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STUDY THE TIRE YAW RATE RESPONSE USING LINEAR ,SATURATED LINEAR AND PACEJKA MODEL APPLIED ON 2 DOF BICYCLE VEHICLE MODEL

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ABSTRACT

A lateral vehicle dynamic system response is investigated using two degrees of freedom Bicycle Model for the Vehicle besides Linear Model, Linear saturation model and Magic Formula for the tires. The Bicycle model is represented by the yaw rate (vehicle rotation around it's z-axis) and lateral acceleration. The Tires models are represented by the lateral force. Five Different Tires characteristics are demonstrated by the three Tire Models using five altered wheel steering and longitudinal velocity inputs (step=30o & u=70mph, step=45o&u=70mph, sine with 0.4Hz, Magnitude=15o & u=70mph, sine with 0.4Hz, Magnitude=15o & u=55mph, sine with 0.4Hz, Magnitude=15o & u=35mph). Time Response, Frequency Response and root locus are performed to analysis the tire characteristic effects on the vehicle yaw rate.

1. INTRODUCTION

Nowadays, car accidents are the most cause of death and serious injurer. Investing in understanding the vehicle behavior plays a significant role to minimize the car accidents and enhance the passenger safety.

The primary focus of the vehicle dynamics field is to demonstrate the vehicle behavior by applying mathematical models and simulation for the vehicle to predict the car behavior under certain driving conditions. One of the most familiar vehicle model is the bicycle model. However, the interaction between the tires and the road is one of the most essential factor to investigate the car stability and performance. So, it is necessary to understand the tire characteristics and their effects on the different driving situations. The tires are responsible for generating the forces and moments that drive and spelling the vehicle. Tires are very complex products due to their complexity design and their characteristics affected by many factors for example (materials, Tread pattern, belts angle, inflation pressure, and Non-skid depth). For the tire Simulation, a semi-empirical tire models are used in the vehicle dynamics, and the two well-known models are Magic Formula and Large Model.

Finally, since doing a real-time vehicle tests will be very expensive and time consuming the prediction of the vehicle performance through mathematical models and simulation arises to the roof so that the main objective of these types of projects is to simulate models that gives as accurate results as possible instead of using real-time tests.

3.MATHEMATICAL MODEL

A. 2 DOF Bicycle model

Assumptions

- Longitudinal velocity (u) is assumed to be constant.
- Small slip angles, i.e. tires operate in the linear region.
- No rear wheel steering.
- No aligning moment in both tires.
- No road gradient or bank angle.
- There are only two wheels, one in the front and one in the rear.
- No lateral and longitudinal load transfer.
- No rolling and pitching motion.
- No chassis or suspension compliance effects.

-Lateral Force Equilibrium

$$m(\dot{v}+ru)=F_{yf}+F_{yr}$$

-Moment Equilibrium

$$I_z\dot{r}=aF_{yf}-bF_{yr}$$

B. For Linear/Linear saturation Model

$$F_{yf}=C_{\alpha f}\alpha_f$$

$$Fyr=C_{\alpha r}\alpha_r$$

$$C_{\alpha f}\alpha_f+C_{\alpha r}\alpha_r=m(\dot{v}+ru)$$

However,

$$\alpha_f=\delta_f-\left(\frac{v+ar}{u}\right)$$

$$\alpha_r=\frac{br-v}{u}$$

Notations:

m : Vehicle total mass.

F_{yf}, F_{yr} : Front/Rear lateral force on the tire.

$\dot{v}$: lateral acceleration of the Vehicle center of gravity in the body frame.

r: Yaw rate.

u: longitudinal velocity.

$C_{\alpha f}, C_{\alpha r}$: Front, Rear tire cornering stiffness.

α_f, α_r : Front, Rear tire slip angle.

δ_f: Front steering Wheel Input.

a, b: Distance from COG to Rear, Front axels.

I_z: Moment of inertia around z-axis.

C.The Magic Formula Tire Model

The Pacejka tire model calculates lateral force and aligning torque based on slip angle and longitudinal force based on percent longitudinal slip. The model parameters are dependent on the normal force F_Z on the tire where the normal force is given in KN.

$$F_y=D_y sin\{C_y\tan^{-1}[B_y\alpha_y-E_y[B_y\alpha_y-\tan^{-1}[(B_y\alpha_y)]]\}+Sv]$$

$$\alpha_y=\alpha+S_{hy}$$

$$C_y=a_0$$

$$Ey = [a_6 F_z + a_7][1 - (a_{16}\gamma + a_{17})sign(\alpha_y)$$

$$K_y = a_3 \sin\left[2\tan^{-1}\left[\left(\frac{F_z}{a_4}\right)\right]\right](1 - a_5|\gamma|)$$

$$B_y = \frac{K_y}{C_y D_y}$$

$$S_{hy} = a_8 F_z + a_9 + a_{10}\gamma$$

$$S_{vy} = a_{11} F_z + a_{12} + (a_{13} F_z + a_{14}) F_z \gamma$$

where, *B* is the stiffness factor.

C is the shape factor.

D Peak Factor.

E Curvature Factor.

S_{hy} ,horizontal and vertical shift factor due to (plysteer, conicity, rolling resistance).

a_o through a_{17} is the tire parameters that can be measured through experiments.

4.SIMULATION

The simulation was run for 5 different wheel steering inputs (5 cases) for each tire (5 tires).

Table1: The 5 Wheel Input cases.

Case No.	1	2	3	4	5
Steering Input	Step=30°	Step=45°	Sine=0.4Hz Mag=15°	Sine=0.4Hz Mag=15°	Sine=0.4Hz Mag=15°
Longitudinal velocity	70mph	70mph	70mph	55mph	35mph

Table2: The Tires Coefficients.

Definition	Tire1	Tire2	Tire3	Tire4	Tire5
	P225/60R16	*P225/55 R16*	*205/55 R16*	*205/55 R16*	*225/45 R17*
ao	1.425	1.458	1.571	1.674	1.372
a1	-16.780	-15.672	-57.091	-33.343	-49.605
a2	-980.600	-1022.181	-1439.877	-1241.698	-1464.384
a3	-2480.617	-1948.961	-2701.618	-3187.508	-3081.318
a4	-11.477	-8.326	-12.275	-17.327	-16.509
a5	0.000	0.000	0.000	0.000	0.000
a6	0.190	0.061	0.279	0.152	0.167
a7	0.816	0.701	1.345	0.579	1.083
a8	-0.016	0.007	-0.003	0.012	0.022
a9	-0.107	-0.105	-0.002	0.053	0.137
a10	0.000	0.000	0.000	0.000	0.000
a11	-17.528	-14.649	-76.107	-62.010	-64.967
a12	-71.954	-75.352	-38.042	7.122	11.790
a13	0.000	0.000	0.000	0.000	0.000
a14	0.000	0.000	0.000	0.000	0.000
a15	0.000	0.000	0.000	0.000	0.000
a16	0.000	0.000	0.000	0.000	0.000
a17	0.235	-0.156	-0.167	0.157	-0.323

Table3: The Vehicle Parameters.

Sprung mass	2257 Kg
Front suspension unsprung mass	125 Kg
Rear suspension unsprung mass	150 Kg
Yaw Inertia	3524.9 Kg.m^2
Distance from COG to Front axels	1.33 m
Distance from COG to Rear axels	1.616 m
Steering Ratio	17.8

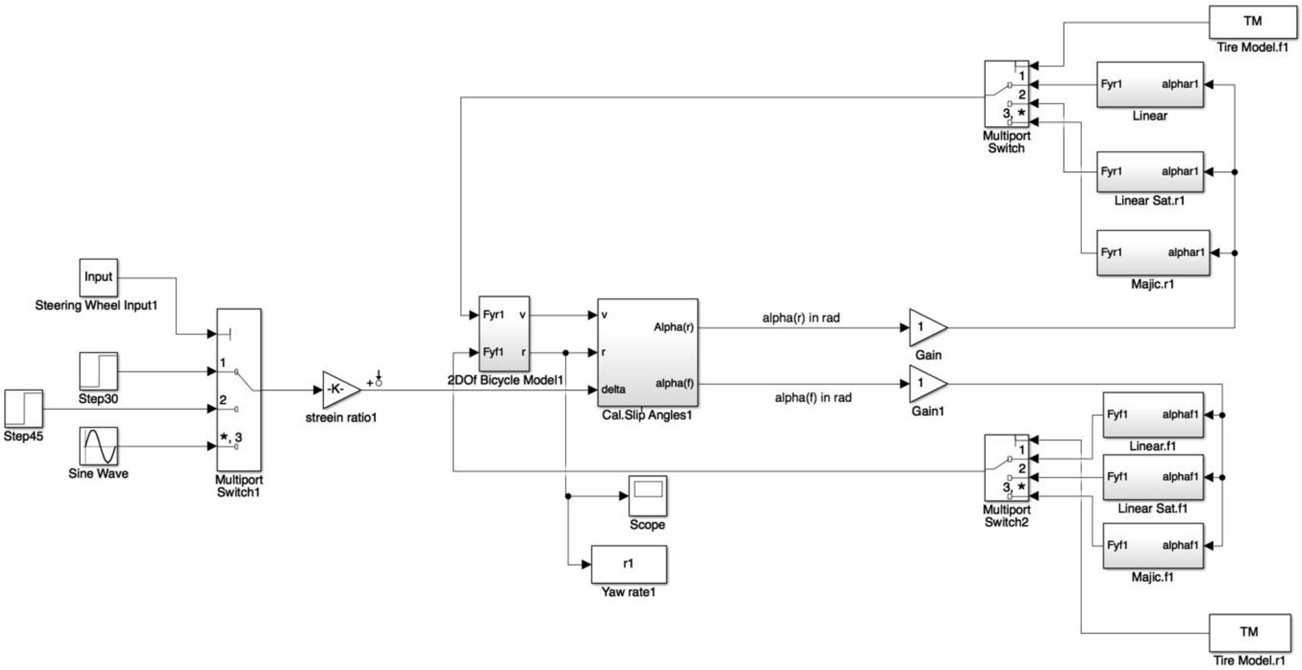

Figure 1: The general Simulink Design that combined the 2 DOF Bicycle model with three different tire models.

-The Simulink Design Combined the 2 DOF Bicycle model with Different Tire Models (Linear, Linear Saturation with 6 degree and Magic formula model)

6.RESULTS AND DISCUSSION

First, The Linear tire model, as it is shown in Figure (1) and Figure (4) for the step wheel steering inputs cases Tire (2) has the highest overshot percentage. Thus, Tire (2) shows less damping magnitude and very quick rising time and longest settling time. And Tire(4) has the lowest overshot percentage, thus high damping magnitude and long rising time and quick settling time. For the sinusoidal wheel steering Inputs, as it shown in Figure (21) and (22) Tire2 has the highest oscillation due to low damping value, and Tire (5) and Tire (4) has the maximum damping values (small rising Time). However, Tire (1) and Tire (3) set in the middle range between high oscillation and high damping values.

Second, the linear saturation model shows the same results as the Linear Model for all cases except for the case (2) (step input=45 degree). Table10 demonstrates Tire(5) has the higher overshot percentage and Tire(3) has the less overshot percentage. Third,Magic Formula Model For case(1) as it shown in the table(4) Tire(3) and Tire(5) has the most reasonable overshot percentage with longer rise time compared to other tires. However, Tire(5) jumps to the largest overshot percentage in case(2) while Tire(3) is placed in the Middle range. For the sinusoidal inputs as it shown in Figure(17) through

Figure(19) In the Linear/Linear Sat. Model The Tire(1) and Tire(3) are almost in the same range but in Magic Formula it is shown that Tire(2) has less damping (high oscillation) and Tires 1,4,5 have the highest damping and Tire(3) is the only one in the Middle range between the high damping and high oscillation. Finally as its shows in Figure(19) for low longitudinal speed Magic Formula does not show a good comparison between all the tires.

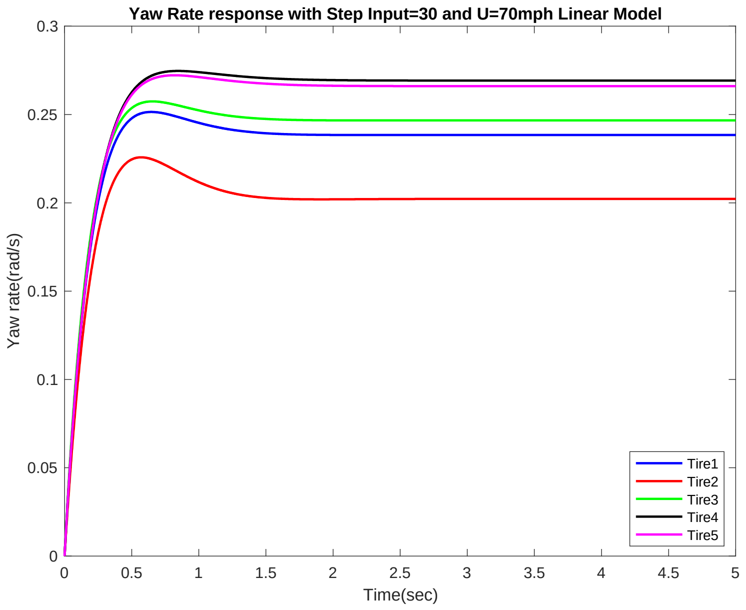

Figure1: Vehicle Yaw rate time response using linear model (case1)

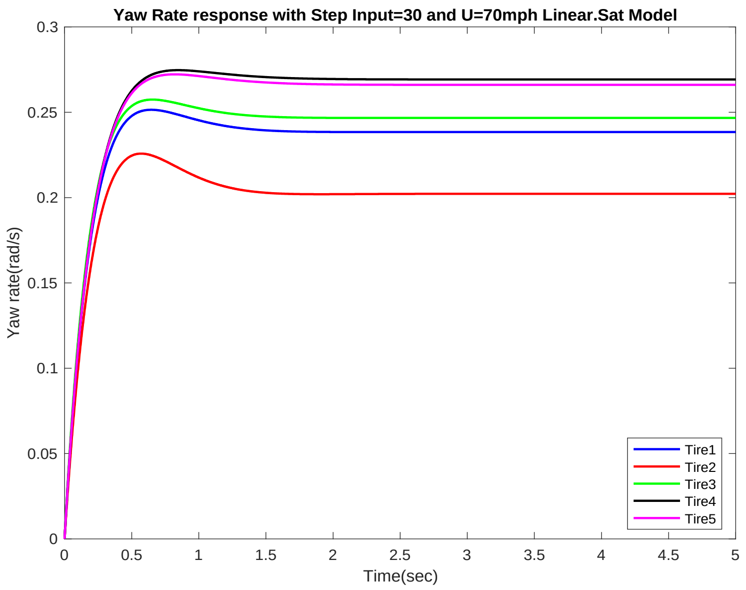

Figure2: Vehicle Yaw rate time response using linear Saturation model (case1)

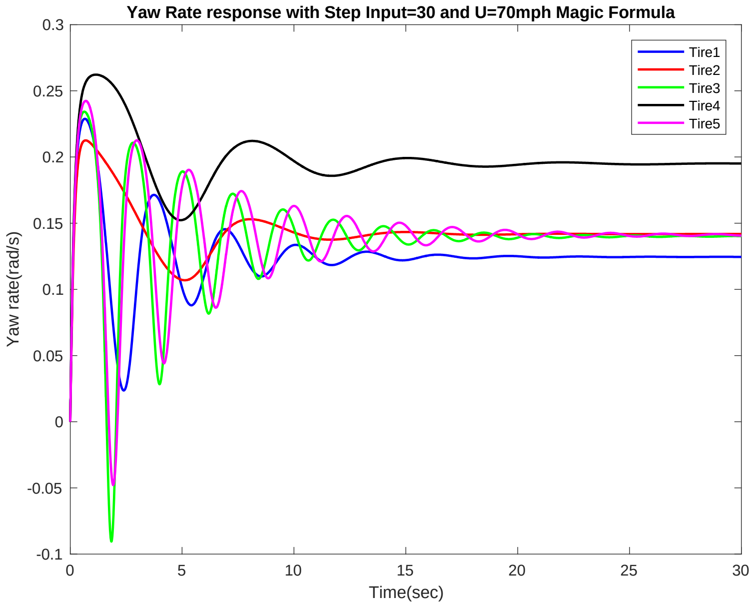

Figure3: Vehicle Yaw rate time response using Magic Formula (case1)

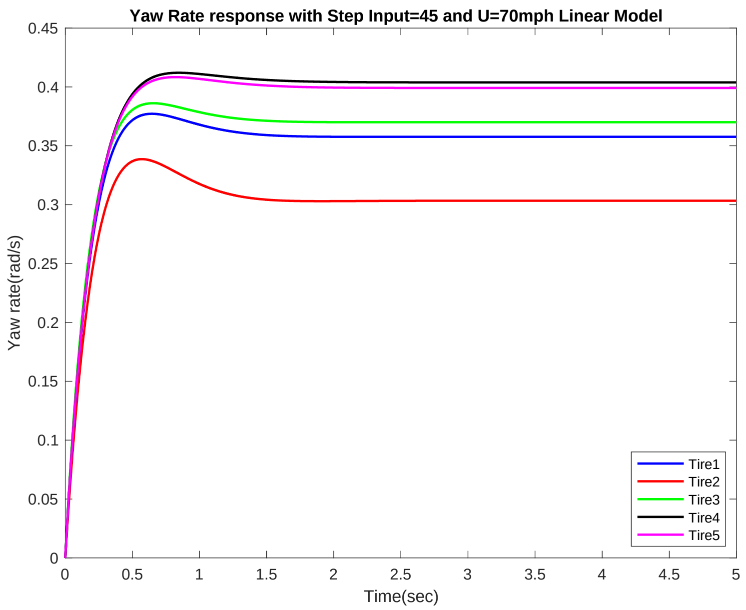

Figure4: Vehicle Yaw rate time response using linear model (case2)

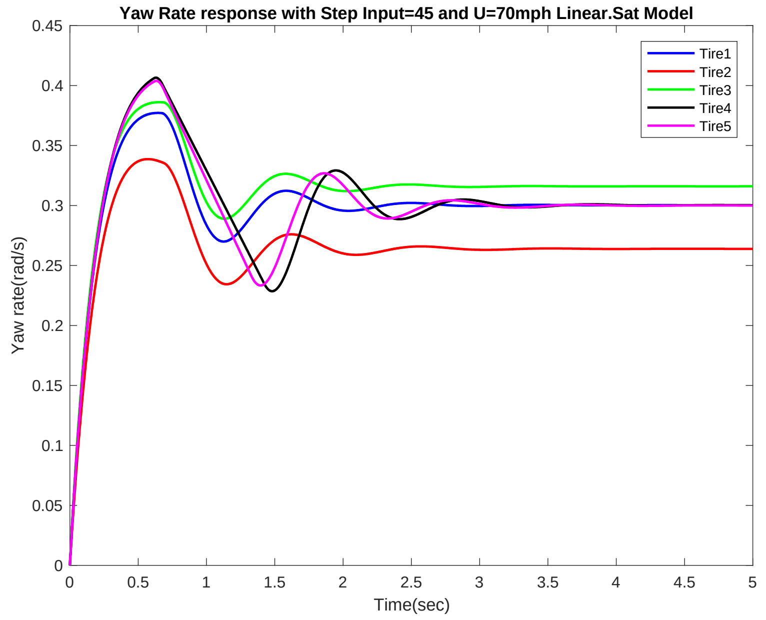

Figure5: Vehicle Yaw rate time response using linear saturation model (case2)

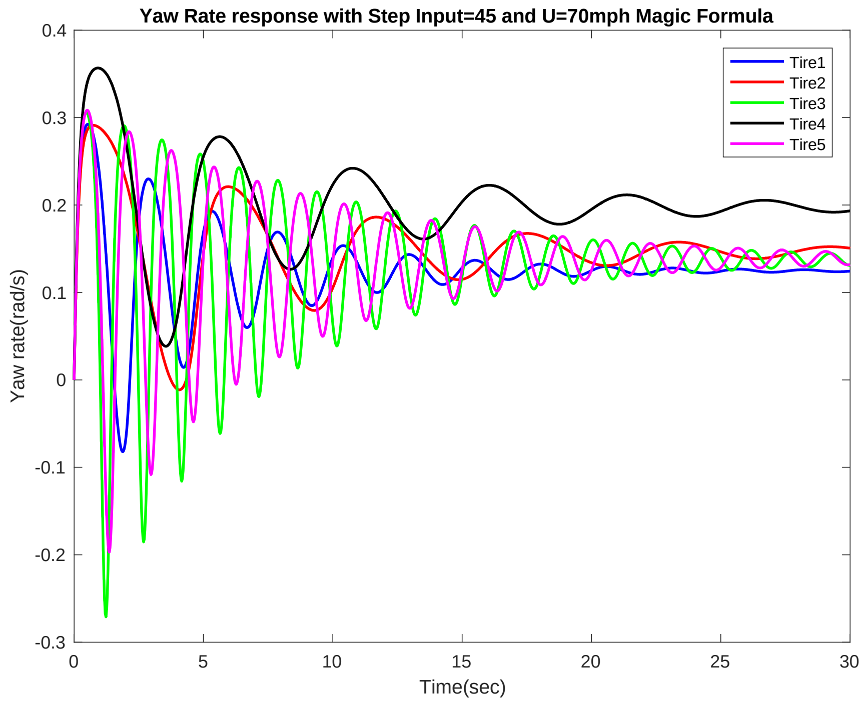

Figure6: Vehicle Yaw rate time response using Magic Formula (case2

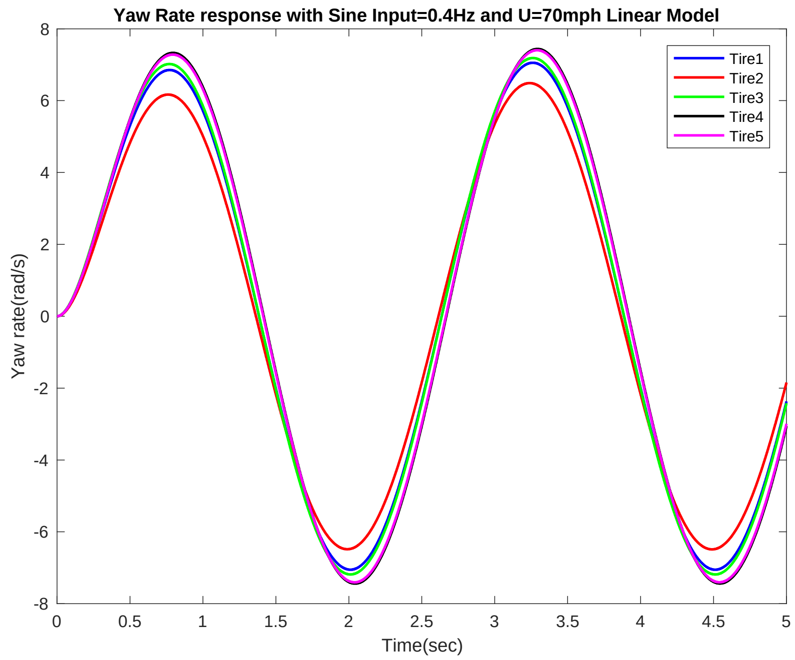

Figure7: Vehicle Yaw rate time response using linear model (case3)

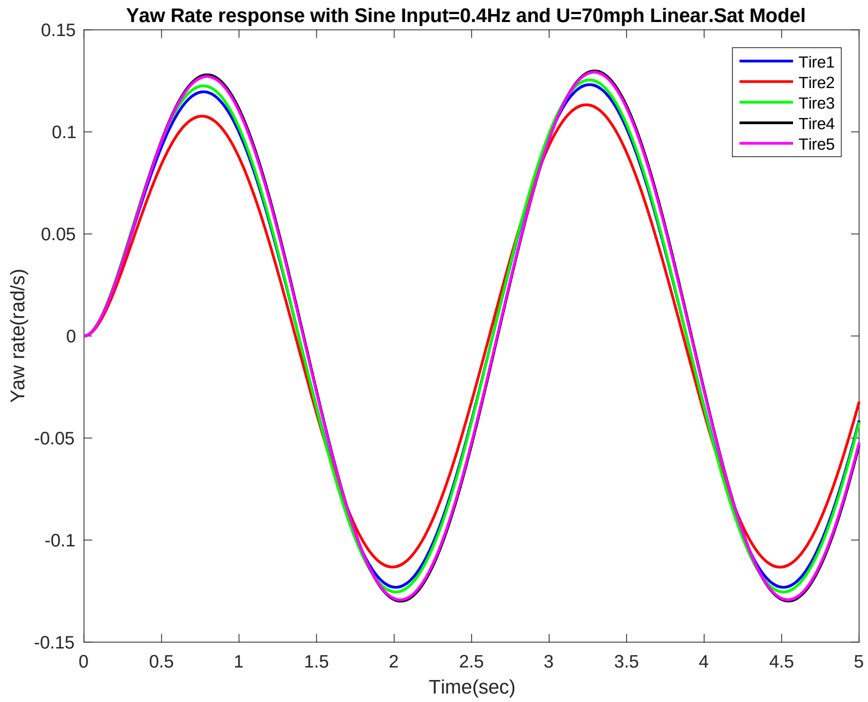

Figure8: Vehicle Yaw rate time response using linear Saturation model (case3)

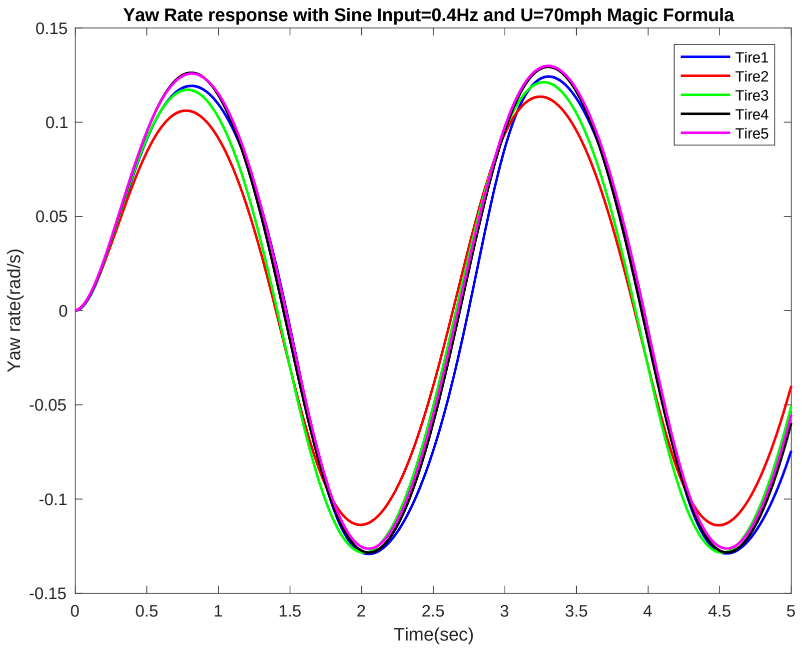

Figure9: Vehicle Yaw rate time response using Magic Formula (case3)

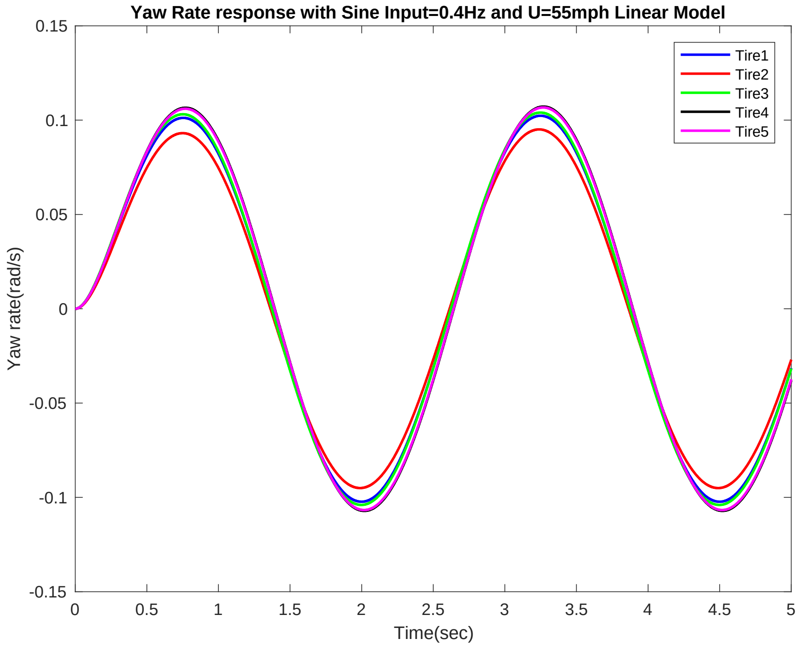

Figure10: Vehicle Yaw rate time response using linear model (case4)

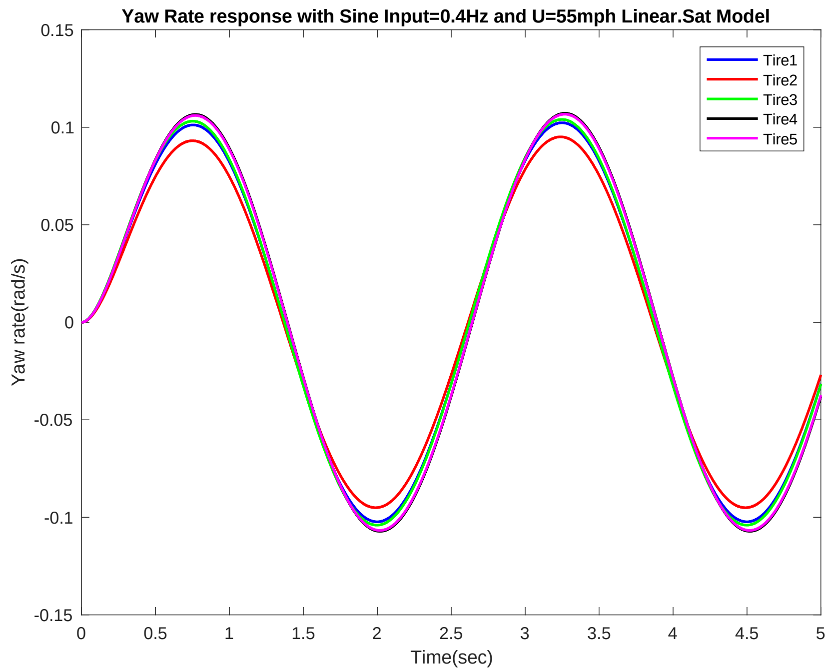

Figure11: Vehicle Yaw rate time response using linear Saturation model (case4)

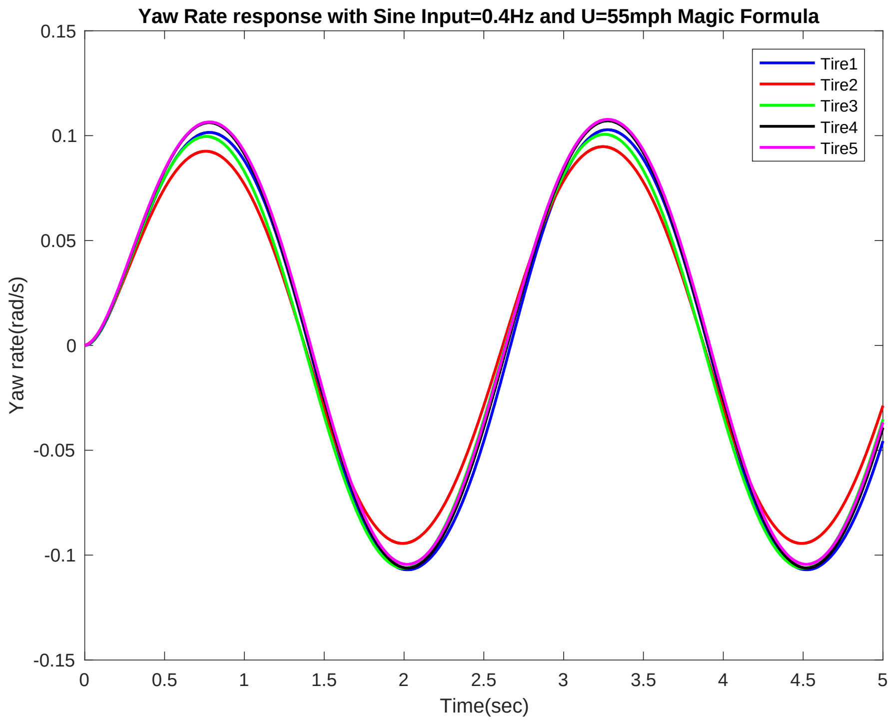

Figure12: Vehicle Yaw rate time response using Magic Formula (case4)

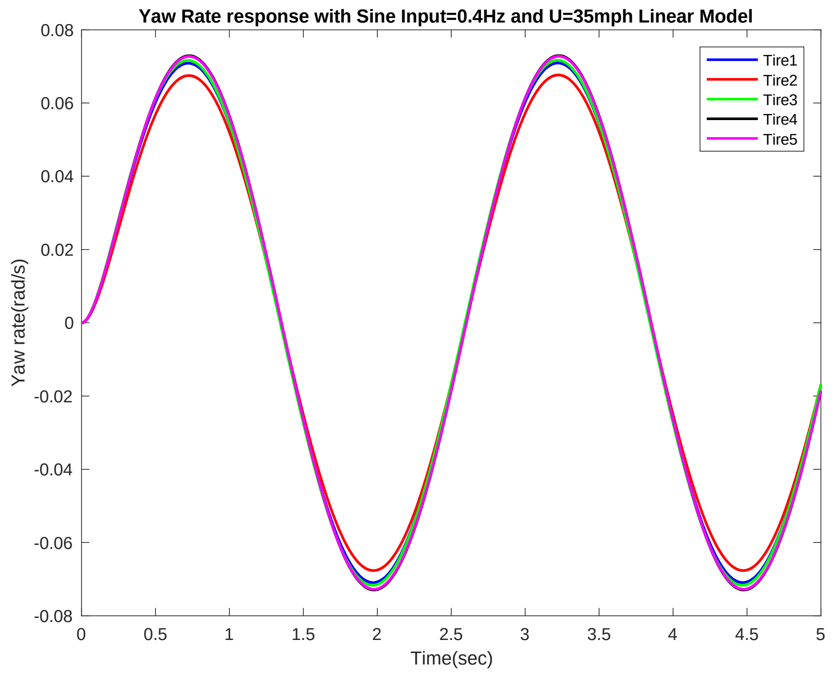

Figure13: Vehicle Yaw rate time response using linear model (case5)

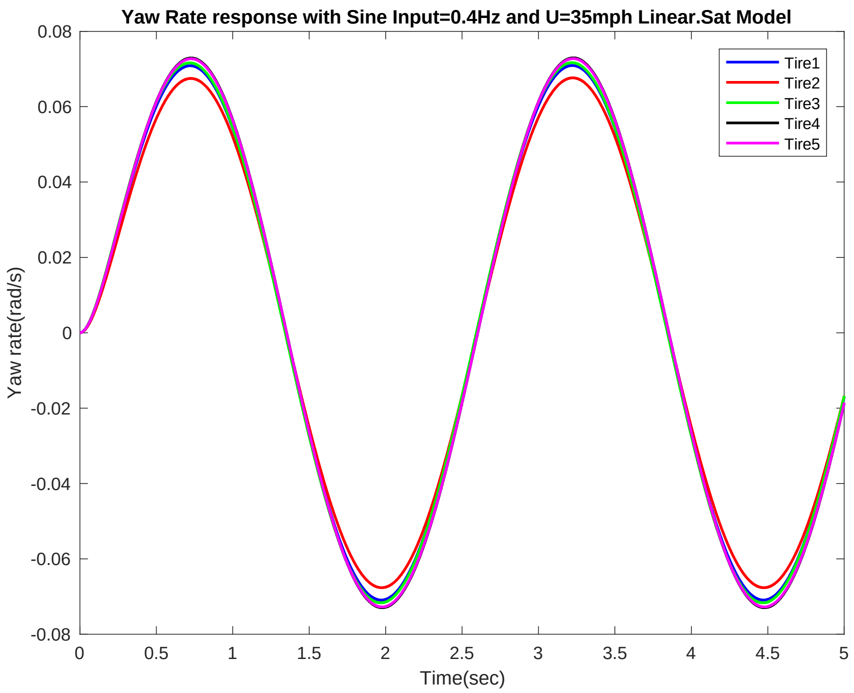

Figure14: Vehicle Yaw rate time response using linear Saturation model (case5)

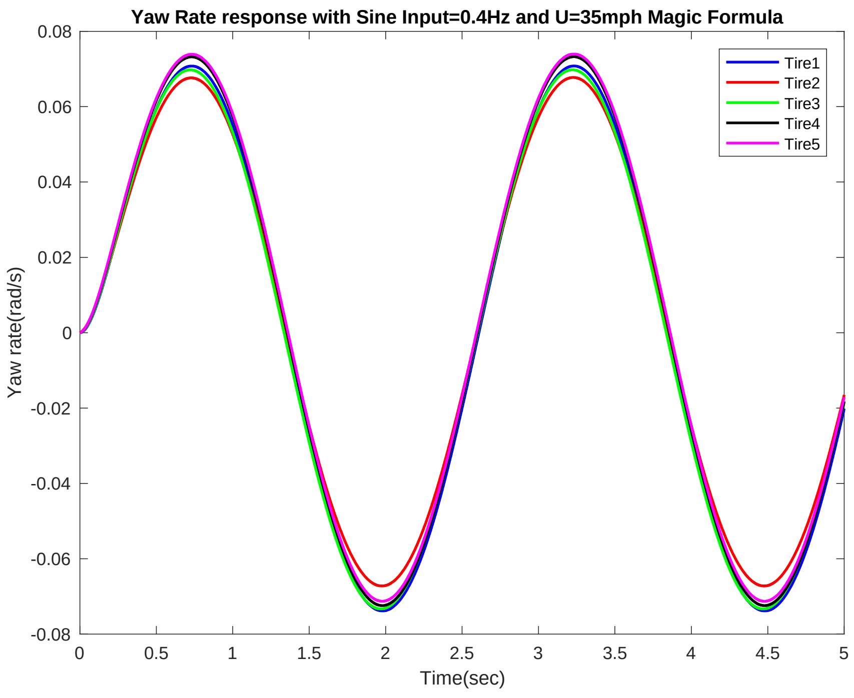

Figure15: Vehicle Yaw rate time response using Magic Formula (case5)

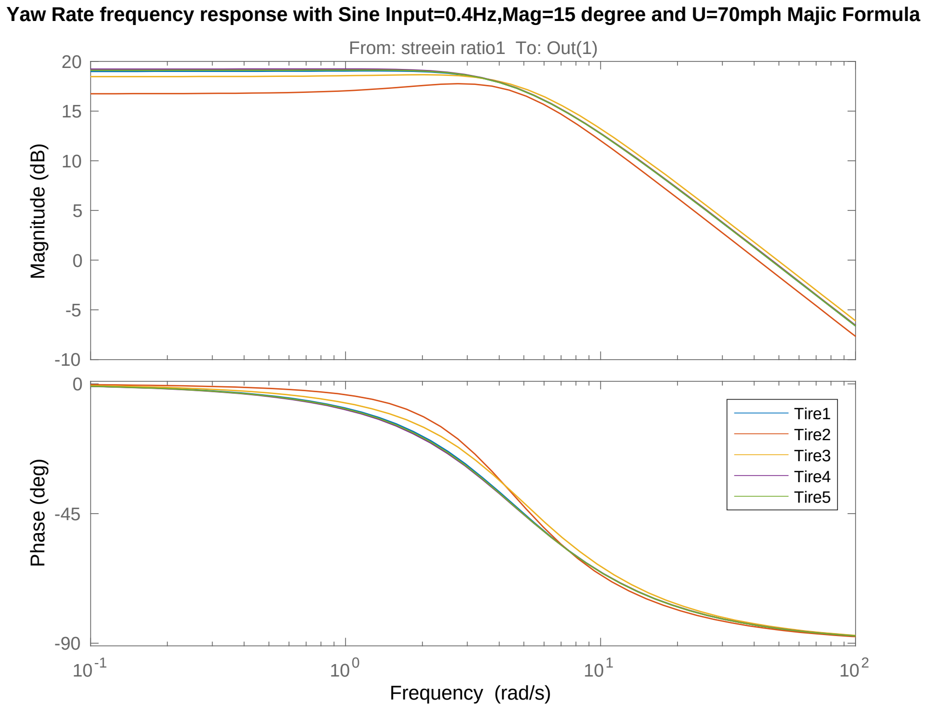

Figure16: Vehicle Yaw rate frequency response using Magic Formula (case3)

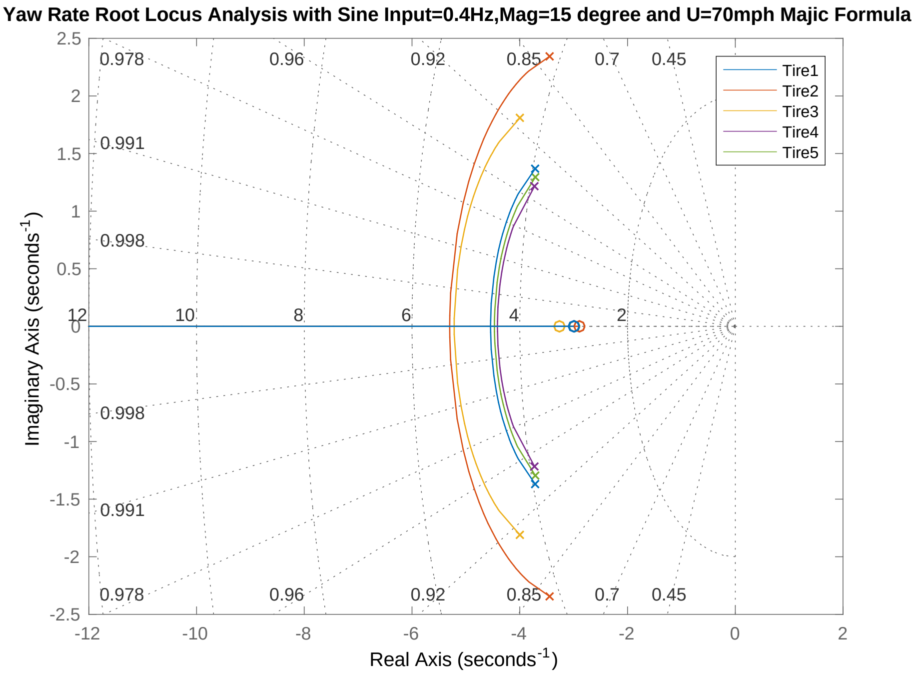

Figure17: Vehicle Yaw rate in Root Locus plot using Magic Formula (case3)

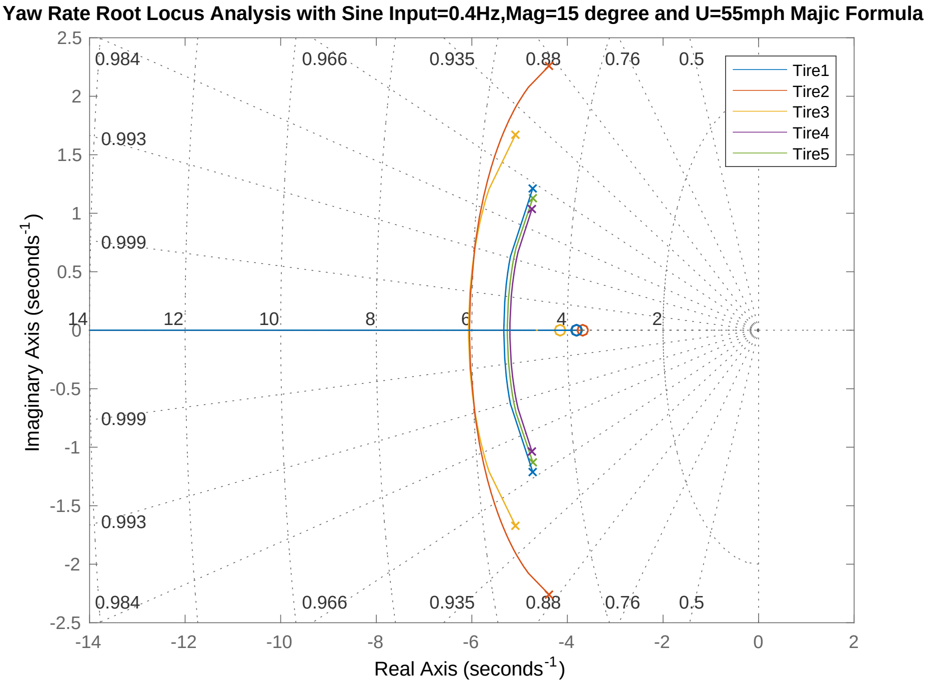

Figure18: Vehicle Yaw rate in Root Locus plot using Magic Formula (case4)

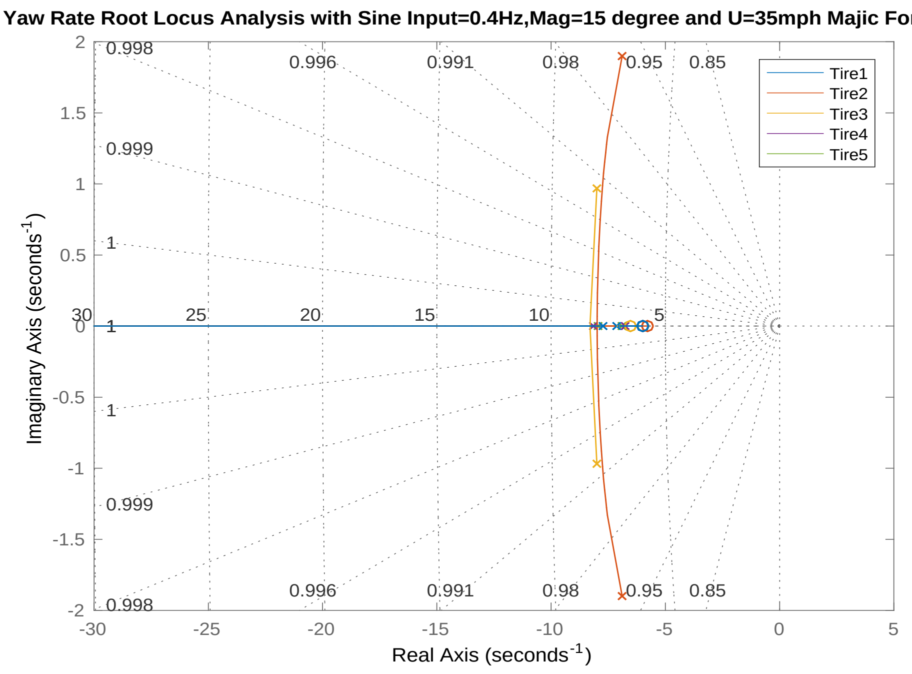

Figure19: Vehicle Yaw rate in Root Locus plot using Magic Formula (case5)

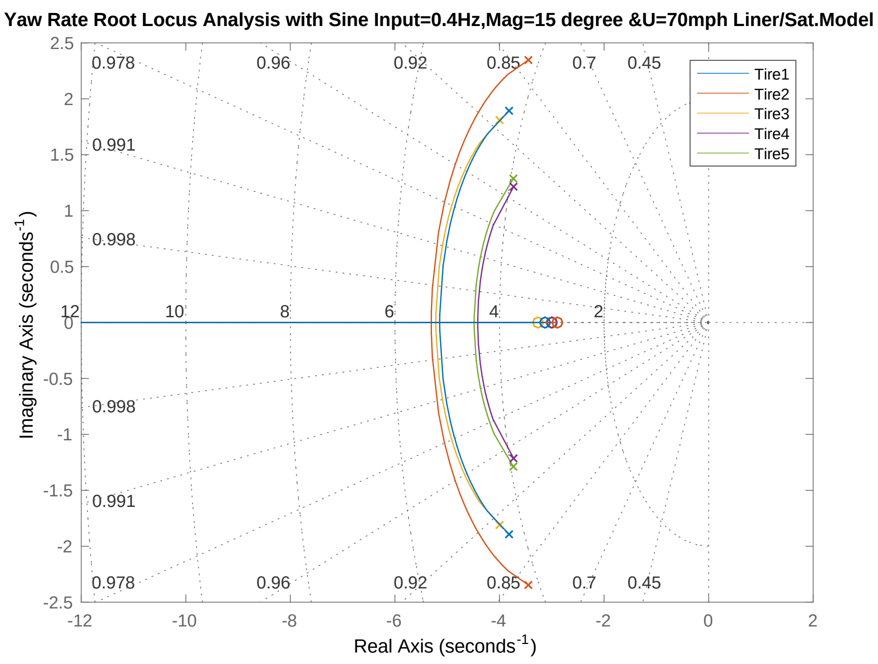

Figure20: Vehicle Yaw rate in Root Locus plot using Linear Saturation Model (case3)

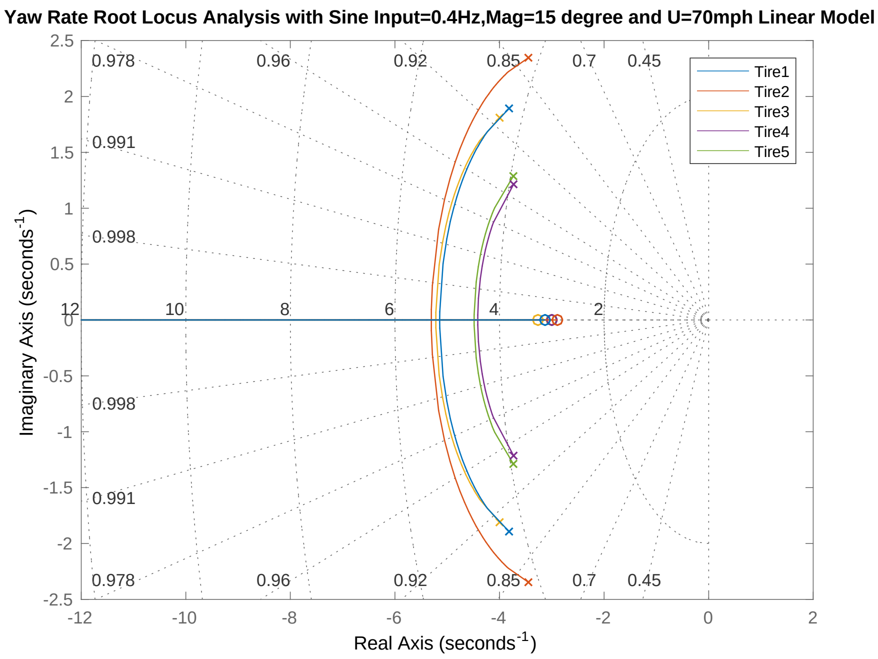

Figure21: Vehicle Yaw rate in Root Locus plot using Linear Model(case3)

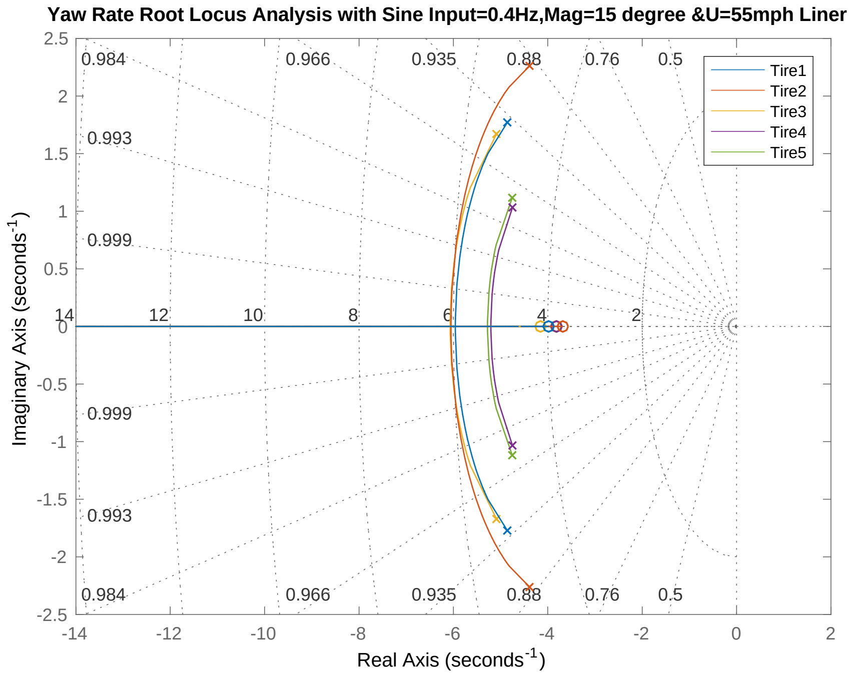

Figure22: Vehicle Yaw rate in Root Locus plot using Linear Model(case4)

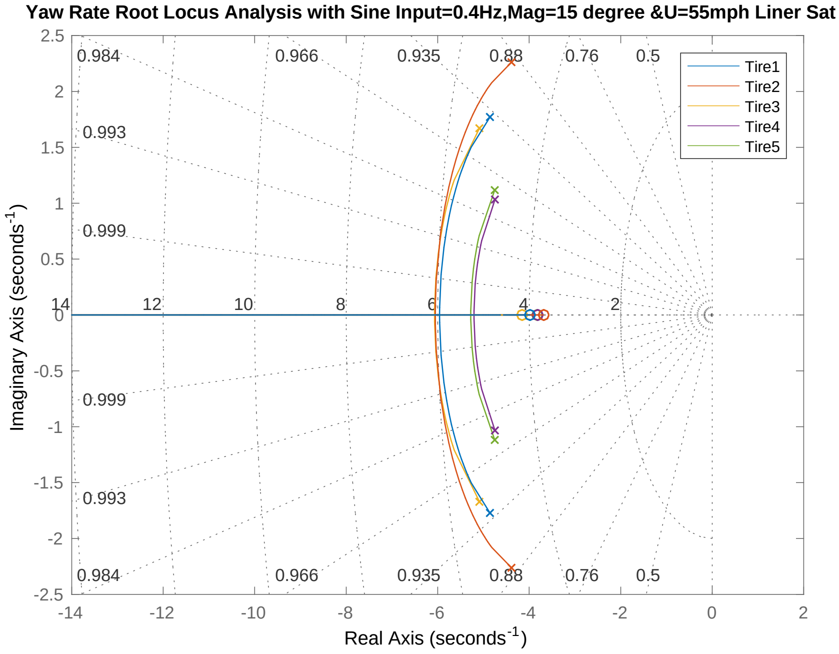

Figure23: Vehicle Yaw rate in Root Locus plot using Linear Sat. Model(case4)

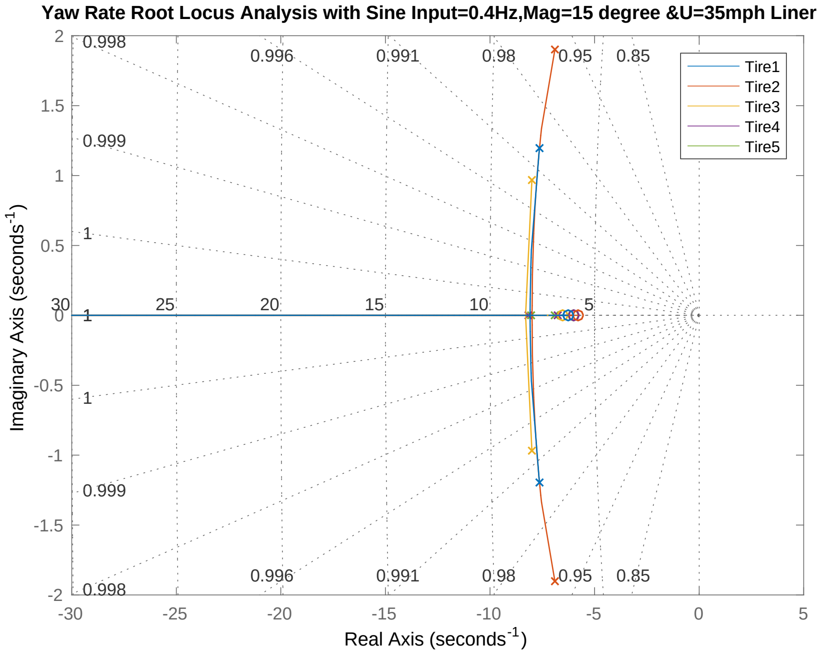

Figure24: Vehicle Yaw rate in Root Locus plot using Linear Model(case5)

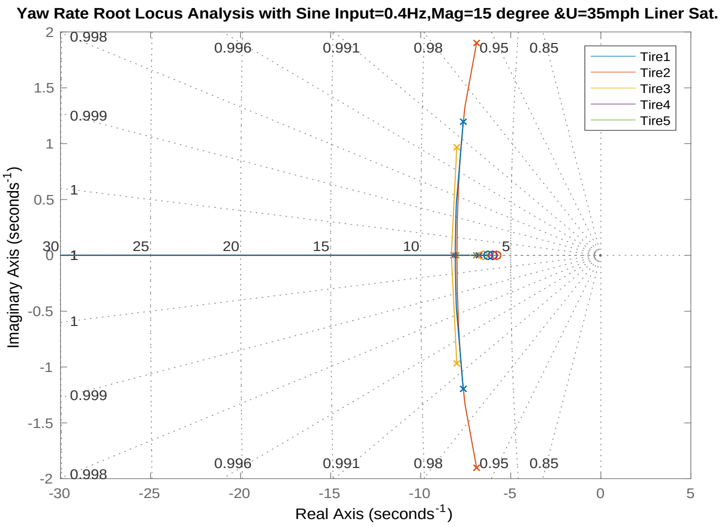

Figure25: Vehicle Yaw rate in Root Locus plot using Linear Sat. Model(case5)

Table4: Time Domain System Response Characteristics using Magic Formula for Case1.

Tire No.	Overshot	Rise Time	Settling Time
Tire1	126	0.0773	4.9583
Tire2	98.36	0.0929	4.7316
Tire3	23.9	0.1804	4.8245
Tire4	72.1294	0.1295	4.5619
Tire5	36.49	0.1719	4.9540

Table5: Time Domain System Response Characteristics using Magic Formula for Case2.

Tire No.	Overshot	Rise Time	Settling Time
Tire1	73.4	0.0996	4.969
Tire2	101.24	0.0859	4.9863
Tire3	22.8683	0.1615	4.7049
Tire4	39.67	0.1681	4.9434
Tire5	116.243	0.0801	4.9878

Table6: Frequency Domain System Response Characteristics using Magic Formula for Case3.

Tire No.	Peak(dB)	At Frequency(rad/s)
Tire1	19	1.35
Tire2	17.8	2.75
Tire3	18.7	2.03
Tire4	19.2	0.844
Tire5	19.1	1.15

Table7: Time Domain System Response Characteristics using Linear Model for Case1.

Tire No.	Overshot	Rise Time	Settling Time
Tire1	5.47	0.2730	1.115
Tire2	11.6487	0.2308	1.2072
Tire3	4.3574	0.2775	1.0467
Tire4	2.0153	0.3497	0.8785
Tire5	2.3035	0.3412	0.9772

Table8: Time Domain System Response Characteristics using Sat.Linear Model for Case1.

Tire No.	Overshot	Rise Time	Settling Time
Tire1	5.47	0.2730	1.115
Tire2	11.6487	0.2308	1.2072
Tire3	4.3574	0.2775	1.0467
Tire4	2.0153	0.3497	0.8785
Tire5	2.3035	0.3412	0.9772

Table9: Time Domain System Response Characteristics using Linear Model for Case2.

Tire No.	Overshot	Rise Time	Settling Time
Tire1	5.47	0.2730	1.115
Tire2	11.6487	0.2308	1.2072
Tire3	4.3574	0.2775	1.0467
Tire4	2.0153	0.3497	0.8785
Tire5	2.3035	0.3412	0.9772

Table10: Time Domain System Response Characteristics using Linear Sat. Model for Case2.

Tire No.	Overshot	Rise Time	Settling Time
Tire1	25.613	0.1889	1.7467
Tire2	28.35	0.1779	1.805
Tire3	22.1946	0.1957	1.7185
Tire4	35.38	0.1838	2.5779
Tire5	34.6647	0.184	2.4821

Table11: Frequency Domain System Response Characteristics using Sat.Linear Model for Case3.

	Peak(dB)	At Frequency(rad/s)
Tire1	18.5	2.28
Tire2	17.8	2.76
Tire3	18.5	2.28
Tire4	19.2	0.845
Tire5	19.2	0.992

-Understeer Gradient

$$Ku = \frac{F_{zf}}{C_{\alpha f}} - \frac{F_{zf}}{C_{\alpha f}}$$

Ku_{Tire1} =2.57 deg/g

Ku_{Tire2} =4.52 deg/g

Ku_{Tire3} =2.21 deg/g

Ku_{Tire4} =1.33 deg/g

Ku_{Tire5} =1.45 deg/g

7.CONCLUSIONS

It is concluded that Linear and Linear Saturation Model give a good result if we compare it with Magic Formula for very small slip angles. The magic formula is more accurate than linear models for large slip angles. Based on the magic formula the Tire (3) is the best tire which gives around (20%) overshot for the step wheel steering inputs and a rising time between 0.16 to 0.18 sec. And for the sinusoidal wheel steering Input, it is placed in the middle range compared to other tires (Not high damping magnitude, Not high oscillation). However, the Tire (4) is the worst Tire that has the largest damping value which may cause instability for other input conditions and rollover.

REFRANCES

- Taheri, Saied. (2017). “Tire Mechanics-ME5647 Lecture Notes”.