A Survey on Rough Intervals Programming Formulation and Methodologies

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Abstract: *This research states a survey on formulation of linear programming problems with rough intervals coefficients in the objective functions and constraints, basic preliminaries about rough intervals, interval method and trust probability constraints for transforming rough intervals to crisp nature and fully rough intervals problems. Finally, presents different operational research models that contain rough intervals coefficients.*

Keywords: *rough intervals, interval method, trust probability constraints and fully rough intervals*.

1. Introduction

The Rough Intervals (RIs), proposed by Robolledo [14] in 2006, are used to deal with partially unknown or ill-defined parameters and variables. RI is introduced to adapt the rough set principles to model continuous variables. It is notable that rough sets were used only to handle discrete objects, initially, and could not represent continuous values. RI is a particular case of rough sets. It fulfils all the rough sets' properties and core concepts, including the upper and lower approximation definitions.

RI which based on the interval analysis has been created as a helpful and basic method to deal with the classificatory analysis of ambiguous concepts; the RI used to deal with partially vague or poorly characterized parameters [13].

RI has two features. First, the results are in the form of intervals. Second, the interval method doesn't ignore any part of solution region. Thus, the interval method gives a solution with high precision [3].

Youness [17] presented a nonlinear programming problem with a rough set of constraints. Also he defined the convex rough set, the local rough optimal solution, the global rough optimal solution, and the roughness measure of optimality.

Xu et al. [16] transformed from random rough nature to the equivalent crisp model and introduced an interactive method to get solution that satisfied decision maker, using a random rough simulation technique which can act with random rough objective functions and constraints, grouping with the genetic algorithm.

Osman et al. [9] introduced a new formulation and classification of the Rough Programming Problems. Also he discussed the rough feasibility, the rough optimality, the rough optimal value, and the rough optimal set.

Lu et al. [6] introduced the concept of RI to express dual uncertain information of many parameters and the related solution method presented to solve RI fuzzy linear programming problems.

Alolyan [1] tackled LP problems with fuzzy parameters in the objective function and the constraints based on preference relations between explored intervals.

Jana et al. [4] handled fuzzy rough multi-item economic production quantity model and developed constant demand. Infinite production rate has adaptability and dependability consideration in the production process, demand dependent unit production cost and shortages under the limitations on the capacity region, by geometric programming technique tackled the problem.

Hamazehee et al. [3] introduced a new class of LP problems in which some or all of the coefficients are RI and showed that each one of them can be transformed into two LP problems with interval coefficients. Also he introduced the surely optimal range, the possibly optimal range, the completely satisfactory solutions, the rather satisfactory solutions, and the rough optimal range.

Ammar and Khalifa [2] applied a new method named, separation method for solving Rough Interval Multi Objective Transportation Problems (RIMOTP), where transportation cost, supply, and demand are RIs. Also discussed that the separation method as an important tool for the DMs when they are handling various types of logistic problems having RI parameters of transportation problems.

Saad et al. [15] presented an algorithm for solving a three-level quadratic programming, where some or all of its coefficients in the objective function are RI.

Omran et al. [7] presented an algorithm for solving a three level fractional programming problem with rough coefficient in constraints.

Osman et al. [8] presented a solution approach for RIMOTP. The concept of solving conventional interval programming combined with fuzzy programming is used to build the solution approach for RIMOTP.

Pandian et al. [11] considered that transportation problem has all or some parameters as rough integer intervals. Also he proposed a new method named, a slice-sum method to solve Rough Integer Interval Transportation Problem, where transportation cost, supply, and demand are rough integer intervals.

2. Formulation of Linear Programming Problem with Rough Interval Coefficients

The linear programming problem with rough interval coefficients can be formulated as follows:

$$Max/\,Min\,f\quad(x)=\sum_{j=1}^{n}\left[\left[\underline{c}_j^L,\underline{c}_j^U\right],\left[\overline{c}_j^L,\overline{c}_j^U\right]\right]x_j,\qquad(1)$$

Subject to

$$G=\sum_{j=1}^{n}\left[\left[\underline{a}_{ij}^L,\underline{a}_{ij}^U\right],\left[\overline{a}_{ij}^L,\overline{a}_{ij}^U\right]\right]x_j\le\left[\left[\underline{b}_i^L,\underline{b}_i^U\right],\left[\overline{b}_i^L,\overline{b}_i^U\right]\right],$$

$$\mathrm{i}=1,2,\dots,\mathrm{m},\ \mathrm{x_j}\ge 0.\qquad(2)$$

In the above Problem (1) – (2), $\left[\left[\underline{c}_j^L,\underline{c}_j^U\right],\left[\overline{c}_j^L,\overline{c}_j^U\right]\right]$ are RI coefficients of the objective function, $\left[\left[\underline{a}_{ij}^L,\underline{a}_{ij}^U\right],\left[\overline{a}_{ij}^L,\overline{a}_{ij}^U\right]\right]$ are RI coefficients of the constraints and $\left[\left[\underline{b}_i^L,\underline{b}_i^U\right],\left[\overline{b}_i^L,\overline{b}_i^U\right]\right]$ are RI of constants.

3. Basic Preliminaries about RI

Conversion of Linear Programming (LP) problem with rough interval coefficients into upper and lower approximation is usually hard work for many cases, but transformation process needs the following definitions to be known:

Definition 1 [3]
RI can be considered as a qualitative value from vague concept defined on a variable x in R.

Definition 2 [3]
The qualitative value A is called a rough interval when one can assign two closed intervals A_* and A^* on R to it where $A_*\subseteq A^*$.

Definition 3 [3]
A_* and A^* are called the Lower Approximation Interval (LAI) and the Upper Approximation Interval (UAI) of A, respectively. Further, A is denoted by $A=(A_*$ and $A^*)$.

Definition 4 [3]
Consider all of the corresponding Linear Programming with Interval Coefficients (LPIC) and LP of Problem (1) - (2):

1) The interval $[f_*^L,f_*^U]([f^{*L},f^{*U}])$ is called the surely (possibly) optimal range of Problem (1) - (2) if the optimal range of each LPIC is a subset (superset) of $[f_*^L,f_*^U]([f^{*L},f^{*U}])$.
2) Let $[f_*^L,f_*^U]([f^{*L},f^{*U}])$ be surely (possibly) an optimal range of Problem (1) - (2), then the rough interval $\left[[f_*^L,f_*^U][f^{*L},f^{*U}]\right]$ is called the rough optimal range of Problem (1) - (2).
3) The optimal solution of each corresponding LPIC of Problem (1) - (2) whose optimal value belongs to $[f_*^L,f_*^U]([f^{*L},f^{*U}])$ is called a completely (rather) satisfactory solution of Problem (1) - (2).

Let D denotes the set of all RIs on the real line R. That $is, D=\{[[b,c],[a,d]], a\le b\le c\le d\ and\ a,b,c,d\ are\ in\ R\}$,

$A=\left[[a_2,a_3],[a_1,a_4]\right]$ and $B=\left[[b_2,b_3],[b_1,b_4]\right]$ $be\ in\ D$.

Definition 5 [11]
Addition:

$A\oplus B=\left[[a_2+b_2,a_3+b_3],[a_1+b_1,a_4+b_4]\right]$.

Definition 6 [3]
Subtraction:

$A-B=\left[[a_2-b_3,a_3-b_2],[a_1-b_4,a_4-b_1]\right]$.

Definition 7 [3]
Negation: $-A=\left[[-a_3,-a_2],[-a_4,-a_1]\right]$,

Definition 8 [11]
Scalar Multiplication:

$KA=\left[[Ka_2,Ka_3],[Ka_1,Ka_4]\right]$,

if k is a positive real number.

Definition 9 [11]
Multiplication:

$A\otimes B=\left[[a_2\,b_2,a_3\,b_3],[a_1\,b_1,a_4\,b_4]\right], if\ A,B\ge 0.$

Definition 10 [11]
A is said to be a rough positive integer if a_i, $i=1,2,3,4$ are positive integers.

Definition 11 [11]

$A\ge B, if\ a_i\ \ge b_i\ ,i=1,2,3,4.$

$A\le B, if\ a_i\ \le b_i\ ,i=1,2,3,4.$

$A=B, if\ a_i\ =b_i\ ,i=1,2,3,4.$

4. Interval Method for Transforming RI Parameters to Crisp Nature

Interval method [3] constructs two LP problems with interval coefficients as shown in Table (1). One of these problems is an LP where all of its coefficients are upper approximations of RI and the other is an LP where all of its coefficients are lower approximations of RI.

Table (1): Two LP Problems with Lower and Upper Approximations of RI

LP with Lower Approximations of RI	LP with Upper Approximations of RI
$Max/\,Min\,\underline{f}\quad(x) = \sum_{j=1}^{n}[\underline{c}_j^L, \underline{c}_j^U]x_j, \qquad (3)$ Subject to $\underline{G} \quad = \sum_{j=1}^{n}[\underline{a}_{ij}^L, \underline{a}_{ij}^U]x_j \le [\underline{b}_i^L, \underline{b}_i^U],$ $i = 1,2,\dots,m, x_j \ge 0.$	$Max/\,Min\,\overline{f}\quad(x) = \sum_{j=1}^{n}[\overline{c}_j^L, \overline{c}_j^U]x_j, \qquad (4)$ Subject to $\overline{G} \quad = \sum_{j=1}^{n}[\overline{a}_{ij}^L, \overline{a}_{ij}^U]x_j \le [\overline{b}_i^L, \overline{b}_i^U],$ $i = 1,2,\dots,m, x_j \ge 0.$

Now, the equivalent problem of the LP problem with lower approximations of RI using interval method [3] can be obtained by getting the surely optimal range of Problems (1) - (2), which resulted in the following two LP problems with crisp parameters in Table (2).

Table (2): Lower Approximations of RI

Lower Approximation Lower Bound (LALB)	Lower Approximation Upper Bound (LAUB)
$Max/\,Min\,\underline{f}^L = \sum_{j=1}^{n}\underline{c}_j^L x_j, \qquad (5)$ Subject to $\underline{G}^L = \sum_{j=1}^{n}\underline{a}_{ij}^U x_j \le \underline{b}_i^L, i = 1,2,\dots,m, x_j \ge 0.$	$Max/\,Min\,\underline{f}^U = \sum_{j=1}^{n}\underline{c}_j^U x_j, \qquad (6)$ Subject to $\underline{G}^U = \sum_{j=1}^{n}\underline{a}_{ij}^L x_j \le \underline{b}_i^U, \quad i = 1,2,\dots,m, x_j \ge 0.$

Now, the equivalent problem of the LP problem with upper approximations of RI using interval method [3] can be obtained by getting the possibly optimal range of Problems (1) - (2), which resulted in the following two LP problems with crisp parameters as shown in Table (3).

Table (3): Upper Approximations of RI

Upper Approximation Lower Bound (UALB)	Upper Approximation Upper Bound (UAUB)
$Max/\,Min\,\overline{f}^L = \sum_{j=1}^{n}\overline{c}_j^L x_j, \qquad (7)$ Subject to $\overline{G}^L = \sum_{j=1}^{n}\overline{a}_{ij}^U x_j \le \overline{b}_i^L, i = 1,2,\dots,m, x_j \ge 0.$	$Max/\,Min\,\overline{f}^U = \sum_{j=1}^{n}\overline{c}_j^U x_j, \qquad (8)$ Subject to $\overline{G}^U = \sum_{j=1}^{n}\overline{a}_{ij}^L x_j \le \overline{b}_i^U, i = 1,2,\dots,m, x_j \ge 0.$

So, the LP problem with RI coefficients in the objective function and the constraints can be converted into four LP problems with crisp parameters.

5. A Trust Probability Constraints for Transforming RI Parameters to Crisp Nature

To convert the LP problem with rough coefficients in the objective functions into the respective crisp equivalents for solving a trust probability constraints, this process is usually hard work for many cases but the transformation process is introduced in the following theorem.

Theorem 1[16]

Assume that random rough variable $\tilde{c}_{ij}$ is characterized by $\tilde{c}_{ij}(\lambda) \sim N(c_{ij}(\lambda), V_i^c)$ where:

$c_{ij}(\lambda)[(c_{ij}(\lambda)_{n\times 1} = (c_{i1}(\lambda), c_{i2}(\lambda), \dots, c_{in}(\lambda))^T)]$ is a rough variable and V_i^c is a positive definite covariance matrix. It follows that$c_i(\lambda)^T x = ([a,b],[c,d])$, (where$c \le a \le b \le d$) is a rough variable and characterized by the following trust measure function:

$$Tr\{c_i(\lambda)^T x \geq t\} = \begin{cases} 0 & if\ \ d \leq t, \\ \dfrac{d-t}{2(d-c)} & if\ \ b \leq t \leq d, \\ \dfrac{1}{2}\left(\dfrac{d-t}{d-c} + \dfrac{b-t}{b-a}\right) & if\ \ a \leq t \leq b, \\ \dfrac{1}{2}\left(\dfrac{d-t}{d-c} + 1\right) & if\ \ c \leq t \leq a, \\ 1 & if\ t \leq c. \end{cases} \quad (9)$$

Then, we have $\mathrm{Tr}\{\lambda | \mathrm{pr}\{c_i(\lambda)^T x \geq F_i(x)\} \geq \delta_i\} \geq \gamma_i$, if and only if

$$\begin{cases} b + R \ \leq F_i \ \leq d - 2\gamma_i\,(d-c) + R & if\ \ b \leq M \leq d, \\ a + R \leq F_i \leq \dfrac{d(b-a) + b(d-c) - 2\gamma_i(d-c)(b-a)}{d-c+b-a} + R & if\ \ a \leq M \leq b, \\ c + R \leq F_i \leq d - (d-c)(2\gamma_i - 1) + R & if\ \ c \leq M \leq a, \\ F_i \leq c + R & if\ \ M \leq c. \end{cases} \quad (10)$$

Where $\mathrm{M} = \mathrm{F_i} - \phi^{-1}(1-\delta_i)\sqrt{\mathrm{x^T V_i^c x}}$ and $\mathrm{R} = \phi^{-1}(1-\delta_i)\sqrt{\mathrm{x^T V_i^c x}}$, where $\phi$is the standardized normal distribution and $\delta_i, \gamma_i \in [0,1]$ are predetermined confidence levels.

6. Fully Rough Intervals Problems

Fully rough intervals problems [11], in which all decision parameters and decision variables in the objective functions and the constraints are RI and the optimal values of decision rough variables and rough objective functions are RI.

Formulation of Linear Programming Problem with Fully Rough Intervals

The linear programming problem with fully rough intervals coefficients can be formulated as follows:

$$Max/\,Min\left[\left[f^{LL}, f^{LU}\right], \left[f^{UL}, f^{UU}\right]\right] = \sum_{j=1}^{n}\left[\left[\underline{c}_j^L, \underline{c}_j^U\right], \left[\overline{c}_j^L, \overline{c}_j^U\right]\right] \otimes \left[\left[x_j^{LL}, x_j^{LU}\right], \left[x_j^{UL}, x_j^{UU}\right]\right], \quad (11)$$

Subject to

$$G = \sum_{j=1}^{n}\left[\left[\underline{a}_{ij}^L, \underline{a}_{ij}^U\right], \left[\overline{a}_{ij}^L, \overline{a}_{ij}^U\right]\right] \otimes \left[\left[x_j^{LL}, x_j^{LU}\right], \left[x_j^{UL}, x_j^{UU}\right]\right] \leq \left[\left[\underline{b}_i^L, \underline{b}_i^U\right], \left[\overline{b}_i^L, \overline{b}_i^U\right]\right], i = 1,2,\dots,m, \ \ [X_j] \geq 0. \quad (12)$$

In the above Problem (11) – (12), $\left[\left[x_j^{LL}, x_j^{LU}\right], \left[x_j^{UL}, x_j^{UU}\right]\right]$ are RI of decision variables, $\left[\left[\underline{c}_j^L, \underline{c}_j^U\right], \left[\overline{c}_j^L, \overline{c}_j^U\right]\right]$ are RI coefficients of the objective function, $\left[\left[\underline{a}_{ij}^L, \underline{a}_{ij}^U\right], \left[\overline{a}_{ij}^L, \overline{a}_{ij}^U\right]\right]$ are RI coefficients of the constraints and $\left[\left[\underline{b}_i^L, \underline{b}_i^U\right], \left[\overline{b}_i^L, \overline{b}_i^U\right]\right]$ are RI of constants.

Slice Sum Method for Transforming Fully Rough Intervals Linear Programming to Crisp Nature

Slice Sum method [11] is a method, in which the problem is sliced into four problems namely, UAUB problem, LAUB problem, LALB problem and UALB problem.

The transformation process is introduced in the following theorem.

Theorem 2 [11]

If $x_j^{*UU}, j = 1,2,\dots n$ is an optimal solution for the UAUB problem of the problem (11) – (12), $x_j^{*LU}, j = 1,2,\dots n$ is an optimal solution for the LAUB problem of the problem (11) – (12), $x_j^{*LL}, j = 1,2,\dots n$ is an optimal solution for the LALB problem of the problem (11) – (12), $x_j^{*UL}, j = 1,2,\dots n$ is an optimal solution for the UALB problem of the problem (11) – (12), then the set of RI $\left[\left[x_j^{*LL}, x_j^{*LU}\right], \left[x_j^{*UL}, x_j^{*UU}\right]\right]$ is an optimal solution for the problem (11) – (12) such that $x_j^{*UL} \leq x_j^{*LL} \leq x_j^{*LU} \leq x_j^{*UU}, j = 1,2,\dots n$.

To prove Theorem 2 above refer to [11].

The solution algorithm can be summarized in the following steps:

Step 1: Construct the UAUB problem of the given problem.

Step 2: Solve the UAUB problem using LP techniques. Let $x_j^{*UU}, j = 1,2,\dots n$ be an optimal solution of the UAUB problem.

Step 3: Construct the LAUB problem of the given problem.

Step 4: Solve the LAUB problem with upper bound constraints $x_j^{LU} \leq x_j^{*UU}, j = 1,2,\dots n$ using LP techniques. Let $x_j^{*LU}, j = 1,2,\dots n$ be an optimal solution of the LAUB problem.

Step 5: Construct the LALB problem of the given problem.
Step 6: Solve the LALB problem with upper bound constraints $x_j^{LL} \le x_j^{*LU}, j = 1,2,\dots n$ using LP techniques. Let $x_j^{*LL}, j = 1,2,\dots n$ be an optimal solution of the LALB problem.
Step 7: Construct the UALB problem of the given problem.
Step 8: Solve the UALB problem with upper bound constraints $x_j^{UL} \le x_j^{*LL}, j = 1,2,\dots n$ using LP techniques. Let $x_j^{*UL}, j = 1,2,\dots n$ be an optimal solution of the UALB problem.
Step 9: The optimal solution of the given problem is $\left[\left[x_j^{*LL}, x_j^{*LU}\right], \left[x_j^{*UL}, x_j^{*UU}\right]\right], j = 1,2,\dots n$.

7. On the Solution of a Rough Interval Three-level Quadratic Programming Problem

A three-level quadratic programming problem is considered where some or all of its coefficients in the objective function are RI [15]. At the first phase of the solution approaches and to avoid the complexity of the problem, two QP problems with interval coefficients have been formulated. One of these problems was a QP where all of its coefficients are upper approximations of RI and the other problem was a QP where all of its coefficients are lower approximations of RI. At the second phase, a membership function is constructed to develop a fuzzy model for obtaining the optimal solution of the three-level quadratic programming problem.

Problem Formulation and Solution Concept

The Three-Level Quadratic Programming Problem with Rough Interval Coefficients (TLQPRIC) in the objective functions may be written as follows:

$[1^{st}\mathrm{Level}]$

$$\max_{x_1} F_1(x) = \sum_{j=1}^{n} \left[\left[\underline{a}_j^L, \underline{a}_j^U\right], \left[\overline{a}_j^L, \overline{a}_j^U\right]\right] x_j + \frac{1}{2} x_j^T \left[\left[\underline{a}_j^L, \underline{a}_j^U\right], \left[\overline{a}_j^L, \overline{a}_j^U\right]\right] x_j, \quad (13)$$

where $x_2, \dots, x_n$ solves

$[2^{nd}\mathrm{Level}]$

$$\max_{x_2} F_2(x) = \sum_{j=1}^{n} \left[\left[\underline{b}_j^L, \underline{b}_j^U\right], \left[\overline{b}_j^L, \overline{b}_j^U\right]\right] x_j + \frac{1}{2} x_j^T \left[\left[\underline{b}_j^L, \underline{b}_j^U\right], \left[\overline{b}_j^L, \overline{b}_j^U\right]\right] x_j, \quad (14)$$

where $x_3, \dots, x_n$ solves

$[3^{rd}\mathrm{Level}]$

$$\max_{x_3} F_3(x) = \sum_{j=1}^{n} \left[\left[\underline{c}_j^L, \underline{c}_j^U\right], \left[\overline{c}_j^L, \overline{c}_j^U\right]\right] x_j + \frac{1}{2} x_j^T \left[\left[\underline{c}_j^L, \underline{c}_j^U\right], \left[\overline{c}_j^L, \overline{c}_j^U\right]\right] x_j, \quad (15)$$

where $x_4, \dots, x_n$ solves

Subject to

$$G = \{x \mid Ax \le d, x \ge 0\}. \quad (16)$$

Where G is the three-level convex constraint set, F_1, F_2 and F_3 are the objective functions of the FLDM, SLDM, TLDM, respectively. Also $\left[\left[\underline{a}_j^L, \underline{a}_j^U\right], \left[\overline{a}_j^L, \overline{a}_j^U\right]\right]$, $\left[\left[\underline{b}_j^L, \underline{b}_j^U\right], \left[\overline{b}_j^L, \overline{b}_j^U\right]\right]$ and $\left[\left[\underline{c}_j^L, \underline{c}_j^U\right], \left[\overline{c}_j^L, \overline{c}_j^U\right]\right]$ are RI coefficients of the objective functions. Let *(j=1,2,…,n)* $, x = (x_1, x_2, \dots, x_n)^T$ denote the vector of all decision variables.

A solution algorithm to solve the TLQPRIC problems (13) - (16) is described in a series of steps as follows [15]:

Step1: Determine the surly random rough interval coefficients range (lower (L) interval problem) in FLDM, SLDM, and TLDM problem, respectively.
Step2: Determine the possible random rough interval coefficients range (upper (U) interval problem) in FLDM, SLDM, and TLDM problem, go to Step3.
Step3: Formulate the corresponding equivalent three-level quadratic programming problem.
Step4: Convert the lower and upper random interval coefficients in the FLDM problem into equivalent crisp models can be solved by classical methods.
Step5: Convert the lower and upper random interval coefficients SLDM, and TLDM problem into equivalent crisp models, go to Step 6.
Step6: Using the fuzzy approach as described in [10] to solve the resulting multi-level decision-making problems in Step 5.
Step7: Build membership functions of the FLDM, SLDM, and TLDM after determining the best and the worst solution of all lower interval coefficients and upper interval coefficients problems.
Step8: Solve a Tchebycheff problem [10] for all DMs level problem.
Step9: Control assumed the FLDM his /her decision by tolerance t_1.
Step10: Control assumed the SLDM his/her decision by tolerance t_2.
Step11: If $\omega < 0$, increase t_1, t_2 then go to Step7; otherwise, go to Step 12.
Step12: The FLDM, SLDM, and TLDM calculating membership function $\grave{\mu}$.
Step13: Compute tolerance functions for x_1, x_2 using t_1, t_2 by [10].

Step14: Solve the Tchebycheff problem defined by [10], then go to Step15.
Step15: If the FLDM isn't satisfied with the solution then go to Step 9 with modifying, $\omega(\underline{\omega}^L, \underline{\omega}^U, \overline{\omega}^L, \overline{\omega}^U)$.
Step16: Stop.

8. On Solving Three Level Fractional Programming Problem with Rough Coefficient in Constraints

A three level fractional programming problem with a random rough coefficient in constraints was considered [7]. At the first phase of the solution approaches and to avoid the complexity of the problem, fractional programming problems were converted into a linear model problem using Charnes & Cooper method. Then interval technique is used to convert the rough nature in constraints into the equivalent crisp model. At the final phase, a membership function was constructed to develop a fuzzy model for obtaining a compromised solution of the three level programming problems.

Problem Formulation and Solution Concept
The Three-Level Fractional Programming Problem with Rough Interval Coefficients (TLFPRIC) in the constraints may be written as follows:
[1st level]

$$\underset{x_1}{Max}\ F_1(x) = \frac{a_1 x + \alpha_1}{b_1 x + \beta_1}, \tag{17}$$

where x_2 solves

[2nd level]

$$\underset{x_2}{\mathrm{Max}}\ \mathrm{F}_2(\mathrm{x}) = \frac{\mathrm{a}_2\mathrm{x} + \alpha_2}{\mathrm{b}_2\mathrm{x} + \beta_2}, \tag{18}$$

where x_3 solves

[3rd level]

$$\underset{x_3}{\mathrm{Max}}\ \mathrm{F}_3(\mathrm{x}) = \frac{\mathrm{a}_3\mathrm{x} + \alpha_3}{\mathrm{b}_3\mathrm{x} + \beta_3}, \tag{19}$$

Subject to

$$G = \left\{\sum_{j=1}^{n} \left([a_j, b_j], [c_j, d_j]\right) x_j \le l, x_j \ge 0, j = 1, \dots, n\right\}. \tag{20}$$

Where F_1, F_1 and F_3 are the objective functions of the FLDM, SDLM, and TLDM, $\left[[a_j, b_j], [c_j, d_j]\right]$ are RI coefficients of the constraints for the three levels.

A solution algorithm to solve the TLFPRIC problems (17) -(20) is described in a series of steps as follows [7]:
Step1: Fractional programming problems in the FLDM, SLDM, and TLDM were converted into a linear model problem using Charnes & Cooper method.
Step2: Determine the surly random rough interval coefficients range (lower (L) interval problem) and the possible random rough interval coefficients range (upper (U) interval problem) in FLDM, SLDM, and TLDM problem, respectively.
Step3: Formulate the corresponding equivalent three level fractional programming problems.
Step4: Convert the lower and upper random interval coefficients in the FLDM problem into equivalent crisp models can be solved by classical methods.
Step5: Convert the lower and upper random interval coefficients in the SLDM and TLDM problem into equivalent crisp models, go to Step 6.
Step6: Using the fuzzy approach as described in [10] to solve the resulting multi-level decision-making problems in Step 5.
Step7: Build membership functions of the FLDM, SLDM, and TLDM after determining the best and the worst solution of all lower interval coefficients and upper interval coefficients problems.
Step8: Solve a Tchebycheff problem [10] for all DMs level problem.
Step9: Control assumed the FLDM his /her decision by tolerance t_1.
Step10: Control assumed the SLDM his/her decision by tolerance t_2.
Step11: If $\omega < 0$, increase t_1, t_2 then go to Step7; otherwise, go to Step 12.
Step12: The FLDM, SLDM, and TLDM calculating membership function $\grave{\mu}$.
Step13: Compute tolerance functions for x_1, x_2 using t_1, t_2 by [10].
Step14: Solve the Tchebycheff problem defined by [10], then go to Step15.
Step15: If the FLDM isn't satisfied with the solution then go to Step 9 with modifying, $\omega(\underline{\omega}^L, \underline{\omega}^U, \overline{\omega}^L, \overline{\omega}^U)$.
Step16: Stop.

9. Fully Rough Integer Interval Transportation Problems

A new method name, a Slice-Sum method for solving fully rough integer interval transportation problems was proposed [11]. The optimal values of decision rough variables and rough objective function for the problem that is obtained by the proposed method, were rough integer intervals.

Problem Formulation and Solution Concept
Consider the following fully rough integer transportation problem:

$$Max/\,Min\left[[f^{LL}, f^{LU}], [f^{UL}, f^{UU}]\right] = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[[\underline{c}_{ij}^L, \underline{c}_{ij}^U], [\overline{c}_{ij}^L, \overline{c}_{ij}^U]\right] \otimes \left[[x_{ij}^{LL}, x_{ij}^{LU}], [x_{ij}^{UL}, x_{ij}^{UU}]\right], \tag{21}$$

Subject to

$$\sum_{j=1}^{n}\left[[x_{ij}^{LL}, x_{ij}^{LU}], [x_{ij}^{UL}, x_{ij}^{UU}]\right] = \left[[\underline{a}_i^L, \underline{a}_i^U], [\overline{a}_i^L, a_i^U]\right], \quad \mathrm{i} = 1,2, \dots, \mathrm{m}, \tag{22}$$

$$\sum_{i=1}^{m}\left[\left[x_{ij}^{LL},x_{ij}^{LU}\right],\left[x_{ij}^{UL},x_{ij}^{UU}\right]\right]=\left[\left[\underline{b}_j^{L},\underline{b}_j^{U}\right],\left[\overline{b}_j^{L},\overline{b}_j^{U}\right]\right],$$
$j = 1,2,\ldots,n$, (23)

$x_{ij}^{LL}, x_{ij}^{LU}, x_{ij}^{UL}, x_{ij}^{UU} \geq 0$ and are integers. $i = 1,2,\ldots,m$and $j = 1,2,\ldots,n$.

In the above Problem (21) – (23), $\underline{c}_{ij}^{L}, \underline{c}_{ij}^{U}, \overline{c}_{ij}^{L}$ and $\overline{c}_{ij}^{U}$ are positive integers, $\underline{a}_i^{L}, \underline{a}_i^{U}, \overline{a}_i^{L}$ *and* $\overline{a}_i^{U}$ are positive integers, $\underline{b}_j^{L}, \underline{b}_j^{U}$, $\overline{b}_j^{L}$ and $\overline{b}_j^{U}$ are positive integers. The above problem is said to be balanced if the total supply is equal to the total demand.

An algorithm for solving the Fully Rough Integer Interval Transportation Problem [11]

Step1: Check that the given problem (21) – (23) is balanced. If not, make it into balanced.

Step2: Construct the UAUB integer transportation problem of the problem (21) – (23) .

Step3: Solve the UAUB integer transportation problem using a transportation algorithm [5] / the zero-point method [12]. Let $x_{ij}^{*UU}, i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ be an optimal solution of the UAUB integer transportation problem with the minimum transportation cost f^{*UU}.

Step4: Construct the LAUB integer transportation problem of the problem (21) – (23).

Step5: Solve the LAUB integer transportation problem with upper bound *constraints* $x_{ij}^{LU} \leq x_{ij}^{*UU}, i = 1,2,\ldots,m$ *and* $j = 1,2,\ldots,n$ using the zero point method [5] / the integer linear programing technique [12]. Let $x_{ij}^{*LU}, i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ be an optimal solution of the LAUB integer transportation problem with the minimum transportation cost f^{*LU}.

Step6: Construct the LALB integer transportation problem of the problem (21) – (23).

Step7: Solve the LALB integer transportation problem with upper bound constraints $x_{ij}^{LL} \leq x_{ij}^{*LU}, i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ using the zero point method [5] / the integer linear programing technique [12]. Let $x_{ij}^{*LL}, i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ be an optimal solution of the LALB integer transportation problem with the minimum transportation cost f^{*LL}.

Step8: Construct the UALB integer transportation problem of the problem (21) – (23).

Step9: Solve the UALB integer transportation problem with upper bound constraints $x_{ij}^{UL} \leq x_{ij}^{*LL}, i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ using the zero point method [5] / the integer linear programing technique [12]. Let $x_{ij}^{*UL}, i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ be an optimal solution of the UALB integer transportation problem with the minimum transportation cost f^{*UL}.

Step10: The optimal solution of the given problem is $\left[\left[x_{ij}^{*LL},x_{ij}^{*LU}\right],\left[x_{ij}^{*UL},x_{ij}^{*UU}\right]\right]$with the minimum transportation cost $\left[\left[f^{*LL},f^{*LU}\right],\left[f^{*UL},f^{*UU}\right]\right]$.

The reader is referred to Theorem 2.

10. Conclusion

This paper stated a literature review on rough intervals formulation and methodologies such as formulation of linear programming problems with rough intervals coefficients in the objective functions and constraints, basic preliminaries about rough intervals, interval method and trust probability constraints for transforming rough intervals to crisp nature and fully rough intervals problems. Finally, presented different mathematical programming models that contain rough intervals coefficients.

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