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MODELING OF LATERALLY SUPPORTED EMBEDDED STRUCTURES

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ABSTRACT

At lateral load-displacement response of embedded structures, the subgrade reaction method is most extensively used because the nonlinearity of soil and variation of subgrade reaction with depth can be considered relatively easily. In this method, the laterally loaded embedded structure is idealized as a beam on an elastic foundation loaded transversely and restrained by independent elastic springs acting along the length of the beam. The actual subgrade reaction distribution and the structure displacements are not dependent only on the soil stiffness but are rather a result of the mutual relationship between the structural element stiffness and the soil stiffness. A simplified distribution of springs and its stiffness coefficients is presented in this study. Through decoupling the embedded structure deformations into rigid body movement and bending deformation, the subgrade reaction modulus is estimated depending on experimental data of rigid wall movement and its accompanier ultimate passive earth pressure from literatures. The produced structural analysis results using the developed model are in a good agreement comparing with the analytical and numerical results. In addition, a parametric study is carried out, which shows that the effect of structure stiffness is more significant than the soil stiffness and with regarding the economical design is fulfilled by using flexible structural elements than the rigid one when the displacement limits is regarded.

Keywords: Beam On Elastic Foundation; Soil-Structure Interaction; Embedded Structures.

1-INTRODUCTION

The vertically embedded structures “underground tanks, building basements, diaphragm walls, abutments and any other laterally supported structures” are influenced with bending deformations in addition to rigid body motion. Both affects the passive earth pressure distribution, in other words, passive supporting reaction. The actual passive supporting reaction depends on the structural system of the embedded structure and its deformations under static and dynamic loading. Thus, it is necessary to investigate a suitable model for simulating of soil-structure interaction of embedded structures, which consider both of structure and soil stiffness within failure zone behind the structural elements. One of these models is the beam on elastic foundation or passive supporting spring model. The concept of constant modulus of the subgrade reaction is not accurate. More accurate model, which considers varied modulus of the subgrade reaction, is therefore needed. The investigated model based on decoupling the total deformation of embedded structure into rigid body motion and bending deformation: i) Rigid body motion is covered by earth pressure theories, which is useful for subgrade reaction modulus determination by applying the stiffness method concept in structural analysis; ii) Bending deformations is covered by beam on elastic foundation theories, which applies the interaction concept.

Embedded structures are widely used to resist lateral loads in building structures, offshore structures, bridges, locks and dams, etc. The lateral loads on structure are derived from earth pressures, inclined loads, wind, waves, earthquakes, etc There are many researches deal with beam on elastic foundation method for modeling of soil-structure interaction as Dobromir Dinev (2012) gave a manner of obtaining a closed-form analytical solution of the problem of bending of a beam on an elastic foundation, K. Frydrýšek (2007) focused on the theory and practice of straight beams on elastic (Winkler's) foundation. Nonlinear dependence for the reaction force on displacement in foundation (i.e. data of experiment) can be described via linear or cubic or linear + cubic approximations.

Also, there are many studies carried out to study the interaction relations of retaining structure and developed earth pressure in supporting soil as Fang et al. (2002) presented experimental data of earth pressure acting against a vertical rigid wall, which moved toward a mass of dry sand with different densities of 38%, 63%, and 80%. The instrumented retaining-wall facility at National Chiao Tung University in Taiwan was used to investigate the effects of soil density on the development of earth pressure. Berhane et al. (2005) present an attempt to develop a soil stiffness dependent mobilization function of the earth resistance of normally consolidated soft soils numerically using the FENL. Mirghasemi et al. (2006) used the Discrete Element Method (DEM) for modelling the failure zone behind retaining wall for determining active and passive earth pressure distribution, and Peng et al. (2012) used the spring supports concept as intermediate step for developing a general analytical method, which proposed to calculate the passive rigid retaining wall pressure based on Coulomb theory.

When the modulus of the subgrade reaction varies as a function of the deflection, the p-y curve method is widely used in the design. The p-y curve method can conveniently and easily incorporate nonlinear soil reaction-deflection relations between the lateral load (p) and the lateral displacement (y) at a point in embedded structure. A number of researches have been proposed to study the lateral load-displacement response of rigid walls such as in Ducan et al. (2001), Wilson et at (2008) & Jesmani et al. (2017) The objective of the present study is developing a finite element beam on elastic foundation model to simulate the soil-structure interaction of embedded structures. The developed model considers soil and wall stiffness effects. Based on rigid body motion, the soil stiffness is not constant as in Winkler theory but varied linearly with embedded length and estimating of its stiffness values depends on experimental data from literatures. The developed model differs from models of Berhane et al. (2005), Mirghasemi et al. (2006) and Peng et al. (2012), that it does not need to predefined deformation mode, which cannot be quite predicted before structural calculations.

2-PROPOSED MODEL

The present study concept, as shown in Fig. 1, the subgrade reaction is assumed as a composed body with one ideal rigid plasticity body and a series of springs, and then the magnitude of stiffness coefficients of subgrade reactions along the embedded length is a linear function of its displacement v. this concept is quite suitable for beam on elastic foundation modelling of vertically embedded structures more than the constant subgrade reaction modulus of Winkler. In this node-spring simulation model, the elastic foundation is idealized by discrete springs and the displacement at any point is assumed to be independent to displacements at any other points. This method can simplify the calculation process and is suitable for manual modeling using universal FE software’s.

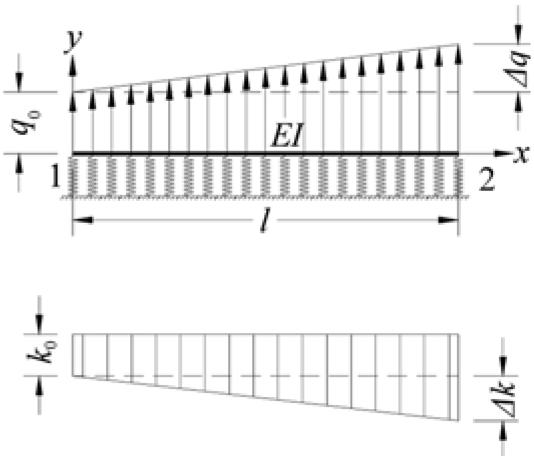


Fig. 1. Beam on an Elastic Foundation with Linearly Distributed Subgrade Reaction Modulus

2.1-BEAM ON ELASTIC FOUNDATION THEORY

Beam lies on elastic foundation when under the applied external loads, the reaction forces of the foundation are proportional at every point to the deflection of the beam at this point, this assumption was introduced first by Winkler in 1867 Dobromir Dinev (2012).
Consider a straight beam supported along its entire length by an elastic medium and subjected to vertical forces acting in the plane of symmetry of the cross-section Fig.2

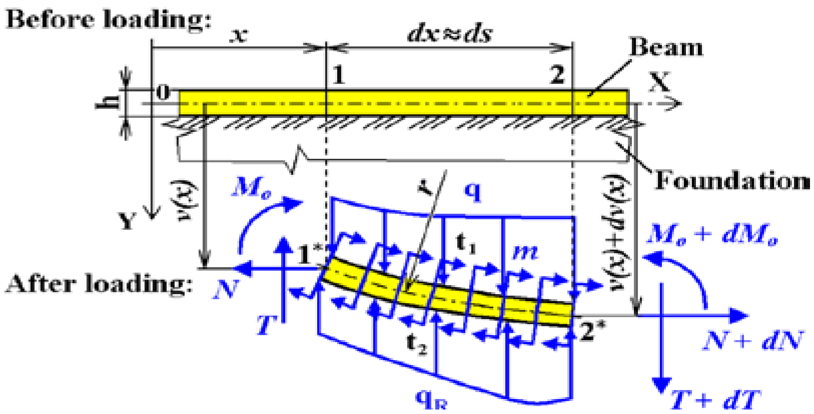


Fig. 2 Element of a straight beam on elastic foundation.

Because of the external loadings the beam will deflect producing continuously distributed reaction forces in the supporting medium. The intensity of these reaction forces at any point is proportional to the deflection of the beam *y*(*x*) at this point via the constant *k*:

$$q_R(x) = k \cdot \mathrm{v}(x) \tag{1}$$

The reactions act vertically and opposing the deflection of the beam. Hence, where the deflection is acting downward there will be a compression in the supporting medium. Where the deflection happens to be upward in the supporting medium tension will be produced which is not possible, in spite of it is assumed that the supporting medium is elastic and is able to take up such tensile forces. In other words, the foundation is made of material which follows Hooke's law. Its elasticity is characterized by the force, which distributed over a unit area, will cause a unit deflection. This force is a constant of the supporting medium called *the modulus of subgrade reaction* k_0.
Assume that the beam under consideration has a constant cross section with constant width *b*

which is supported by the foundation. A unit deflection of this beam will cause reaction equal to $k_0 \cdot b$ in the foundation, therefore the intensity of distributed reaction (per unit length of the beam) will be:

$$R(x) = b \cdot k_0 \cdot v(x) = k \cdot v(x) \qquad (2)$$

where $k = k_0 \cdot b$ is the ***constant of the foundation***, known as ***Winkler's constant***, which includes the effect of the width of the beam.

For linearly varied modulus of subgrade reaction in Fig.1 k is equal to $k(x)$, which varied linearly along the beam length (L) as follows

$$k(x) = k_0 + \frac{\Delta k}{L} . x \qquad (3)$$

Using the last relation and the beam theory, one can generate the governing differential equations for the centroidal line of the deformed beam resting on one-parameters elastic foundation Karmvir et al (2014) as:

$$EI \frac{d^4 v}{d^4 x} + k(x)\, v = F \qquad (4)$$

Where E is elastic modulus and I is beam cross section moment of inertia.

Equation 5 consists of three terms:

First term: represents the bending stiffness of beam,

Second term: represents the spring supporting stiffness of soil,

Third Term: represents the external load.

By applying the weighted residual method on equation 5, the following equilibrium equation in finite element form is presented as:

$$[K_B] . \{v\} + [K_S] . \{v\} = \{F\} \qquad (5)$$

Where $[K_B]$, $[K_S]$ are element stiffness matrix of beam and spring element respectively and {v}, {F} are displacement and force vectors of element.

2.2-STIFFNESS METHOD CONCEPT IN EVALUATION OF SUBGRADE REACTION MODULUS

As mentioned above, the investigated model based on decoupling the total deformation of embedded structure into two parts, namely, rigid body motion and bending deformation: i) Rigid body motion is covered by earth pressure theories, which is useful for subgrade reaction modulus determination by applying the stiffness method concept in structural analysis; ii) Bending deformations is covered by beam on elastic foundation theories, which applies the interaction concept. Thus, the subgrade reaction modulus k depends on soil stiffness and its value is sensitive to soil properties. So rigid body translation mobilizes the earth pressure and bending deformation of structure develops the structure element's internal forces. The first part mobilizes the passive earth pressure $P_{passive,}$ and its extreme value and its corresponding displacement value are estimated according to experimental results. Once they are known experimentally, the subgrade reaction modulus k is expressed as relation of passive earth pressure $P_{passive}$ to its corresponding displacement s as follows:

$$k = \frac{p_{passive}}{s} \quad \text{(Force per cubic meter)} \qquad (6)$$

By this way the passive earth pressure zone is modelled as series of spring supports like Winkler theory. But, the present model differs from Winkler theory that the soil spring coefficient is not constant but varied linearly along the embedded length of structure.

By representing the embedded structure as unit width wall and decoupling the wall deformation in two parts; i) rigid wall translation, which depends only on soil stiffness and ii) Wall bending deformation, which depends on the bending stiffness (EI) of wall, then, the determination of supporting spring can be easier proceeded by considering the passive earth pressure distribution of critical state concept after either Terzaghi or Coulomb theory and its accompanier rigid wall translation (s). The determination of displacement value s depends on the experimental results after Fang et al. (2002), which is illustrated in Fig. 3. It shows the relation between the passive earth pressure coefficient K_h and the ratio of rigid wall translation (s) to wall height for different backfill densities as follows:

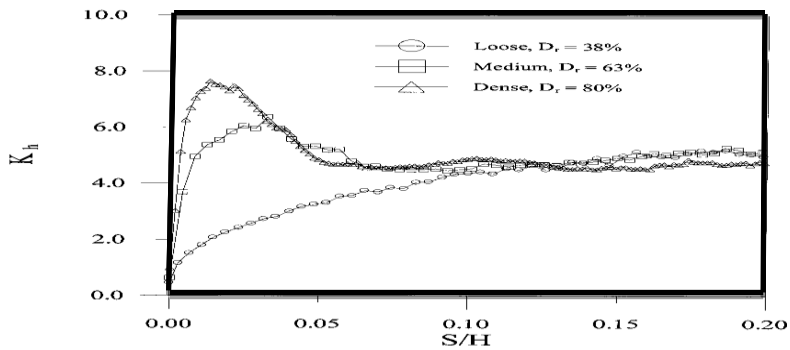


Fig. 3. Variation of K_h with wall movement for loose, medium dense, and dense backfill after Fang et al. (2002)

i. For the wall with dense backfill, the earth pressure coefficient K_h increased with increasing wall movement. After reaching a peak value about S/H=0.01, K_h decreased with increasing wall movement, and finally reached the critical state.

ii. For the wall with medium dense backfill, the earth pressure measured increased with increasing wall movement before reaching a peak value at S/H=0.03. After the peak, K_h decreased with further wall displacement, and finally reached the critical state.

iii. For the wall with loose backfill, the earth pressure increased with increasing wall movement, and eventually reached a limiting passive pressure at S/H=0.17

iv. As the wall movement S exceeded 12% of the wall height H, the passive earth pressure would reach a constant value, regardless of the initial density of backfill. It may be deduced that soils along the rupture surface had reached the critical state. Also, it is found that the ultimate passive earth pressure was successfully estimated by adopting the critical state concept to either Terzaghi or Coulomb theory, which represented with the residual friction angle φ_r.

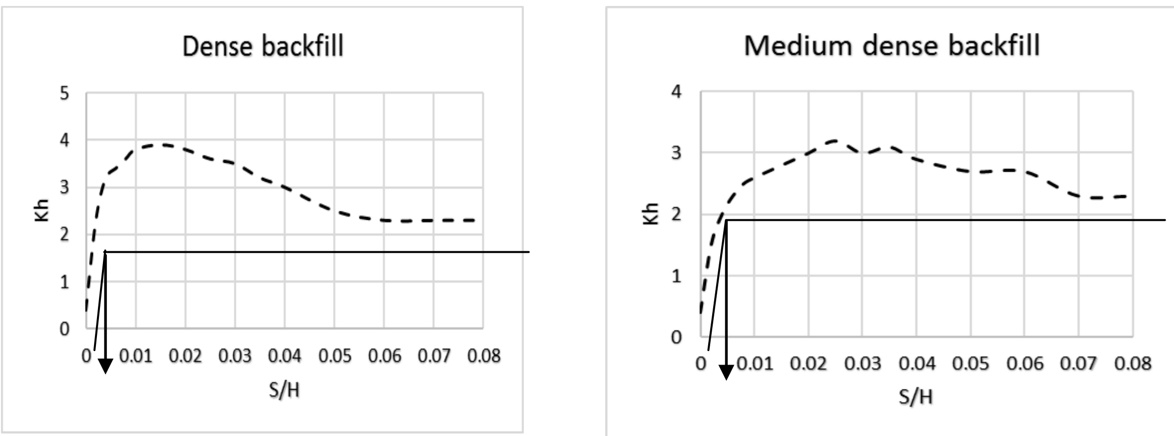


Fig. 4. linearized variation of K_h with wall movement for dense and medium dense backfill

However, using of loose backfill is not desirable. So, medium dense and dense backfill are strongly recommended, Therefore, the experimental data of Fang et al. (2002) study is linearized for dense and medium dense backfill in idealized diagram shown in Fig. 4 through the following steps:

i. Represent the limiting critical state as horizontal line
ii. Extending the tangent of linear part of diagrams for dense and medium dense sand till the intersection with horizontal line represented the limiting critical state
iii. According to the idealized variation shown in Fig. 4, the maximum allowable rigid wall translation *s* is as shown in Table 1:

Table 1: the wall movement according to backfill type

backfill	displacement s (m)
dense sand	0.002H
medium sand	0.004H

By using the average value of *S/H* for medium dense and dense backfill, then, the spring stiffness in Equ. 6 is determined as follows:

$$k = 300 \left(\frac{p_{passive}}{H}\right) \quad \text{(Force per cubic meter)} \tag{7}$$

So, for embedded retaining structures the height H in Equ. 7 could be replaced with wall penetration depth D_p. Also, the penetration depth D_p should be controlled; that the passive spring pressure at any point should not exceed the limiting critical state of passive earth pressure at this point with regarding reasonable factor of safety.
So, the present model is applied through 2D finite element discretizing of the retaining structure, considering its mechanical properties, estimating the stiffness of its spring supports, and representing any acting loading system as loading action. After structural calculations at any finite element structural analysis program, the straining actions and the actual passive springs reaction distribution considering soil-structure interaction effects are produced automatically.

3- ANALYZING THE MODEL RESULTS

3.1- Application example

A concrete cantilever diaphragm wall of 11m height and 0.50m thick is modeled by 2D finite element model as beam on elastic foundation. The wall retains 5m excavation height and penetrates 6m depth in soil. The in-situ soil has residual internal friction angel φ_r=33° and density γ=1.8 t/m^3. The wall friction angel δ=0. The stiffness of supporting springs k is calculated per meter of 6m penetration depth according to Equ. 7. The active earth pressure is considered as loading action Fig. 5a. The results are illustrated in Fig. 5b,c,d,e. It is clear from results that the passive earth pressure distribution affected by wall deformation. The results show a translation and rotation mode Fig. 5c and its accompanier passive reaction distribution is parabolic curve Fig. 5b. So, the maximum shear force is 8.05 t Fig. 5d and the maximum bending moment is 17.23 m.t as shown in Fig. 5e.
Based on manual calculation of fixed earth support method, the maximum bending moment equal to 22.58 m.t, which exceeds 5.35 m.t comparing with present 2D finite element modeling in Fig. 5e, this difference produced from the flexibility of wall as illustrated in section 3.2.1. By using the Plaxis Program in analysis considering wall thickness equal to 60 cm the difference is 2.34 m.t through comparing Fig. 6d with Fig. 7c, which is acceptable within ± 20%.
Moreover Fig. 5f shows comparing of passive earth pressure distribution along the length of penetration depth, between actual passive earth pressure distribution of present work, passive earth pressure distribution of Coulomb theory and the analytical passive earth pressure distribution according the formula after Peng et al. (2012). It is noticeable that: i) The

integrated area under actual passive reaction distribution is approximately one third of others, which means that the actual reaction depends on acting force and stiffness of retaining structure in addition to the supporting soil stiffness, not on the supporting soil stiffness only as in the other two methods, in other words, it does not assume a failure situation in the soil in which the deformations are so large that maximum shear stresses can be developed ; ii) The maximum horizontal displacements are less than 0.5% of excavation height, that suggested by Clough and O'Rourke (1990).

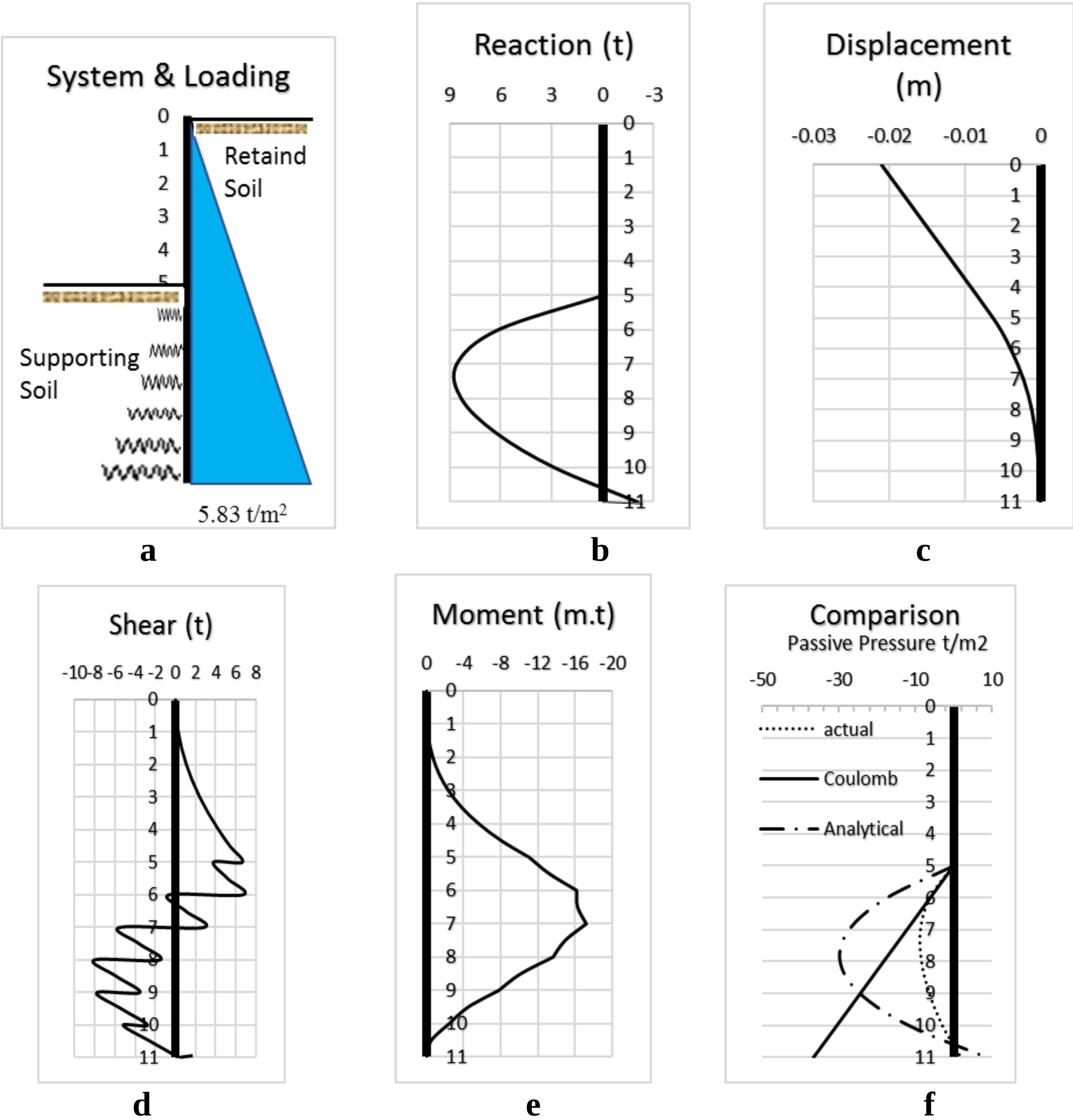


Fig. 5 Load and straining action of equivalent cantilever beam on elastic foundation

3.2 -Parametric Study

The Straining actions "spring reaction distribution, deformations and internal forces" are not only depend on supporting soil stiffness but are rather a result of the mutual relationship between the wall bending stiffness (EI) and the supporting soil stiffness (k). So, the effect of wall stiffness and supporting soil stiffness on the straining actions of cantilever diaphragm wall are studied in the following:

3.2.1 -Effect of wall thickness

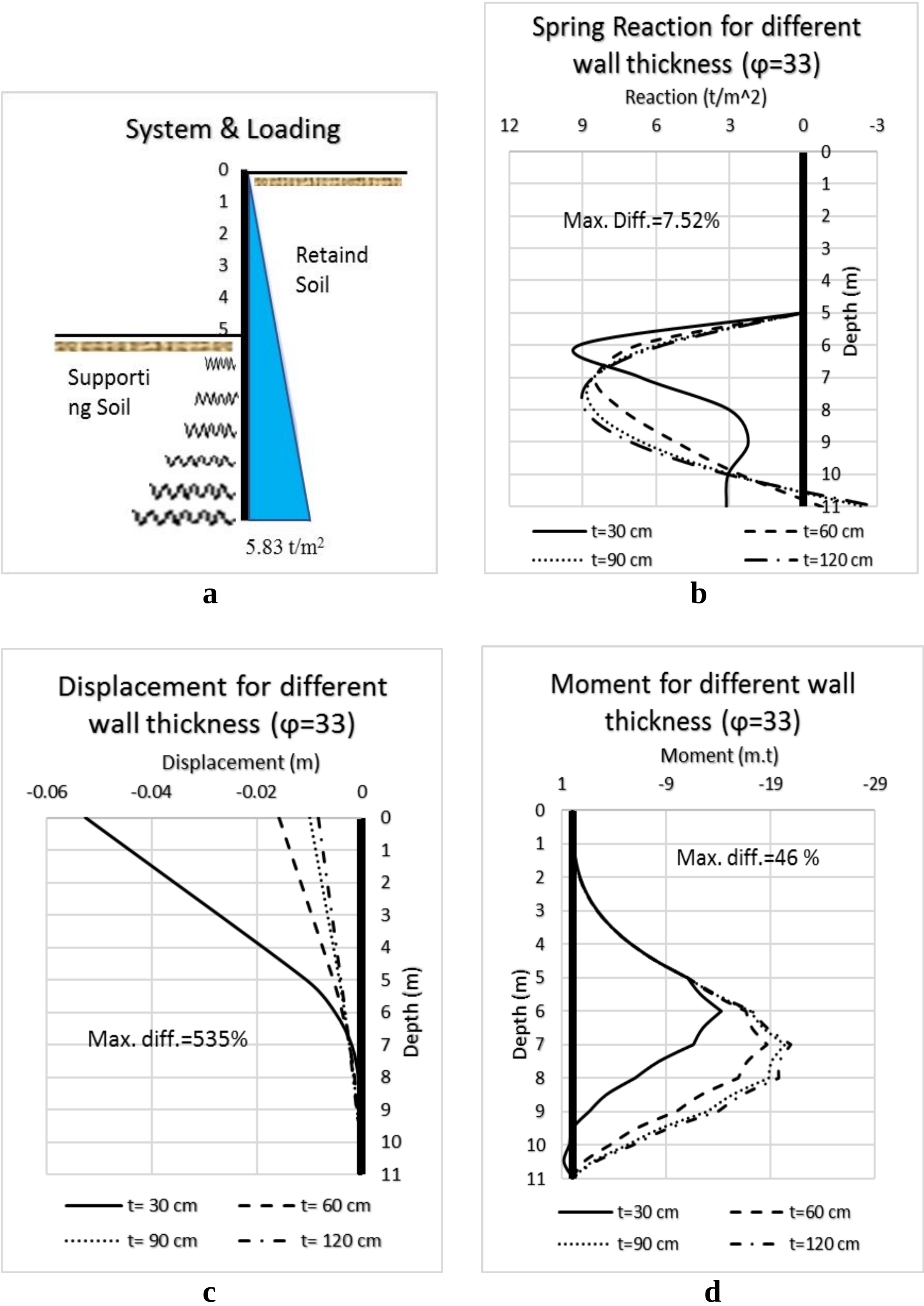


Fig. 6 Load and straining action of equivalent cantilever beam on elastic foundation for different wall thickness

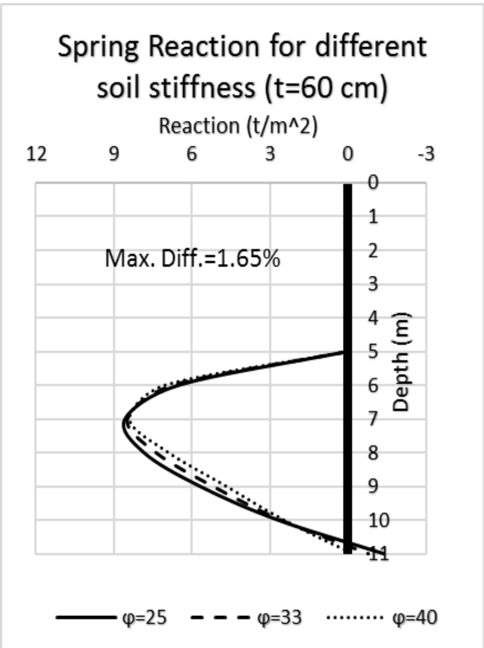
The studied case in section 3.1.1 is examined for different wall thickness as illustrated in Fig.6, which shows the structural system and loading action Fig. 6a, passive spring reaction distribution Fig. 6b, diaphragm wall displacement Fig. 6c, bending moment distribution along the wall height Fig.6d. So, the following conclusions are given:

- The distribution of springs reaction varies with changing the wall thickness, the maximum difference between its beak values is small and its resultant values are equal but differ in application points.
- The maximum displacement increases when the wall thickness decreases, also end fixation zone increases.
- When the wall thickness decreases the beak value of bending moment decreases also. So economical design is achieved when the wall thickness is smaller than 1/10 of excavation height.

3.2.2 -Effect of supporting soil stiffness
The studied case in section 3.1.1 is examined for different supporting soil stiffness in Fig.7, which shows the passive spring reaction distribution Fig. 7a, diaphragm wall displacement Fig. 7b, bending moment distribution along the wall height for wall thickness t=60 cm and t= 30 in Fig.7c&d respectively. So, the following conclusions are given:

- The distribution of springs reactions is seemly identical, the maximum difference between its beak values is very small and its resultant values are equal and approximately coincide.
- The maximum displacement is significantly increases when the wall thickness decreases.
- The change in bending moment distribution and its peak value is small for both partially rigid and flexible wall. As well the end fixation zone increases when wall thickness decreases. it means that the effect of structure stiffness is more significant than the soil stiffness.

As regards the structural analysis with Plaxis Program in Fig. 8 using drained Mohr-Coulomb Model with dilatancy ψ=10, ν=0.20, E_{ref} = 30000 kN/m^2, c_{ref} =1 and interface rigidity R=0.20. Its noticeable that the change in maximum displacement and bending moment distribution beak values is bigger than the developed beam on elastic foundation model, this refers to the developing deformations and stresses in around wall discretized soil field.



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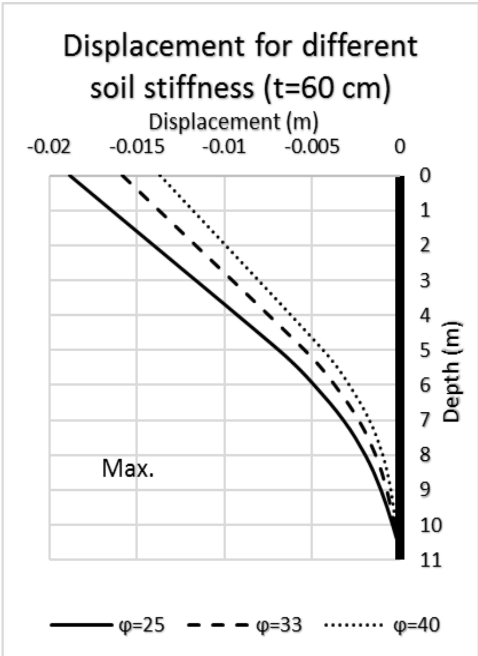

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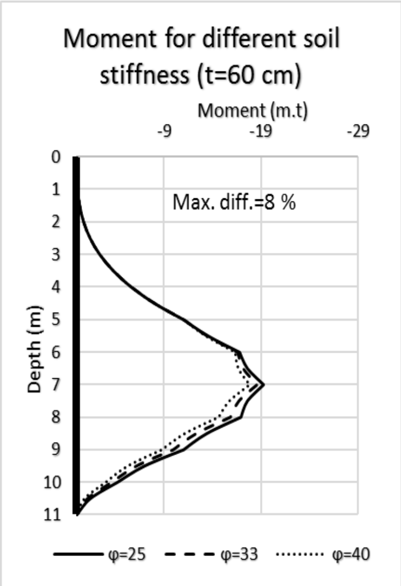

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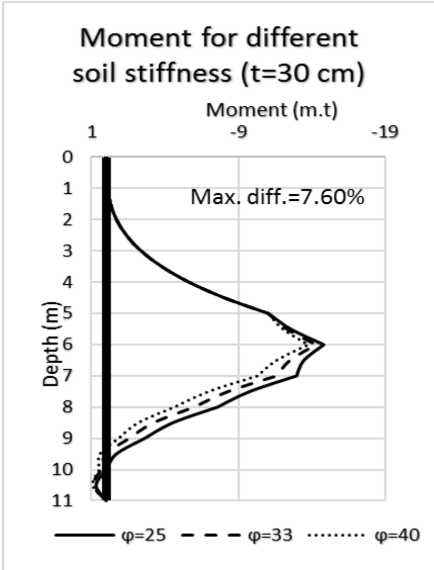
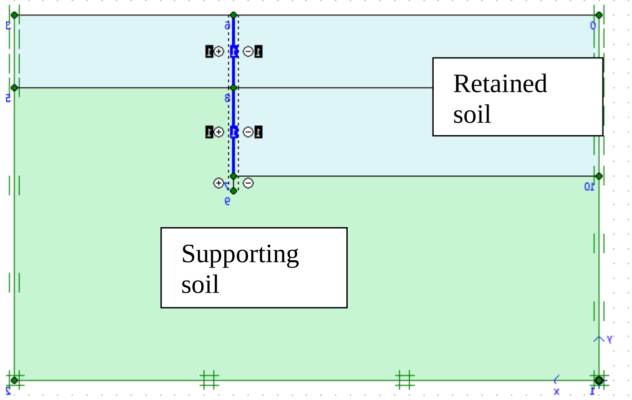


d

Fig. 7 Straining action of equivalent cantilever beam on elastic foundation for different supporting soil stiffness



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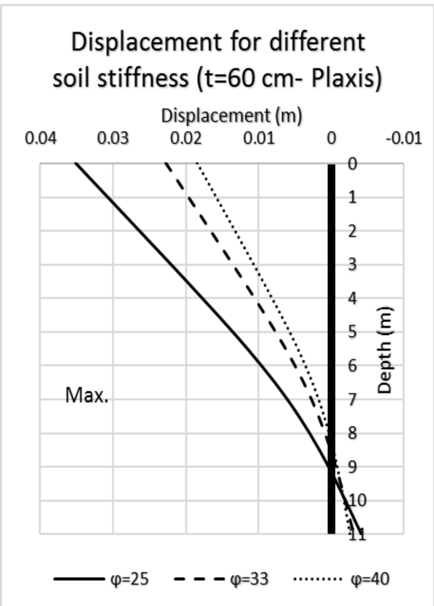

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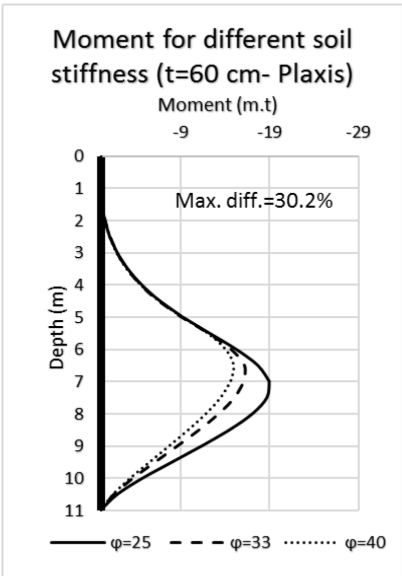

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Fig. 8 Straining action of equivalent cantilever beam on elastic foundation for different supporting soil stiffness (Static Analysis Using Plaxis Program)

4- CONCLUSIONS

1. For equivalent beam on elastic foundation BEF it is found that the calculated spring reaction distribution over the embedded length is usually closer to reality. The spring reaction distribution and the displacements are not only depending on the soil spring stiffness but are rather a result of the mutual relationship between the structure stiffness (EI) and the subgrade reaction modulus (k). The analysis results of present spring modelling are in a good agreement with analytical methods and gives conservative and economical results. Also, it does not need all the soil parameter required in Plaxis or any geotechnical analysis Program.
2. The straining actions at any embedded structure are affected with structural bending stiffness more than the supporting soil stiffness. So, when the maximum limits of structure displacement are considered, the economical design is achieved for flexible structures than the rigid one.
3. Considering the passive earth pressure zone as known supporting reaction by conventional methods in design of embedded structure is not accurate, because, it assumes a failure situation in the soil in which the deformations are so large that maximum shear stresses can be developed. So, this method uses calculations with minimum active and maximum passive earth pressures.
4. The present model is applied through using 2D finite element discretization of the embedded structure, considering its mechanical properties, estimating the stiffness of its spring supports, and representing any acting loading system as loading action, then 2D finite element model is easily ready for structural calculation at any finite element analysis program, after calculations the straining action and the actual passive spring reaction distribution and deformations of embedded structure considering soil-structure interaction is produced automatically.
5. The developed finite element beam on elastic foundation model simulates the soil-structure interaction of embedded structures and their straining actions depend on soil-structure interaction of Winkler theory. The developed model considers soil and embedded structure stiffness effects. The subgrade reaction modulus is not constant as in Winkler theory but varied linearly with embedded length and estimating of its values depends on experimental data from literatures. The developed model does not need to predefined deformation mode, which cannot be quite predicted before structural calculations. In addition, it does not need an interface element between structural element and soil, also it does not need many soil parameters.

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