural modes of the sphere S_o as $\bar{K}_n$. Equation (5) is an integral equation in the unknown function $\bar{I}$. Solving this equation we can get the induced current on S and the field over the compelete surface Σ. The main advantage of the present method is that it combines the evaluation of the surface current and the aperture field in single problem.

III. MODAL SOLUTION OF THE INTEGRAL EQUATION

In the modal solution of the integral equation (5), the unknown function $\bar{I}$ is replaced by an appropriate series expansion. The basis functions of this expansion will be chosen so as to satisfy the boundary and edge conditions on S. In this way $\bar{I}$ is forced to satisfy these conditions. The problem of determining $\bar{I}$ can be regarded as one of finding a vector function which is an element of a vector space whose coordinates are the basis functions. To facilitate computation, it is recommended that this space be a Hilbert Space. Scalar product and norm in this space are defined by the following formulae:

$$(\bar{A} \ , \bar{I}) = \int_s \bar{A} \ (\bar{R} \ \bar{I})^* \ ds \qquad (6)$$

$$\|I\| = \int_s \left[\bar{I} \ (\bar{R} \ \bar{I})^* \ ds\right]^{1/2} \qquad (7)$$

where $\bar{R}$ is a linear operator chosen such that the axioms of Hilbert Space are satisfied and all the elements of $L^2_R(S_o)$ satisfy edge conditions for the current as one approaches $\mathcal{L}$ from the S side and for the electric field as one approaches from the Σ side. The asterisk denotes complex conjugate. It is convenient to take as the operator $\bar{R}$ a matrix having a diagonal form with positive elements on A and

$$\bar{R} = \begin{vmatrix} R_{11} & 0 \\ 0 & R_{22} \end{vmatrix} \qquad (8)$$

specific values of R_{11} and R_{22} will be given later.

In terms of the above notations, our fundamental equation (5) can be rewritten as follows:

$$(\bar{I} \ , \bar{R}^{-1} \ \bar{F}_n^*) = a_n \qquad (9)$$

where

$$a_n = - \int_s \bar{K}_n \ \bar{E}^o \ ds \qquad (10)$$

It can be shown that the set $\bar{R}^{-1} \ \bar{F}_n^*$ is a complete set in $L^2_R(S_o)$ (1). Hence this set can be used as basis functions to expand the function $\bar{I}$ belonging to $L^2_{R^{-1}}(S_o)$ in a uniformly convergent series. However, the set $\bar{R}^{-1} \ \bar{F}_n^*$ is in general not orthonormal. To simplify to evaluation of the expansion coefficients, we at first convert $\bar{R}^{-1} \ \bar{F}_n^*$ to an orthonormal family using the Gram-Schmidt process. The new basis functions $\bar{u}_m$ are found to be given by

$$\bar{u}_m = \sum_{n=1}^{m} a_n^m \bar{R}^{-1} \bar{F}_n^* \qquad m = 1,2,3,\dots \tag{11}$$

where a_n^m are constants given by:

$$a_n^m = \frac{A_{mn}}{\sqrt{\Gamma_{mm} - \sum_{h=1}^{m-1} |B_{mh}|^2}} \tag{12}$$

(note that $\sum_{h=1}^{m-1} |B_{mh}|^2 = 0$ when $n=m=1$, hence $a_1^1 = \frac{1}{\sqrt{\Gamma_{11}}}$)

where

$$A_{mn} = \begin{cases} 1 & \text{for } n = m \\ -\sum_{h=n}^{m-1} a_n^h B_{mh} & \text{for } n \quad m \end{cases} \tag{13}$$

$$B_{mh} = \sum_{p=1}^{h} a_p^{*h} \Gamma_{mp} \tag{14}$$

$$\Gamma_{mp} = (\bar{R}^{-1} \bar{F}_m^* , \bar{R}^{-1} \bar{F}_p^*) \tag{15}$$

Rewriting equation (9) in terms of $\bar{u}_m$ we get :

$$(\bar{I} , \bar{u}_m) = c_m$$

where c_m is the expansion coefficient of the function $\bar{I}$, and is given by:

$$c_m = \sum_{n=1}^{m} a_n^{*m} a_n \tag{16}$$

In terms of the complete orthonormal system $\bar{u}_m$

$$\bar{I} = \sum_{m=1}^{\infty} c_m \bar{u}_m \tag{17}$$

This series converges along the norm of the space $L_R^2(S_o)$ to some function in this space, and defines the current $\bar{K}$ on S and the tangential field $\bar{E}_t$ on .

IV. COMPUTATION OF SECONDARY FIELDS

The integral representation of the field due to the induced current $\bar{K}$ on S can be obtained by using the well-known relations of electromagnetic field theory:

$$\bar{E} = \sum_{m=1}^{\infty} c_m \bar{U}_m \tag{18}$$

where

$$\bar{U}_m = \frac{1}{i\epsilon\omega} \text{rot rot} \int_S \bar{u}_m(p) \frac{e^{-ik|p-q|}}{4\pi |p-q|} ds_p \tag{19}$$

and

$$\bar{H} = \frac{i}{\mu\omega} \text{rot } \bar{E} \tag{20}$$

where $|p-q|$ is the distance between the observation point q and the integration point p.

The far-zone electric field of the reflector is given to a good approximation by:

$$\bar{E} = \frac{-i(Ka)^2}{4\pi\epsilon\omega} \frac{e^{-ikr}}{r} \sum_{m=1}^{\infty} c_m \int_{S}^{2\pi}\int^{\pi} \bar{u}_m(p)\, e^{ika\cos\gamma} \sin\theta_p\, d\phi_p\, d\theta_p \tag{21}$$

where γ is the angle between the lines drawn from the origin of the coordinates to the points of observation and integration as in Fig. (1).

V. SPHERICAL REFLECTOR ILLUMINATED WITH AN ELECTRIC DIPOLE

In this section we consider the specific case of a spherical reflector illuminated with an electric dipole. The origin of the coordinates is taken to be at the center of the sphere. The dipole is situated at the focal point of the reflector and directed perpendicular to the axis. The focus is at a distance "a/2" from the origin as in Fig. (1). In this system of coordinates the electric (primary) field components of the dipole are given by:

$$\bar{E}^{o}_{\theta} = \frac{pk^3}{4\pi\epsilon} \frac{e^{-ikR}}{KR} \sin\phi \left[(1 - \frac{i}{KR})\cos\theta_1 \cos\theta_2 - \frac{2i}{KR}\sin\theta_1 \sin\theta_2 \right] \tag{22}$$

$$\bar{E}^{o}_{\phi} = \frac{pk^3}{4\pi\epsilon} \frac{e^{-ikR}}{KR} \cos\phi \,(1 - \frac{i}{KR}) \tag{23}$$

where

$$R = \sqrt{(\frac{a}{2})^2 + r^2 + ra\cos\theta} \tag{24}$$

$$\cos\theta_1 = \frac{a/2 + r\cos\theta}{R}, \quad \cos\theta_2 = \frac{r + a/2\cos\theta}{R}$$
$$\sin\theta_1 = \frac{r\sin\theta}{R}, \quad \sin\theta_2 = \frac{a/2\sin\theta}{R} \tag{25}$$

and : P is the electric dipole moment, ϵ dielectric constant and $K = 2\pi/\lambda$.

As mentioned above, the current $\bar{K}_n$ and the associated tangential field $\bar{E}_t(\bar{K}_n; S_o)$ are the normal modes of a spherical surface:

$$\bar{K}_n = \begin{cases} i_\theta \sin\phi \dfrac{dp^1_{n+1/2}}{d\theta} + i_\phi \cos\phi \dfrac{p^1_{n+1/2}}{\sin\theta} & n=1,3,5,\dots \\ i_\theta \sin\phi \dfrac{p^1_{n/2}}{\sin\theta} + i_\phi \cos\phi \dfrac{dp^1_{n/2}}{d\theta} & n=2,4,6,\dots \end{cases} \tag{26}$$

and

$$\bar{E}_t(\bar{K}_n;S_o) = -\rho_o \Psi'_{n+1/2}(Ka)\, \mathcal{E}'_{n+1/2}(Ka)\left[i_\theta \sin\phi \frac{dp^1_{n+1/2}}{d\theta} + i_\phi \cos\phi \frac{p^1_{n+1/2}}{\sin\theta}\right] \qquad n=1,3,5,\dots$$

$$= -\rho_o \Psi'_{n/2}(Ka)\, \mathcal{E}_{n/2}(Ka)\left[i_\theta \sin\phi \frac{p^1_{n/2}}{\sin\theta} + i_\phi \cos\phi \frac{dp^1_{n/2}}{d\theta}\right] \qquad n=2,4,6,\dots \tag{27}$$

where $p^1_n(\cos\theta)$ is the first associated Legendre function

$$\mathcal{E}_n(x) = \sqrt{\frac{\pi x}{2}}\; H^{(2)}_{n+\frac{1}{2}}(x)$$

$$\Psi_n(x) = \sqrt{\frac{\pi x}{2}}\; J_{n+\frac{1}{2}}(x)$$

$\mathcal{E}'_n(x)$ and $\Psi'_n(x)$ are the first derivatives of $\mathcal{E}_n(x)$ and $\Psi_n(x)$, respectively;

and ρ_o is the characteristic impedance of the medium.

The matrix $\bar{R}$ is taken as;

$$R = \begin{vmatrix} R_{11} & 0 \\ 0 & R_{22} \end{vmatrix} = \begin{vmatrix} (\cos\beta - \cos\theta)^{-0.5} & 0 \\ 0 & (\cos\beta - \cos\theta)^{0.5} \end{vmatrix} \quad \text{on } S$$

$$= \begin{vmatrix} (\cos\theta - \cos\beta)^{0.5} & 0 \\ 0 & 1 \end{vmatrix} \quad \text{on } \Sigma \tag{28}$$

With this choice of R_{11} and R_{22}, every term of the expansion of equation (17) will satisfy Meixner's current conditions on the contour $\mathcal{L}$ (the current component which is parallel to $\mathcal{L}$ approaches infinity as $\rho^{-0.5}$ and the current component which is perpendicular to $\mathcal{L}$ approaches zero as $\rho^{0.5}$ when ρ approaches zero, where ρ is the distance to $\mathcal{L}$)[7].

The steps of computations are as follows:

1) The values of Γ_{mp} are computed using equation (15)

ii) The value of a_1^1 is obtained from equation (12)
iii) B_{mp} is computed using equation (14)
iv) A_{mp} is computed using equation (13)

Another a_n^m is obtained from equation (12) and the above steps are repeated until all the coefficients a_n^m are evaluated. These coefficients are then used to compute the expansion coefficient c_m using equation (16). In This way the current induced on the reflector is obtained in a series form. The aperture field is obtained when equation (17) is applied to the complementary surface Σ. The far-zone secondary fields are evaluated from equation (21).

VI. RESULTS AND DISCUSSION

Fig. (2) shows the results of computing the normalized current density for Ka = 37. It is to be noted that the curves of Fig. (2) are plotted only for factors which depend on θ. The dependence on ϕ, which is of theform $\sin\phi$ and $\cos\phi$ has been discarded. It is clear from the above - mentioned curves that current flows mainly near the apex of the reflector. The θ-component of the current vanishes near the edge of the reflector, while the ϕ-componet increases infinitely in a very thin region. We have plotted, in the same figure, the current density $\bar{J}=2\,\bar{n} \times \bar{H}$ used in geometrical optics approximation method. Evidently this current cannot satisfy the edge conditions. However, the θ-component of this current does not deviate too much from a θ-component of the true current calculated by the present method. On the other hand, the deviation between the θ-components of the two currents is large. In our opinion, this deviation explains why the geometrical optics does not give accourate results for the diffracted field in the shadow region. Although some attempts have been made to improve the accuracy of this approximation by adding ring currents on the rim of the reflector [5], it is diffecult to find a physical interpretation for this additional current. We expect that the true currents calculated by the present method will give the exact field distribution in all regions of space. Fig. (3) shows the results of computing the secondary field for the present case of Ka = 37. The total field is obtained by adding the primary field of the feeder to the secondary field shown in the figure. In the front region the contribution of the primary field to the total field may be neglected since the feeder is oriented such that its main beam is directed toward the reflector. In the shadow region and near the axis of the reflector, the primary and secondary fields add almost out of phase. In the case of a parabolic reflector, of approximately the same size as the present spherical reflector, the field at $\theta = \pi$ (on the axis behind the reflector) is found to be (-40 db) relative to the main beam [3].

Another advantage of the present method is that it gives directly the tangential field over the complementary surface using equation (17). The results of these fields are shown in figures (4,5). These results give an estimation of the fields in the near zone of the reflector. They can also be used to evaluate the far zone fields using appropriate integral relations using equation (7) .

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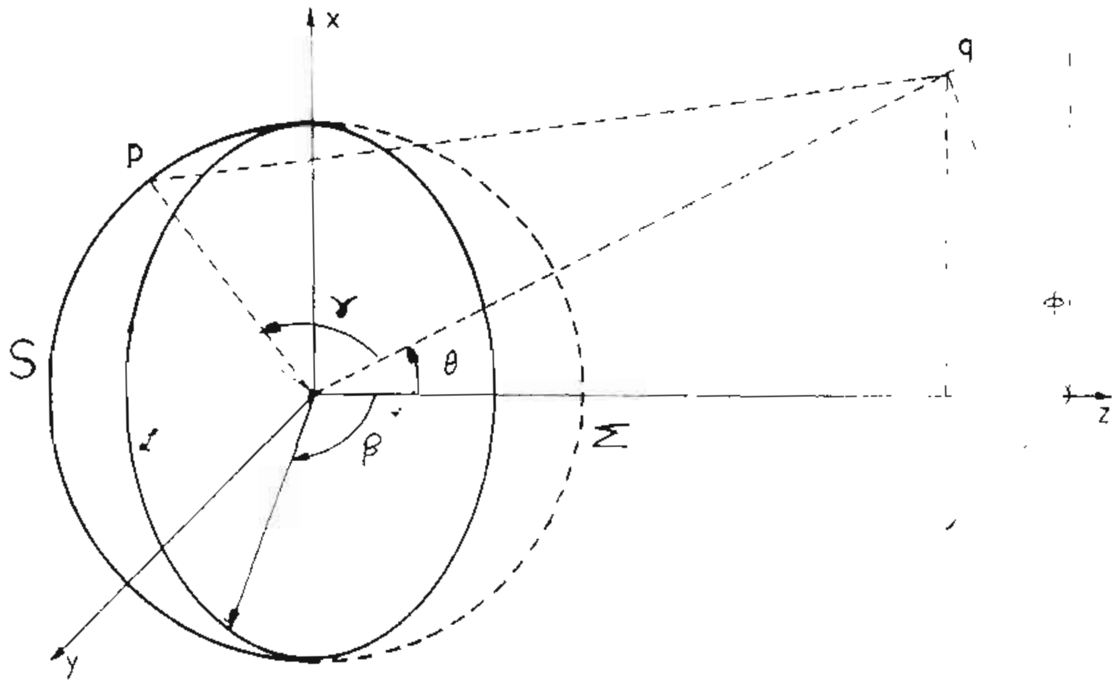
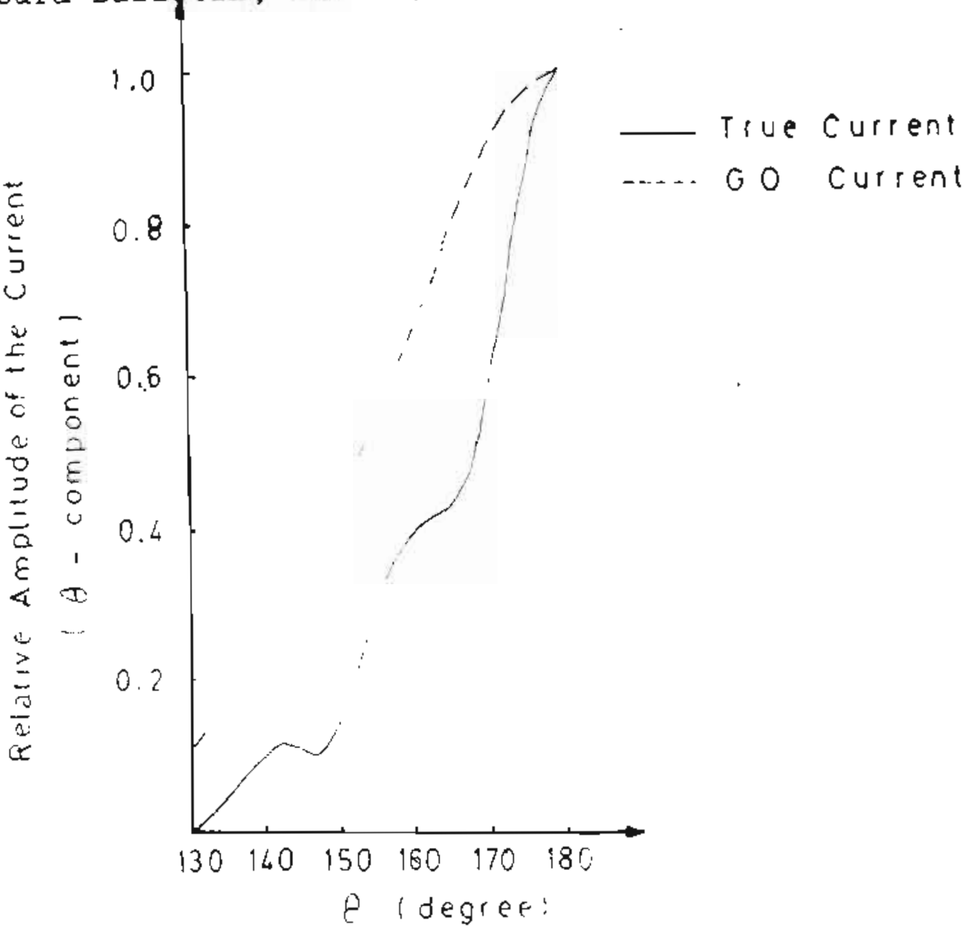


Fig.(1) Spherical Reflector



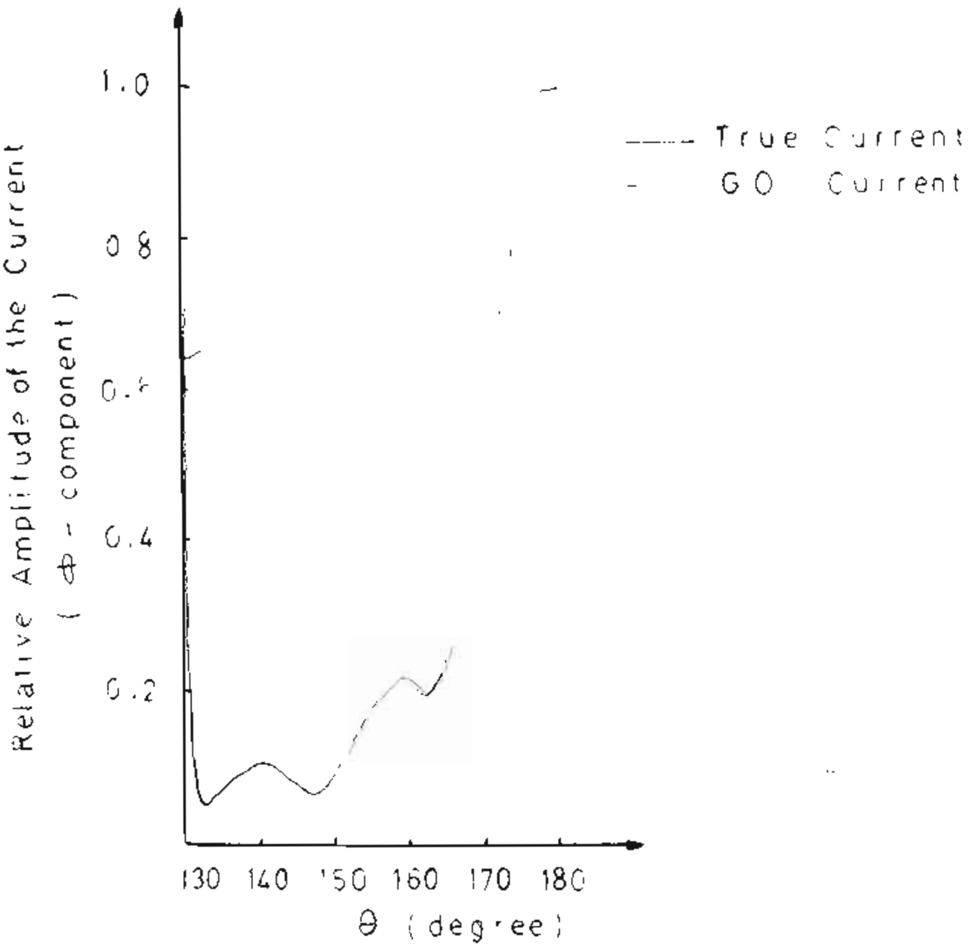


Fig.(2) The induced surface current distribution
$\beta = 130^{\circ}$, $ka = 37$

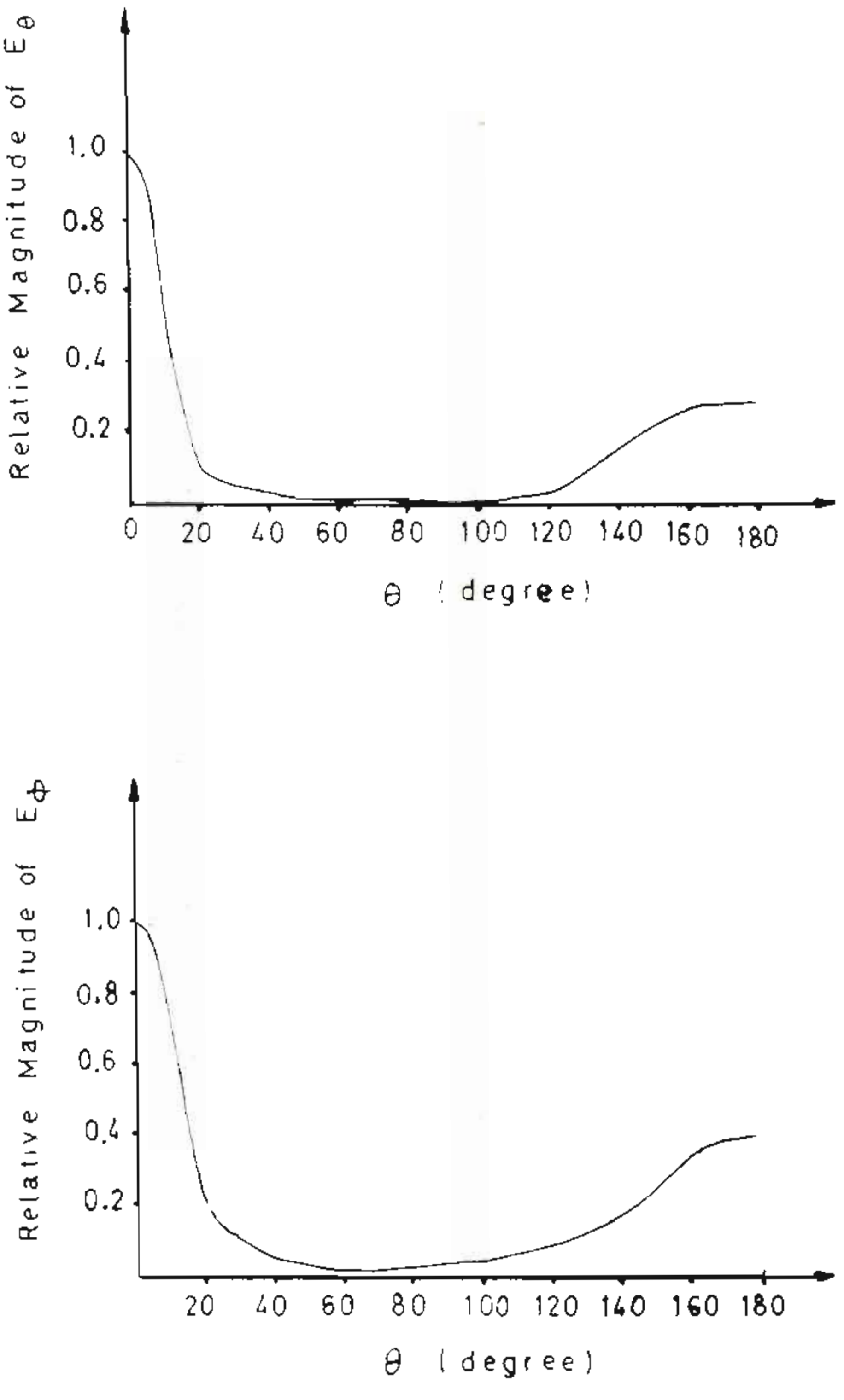


Fig (3 -) The normalized components of the radiation pattern; $\beta=130$, ka=37

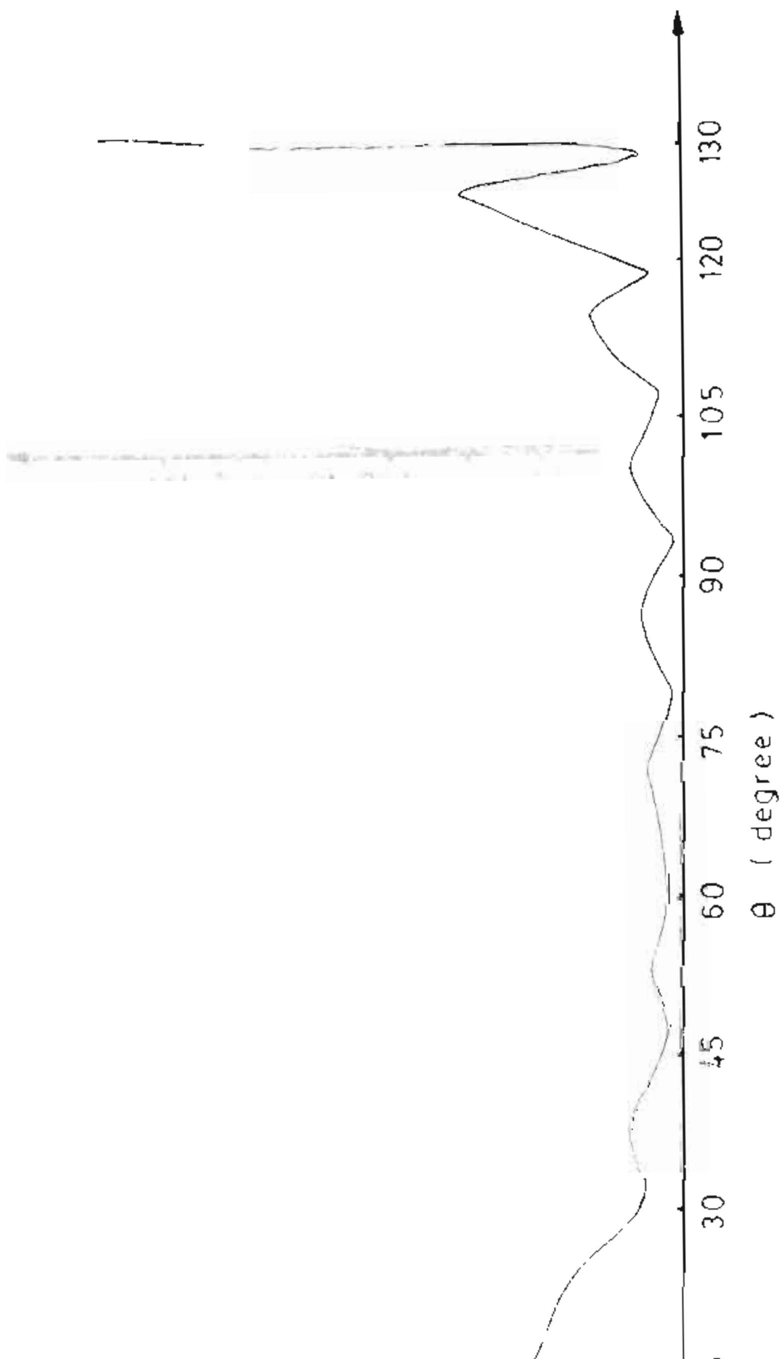


Fig (4) Θ -component of normalized aperture field ; β=130 , ka=37

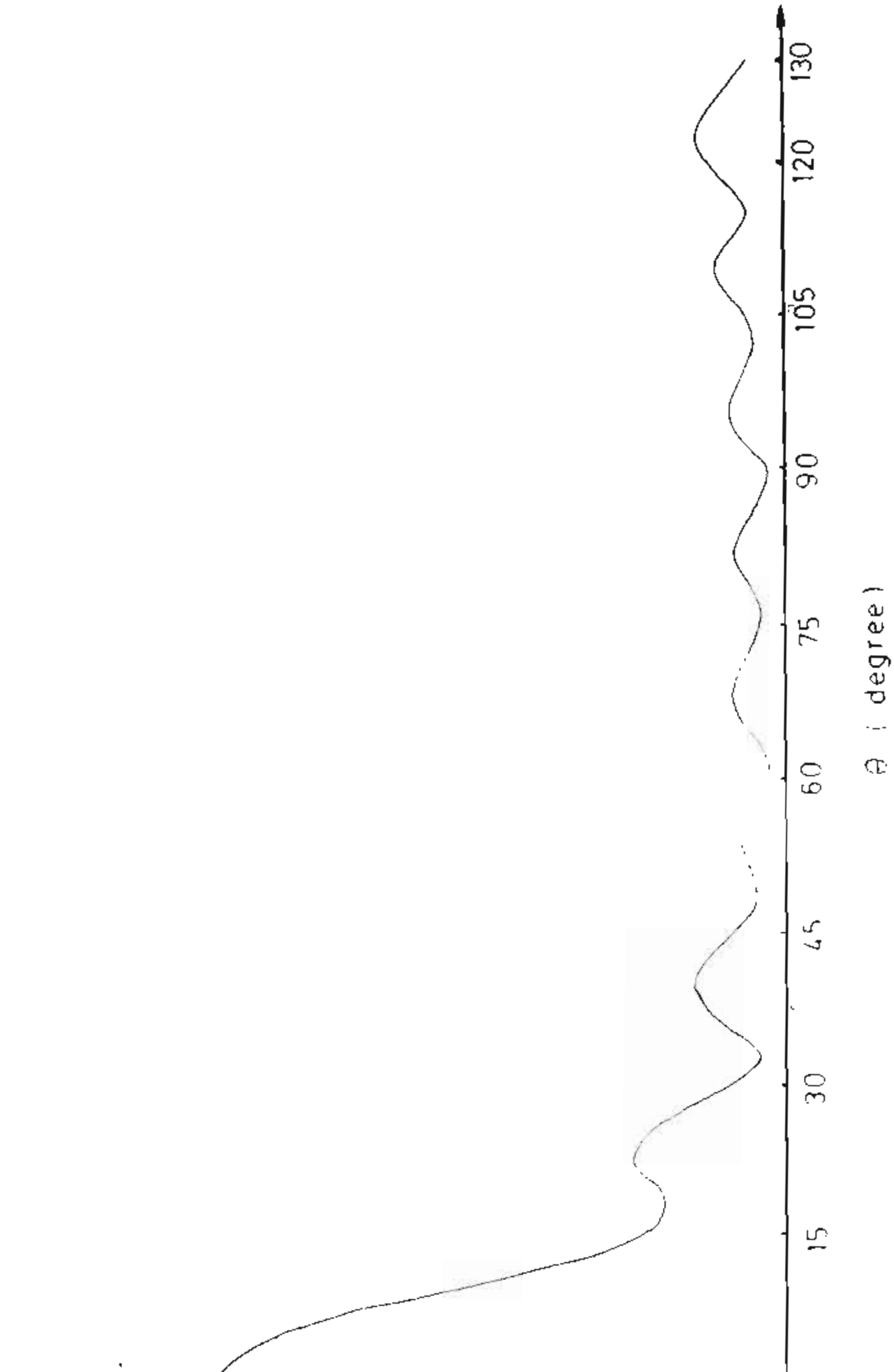


Fig.(5) ϕ-component of normalized aperture field; β=130, ka=37