



OBSERVABILITY ANALYSIS AND STATE ESTIMATION APPROACH OF POWER SYSTEM

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ABSTRACT

This paper introduces a new approach to solve the problem of power system state estimation and check the observability of the measurement scheme. The state estimation is a powerful technique for system monitoring and control. Through a numerical method for observability analysis in power system state estimation based on weighted least square (WLS), the system state will be implemented and the measurement scheme will be judged. In addition to that, investigation for the effects of the measurements redundancy on the accuracy of the estimated variables is also discussed. The technique is applied to IEEE 30 bus system. Test results show the effectiveness of the proposed method to reach the best of the state estimator solution.

Keywords: State Estimation, Weighted Least Square (WLS), Observability Analysis.

I. INTRODUCTION

The electric power grid is a complex interconnected system that may be subjected to blackouts and external disasters like hurricanes. It is necessary for utilities to repair and restore their power system as quickly as possible during extreme conditions. State estimation helps to get a better picture of the power system with an available set of measurements. State Estimation (SE) is defined as a statistical procedure for deriving, from a set of system measurements, a "best" estimate of the system state [1]. In the field of power systems, the objective is to provide a reliable and consistent data base for security monitoring, contingency analysis and system control. To meet the above objectives, SE is required to:

- Produce a "best" estimation of the bus voltage magnitudes and angles.
- Detect, identify and suppress gross measurement errors.
- Produce an estimate of non-metered or lost data points.

The observability problem in power system state estimation was introduced by F. Schweppe and Wildes in [2]. The simplest definition of an observable system is the one that all state variables (bus voltage magnitudes and angles) can be obtained, (calculated, estimated) from the given measurements set. In real-time operation it may happen that the system be formed by observable islands and also by non-observable parts, due to the temporary loss of measurements[3]. Analysis of network observability is one of the main functions that needs to be executed in a power system state estimator. The state estimator cannot be executed unless the observable islands are identified ahead of time. The problem of network observability has been studied thoroughly by various researchers in the past. There have been essentially two different approaches taken in solving this problem. The first approach uses a topological model of the power system and its associated measurement set [2, 4, 5]. This approach is fast due to the lack of floating point

operations. It classifies the measurements as critical or redundant, determines the observable islands, and also identifies the residual spread components. However, it uses the decoupled measurement model where it is assumed that the real and reactive measurements appear in pairs and that there are no ampere measurements (even if they exist, they will be ignored during the observability analysis). Extension of the topological observability method to address multiplicity of solutions in the presence of ampere measurements has not yet been reported in the literature. The second approach is based on numerical methods, either using the information (or gain) matrix [6, 7], or the measurement Jacobean [8] as the model reflecting the configuration of the system and the measurement set. This approach is rather easy to implement due to its use of already built and factorized matrices of the state estimation problem, and well established sparse factorization and repeat solution routines. While the method is presented for decoupled measurement model [9], extension to the full coupled model is possible. The method can identify observable islands, however classification of the measurements and identification of residual spread components require building the residual sensitivity matrix. Both methods have been implemented and used by various state estimators and they have proved to be efficient and effective alternatives. Recently, it has been demonstrated that uniqueness of the state estimate may be questionable when line current magnitude (ampere) measurements are used in extending the observable islands in power system [9, 10]. Integration of ampere measurements into the measurement system prohibits the use of decoupled measurement models both for state estimation and for observability analysis. Also, several definitions related to the classification of measurements, such as critical, redundant, etc. should be revisited to account for uniquely and non-uniquely observable cases [11, 12]. A system is said to be non-uniquely observable, if more than one solution can be found for the state estimate based on the given set of measurements. Those branches, whose flows can assume more than one value that satisfies all the system measurements, will be labeled as non-uniquely observable branches. On the other hand, uniquely observable systems will have a unique state estimate and a unique set of branch flows through every system branch. It has been observed that certain conditions on the measurement criticality and measurement residual spread components should be satisfied to guarantee unique observability [11-14]. It was also shown that identification of unobservable and non-uniquely observable branches could be carried out by formulating the problem in newly defined branch variables. The redundancy factor is defined as the ratio of number of measurements to number of state variables in most practical cases. This paper investigates the effects of the redundancy on the accuracy of the estimated variables. If some measurements are added to the state estimation, the state estimation accuracy will be affected. In addition to the reconciliation of our previous results in form of a generalized network observability analysis method, this paper's contributions are a numerical method of classifying measurements, and a non-iterative numerical method of determining non-uniquely observable and unobservable branches. The presented observability method is numerical and it allows determination of network observability, unobservable and, critical measurements, redundancy factor, and residual spread components all within the same framework.

II. Weighted Least Squares Approach

The development of the notion of state estimation may be processed along several lines depending on the statistical criterion selected. Of the many criteria that have been examined and used in various applications, WLS criterion is the most commonly encountered method. WLS estimate of our unknown parameter is always expressed as that value of the parameter that gives the minimum of the sum of the squares of the difference between each measured value Z and the estimated measurements \hat{z} (the true value being measured, expressed as a function of our unknown parameter) with each squared difference divided or "weighted" by the variance of the meter error [15]. This technique is the weighted least squares based on the successive elimination, which relies on the calculations of the normalized or weighted residuals performed after two to four iterations of the WLS estimation.

A measurement vector \underline{Z} may be created which contains m measurements from the power system. Measurements include real and reactive power line flows and bus injections, voltage magnitudes at buses, tap ratios for transformers, and more recently, phase angle measurements. Those used in this paper are power flows and injections and voltage magnitudes. The $2N-1$ state variables constitute the state vector \underline{X} , which may be related to the measurements \underline{Z} as following:

$$\begin{aligned} \underline{Z}^{\text{meas}} &= \underline{Z}^{\text{true}} + \mathbf{e} \quad \text{or,} \\ \underline{Z} &= \mathbf{h}(\underline{X}) + \mathbf{e} \end{aligned} \quad \dots\dots\dots (1)$$

By deciding upon a criterion for calculating the state estimates $\hat{\underline{X}}$ from which the estimated measurements and estimated errors are to be computed. It is preferable to minimize the direct sum of the squares of the errors. However, to ensure that the measurements from meters of known greater accuracy are treated more favorably than less accurate measurements, each term in the sum of squares is divided by an appropriate weighting factor σ^2 (the variance of the measurement) to give the objective function.

$$J(\underline{X}) = \left(\sum_{i=1}^m \frac{[\underline{Z}_i^{\text{meas}} - \hat{\underline{Z}}_i]^2}{\sigma_i^2} \right) \quad \dots\dots\dots (2)$$

The best estimates of the state variables are the estimates that cause the objective function $J(\underline{X})$ to take on its minimum value, which can be formed by the expression:

$$\begin{aligned} \min_x J(\underline{X}) &= \min_x \left(\sum_{i=1}^m \frac{[\underline{Z}_i^{\text{meas}} - \hat{\underline{Z}}_i]^2}{\sigma_i^2} \right) \quad \text{or,} \\ \min_x J(\underline{X}) &= \min_x \left(\sum_{i=1}^m \frac{[\underline{Z}_i^{\text{meas}} - h_i(\underline{X})]^2}{\sigma_i^2} \right) \end{aligned} \quad \dots\dots\dots (3)$$

III. MEASUREMENTS OBSERVABILITY ANALYSIS

Consider the measurements Jacobean given by H , Jacobean matrix and it's equal to $\frac{\partial h(\underline{X}_i)}{\partial \underline{X}_i}$.

Using row pivoting and applying Peters-Wilkinson decomposition [16, 17], H can be decomposed into its factors as:

$$H = L \cdot R \quad \dots\dots\dots (4)$$

Where:

- H : is $m \times n$ rectangular Jacobean
- L : is $m \times n$ lower trapezoidal factor
- m : is the total number of measurement and loop equations
- n : is the total number of branch variables

Provided the system is fully observable, this factorization can be carried out numerically by proper row and column pivoting. Otherwise, zero pivots may be encountered during the factorization process at those columns corresponding to some of the existing unobservable branches. Anytime such a zero pivot is encountered, it will be replaced by a value of 1.0 and the corresponding column number will be recorded. The resulting decomposition will appear as below:

$$H = \begin{bmatrix} H_1 \\ \dots \\ H_2 \end{bmatrix} = \begin{bmatrix} L_1 \\ \dots \\ M \end{bmatrix}^{[R]} \quad \dots\dots\dots (5)$$

Where:

L_1 is a $n \times n$ lower triangular matrix, R is a $n \times n$ upper triangular matrix, H_2 is a $(m - n) \times n$ rectangular matrix, M is a $(m - n) \times n$ rectangular matrix.

Following the network observability theory described in [18,19], a set of zero measurements should yield zero branch variables for a fully observable system. If a zero pivot is encountered during the factorization of H , and it is replaced by 1.0, then the measurement in the corresponding row will be assigned an arbitrary but nonzero value. This is equivalent to adding a critical pseudo measurement for the corresponding branch variable. The remaining unobservable branches, those in addition to the already identified ones through the zero pivot columns, can now be determined by carrying out back substitutions for X in the following equation:

$$R \cdot X = Y \quad \dots\dots\dots (6)$$

Where $Y_i = \begin{cases} i & \text{if } i\text{-th pivot was zero} \\ 0 & \text{otherwise} \end{cases}$

After back substitution:

$$X_j = \begin{cases} 0 & \text{if } j\text{-th branch is observable} \\ \neq 0 & \text{if } j\text{-th branch is unobservable} \end{cases} \dots\dots\dots (7)$$

If there were no current magnitude (ampere) measurements, observability analysis would have ended here. However, in the presence of ampere measurements, those branches that are identified as *observable* in the above analysis, may not all be *uniquely observable*? This means that, there is a chance that more than one state estimation solution, which satisfies all the available measurements, can be reached for the given measurement configuration. Hence, the conventional observability analysis should be extended to determine if such multiple solutions are likely. If they are found to be likely, then those branches, whose flows can assume multiple solutions for the same measurement set, will have to be identified and labelled as *non-uniquely observable* branches.

The redundancy factor is defined as the ratio of number of measurements to number of state variables in most practical cases.

The paper also investigates the effects of the redundancy on the accuracy of the estimated variables. If some measurements are added to the state estimation, the state estimation accuracy will be affected.

The network is observable because only one pseudo measurement is equal zero during the triangular factorization

$$U = \begin{bmatrix} 1 & \dots & \dots \\ \dots & 1 & \dots \\ \dots & \dots & 0 \end{bmatrix}$$

If the result of this technique shows multi pseudo measurements are equal to zero during the triangular factorization, it means the system is un-observable

$$U = \begin{bmatrix} 1 & \dots & \dots \\ \dots & 0 & \dots \\ \dots & \dots & 0 \end{bmatrix}$$

IV. CASES STUDY

In this study, two cases will be introduced here based on the proposed technique of the WLS estimator and observability analysis. The system used in the study is the IEEE 30-bus and 41-branch system. The first case uses 75 number of measurements which represent the unobservable system, the second case is the same system of measurements with additional 4 measurements to change it to observable system of measurements.

Case One

The measurements placement used for this case is shown in figure (1), which is summarized as follow:

Voltage measurements = 1

Injection measurements (active and reactive) = $12 \times 2 = 24$

Power flow measurements (active and reactive) = $25 \times 2 = 50$

Total number of measurements (m) = 75

The global redundancy (η) = $m / n = 75 / 59 = 1.27$

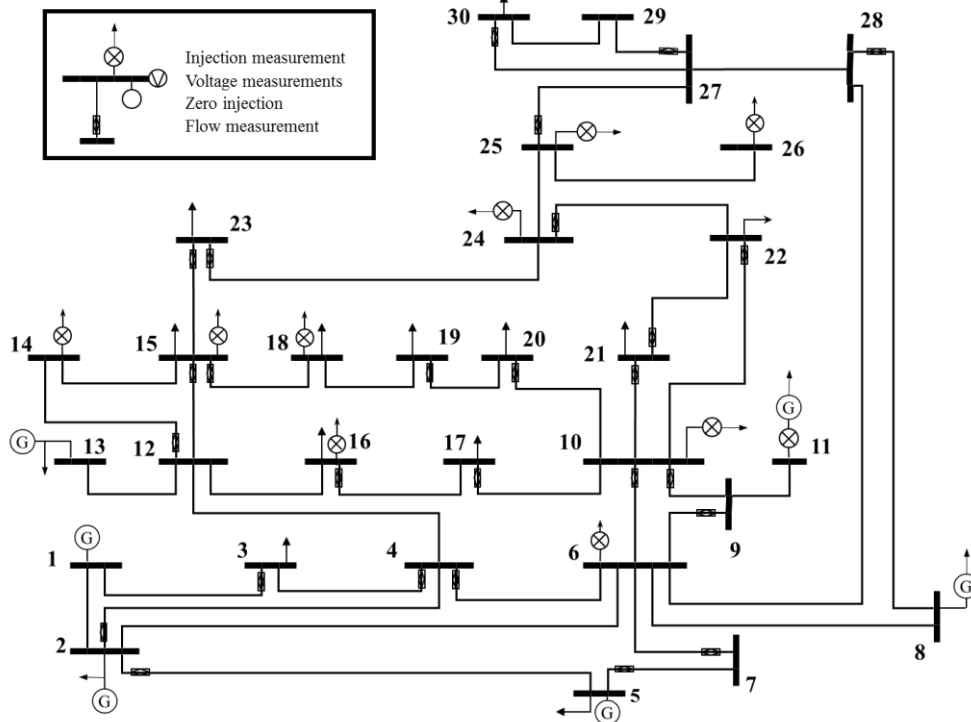


Figure 1: IEEE 30 nodes network with measurements of case one

Running the Matlab program on the proposed method to estimate the system state and check the observability of the measurements configuration, it results 4 zerospivot in the U matrix in the factorization of the Jacobean matrix.

The result is the system **un-observabel system** and cannot continue for **state estimation procedure**.

Case two

The measurements placement used for this case is shown in figure (2), which are the measurements of case one adding voltage measurements at buses 11 and 13 and power flow measurements at line 6-8. It can be summarized as follow:

Voltage measurements = 3

Injection measurements (active and reactive) = $12 \times 2 = 24$

Power flow measurements (active and reactive) = $26 \times 2 = 52$

Total number of measurements (m) = 79

The global redundancy (η) = $m / n = 79 / 59 = 1.34$

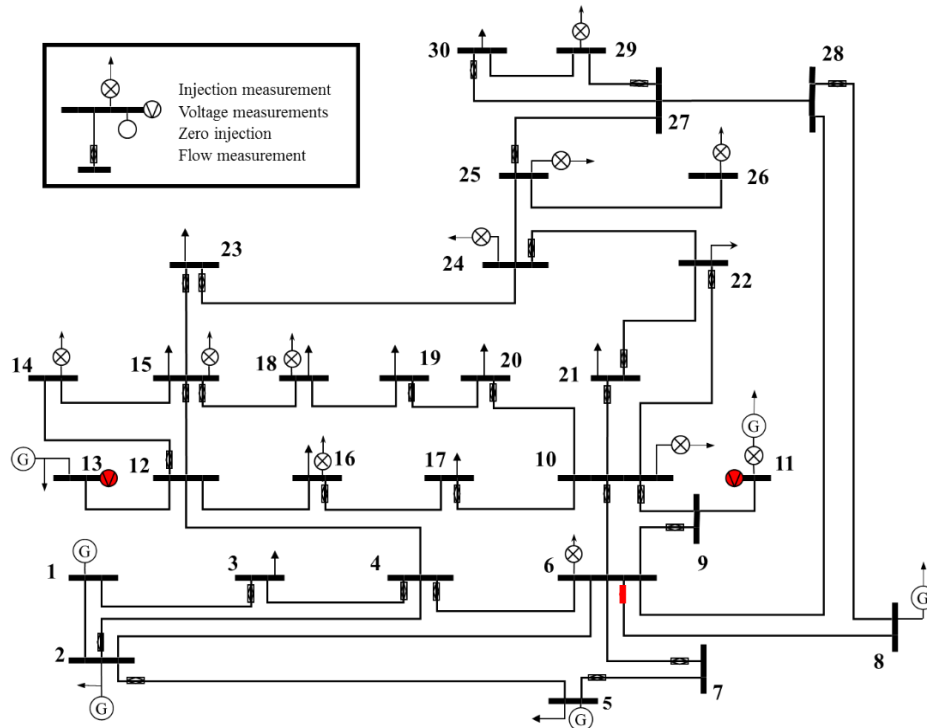


Figure 2: IEEE 30 nodes network with measurements of case two

Running the Matlab program on the proposed method to estimate the system state and check the observability of the measurements configuration, it results **non-zero** pivot in the U matrix in the factorization of the Jacobean matrix.

The result is the system **observabel** and the program continue for **state estimation procedure**.

The results of the state estimation program are as shown in Table 1

Table 1: State Estimation results of case study 2

Voltages in p.u.			Angles in deg.		
J	Estimated	Actual	J	Estimated	Actual
1	1.0594	1.0600	1	0.0000	0.0000
2	1.0401	1.0330	2	-5.3672	-5.2195
3	1.0163	1.0135	3	-7.5430	-7.5125
4	1.0058	1.0028	4	-9.2928	-9.2687
5	1.0046	1.0000	5	-14.2210	-14.2114
6	1.0033	1.0005	6	-11.0566	-11.0433
7	0.9957	0.9924	7	-12.8910	-12.8815
8	1.0025	1.0000	8	-11.8002	-11.8040
9	1.0332	1.0302	9	-14.0810	-14.0671
10	1.0165	1.0131	10	-15.6956	-15.6814
11	1.0744	1.0720	11	-14.0810	-14.0671
12	1.0479	1.0455	12	-15.1939	-15.2355

Voltages in p.u.			Angles in deg.		
J	Estimated	Actual	J	Estimated	Actual
13	1.0738	1.0710	13	-15.1939	-15.2355
14	1.0296	1.0276	14	-16.0733	-16.1303
15	1.0222	1.0203	15	-16.0634	-16.1420
16	1.0273	1.0245	16	-15.6794	-15.6945
17	1.0143	1.0111	17	-15.9028	-15.8994
18	1.0078	1.0053	18	-16.6572	-16.7061
19	1.0025	0.9997	19	-16.8157	-16.8427
20	1.0052	1.0022	20	-16.5943	-16.6109
21	1.0036	1.0006	21	-16.2551	-16.1406
22	1.0069	1.0013	22	-16.0110	-16.1217
23	1.0041	1.0039	23	-16.2665	-16.4357
24	0.9940	0.9906	24	-16.3327	-16.4691
25	0.9972	0.9942	25	-16.0907	-16.1959
26	0.9792	0.9761	26	-16.5088	-16.6356
27	1.0081	1.0053	27	-15.6716	-15.7481
28	0.9994	0.9985	28	-11.7034	-11.7133
29	0.9880	0.9850	29	-16.9325	-17.0233
30	0.9764	0.9733	30	-17.8416	-17.9398

V. CONCLUSION

This dissertation is concerned to provide a new approach for robust state estimation of power system using WLS. The dissertation also proposed technique to check the observability of the network and determine whether the set of measurements are sufficient in number and location to make state estimation possible or not. The result of state variable in the cases study show the effect of redundancy in state estimation and show that The IEEE 30-bus grid attains improved estimation accuracy due to its increased redundancy ratio. The proposed technique was applied to the IEEE 30 buses test network, and the tabulated results achieved its reliability.

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