Staffing Optimization Problem Based On Queueing Models Of Multi-Skill Call Center With Patient Customers

N. A. Hassan , R. A. Atwa , N. M. S. Abdallah
Department of Mathematics, Faculty of Science, Zagazig University
nahassan@zu.edu.eg
*rasha_atwa*75@hotmail.com
noura.msa@zu.edu.eg

Abstract

The call center considers an interesting area of the application of queueing models. In a call center's queueing model, the customers are the callers and the servers are the call agents. The call center, which effectively running must get to the balance between service level and service costs. The quality of the service presented is of essential importance. So, the call center must guarantee at any time an appropriate number of servers with appropriate multi-skills according to the expected level of the demand, this issue is ordinarily called the staffing problem. Thus, the paper has the aim to explain how to apply the queueing model method for evaluating the performance of multi-skill call center's index and computing the formula of service level through presenting the optimization of the staffing problem for the optimal number of agents in each group. Finally, we used the deduced results through a numerical example to calculate the steady-state probabilities, service levels, the optimal number of agents in each group, performance measures, and how these influence factors in the whole system.

Keywords: Queueing model; Multi-skill call center; Steady-state probabilities; Service level; Staffing problem; Performance measure.

1 Introduction

A call center is defined as a sort of service network where customers wait in queues to be served by the service providers (agents). Nowadays, the industry of call centers is becoming a gradually significant part of the environment of business. The business purpose is to satisfy the customers and to make a standard name for the company. One of the most popular utilized ways in business is call centers provide customer service and technical support. Call centers concentrated on the satisfaction of customers with the service. The company utilizes many ways, such as saving money and reducing waiting time by providing a better service to the customers. With the fast progress in technology, other channels appeared in call centers and these make the servers busy and have to grasp various skills to handle several different classes of customers within a specified time.

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In multi-skill call centers, customers can be handled in different ways, the agents can be partitioned into groups according to their skills, and skills-based routing can be utilized to direct customers to appropriate agents. Multi-skill call centers make the process of estimation of the system extra complicated. So, with the complexity of modern call centers, the prioritization and routing of callers are now imperative. Staffing in a multi-skill call center is a complicated mission. Many factors govern the satisfaction of the required service level like agent salaries, skill availability, and work standards. An important related survey on telephone call centers is introduced by Koole and Mandelbaum (2002). Also, we refer to Gans et al. (2003) for more featured information on the telephone call centers, skill-based routing, Erlang C, staff hiring, and queueing. Aksin et al. (2007) supplied a survey of the recent literature on call center operations management, interested in modern management challenges that have been caused by emerging technologies, issues associated with both customers and servers, and showed the interface between the operations of a call center and sales and sales and marketing. Ormeci (2004) discussed the dynamic admission control of a loss system with one shared and two dedicated stations. Li and Yue (2016) studied the N-design multi-skill call center. This paper's structure is organized as follows: The next section describes the multi-skill call center system model. Section 3 presents the division of the system's state space, the calculation of the state-transition rates based on the results of $M/M/c/c$ and $M/M/c$ queueing models, the calculation of the steady-state probabilities of the system through the establishment of the equilibrium equations, and the calculation of the call center's service level . Section 4 introduces the computational technique of the staffing problem. Section 5 explains the previous results through an example. Finally, concluding comments are given in Section 6.

2 Model Description

In our model, the calls (customers) are classified into two types (Call Type 1 and Call Type 2) and agents (servers) are divided into three groups (Group 1, Group 2, and Group 3) with different skills. This queueing model is fully characterized by customer profiles (Arrival Process), agent properties (Service Process), routing policies, and the limitation of the waiting queue (Queueing Discipline). Our methodology as follows:

2.1 Customer Profiles

Two customer types arrive according to the Poisson process with arrival rates λ_1 and λ_2, respectively. There are two queues (Queue 1 and Queue 2), the arriving calls are lined with it, which contain calls of Type 1 and Type 2, respectively. There are infinite waiting spaces for two queues.

2.2 Agent Properties

There are three categories of agents. Group 1 (has skill 1), Group 2 (has skill 2) and Group 3 (have skill 1 and skill 2) consist of N_1 agents, N_2 agents and N_3 agents, respectively. Group 1 (who can only serve calls of Type 1) is of specialized agents with N_1 agents and with mean μ_1. Group 2 (who can only serve calls of Type 2) is also specialized agents with N_2 agents and with mean μ_2. Group 3 (who can serve calls of both Type 1 and Type 2) is of flexible agents with N_3 agents and with mean μ_3. The service times of agents in Group 1, Group 2, and Group 3 are all exponentially distributed.

2.3 Limitation of the Waiting Queue (Queue Discipline)

The waiting spaces for both two queues are infinite. Each type of calls has its own queue. The queues of both two call types (Call Type 1 and Call Type 2) are independent of each other. For the same type of waiting calls (Call Type 1 or Call Type 2), they are served in FCFS (First-come First-served) discipline by a free agent of its own group (Group 1 or Group 2) and also, a free agent in Group 3 serve the waiting calls in Queue 1 or Queue 2 according to FCFS discipline. If all agents in both Group 1 and Group 2 are busy and there are waiting for calls both in them, a free agent in Group 3 will pick out a Call Type 1 and Call Type 2 for service at random (i.e., with equal probability).

2.4 Routing Policy

The routing policy in our model is skill-based routing (i.e., it is based on skills). There are priorities for various call types. The waiting calls (customers) of Type 1 have priority to be served by the agent in Group 1 if there are free agents in Group 1 and free agents in Group 3. Also, the waiting calls of Type 2 have priority to be served by the agent in Group 2 if there are free agents in Group 2 and free agents in Group 3. The calls will be serviced by a free agent in Group 3 if all agents in Group 1 or Group 2 are busy. The customer must wait in Queue 1 or Queue 2 if all agents are busy in Group 3.

3 Steady-State Probabilities

In this section, we firstly defined the states of our system model, then obtained the state-transition rates in two cases (call arrival and service completion), and established the equilibrium equations for the steady-state probabilities of the model when the model is stationary. Finally, we computed the service level.

3.1 State Space Description

We have three groups with various skills in our model wherein each of them there are three states (idle, busy, and overload).

I- An Idle State: In this case, at least one agent is idle. This state symbolized by 1.

II- A Busy State: In this case, all agents in the group are busy and there no calls waiting for service served by this group. This state symbolized by 2.

III- An Overload State: In this case, all agents in the group are busy, and there at least one call waiting for service by this group. This state symbolized by 3.

We note that the state space of the system model consisted of 12 states, according to the routing policy assumed above. Thus, the state space is given by

$$E = \{(111), (121), (122), (132), (211), (212), (222), (221), (232), (312), (322), (332)\}$$

Let $S_i, (i = 1, 2, 3, ..., 12)$ be the i^{th} state in the state space E. let n_i be the number of calls waiting in a queue that are assumed to be served by Group i plus the busy agents in Group i. For example, n_1 be the number of waiting calls (customers) for service involving those being serviced by agents (servers) of Group 1, n_2 be the number of waiting calls for service involving those being serviced by agents of Group 2, and n_3 be the number of calls being serviced by agents of Group 3 as no waiting calls in a queue according to routing policy.

3.2 The construction of the State-Transition Rates

We derive the state-transition rates by using results of M/M/c/c and M/M/c queueing systems (which are given in [6]). There are only two events that can make the state transferred: call arrival or service fulfillment. We will be debating the two cases separately to obtain how the state transition rates are calculated.

I- The transfer of states due to the call arrival.

We assume the state $S_1 = (111)$, for example, which means the agents in the three groups (Group 1, Group 2, and Group 3) are in the idle state. Let $q_{S_i - S_j}, (i, j = 1, 2, 3, ..., 12)$ denote the state-transition rate. The trigger for the transfer from the state S_1 to the state S_5 is due to Call Type 1. Thus, the transition rate from the state S_1 to the state S_5 is obtained as follows:

$$q_{1-5} = \lambda_1 P(n_1 = N_1 - 1) \quad (1)$$

Where $P(n_1 = N_1 - 1)$ is the probability that the number of Call Type 1 needing to be serviced by the agents in Group 1 is $n_1 = N_1 - 1$ for state S_1. Note that $n_1 < N_1$ and $n_2 < N_2$ (i.e., if the operation is in state S_1 then the number of calls of either Type 1 or Type 2 is less than the number of agents either in Group 1 or Group 2), and that the two queues are independent of each other so the results of the $M/M/c/c$ loss queueing system can be used. Consequently, we have

$$P(n_1 = N_1 - 1) = \rho_1^{N_1 - 1} / \left[(N_1 - 1)! \sum_{j=0}^{N_1} \rho_1^j / j! \right] \quad ; \rho_1 = \lambda_1 / N_1 \mu_1 \quad (2)$$

In a similar way, we can acquire the other transition rates q_{i-j} caused by the arrival of calls as follows:

$$q_{1-5} = q_{2-8} = q_{3-7} = q_{4-9} = \lambda_1 P(n_1 = N_1 - 1) \quad (3)$$

$$q_{1-2} = q_{6-7} = q_{5-8} = q_{10-11} = \lambda_2 P(n_2 = N_2 - 1) \quad (4)$$

$$q_{9-12} = q_{7-11} = q_{6-10} = \lambda_1 \quad (5)$$

$$q_{7-9} = q_{3-4} = q_{11-12} = \lambda_2 \quad (6)$$

$$q_{5-6} = \lambda_1 P^2(n_3 = N_3 - 1) \quad (7)$$

$$q_{2-3} = \lambda_2 P^1(n_3 = N_3 - 1) \quad (8)$$

$$q_{8-11} = q_{8-9} = (\lambda_1 + \lambda_2)P^3(n_3 = N_3 - 1) \quad (9)$$

$$q_{1-8} = \lambda_1 P(n_1 = N_1 - 1) + \lambda_2 P(n_2 = N_2 - 1) \quad (10)$$

$$q_{7-12} = \lambda_1 + \lambda_2 \quad (11)$$

$$q_{3-9} = \lambda_1 P(n_1 = N_1 - 1) + \lambda_2 \quad (12)$$

$$q_{6-11} = \lambda_2 P(n_2 = N_2 - 1) + \lambda_1 \quad (13)$$

$$q_{2-7} = \lambda_1 P(n_1 = N_1 - 1) + \lambda_2 P^1(n_3 = N_3 - 1) \quad (14)$$

$$q_{5-7} = \lambda_2 P(n_2 = N_2 - 1) + \lambda_1 P^2(n_3 = N_3 - 1) \quad (15)$$

Where

$$P(n_2 = N_2 - 1) = \rho_2^{N_2-1} / \left[(N_2 - 1)! \sum_{j=0}^{N_2} \rho_2^j / j! \right] \quad ; \rho_2 = \lambda_2 / N_2\mu_2 \quad (16)$$

$$P^1(n_3 = N_3 - 1) = \rho_3^{N_3-1} / \left[(N_3 - 1)! \sum_{j=0}^{N_3} \rho_3^j / j! \right] \quad ; \rho_3 = \lambda_1 / N_3\mu_3 \quad (17)$$

$$P^2(n_3 = N_3 - 1) = \rho_4^{N_3-1} / \left[(N_3 - 1)! \sum_{j=0}^{N_3} \rho_4^j / j! \right] \quad ; \rho_4 = \lambda_2 / N_3\mu_3 \quad (18)$$

$$P^3(n_3 = N_3 - 1) = (\rho_3 + \rho_4)^{N_3-1} / \left[(N_3 - 1)! \sum_{j=0}^{N_3} (\rho_3 + \rho_4)^j / j! \right] \quad (19)$$

II- The transfer of states due to the fulfillment of service.

We consider the state $S_2 = (121)$, for example. If the call of Type 2 finished the service, then the set of states will be varied from state S_2 to state S_1. In the state S_2, $n_2 = N_2$ (i.e ., all N_2 agents are busy) and the service rate for the call of Type 2 is $N_2\mu_2$. The trigger for the transfer from state S_2 to state S_1 is obtained as follows:

$$q_{2-1} = N_2\mu_2 \quad (20)$$

In a similar way, we can acquire the other transition rates q_{i-j} caused by the completion of service of a call as follows:

$$q_{5-1} = q_{8-2} = q_{7-3} = q_{9-4} = N_1\mu_1 \quad (21)$$

$$q_{2-1} = q_{7-6} = q_{8-5} = q_{11-10} = N_2\mu_2 \quad (22)$$

$$q_{3-2} = q_{6-5} = N_3\mu_3 \quad (23)$$

$$q_{12-9} = q_{11-7} = q_{10-6} = N_1\mu_1 P(n_1 = N_1 + 1) \quad (24)$$

$$q_{9-7} = q_{4-3} = q_{12-11} = N_2\mu_2 P(n_2 = N_2 + 1) \quad (25)$$

$$q_{9-3} = N_1\mu_1 + N_2\mu_2 P(n_2 = N_2 + 1) \quad (26)$$

$$q_{11-6} = N_2\mu_2 + N_1\mu_1 P(n_1 = N_1 + 1) \quad (27)$$

$$q_{7-2} = N_1\mu_1 + N_3\mu_3 \quad (28)$$

$$q_{8-1} = N_1\mu_1 + N_2\mu_2 \quad (29)$$

$$q_{12-7} = N_1\mu_1 P(n_1 = N_1 + 1) + N_2\mu_2 P(n_2 = N_2 + 1) \quad (30)$$

$$q_{11-8} = (N_1\mu_1 + \ 1/2\ N_3\mu_3) P(n_1 = N_1 + 1) \quad (31)$$

$$q_{9-8} = (N_2\mu_2 + \ \frac{1}{2}\ N_3\mu_3) P(n_2 = N_2 + 1) \quad (32)$$

The results of the $M/M/c$ queueing system can be used to obtain the probabilities of $P(n_1 = N_1 + 1)$ and $P(n_2 = N_2 + 1)$ as follows:

$$P(n_1 = N_1 + 1) = \frac{\rho_1^{N_1+1}}{N_1(N_1)!} P_0^1 \ , \ P_0^1 = [\sum_{j=0}^{N_1-1} \rho_1^j/j! + \frac{N_1\rho_1^{N_1}}{N_1!(N_1 - \rho_1)}]^{-1} \quad ; (33)$$

$$P(n_2 = N_2 + 1) = \frac{\rho_2^{N_2+1}}{N_2(N_2)!} P_0^2 \ , \ P_0^2 = [\sum_{j=0}^{N_2-1} \rho_2^j/j! + \frac{N_2\rho_2^{N_2}}{N_2!(N_2 - \rho_2)}]^{-1} \quad ; (34)$$

3.3 The Computation of steady-state probabilities

The steady-state probabilities of each state are denoted by P_i ; $i = 1, 2, 3, ..., 12$ and the equations of the steady-state probabilities of the system are shown as follows:

$$P_1(q_{1-2} + q_{1-5} + q_{1-8}) = P_2q_{2-1} + P_5q_{5-1} + P_8q_{8-1} \quad (35)$$

$$P_2(q_{2-1} + q_{2-3} + q_{2-7} + q_{2-8}) = P_1q_{1-2} + P_3q_{3-2} + P_7q_{7-2} + P_8q_{8-2} \quad (36)$$

$$P_3(q_{3-4} + q_{3-7} + q_{3-9}) = P_4q_{4-3} + P_7q_{7-3} + P_9q_{9-3} \quad (37)$$

$$P_4(q_{4-3} + q_{4-9}) = P_3q_{3-4} + P_9q_{9-4} \quad (38)$$

$$P_5(q_{5-1} + q_{5-6} + q_{5-7} + q_{5-8}) = P_1q_{1-5} + P_6q_{6-5} + P_7q_{7-5} + P_8q_{8-5} \quad (39)$$

$$P_6(q_{6-5} + q_{6-7} + q_{6-10} + q_{6-11}) = P_5q_{5-6} + P_7q_{7-6} + P_{10}q_{10-6} + P_{11}q_{11-6} \quad (40)$$

$$P_7(q_{7-2} + q_{7-3} + q_{7-5} + q_{7-6} + q_{7-9} + q_{7-11} + q_{7-12}) = P_2q_{2-7} + P_3q_{3-7} + P_5q_{5-7} + P_6q_{6-7} + P_9q_{9-7} + P_{11}q_{11-7} + P_{12}q_{12-7} \quad (41)$$

$$P_8(q_{8-1} + q_{8-2} + q_{8-5} + q_{8-9} + q_{8-11}) = P_1q_{1-8} + P_2q_{2-8} + P_9q_{9-8} + P_{11}q_{11-8} \quad (42)$$

$$P_9(q_{9-3} + q_{9-4} + q_{9-7} + q_{9-8} + q_{9-12}) = P_3q_{3-9} + P_4q_{4-9} + P_7q_{7-9} + P_8q_{8-9} + P_{12}q_{12-9} \quad (43)$$

$$P_{10}(q_{10-6} + q_{10-11}) = P_6q_{6-10} + P_{11}q_{11-10} \quad (44)$$

$$P_{11}(q_{11-6} + q_{11-7} + q_{11-8} + q_{11-10} + q_{11-12}) = P_6q_{6-11} + P_7q_{7-11} + P_8q_{8-11} + P_{10}q_{10-11} + P_{12}q_{12-11} \quad (45)$$

$$P_{12}(q_{12-7} + q_{12-9} + q_{12-11}) = P_7q_{7-12} + P_9q_{9-12} + P_{11}q_{11-12} \quad (46)$$

$\sum_{i=1}^{12} P_i = 1$. Since the equations are linear, so by solving these equations from (35) to (46), (using MATLAB software), we can obtain all the steady-state probabilities P_i ; $i = 1, 2, 3, ..., 12$ as shown in Table (1).

3.4 The Computation of Service Level

Calculating service level is considered as one of the most important works in our paper to evaluate the performance of a multi-skill call center and is calculated by using the steady-state probabilities. The service level can be expressed as the percentage of the calls should be serviced within a given waiting time T_i, denoted as P_i/T_i. Let $P_{sl}^i = 1 - P_{ns}^i$; $i = 1, 2$ be the probability that the call of Type i is serviced in a fixed time T_i.

Consider the call of Type 1, for example. Calls of Type 1 has a queue only occur in the states $S_{10} = (3\ 1\ 2), S_{11} = (3\ 2\ 2)$, and $S_{12} = (3\ 3\ 2)$. The service rate for the call of Type 1 in each state of S_{10} and S_{11} is $N_1\mu_1 + N_3\mu_3$, the number of calls could be served in time T_1 is $T_1(N_1\mu_1 + N_3\mu_3)$. Also, the service rate in state S_{12} is $N_1\mu_1 + N_3\mu_3/2$ according to the routing policy and the number of calls could be served in this state in time T_1 is $T_1(N_1\mu_1 + N_3\mu_3/2)$. We can obtain the probability P_{ns}^1 via the above analysis as follows:

$$P_{ns}^1 = (P_{10} + P_{11}) \sum_{i=K_1}^{\infty} P(n_1 = i) + P_{12} \sum_{i=K_2}^{\infty} P(n_1 = i) \quad (47)$$

Where,

$$K_1 = N_1 + N_3 + T_1(N_1\mu_1 + N_3\mu_3), K_2 = N_1 + N_3/2 + T_1(N_1\mu_1 + N_3\mu_3/2) \quad (48)$$

Similarly, for a call of Type 2, calls have a queue only occur in the states $S_4 = (1\ 3\ 2)$, $S_9 = (2\ 3\ 2)$, and $S_{12} = (3\ 3\ 2)$. Thus, we can obtain the probability P_{ns}^2 via the same analysis method above as follows:

$$P_{ns}^2 = (P_4 + P_9) \sum_{i=K_3}^{\infty} P(n_2 = i) + P_{12} \sum_{i=K_4}^{\infty} P(n_2 = i) \tag{49}$$

Where,

$$K_3 = N_2 + N_3 + T_2(N_2\mu_2 + N_3\mu_3), K_4 = N_2 + N_3/2 + T_2(N_2\mu_2 + N_3\mu_3/2) \tag{50}$$

And also, $P(n_1 = i)$ and $P(n_2 = i)$ are the probabilities that there are i customers in the$M/M/N_1$ and $M/M/N_2$ queueing system with arrival rate λ_1 and λ_2 and service rates μ_1 and μ_2 , respectively, which their formulas are given in ([6]).

By using MATLAB software, we can get the service levels P_{sl}^1 and P_{sl}^2 as shown in Table (2).

4 Staffing Optimization

In this section, we offered the optimization model of the staffing problem to find the optimal numbers of the agents in each group to minimize the cost of the system. We suppose that the cost of the agents' Group 1 is C_1, the cost of the agents' Group 2 is C_2, and the cost of the agents' Group 3 is C_3. We seek to get the optimal number of agents N_1, N_2, and N_3 subject to the conditions of the constrain to minimize the cost of the system model. The staffing optimization can be expressed as follows:

$$\min Z = C_1N_1 + C_2N_2 + C_3N_3 \tag{51}$$

$$\text{s.t. } P_{sl}^1 \geq \alpha_1$$

$$P_{sl}^2 \geq \alpha_2$$

$$N_i \in Z^+; i = 1, 2, 3$$

We note that the objective function is linear and the conditions of constraint represent that the probabilities of every type calls can be serviced are less than or equal α_1 and α_2, respectively. where α_1 and α_2 are the given service rate of the Call Type 1 and the Call Type 2, respectively, Z^+ denote the set of positive integers and N_1, N_2 , and N_3 are unknown positive integers for solving, we can find these unknown variables by using EXCEL as shown in Table (3).

5 Example

To explain how we can utilize the previous outcomes to calculate the steady-state probabilities, the service levels, the optimal number of agents in each group and the important measures of performance like W_q , L_q , W_s , and L_s which may effect on the system's efficiency under consideration.

let us assume the following example ($M/M/36/120/PRT$). Suppose that our system is worked at first with three groups of servers and each type of customer with being serviced by its specific group if there are free servers in each group and when one of two groups of servers (specific group) is busy, the system is operated with the the third group (flexible group) but if all servers are busy in the third group, the customer of any type will wait in its specific queue. After the customer finished his service, the system re-operates with three groups of servers again. Hence the system alternates between two types of customers of system operation. The capacity of the system is 120 customers only.

The results explained in tables 1, 2, 3, 4 and 5 were gotten by utilizing personal computer, MATLAB software, Excel, and the FORTRAN program, respectively. For the generated data utilized in this example were gotten by giving the FORTRAN program the various input in each case $\frac{1}{\lambda_1}$, $\frac{1}{\lambda_2}$, $\frac{1}{\mu_1}$, $\frac{1}{\mu_2}$, $\frac{1}{\mu_3}$, N_1, N_2, and N_3, then from IMSL library calling the routine RNEXP and running the FORTRAN program. Tables (1-5) contain different service and arrival rates to represent the the variation effect on the system. Where λ_1 is the arrival rate of call 1 to the system and λ_2 is the arrival rate of call 2 to the system. To show how the results in each table were obtained, let us take the first case, for example, when $N_1 = 11$, $N_2 = 16$, and $N_3 = 9$ and the number of customers of type 1 = 60 and the number of customers of type 2 = 60. By giving the FORTRAN program, the input means $\frac{1}{\lambda_1} = 6$, $\frac{1}{\lambda_2} = 7$, $\frac{1}{\mu_1} = 170$, $\frac{1}{\mu_2} = 175$, and $\frac{1}{\mu_3} = 180$ unit time and running the program , we calculated all observed rates $\lambda_1 = 0.2271$, $\lambda_2 = 0.1923$, $\mu_1 = 0.0089$, $\mu_2 = 0.0079$, and $\mu_3 = 0.0061$.

Then by putting $\lambda_1 = 0.2271$, $\lambda_2 = 0.1923$, $\mu_1 = 0.0089$, $\mu_2 = 0.0079$, and $\mu_3 = 0.0061$ and utilizing the equations (1-34) we get all the state-transition rates of both call arrival and service fulfilment, then by substituting in equations (35-46), we get all the steady-state probabilities, and by using equations (47-50), we calculate the service levels, and from equation (51) we can get the optimal number of servers in each group.

6 Conclusion

In this paper, we discussed the multi-skill call center based on queueing model (the exponential model in a multi-skill call center). The model system has two customer classes and three groups of agents where specialized agents can treat only their own customer type while flexible agents manage all types. We showed the state space of the system and by using the results of the $M/M/c/c$ and $M/M/c$ queueing system, we obtained the transition rates of the state sets, and then we created the equilibrium equations for the steady-state equations of the system. Also, we offered the computational formula of the service level, debated the computational technique of the staffing problem to calculate the optimal number of agents in each group, and then compute the performance measures of the system.

Table 1. Numerical results of the steady-state probabilities

Case	1	2	3	4	5
P_1	0.0105	0.0975	0.0265	0.0233	0.9805
P_2	0.0000	0.0000	0.0000	0.0000	0.0000
P_3	0.0000	0.0000	0.0000	0.0000	0.0000
P_4	0.0087	0.0091	0.0092	0.0082	0.0002
P_5	0.0000	0.0001	0.0000	0.0000	0.0000
P_6	0.0000	0.0000	0.0000	0.0000	0.0000
P_7	0.0000	0.0000	0.0000	0.0000	0.0000
P_8	0.0000	0.0000	0.0000	0.0000	0.0000
P_9	0.0000	0.0000	0.0000	0.0000	0.0000
P_{10}	0.4395	0.3329	0.3935	0.0325	0.0010
P_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
P_{12}	0.5413	0.5604	0.5708	0.9360	0.0183

References

[1] Koole, G., and Mandelbaum, A. (2002). Queueing models of call centers: an introduction. Annals of Operations Research, 113(1), 41-59.

[2] Gans, N., Koole, G., and Mandelbaum, A. (2003). Telephone call centers: tutorial, review, and research prospects. Manufacturing and Service Operations Management, 5(2), 79-141.

[3] Ormeci, E. L. (2004). Dynamic admission control in a call center with one shared and two dedicated service facilities. IEEE Transactions on Automatic Control, 49(7), 1157-1161.

[4] Aksin, Z., Armony, M., and Mehrotra, V. (2007). The modern call center: a multi-disciplinary perspective on operations management research. Production and Operations Management, 16(6), 665-688.

[5] Li, C., and Yue, D. (2016). The staffing problem of the N-design multi-skill call center based on queueing model. In 3rd International Conference on Wireless Communication and Sensor Networks (WCSN 2016), Atlantis Press, 427-432.

[6] Shortle, J. F., Thompson, J. M., Gross, D., and Harris, C. M. (2018). Fundamentals of queueing theory. John Wiley and Sons, 399, 89-104.

Table 2. Numerical results of the service levels

Case	N_1	N_2	N_3	P^1_{sl}	P^2_{sl}
1	11	16	9	0.0192	0.4500
2	11	14	11	0.1067	0.4305
3	11	15	10	0.0357	0.4200
4	15	15	10	0.0315	0.0558
5	15	15	15	0.9807	0.9815

Table 3. Numerical results of the optimal N_1, N_2, and N_3 at $C_1 = 10$, $C_2 = 15$, and $C_3 = 20$

Case	N_1	N_2	N_3	*Cost*
1	11	16	9	530
2	11	14	11	540
3	11	15	10	535
4	15	15	10	575
5	15	15	15	675

Table 4. Expected Average Waiting Time in the queue and in the system for type 1 and type 2.

Case	W_{q_1}	W_{s_1}	W_{q_2}	W_{s_2}
1	26.441	155.7	21.204	151.507
2	25.762	154.533	22.501	153.137
3	26.076	155.406	21.969	152..398
4	10.953	138.612	11.814	142.446
5	3.425	128.346	1.625	132.718

Table 5. Expected Average Number in the queue and in the system for type 1 and type 2.

Case	L_{q_1}	L_{s_1}	L_{q_2}	L_{s_2}
1	4.875	28.707	3.522	25.166
2	4.170	28.252	3.780	25.727
3	4.808	28.652	3.649	25.314
4	2.184	27.643	2.213	26.686
5	0.717	26.868	0.335	27.389