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EFFECTIVE STARTING METHODS FOR THE EVALUATION OF INITIAL ESTIMATES FOR NEWTON-RAPHSON LOAD FLOWS

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Abstract- Load flow calculations are performed on any electric energy system to enable system engineers to satisfy the needs of the society from electric energy, and are necessary in making projects and research work connected with system planning and development.

The most popular and widely used method for these calculations is the Newton-Raphson method which is advantageous to other methods by its quadratic convergence property. There are cases, however, where the Newton-Raphson method is divergent. One of the main reasons for this divergence is its high sensitivity to the initial estimates of the unknowns.

This paper presents two starting methods for the evaluation of the initial estimates for Newton-Raphson load flows. The initial estimates calculated by these methods are found to guarantee a reliable convergence and increased speed of solution by the Newton-Raphson method. The reduction in computer time has amounted to as high as 50% of the time without starting methods.

1. INTRODUCTION

Load flow calculations are performed on any electric energy system to enable system engineers at the dispatching center to satisfy the needs of the society from electric energy. They are also necessary in making projects and research work connected with system planning, expansion, and development. For these reasons, load flow is the most often carried out of routine computer power network analyses.

The earliest known practical approach to load flow calculations was the Gauss-Seidal iterative method [1] using the system nodal admittance matrix. This method was well-suite to the early generation of computers since it requires minimal computer storage. Although it performs satisfactorily on many problems, it converges slowly; and too often not at all. The incentive to overcome this deficiency led to the development of many other methods, most popular of which is the Newton-Raphson (N-R) method [2-4]. Since then , mathematicians and engineers have been involved in intensive developments to this method to improve its reliabilty, speed, and computation ability [5-9].

With its quadratic convergence ability, there are cases however where the N-R method is divergent. One of the main reasons for divergence, or for slowing down the convergence, is its high sensitivity to the values of the initial estimates of the unknown quantities, namely, busbar voltage moduli and phase angle values. For instance, if these estimates are outside "the area of sol uti on gravity" the convergence is not ensured [10]. The importance of good initial estimates has also been stressed recently [8].

Some programs perform one or two Guass-Seidal iterations before the N-R process [2]. This is beneficial only for cases which could be comnpletely solved by the relatively weak Gauss-Seidel method. Other programs may use the d.c. load flow for calculating the initial angular estimates [4] and a similar technique for the voltage magnitude initial estimates [11]. Authors' experience has shown that, the initial voltage estimates calculated by this process are of values which are very close to the nominal voltage moduli contradicting thereby the idea about the necessity of the starting methods and in the majority of study cases, this process has contributed only slightly in reducing the number of iterations executed by the main N-R algorithm.

This paper presents two starting methods developed to guarantee a reliable convergence and increased speed of calculation by the N-R load flows. They are superior to previously known processes [2,11] in that; they fulfil two main requirements:

a- Simplicity of the algorithm which is demonstrated in the smaller number of calculating operations in comparison with those of one N-R iteration cycle; and

b- obtaining initial estimates that ensure a reliable solution with appreciably smaller number of iterations for the main N-R process.

In either method, the mathematical expressions for evaluating the initial angular estimates are first set up. These will then be used as variables in setting up the mathematical formulations for the calculation of the voltage magnitudes initial estimates.

2. MATHEMATICAL FORMULATIONS FOR THE DEVELOPED STARTING METHODS

2.1 First Method:

Consider a vector C whose elements are constructed by the specified powers, P^{sp}, and the nominal voltage values, V_{nom}, i.e. for busbar i;

$$C_i = P_i^{sp} / V_{i_{nom}}$$

Then, the initial angular estimates, Θ^{ie}, can easily be determined from the base-case load flow matrix equation

$$C = D \cdot \Theta^{ie}$$

i.e. $$\Theta^{ie} = D^{-1} \cdot C \qquad \ldots\ldots (1)$$

Matrix D represents constant approximations to the slopes of the tangent hyperplanes of the vector C. Its elements are given by

$$D_{ik} = -1 / X_{ik} \qquad \text{for } i \neq k$$

and $$D_{ii} = \sum_{k \in i} 1 / X_{ik} \qquad \ldots\ldots (2)$$

where X_{ik} is the reactance of the branch connecting bus i to bus k ; and

$k \in i$ denotes a bus k directly connected to bus i.

Supercripts ^{ie} and ^{sp} denote initial estimate and specified value, respectively.

In effect, the matrix D is very close to being the Jacobian submatrix J_1 in the well known linearised system of equations

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \Theta \\ \Delta V \end{bmatrix} \qquad \ldots\ldots (3)$$

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but evaluated at system no-load.

The part of equation (3) regarding the reactive power mismatch is used for developing the algorithm for calculating the initial estimates of the busbar voltage moduli. With some tolerance, this may be written in the form

$$(\Delta Q / V) - (J_3 / V) \cdot \Delta\Theta = M \cdot V \qquad \ldots\ldots (4)$$

The elements of the matrix M are derived from those of J_4 after the division by V and using the approximations

$$\cos\Theta_{ik} = \cos(\Theta_i - \Theta_k) = 1 \; ;$$

$$Q_i / V_i \ll V_i B_{ii} \quad ; \text{ and}$$

$$G_{ik} \sin\Theta_{ik} \ll B_{ik}$$

which are in line with, and much more reflect the actual operating conditions in an electric energy system when compared with those employed in Ref. [1]. G_{ik} and B_{ik} are the elements of the system nodal admittance matrix $Y = G + jB$. Hence, the elements of M are:

$$M_{ik} = -B_{ik} \; , \text{ for } i \neq k$$

and

$$M_{ii} = \sum_{k \in i} B_{ik}$$

which clearly indicate that M is effectively the imaginary parrt of Y.

Detailed study of equation (4) with the simultaneous consideration of Θ^{ie} in calculating the terms $(\Delta Q / V)$ and (J_3 / V) will result in rewriting it in the form:

$$E - (J_3 / V^o) \cdot \Theta^{ie} = M \cdot \Delta V \qquad \ldots\ldots (5)$$

where E is a column vector whose elements are defined by

$$E_i = Q_i^{sp} / V_i^o - V_i^o B_{ii} + \sum_{k \in i} V_k^o (B_{ik} \cos\Theta_{ik}^{ie} + G_{ik} \sin\Theta_{ik}^{ie}) \qquad \ldots\ldots (6)$$

J_3 / V^o is a square, highly sparse matrix with the elements

$$J_{3ik} / V_i^o = V_k^o (G_{ik} \cos\Theta_{ik}^{ie} - B_{ik} \sin\Theta_{ik}^{ie}) \; , \; i \neq k \; ; \text{ and}$$

$$J_{3ii} / V_i^o = \sum_{k \in i} V_k^o (B_{ik} \sin\Theta_{ik}^{ie} - G_{ik} \cos\Theta_{ik}^{ie}) \qquad \ldots\ldots (7)$$

Solving equation (5) for ΔV, the initial estimate for voltage magnitude at bus i, is

$$V_i^{ie} = V_i^o + \Delta V_i \qquad \ldots\ldots (8)$$

The superscript ^{o} denotes an input value given to the starting algorithm.

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It should be noted that the presence of the correction term (J_3 / V^0) . Θ^{ie} in the left-hand side of equation (5) does not represent a calculating difficulty since its elements are similar to those of E and can be calculated simultaneously with them.

2.2 Second Method:

In this method, the initial angular estimates are calculated from the equation

$$\Theta^{ie} = D^{-1} \cdot F \qquad \ldots\ldots (9)$$

which differs from equation (1) in one respect: the elements of the vector F are closely related to those of $\Delta P / V$ than those of C. Whence, for bus i

$$F_i = P_i^{sp} / V_i^0 - G_{ii} V_i^0 + N_i \qquad \ldots\ldots (10)$$

where

$$N_i = \sum_{k \in i} G_{ik} V_k^0 \; ;$$

and D is as already defined by (2).

Equation (9) is best applied where load flow calculations are performed for the first time. If however the results of a previous solution are availab, they are advised to be used as the elements of the vector Θ^0 in the equation

$$\Theta^{ie} = \Theta^0 + \Delta\Theta \qquad \ldots\ldots (11)$$

where the correction terms are determined from

$$\Delta\Theta = D^{-1} \cdot F \qquad \ldots\ldots (12)$$

Here the vector F is effectively $\Delta P / V$, i.e. as defined by equation (10) with N_i as

$$N_i = \sum_{k \in i} V_k^0 (G_{ik} \cos \Theta_{ik}^0 - B_{ik} \sin \Theta_{ik}^0)$$

The initial estimates of the voltage magnitudes are to be calculated as for the first method, using equations (5) and (8).

The solutions to the system of equations (1), (5), (9) and (12) are obtained by an exact method with optimal ordered elimination of the unknowns and taking into cosideration the sparsity of matrices D and M. The calculation process involved is found to be less than that necessary for only one iteration cycle of the main N-R algorithm.

3. LOAD FLOW TESTS

The effectiveness of the starting methods presented above has been investigated by solving many load flow problems, using the polar power mismatch version of the N-R method for the main algorithm, with and without starting processes. Included also in the investigation, for comparison purposes, the starting method previously presented by B. Stott [11]. The comparison between the various starting methods is based on the number of iteration cycles executed by the main algorithm for solving each problem

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with a predetermined accuracy of 0.05 MW and MVAr. The results are listed in Table.I and show that, in general, a starting process leads to a reduction in the number of iteration cycles. The starting processes developed in this paper, however, are advantageous to that due to B. Stott for two reasons:

(i) a reduction in the number of iteration cycles is always ensured, which is not the case for Stott's process; and

(ii) the resulting reduction is generally greater.

Table I. Number of iteration cycles for load flow solutions with and without starting methods.

Test prob. No.	No. of nodes	No. of branches	Number of iteration cycles			
			N-R without starting methods	N-R method with starting methods		
				B. Stott's Ref. [11]	Developed methods	
					First	Second
1	4	3	3	2	2	2
2	5	7	6	5	5	5
3	11	11	3	2	2	2
4	15	17	3	3	2	2
5	17	20	3	3	2	2
6	17	19	9	8	7	7
7	17	19	8	7	6	6
8	17	19	11	10	9	9
9	20	24	12	12	11	11
10	21	28	7	6	5	5

Investigations concering the effect on solution speed, convergence property, and reliability of these two methods are also carried out. For this purpose, one of the 10 test-systems, namely no. 5 in Table I, is considered. The circuit diagram, together with busbar data of this system, are given in Fig. 1, while its branch data are given in the Appendix, Table A1.

The effect on the speed of solution may be examined by considering the absolute angular errors, defined as the differences between the values obtained after a complete solution and those resulting from the starting method alone. It is clear from Table 2 that, for the majority of system nodes, our starting methods give angular assessments which are nearer to the final solution. This is also confirmed by calculating, from the results of Table 2, the arithmetic mean error which is found to be smaller in the cases of our methods than in the case of Ref.[11].

The corresponding percentage errors in busbar voltage magnitudes are listed in Table 3. These results emphesise additional advantage to the developed methods. This fact is further clarified when the arithmetic mean value of the percentage error is considered; it is 0.15% for the first method, 0.45% for the second, but as much as 2.73% for that of Ref. [11].

The effect on the convergence pattern of the developed methods is best illustrated by the logarithmic scaled curves of Figs. 2 and 3. The curves show that, convergence is approached faster in the case of the starting methods developed in this paper.

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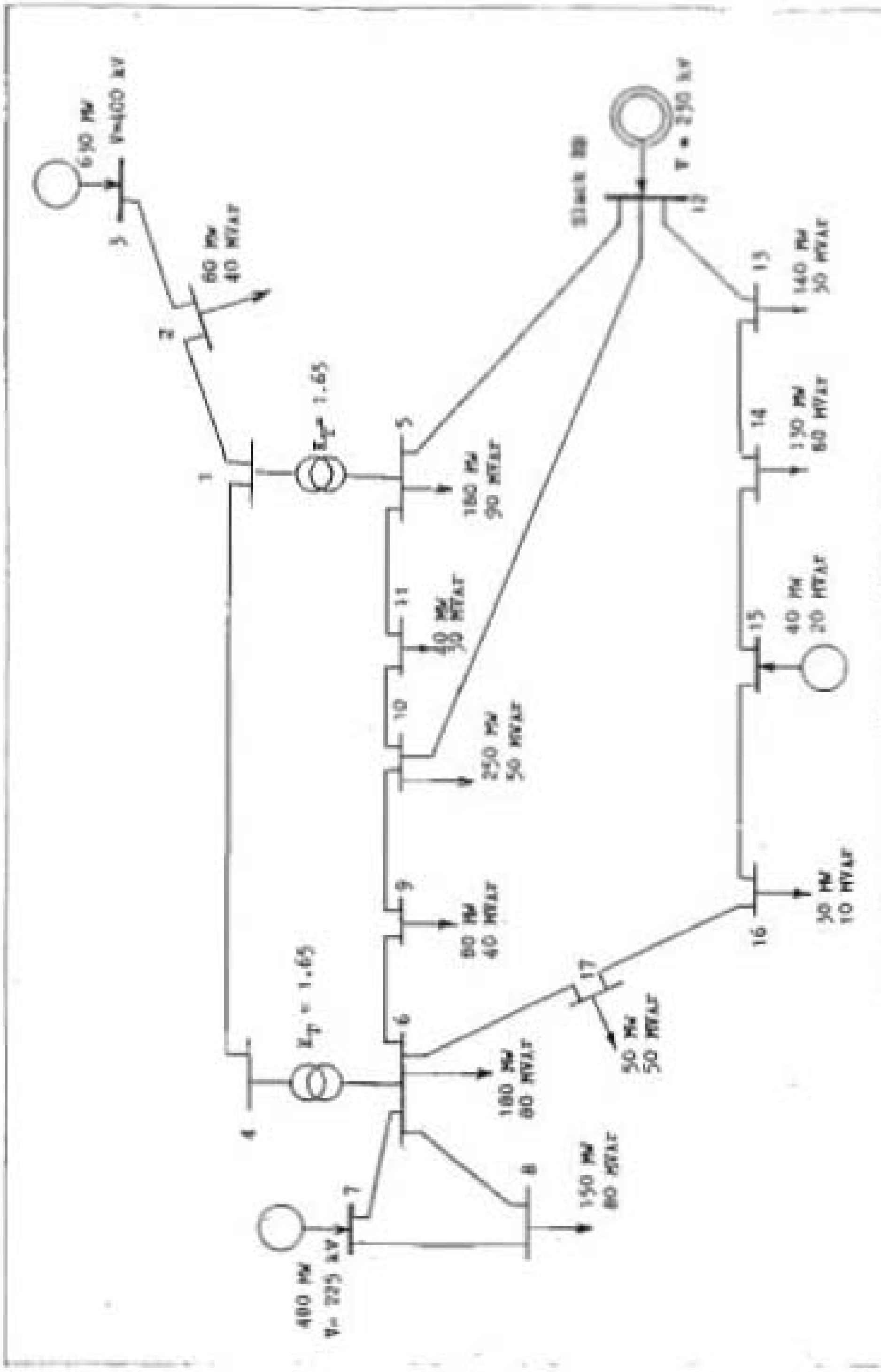

Fig. 1 Test system with 17 busbars and 20 branches.

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Table 2. Absolute angular errors for the system of Fig. 1 in electrical degrees.

Bus No.	Method of Ref. 11	Developed methods	
		First	Second
1	1.737	1.711	1.574
2	1.791	1.780	2.516
4	1.599	1.295	1.297
5	0.862	0.862	0.643
6	0.817	0.813	0.724
8	0.888	0.860	0.823
9	0.863	0.857	0.685
10	0.681	0.681	0.294
11	0.957	0.950	0.786
13	0.240	0.236	0.184
14	0.691	0.691	0.326
15	0.692	0.692	0.553
16	0.706	0.748	0.656
17	0.825	0.819	0.713

Table 3. Percentage errors in voltage magnitudes for the system of Fig. 1.

Bus No.	Method of Ref. 11	Developed methods	
		First	Second
1	6.49	0.37	1.20
2	2.53	0.23	0.27
4	7.23	0.18	0.54
5	4.50	0.42	1.13
6	0.03	0.08	0.30
8	0.11	0.02	0.17
9	0.05	0.08	0.34
10	0.04	0.07	0.04
11	4.46	0.52	0.49
13	4.18	0.02	0.02
14	2.06	2.07	0.17
15	2.13	0.06	0.50
16	2.16	0.10	0.33
17	2.16	0.10	0.31

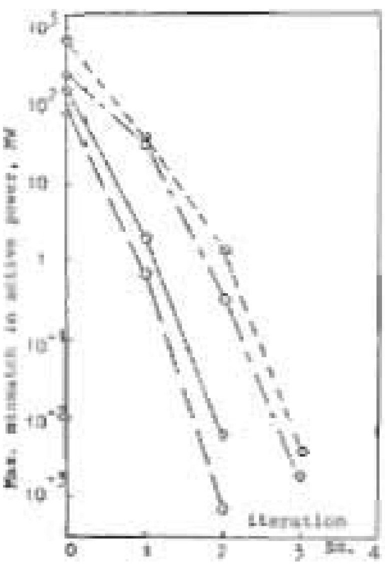

Fig. 2 Active power convergence characteristics for the system of Fig. 1.

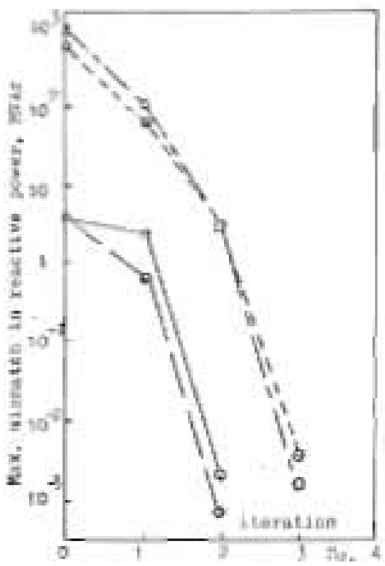

Fig. 3 Reactive power convergence characteristics for the system of Fig. 1.

— — — N - R method alone, ——— N - R with 1st. start. meth. — - — N - R with start. meth. of [11], —— —— N - R with 2nd start. method.

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To illustrate the effect on the solution reliability enforced by the developed methods, a load flow solution is carried out for the system of Fig. 1, this time however, with a large error included in the busbar data. Instead of 220 kV, a value of 420 kV is specified for busbar numbered 15. The results of this study are represented by the curves of Fig. 4. It is seen that, while no solution is possible by the Newton-Raphson method alone, a complete solution is guaranteed after only 2 iteration cycles when our first starting method is entered in the process. This means that, neither the reliability nor the solution speed, gained by the application of our starting methods, are affected by the introduction of errors in the system input data.

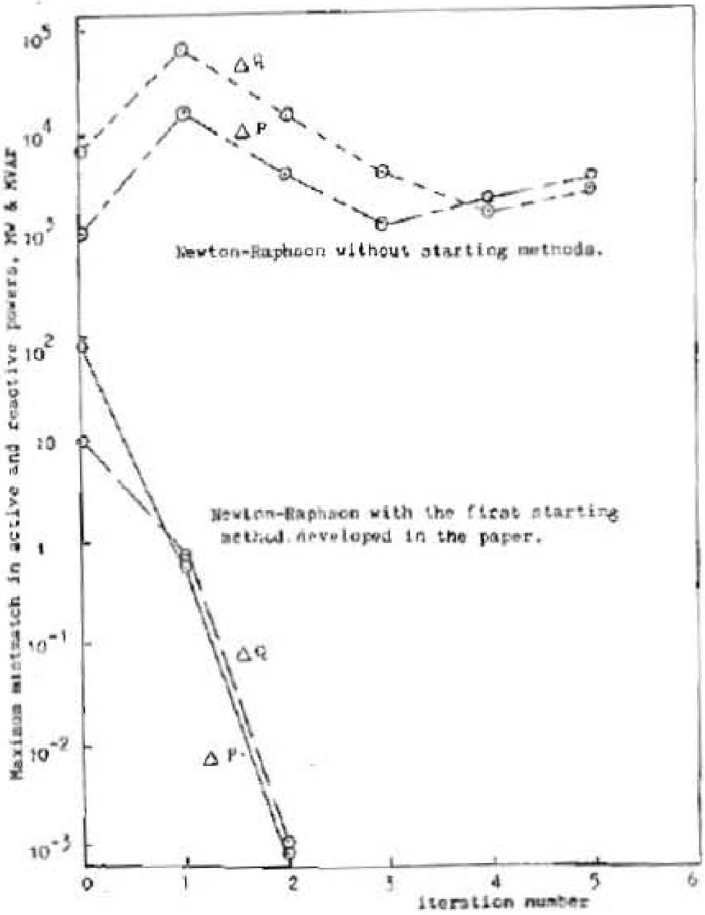

Fig. 4 : Convergence characteristics for the case when an error of 200 kV is included in the data of bus 15 of the system of Fig. 1.

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9. APPLICATIONS TO LARGE PRACTICAL SYSTEMS

Tests involving large practical systems have also been carried out using only the first of the two developed starting methods for its relatively better performance. Fig. 5 shows the circuit diagram for one of such systems. It is a part of the Bulgarian 110, 220, and 400-kV interconnected electric energy network. This system consists of 36 busbars, 54 branches of which 38 are transformers, and 13 reactive power sources. Two of the transformers have complex turns ratio and the busbar and branch input data are given in the Appendix, Tables A2 and A3. The slack busbar is numbered 30 in Fig. 5.

Fig. 6 gives the variations with the number of iteration cycles of the maximum mismatch in both active and reactive powers (convergence characteristics) for different solution conditions to the system of Fig. 5. These conditions include that of introducing some large errors in the input data and the execution of the solution with and without the starting method. All solutions are completed with a predetermined tolerance of 1 MW and 1 MVAr.

The curves show that, for no error introduced in the input data, a solution is possible after 6 iterations by the main N-R algorithm alone. Should an error be introduced in the input data, no solution is obtained without the use of the starting method.

Conditions are however much improved, even where errors are introduced in the input data, when the action of the starting method is included in the calculations. For example, with an error of -93.5 kV is entered in the voltage data of busbar number 1 (i.e. 17.5 kV instead of 111 kV) a complete solution is obtained after only 4 iterations, which means that our starting method not only ensured a solution, but reduces the number of iteration cycles as well; by at least 33% in this case.

A much larger system has also been included in the study. The system has the following components:

- No. of busbars = 293 ;
- No. of branches = 406 ;
- No. of reactive power sources = 68 ; and
- No. of transformers = 61 of which 3 have complex turns ratio for voltage control under loading conditions.

Solving this sytem without the starting method, a solution is obtained after 6 iteration cycles. Using the first of our developed starting methods, the complete solution is obtained after only 3 iteration cycles, even with some errors introduced in the input data. This represents a 50% reduction in the number of iteration cycles and, consequently, in the computing time.

It might be of interest to learn that, when our first starting method was included in optimisation calculations, the same effect on reliability and solution speed was also obtained. The system considered consisted of 299 busbars, 412 branches of which 63 are transformers, and 67 reactive power sources. Three of the transformers have complex turns ratio. In calculating an optimal regime without the starting method, no acceptable solution was obtained after 36 optimisation iterations. The target function was found to change very little for a wide range of change in the busbar voltage values. With large errors introduced in the input data, the optimisation process without the starting method was iterrupted, by conditional informations, at the 11th iteration cycle. Conditions are much improved when the starting method is inserted in the process, and an optimal solution was attained after only 34 iteration cycles.

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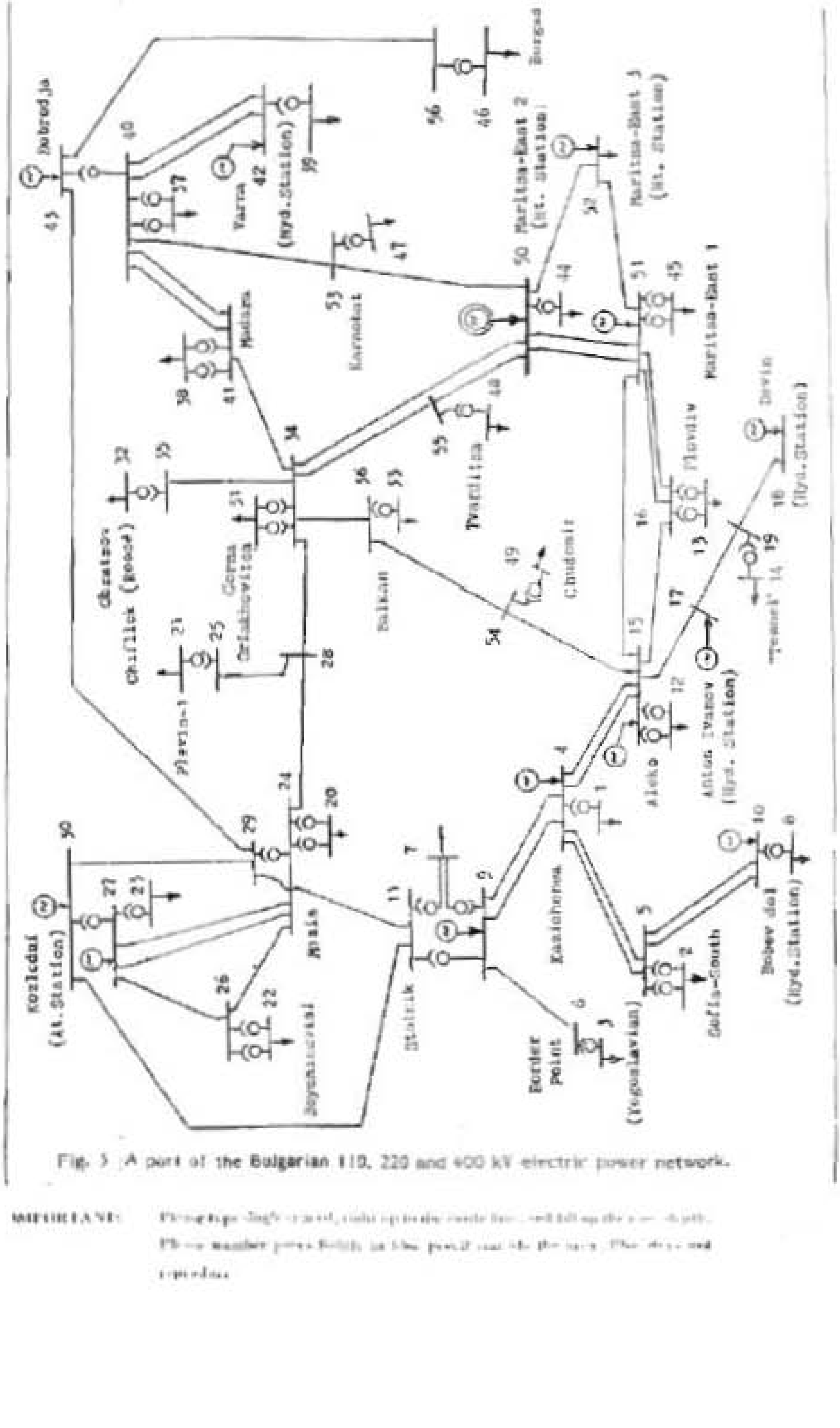

Fig. 3 A part of the Bulgarian 110, 220 and 400 kV electric power network.

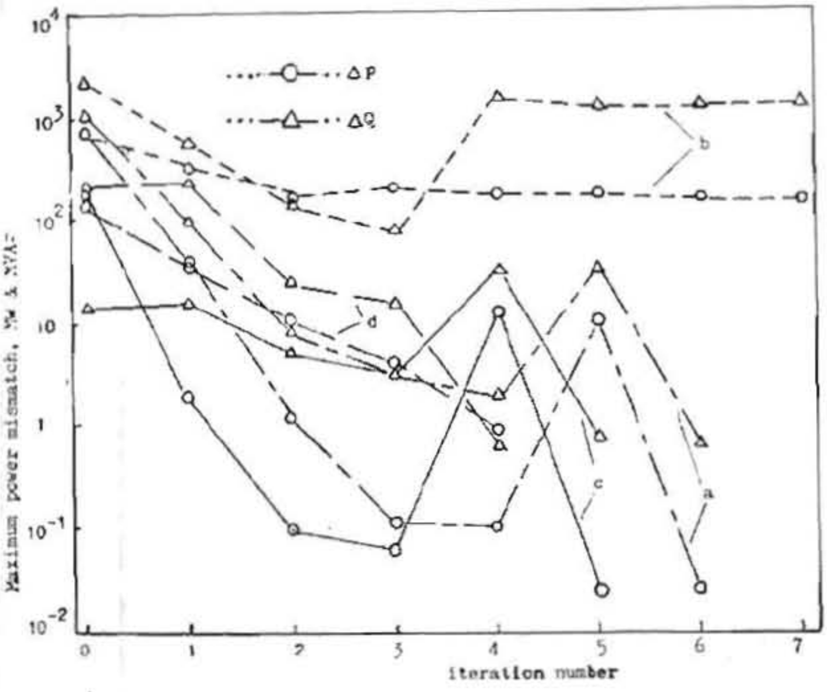

Fig. 6 Convergence characteristics for different solution conditions to the system of Fig. 5.

a ——- - —— N - R method with correct data and without starting method,
b - - - - - - N - R with an error of -100 kV in the voltage data of bus number 13 but without starting method,
c ———— As b but with the starting method, and
d ——— —— N - R with an error of -93.5 kV in the voltage data of busbar number 1 and with the starting method.

5. CONCLUSIONS

Two starting algorithms for the Newton-Raphson load flows have been developed and intensively tested in this paper. The results of the tests are so consistant as to make precise conclusions possible.

In comparison with previously known starting methods, those developed here ensure better initial estimates for both magnitudes and phase angles of busbar voltages. Consequently, the reliability of obtaining a solution is increased and the solution time is reduced appreciably. For instance, for a large system with 293 busbars and 406 branches, the reduction has amounted to 50% of that necessary for the same solution without a starting method. In general, a pronounced reduction in the solution time shall always be guaranteed by the use of our starting methods.

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In addition, for system conditions where the Newton-Raphson method alone has failed, the use of the starting algorithms presented here has indeed proved very useful. Same advantages have also been ensured in optimisation calculations.

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8. APPENDIX

Table A1. Branch data for the test system of Fig. 1

Branch from - to	R (Ω)	X (Ω)	Y_c (μs)	Branch from - to	R (Ω)	X (Ω)	Y_c (μs)
1 - 2	7.7	14.0	968	6 - 17	2.96	20.96	336
1 - 4	9.36	19.6	1040	7 - 8	3.24	20.0	136
1 - 5	0	25.4	0	9 - 10	5.94	36.9	254
2 - 3	3.47	48.6	500	10 - 11	7.66	46.7	330
4 - 6	0	25.4	0	10 - 12	3.45	21.0	586
5 - 11	3.6	22.0	151	13 - 12	1.36	8.4	230
5 - 12	12.4	76.0	520	13 - 14	6.15	23.3	488
6 - 7	1.06	10.6	477	14 - 15	3.67	46.3	104
6 - 8	4.87	30.7	497	15 - 16	2.96	18.2	530
6 - 99	2.45	15.1	105	16 - 17	0.87	5.0	136

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Table A2. Busbar data for the actual system of Fig. 3.

Bus No.	V_{nom}, kV	P_L, MW	Q_L, MVAr	P_G, MW	Q_G, MVAr	|V|, kV	Q_c, MVAr max.	min.
1	110	127	67			115		
2	110	272	72			115		
3	110	150	61			114		
4	220			202	106	224		
5	220	0	0	0	0	224		
6	220	0	0	0	0	223		
7	110	279	54			117		
8	110	109	36			118		
9	220			118	39	225		
10	220			217		230	150	200
11	400	0	0	0	0	397		
12	110	100	18			116		
13	110	252	105			113		
14	110	46.5	20.5			112		
15	220			74.4	27.8	219		
16	220	0	0	0	0	214		
17	220			89.6	69.8	222		
18	220			58.6	15.8	223		
19	220	0	0	0	0	222		
20	110	0	0			116		
21	110	0	0			108		
22	110	0	0			117		
23	110	0	0			120		
24	220	0	0	0	0	224		
25	220	0	0	0	0	209		
26	220	0	0	0	0	222		
27	220			797		234	250	350
28	220	0	0	0	0	211		
29	400	0	0	0	0	392		
30	400			417	134	399.7		
31	110	215	72			105		
32	110	100	36			101		
33	110	111	46			101		
34	220	0	0	0	0	200		
35	220	0	0	0	0	190		
36	220	0	0	0	0	196		
37	110	204	18			111		
38	110	240	71			107		
39	110	115	3			113		
40	220	0	0	0	0	213		
41	220	0	0	0	0	205		
42	220			571		215.4	100	200
43	400			820	-100	376		
44	110	102	28			116		
45	110	273	43			117		
46	110	250	-4			100		
47	110	52	62			111		
48	110	41	31			108		
49	110	95	29			107		
50	220 slack busbar					218.2	50	200
51	220			0	0	222		
52	220			0	0	242		
53	220	0	0	0	0	207		

Cont./

[illegible]

Continued from Table A2.

54	220	0	0	0	0	198		
55	220	0	0	0	0	208		
56	400	0	0	0	0	372		

Table A3. Branch data for the actual system of Fig. 5.

From bus.	To bus.	R, ohms	X, ohms	Y_c, µS	X_{TD}, p.u.	X_{TQ}, p.u.	#
43	29	9.39	95.0	960			
43	56	6.14	93.9	459			
29	11	3.12	31.98	330			
29	30	1.498	20.21	270.9			
30	11	5.24	56.3	696			
40	41	3.04	18.46	128			1
40	41	2.96	18.06	128			2
40	53	5.825	37.15	251.8			
40	42	0.7	6.54	58			1
40	42	0.7	6.54	58			2
41	34	7.659	46.66	330			
53	50	5.463	34.85	236.2			
34	28	6.07	37.14	258			
34	36	4.704	19.2	118			
34	50	6.838	42.04	304			
34	35	5.26	32.341	224			
34	55	3.522	21.65	157			
55	50	3.316	20.39	147			
54	36	2.93	12.0	73			
28	25	0.28	1.76	12			
28	24	2.258	15.28	108			
24	26	4.87	30.8	210			
24	27	2.218	22.2	229.5			1
24	27	2.218	22.2	229.5			2
27	26	3.313	20.96	139			
6	9	0.7	4.35	34			
4	9	1.6	9.85	61			1
4	9	1.6	9.85	61			2
4	15	5.884	35.51	254			1
4	15	5.884	35.51	254			2
5	4	1.084	6.55	43.5			1
5	4	1.084	6.55	43.5			2
10	5	2.83	17.07	123			1
10	5	2.83	17.07	123			2
15	51	4.811	48.55	460			
15	16	1.223	12.4	127			
15	17	1.756	10.727	71			
15	54	10.3	42.16	259			
17	19	2.318	11.69	79.6			
19	18	1.179	4.523	29.3			
16	51	5.87	37.4	253			1
16	51	9.75	39.8	244			2
51	50	2.56	15.58	110			1
51	50	2.89	18.35	124			2
50	52	2.584	24.725	398			
51	52	0.404	3.83	63			

Cont./

IMPORTANT: [illegible]

Continued from Table A3.

From bus.	To bus.	R, ohms	X, ohms	$1/X_m$, μS	K_{TD}, p.u.	K_{TQ}, p.u.	#
40	37	0.59	26.45	-12.1	0.526		1
40	37	0.59	26.45	-12.1	0.526		2
43	40	0.618	25.14	- 4	0.574		
41	38	0.57	28.5	-12	0.537		1
41	38	0.57	28.5	-12	0.537		2
32	35	0.2	8.83	-93.4	1.818		
34	31	0.571	25.31	-12	0.537		1
34	31	0.571	25.31	-12	0.537		2
36	33	0.59	26.45	-12.1	0.537		
54	47	0.585	31.74	-12.1	0.558		
25	20	0.562	27.2	-12	0.537		
24	20	0.59	26.45	-12.1	0.526		1
24	20	0.59	26.45	-12.1	0.526		2
29	24	0.556	25.4	- 4	0.581	0.661	
27	23	0.566	26.71	-13	0.537		
30	27	0.335	23.46	- 5	0.527		
11	9	0.556	25.4	- 4	0.589	1.957	
26	22	0.715	30.6	-15	0.537		1
26	22	0.715	30.6	-15	0.537		2
7	9	0.2	8.83	-93.4	1.868		
5	6	0.2	8.83	-93.4	1.868		
1	4	0.2	8.83	-93.4	1.868		
2	5	0.2	8.83	-93.4	1.868		1
2	5	0.2	8.83	-93.4	1.868		2
10	18	0.547	27.2	-13	0.526		
15	12	0.59	26.45	-12.1	0.537		1
15	12	2.38	77.59	-12.15	0.532		2
14	19	1.33	42.6	-22	1.818		
16	13	0.546	27	-26.4	0.547		1
16	13	0.546	27	-26.4	0.547		2
48	55	0.791	31.06	-18.3	1.764		
53	47	1.519	42.75	-22.56	0.591		
42	46	0.59	26.45	-12.1	0.526		
44	50	0.2	8.83	-93.4	1.843		
46	56	0.077	1.025	-102.4	3.43		
51	45	0.59	26.45	-12.1	0.537		1
51	45	0.59	26.45	-12.1	0.537		2
11	7	1.59	7.34	-64	0.315		

In the Table above;

X_m is transformer magnetising reactance;

K_{TD} is transformer turns ratio in the direct axis;

K_{TQ} is transformer turns ratio in the quadrature axis; and

\# denotes branches in parallel.

IMPORTANT: [illegible]