Fractional Order Sliding Mode Controllerfor Coupled Tank System

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***Abstract*—In this paper, a fractional order sliding mode controller (FOSMC) is proposed for nonlinear coupled tank system. The proposed controller integrates the advantages of the fractional order control and sliding mode control. Fractional order controller has an extra flexibility to meet the specifications of the controller design and sliding mode control is a robust controller, which is able to respondtoexternal disturbances combined with nonlinear systems.The objective of the controller is to allow the system states to move to the sliding surface and remain on it so as to ensure the asymptotic stability of the closed-loop system. The stability analysis for the proposed controller is studied based on the Lyapunov stability theorem. Simulation results show the ability of the proposed controller to improve the system performance especially when the controlled system is exposed to external disturbances and variation of reference trajectory compared with the conventional proportional-integral-derivative controller (PID) and conventional sliding mode controller (SMC), which is based on integer order derivatives.**

***Keywords*—Fractional control, Sliding mode control, Lyapunov function,Nonlinear systems.**

I. INTRODUCTION

Fractional calculus (FC) is an ancient branch of mathematical science that deals with derivatives and integrals from non-integer orders [1-3]. Due to the higher degree of freedom that can be provided with the fractional calculus, it has been applied in many areas of engineering and science in recent decades [4]. Several research papers have been published on the topic of fractional order(FO) calculus, which is distinguished by some non-classical phenomena in many applications found in engineering and natural sciences [5-8]. FO calculus provides perfect mathematical models for complex and proportional systematic procedures and events. So, it has much contribution to many fields such as physics, biology, and control theory[9-11].

Theory and application of the FO controllers have attracted the focus of many studies in the past decades especially in science and engineering areas. Fractional order control (FOC) has been introduced in different schemes such as fractional order PID control, which is used for controlling FO plants with dead time [12], fractional order adaptive control, which is applied for non-identical chaotic FO systems [13] and FO optimal control, which is designed for time invariant and time varying problems [14].

Most practical engineering systems are complex with nonlinearity. These systems contain a time-delay, an external disturbance and modeling error. These problems reduce the performance of the systems [15, 16]. It is undesirable to be efficiently controlled these systems by the conventional controllers. One of the robust and effective control approaches that deal with disturbances is the sliding mode control (SMC) [17]. SMC is an effective approach for the control of some nonlinear systems that are not stable using continuous state feedback approaches and it is basically considered as a consequence of discontinuous control. SMC is superior to other control schemes in the sense of design[18]. SMC techniques contain two steps; firstly, the desired system performance in the sliding mode is provided using an arbitrary switching surface. Secondly, a control law is designed based on the sliding mode theory and Lyapunov stability theory to ensure the sliding motion [19].

The asymptotic stability concepts are the main concern using these two steps. Using such control algorithm, the desired performances are maintained, and the system states can reach the equilibrium points. SMC technique has been improved based on different methods and it is proposed based on linear matrix inequality (LMI) [20], Riccati approach [21], adaptive technique [22], and neural network [23]. Integral sliding mode control (I-SMC) designs an integral sliding surface for a class of nonlinear FO systems [24–26]. Furthermore an improved stable sliding surface is given in [27]. However, the best knowledge of the prior works shows that there are many researches in control of nonlinear dynamical systems in which the authors have used the FC for designing the SMC such as [28–30].

The utilization of the FO integrals and derivatives in designing of the SMC can cause robust and finite time stability, reduction the chattering and fast response of the closed loop system in presence of disturbances. A novel fractional order sliding mode control (FOSMC) is designed for output tracking of a broad class of FO systems [31]. Different applications have been proposed for FOSMCs such as heat transfer [32], economic system [33] , speed control of permanent magnet synchronous motor [34], secure communication [35] and antilock braking systems [36] .However, the research about FOSMC for controlling nonlinear systems still exists.

In this paper, a FOSMC is proposed for nonlinear systems. In the proposed algorithm, an appropriate sliding surface is designed and the finite-time stability of this surface is proved. It means that all the state trajectories of the system reach to this surface in a finite time and stay on it for all future times. Therefore, the proposed control law is able to overcome disturbances and guarantees the asymptotic convergence of the system's output toward the desired reference signal. Finally, in order to verify the theoretical results and show the applicability and performance of the proposed design scheme, it is applied to two coupled tanks system. Simulation results show the efficiency of the proposed controller to overcome the system disturbances and compared to other existing schemes.

This study has several contributions, which summarized as follows:

- Proposing a FOSMC for nonlinear systems.
- The stability analysis of the proposed controller is derived based on Lyapunov theorem.
- The proposed controller is able to improve the system performance compared with other existing controllers.

The rest of the paper is organized as follows: In Section 2, the basic definitions of FC are introduced. Section 3 presents the proposed FOSMC. Section 4 shows the simulation results for nonlinear system. Section 5 is devoted to conclusions.

II. Preliminaries and Problem Formulation

A. *Basic definitions of fractional calculus*

There are different basic definitions of the fractional order integration and differentiation introduced in this section. The main fractional derivatives definitions are Grunwald – Letnikov (GL), Riemann – Liouville (RL) and Caputo, which are defined as follows [37]:

Definition 1: The Grünwald – Letnikov definition is given as:

$$ {}_bD_t^{\gamma} f(t) = \lim_{h\to 0} \frac{1}{h^{\gamma}} \sum_{j=0}^{[(t-b)/h]} (-1)^j \binom{\gamma}{j} f[t-jh] \qquad (1)$$

$$\binom{\gamma}{j} = \frac{\Gamma(\gamma+1)}{j!\Gamma(\gamma-j+1)} \qquad (2)$$

where ${}_bD_t^{\gamma}$ is the fractional derivative of an order γ with terminals b and t, $[(t-b)/h]$ is integer part, γ represents the FO operator and h denotes the step size. The Gamma function used in the above equation can be defined by the following equation:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{t} dt \qquad (3)$$

Definition 2: This definition is denoted by Riemann-Liouville as:

$$ {}_bD_t^{\gamma} f(t) = \frac{1}{\Gamma(n-\gamma)} \frac{d^n}{dt^n} \int_b^t (t-\tau)^{n-\gamma-1} f(\tau) d\tau, \quad n-1 \le \gamma < n \qquad (4)$$

where n is an integer.

Definition 3: Caputo's definition is defined as:

$$ {}_bD_t^{\gamma} f(t) = \frac{1}{\Gamma(\gamma-n)} \int_b^t \frac{f^{(n)}(\tau)}{(t-\tau)^{(\gamma-n+1)}} d\tau \qquad (5)$$

where the order of differential operator; γ is limited as $\gamma \in (n-1, n)$.

B. *FO operators' approximation*

To find the integer transfer functions that approximate the dynamic behavior of a given fractional transfer function. Grünwald – Letnikovapproximation is one of the most used methods to approximate the FO transfer function as [38]:

$$ s^{\gamma} \approx \sum_{k=0}^{N} \frac{(-1)^k \binom{\gamma}{k}}{T_s} z^{-k} \qquad (6)$$

where T_s is the sampling time and z^{-k} is transform of t in the Laplace transform domain.

C. *Theproblem formulation*

The hydraulic system with coupled tank is composed of two transparent tanks made in Fig. 1. In the horizontal one (Tank 2), an electrical water pump is used to transfer the liquid, water, from this tank to the vertical one (Tank 1). On the concerned process there are two transducers: flow and level. The flow transducer is a "blade or turbine flow-meter" that provides the system output with a pulse voltage which frequency is proportional to the liquid flow. The level transducer uses the pressure exerted by the water in the first tank (Tank1) to generate an elementary deformation on the inbuilt strain gages. Then the liquid level can be deduced from a signal conditioner proportional to this pressure.

The system with coupled tank is considered as a benchmark for the study and the analysis of the hydraulic systems control problems [39]. This designed device allows us to examine the control level of the liquid in a tank by varying the flow of the pump; P using the valve; V1 or by applying a disturbance using the valve; V2 (Fig.1).

Using Table 1, the relation between the supply voltage; u of the pump and the input flow; q can be written as:

$$\begin{cases} q = k_p\, u \\ q_2 = S_2\, a_2 \sqrt{2g\,(h_1 - h_2)} \\ q_3 = S_3\, a_3 \sqrt{2g\, h_2} \end{cases} \qquad (7)$$

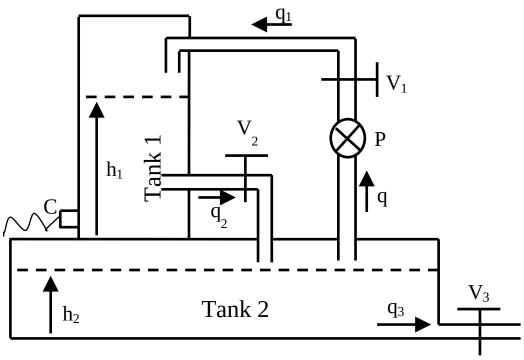

Fig. 1. Coupled tank system.

Table 1 presents the various parameters of this coupled tank system

A_1	Section of tank 1	0.03 m^2
A_2	Section of tank 2	0.08 m^2
S_1	Variable section of valve 1	00008 m^2
S_2	Variable section of valve 2	00008 m^2
S_3	Variable section of valve 3	00008 m^2
a_1	Discharge coefficient	1
a_2	Discharge coefficient	1
a_3	Discharge coefficient	1
h_1	Tank 1 level	0.6 m
h_2	Tank 2 level	0.2 m
u	Input power	12 v
k_p	Pump gain	7.5 m3 /s.v
k_s	Transducer gain	40 v/m
g	Gravity constant	9.8 m/sec

The vertical tank (Tank 1) can be filled from the horizontal tank (Tank 2) via the variable channel 1 by opening the valve; V_1 placed after the pump which changes the incoming flow. A second valve; V_2 (channel 2) inserted below the vertical tank (Tank 2) may change the output flow; q_2. When we consider only the case of level control, we will have $q_1 = q$. By using the flow equilibrium equation we obtain:

$$\begin{cases} \dot{h}_1 = \dfrac{1}{A_1}(q_1 - q_2) \\ \dot{h}_2 = \dfrac{1}{A_2}(q_2 - q_3) \end{cases} \tag{8}$$

Finally, the coupled tank system is modeled by the following state representation:

$$\begin{cases} \dot{h}_1 = \dfrac{1}{A_1}\left(k_p\, u - S_2\, a_2 \sqrt{2\, g\,(h_1 - h_2)}\right) \\ \dot{h}_2 = \dfrac{1}{A_2}\left(S_2\, a_2 \sqrt{2\, g\,(h_1 - h_2)} - S_3\, a_3 \sqrt{2\, g\, h_2}\right) \\ y = k_s\, h_1 \end{cases} \tag{9}$$

We consider $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ with $x_1 = h_1$ and $x_2 = h_2$, equation (9) gives (10).

$$\begin{cases} \dot{x}_1 = \dfrac{1}{A_1}\left(k_p\, u - S_2\, a_2 \sqrt{2\, g\,(x_1 - x_2)}\right) \\ \dot{x}_2 = \dfrac{1}{A_2}\left(S_2\, a_2 \sqrt{2\, g\,(x_1 - x_2)} - S_3\, a_3 \sqrt{2\, g\, x_2}\right) \\ y = k_s\, x_1 \end{cases} \tag{10}$$

From Eq. (10), we can represent this nonlinear system by the following form:

$$\begin{cases} \dot{x}_1 = f_1(x) + B u \\ \dot{x}_2 = f_2(x) \\ y = C\, x_1 \end{cases} \tag{11}$$

with

$$f_1(x) = \frac{-S_2\, a_2}{A_1}\sqrt{2\, g\,(x_1 - x_2)}\ , C = k_s \text{ and } B = \frac{k_p}{A_1}$$

$$f_2(x) = \frac{S_2\, a_2}{A_2}\sqrt{2\, g\,(x_1 - x_2)} - \frac{S_3\, a_3}{A_2}\sqrt{2\, g\, x_2}$$

III. Fractional Order Sliding Mode Control

The study of the FOSMC becomes an active area in recent years as a controller to respond the effect of external disturbances and system uncertainties for nonlinear systems. To design FOSMC, a fractional order sliding surface and an appropriately control signal are designed so that the state trajectories reach to the sliding surface and remain on it. Let the FO sliding surface; $S(t)$ be selected as:

$$S(t) = k_1\, e(t) + k_2\, D^{-\mu} e(t) + k_3\, D^{\beta} e(t) \tag{12}$$

$$S(t) = k_1\,(x_d - x_1) + k_2\, D^{-\mu}(x_d - x_1) + k_3\, D^{\beta}(x_d - x_1) \tag{13}$$

where $e(t) = x_d - x_1$ denotes the tracking error with output desired value x_d and x_1 is the system output. k_1, k_2 and k_3 are design parameters and μ, β denote the FO parameter.

The derivative of the FO sliding surface, which is defined in Eq. (13), is obtained as:

$$\begin{aligned} \dot{S}(t) = \big(&k_1\dot{x}_d - k_1\dot{x}_1 + k_2\, D^{-\mu}\dot{x}_d - k_2\, D^{-\mu}\dot{x}_1 \\ &+ k_3\, D^{\beta}\dot{x}_d - k_3\, D^{\beta}\dot{x}_1\big) \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{S}(t) = \big(&k_1\dot{x}_d - k_1\dot{x}_1 + k_2\, D^{-\mu}\dot{x}_d - k_2\, D^{-\mu}x_2 \\ &+ k_3\, D^{\beta}\dot{x}_d - k_3\, D^{\beta}x_2\big) \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{S}(t) = \big(&k_1\dot{x}_d - k_1\dot{x}_1 + k_2\, D^{-\mu}\dot{x}_d - k_2\, D^{-\mu}D^{-1}Dx_2 \\ &+ k_3\, D^{\beta}\dot{x}_d - k_3\, D^{\beta}D^{-1}Dx_2\big) \end{aligned} \tag{16}$$

$$\begin{aligned} \dot{S}(t) = \big(&k_1\dot{x}_d - k_1\dot{x}_1 + k_2\, D^{-\mu}\dot{x}_d - k_2\, D^{-(\mu+1)}\dot{x}_2 \\ &+ k_3\, D^{\beta}\dot{x}_d - k_3\, D^{(\beta-1)}\dot{x}_2\big) \end{aligned} \tag{17}$$

Substituting from Eq. (11) into Eq.(17), we obtain:

$$\dot{S}(t) = \left(k_1\dot{x}_d + k_2 D^{-\mu}\dot{x}_d + k_3 D^{\beta}\dot{x}_d - k_1\left(f_1(x) + Bu\right) - k_2 D^{-(\mu+1)} f_2(x) - k_3 D^{(\beta-1)} f_2(x)\right) \tag{18}$$

A necessary condition to maintain the output trajectory on the FO sliding surface is $\dot{S}(t)=0$. Setting $\dot{S}(t)=0$, we get the equivalent control law; $u_{eq}(t)$ as the following:

$$u_{eq}(t) = \frac{1}{Bk_1}\left(k_1\dot{x}_d + k_2 D^{-\mu}\dot{x}_d + k_3 D^{\beta}\dot{x}_d - k_1 f_1(x) - k_2 D^{-(\mu+1)} f_2(x) - k_3 D^{(\beta-1)} f_2(x)\right) \tag{19}$$

In order to satisfy the sliding condition, the reaching law is

$$u_{sw}(t) = \begin{cases} L\, sign(S) & if\ |S| > \Omega \\ L\left(\dfrac{S}{\Omega}\right) & if\ |S| \le \Omega \end{cases} \tag{20}$$

where L, Ω are constant gains.

The controllaw for the FOSMC; $u(t)$ is given as:

$$u(t) = u_{eq}(t) + u_{sw}(t) \tag{21}$$

Theorem 1. For the general system (11), the fractional sliding order surface (12) and the controller (21), the closed-loop system is stable if $L, \Omega \ge 0$.

Proof. The Lyapunov function is defined as $v(t) = \frac{1}{2} S(t)^2 > 0$, and its time derivative isobtained as:

$$\dot{v} = S\,\dot{S} \tag{22}$$

To ensure the reaching condition, the following inequality should be satisfied $S\,\dot{S} < 0$

$$\dot{v} = S\left(k_1\dot{x}_d + k_2 D^{-\mu}\dot{x}_d + k_3 D^{\beta}\dot{x}_d - k_1\left(f_1(x) + Bu\right) - k_2 D^{-(\mu+1)} f_2(x) - k_3 D^{(\beta-1)} f_2(x)\right) \tag{23}$$

$$\dot{v} = S\left(k_1\dot{x}_d + k_2 D^{-\mu}\dot{x}_d + k_3 D^{\beta}\dot{x}_d - k_1 f_1(x) - k_1 Bu - k_2 D^{-(\mu+1)} f_2(x) - k_3 D^{(\beta-1)} f_2(x)\right) \tag{24}$$

$$\dot{v} = S\left(k_1\dot{x}_d + k_2 D^{-\mu}\dot{x}_d + k_3 D^{\beta}\dot{x}_d - k_1 f_1(x) - k_1 B \left(u_{eq}(t) + u_{sw}(t)\right) - k_2 D^{-(\mu+1)} f_2(x) - k_3 D^{(\beta-1)} f_2(x)\right) \tag{25}$$

$$\dot{v} = S\Bigg(k_1\dot{x}_d + k_2 D^{-\mu}\dot{x}_d + k_3 D^{\beta}\dot{x}_d - k_1 f_1(x) - k_1 B \left(\frac{1}{Bk_1}\left(k_1\dot{x}_d + k_2 D^{-\mu}\dot{x}_d + k_3 D^{\beta}\dot{x}_d - k_1 f_1(x) - k_2 D^{-(\mu+1)} f_2(x) - k_3 D^{(\beta-1)} f_2(x) + u_{sw}\right)\right) - k_2 D^{-(\mu+1)} f_2(x) - k_3 D^{(\beta-1)} f_2(x)\Bigg) \tag{26}$$

$$\dot{v} = S\left(-k_1 B u_{sw}\right) \tag{27}$$

$$\dot{v} = S\left(-k_1 B \begin{cases} L\, sign(S) & if\ |S| > \Omega \\ L\left(\dfrac{S}{\Omega}\right) & if\ |S| \le \Omega \end{cases}\right) < 0 \tag{28}$$

according to Eq. (28), if $L \ge 0, \Omega \ge 0$ make $\dot{v} < 0$, which implies that the sliding surface can be attained in finite time.

IV. SIMULATION RESULTS

To verify that the proposed FOSMC could obtain better robustness, hydraulic system with coupled tank is simulated in this section. The simulation results of the proposed FOSMC are performed with sampling period equals 5 msec and they are compared with the results of SMC [40] and PID controller [41]. The performance of control system will be compared in terms of the root-mean-squared errors (RMSE), the mean absolute errors (MAE) and the mean absolute percentage errors (MAPE), which are defined in the following equations, are measured and compared with the obtained performance indices of other controllers [42]:

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N} e(k)^2} \tag{29}$$

$$MAE = \frac{1}{N}\sum_{k=1}^{N} \left|y_d(k) - y(k)\right| \tag{30}$$

$$MAPE = \frac{1}{N}\sum_{k=1}^{N} \frac{y_d(k) - y(k)}{y_d(k)} * 100\% \tag{31}$$

A. Task 1: Normal case

Fig. 2 shows the output response of the nonlinear system under the PID, SMC and FOSMC. Fig. 3 shows the control signal response under the three controllers. Fig. 4 shows the MAE response under the controllers. From the figure, it is obvious that the proposed FOSMC has better performances than the other controllers.

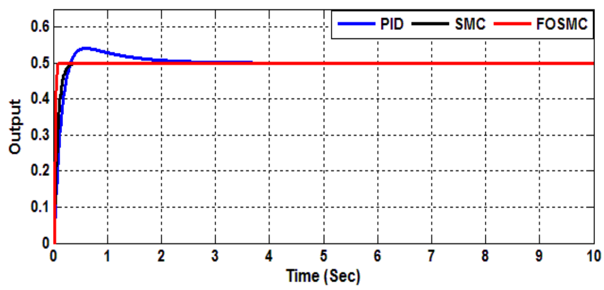

Figure 2: Output response of coupled tank system for normal case.

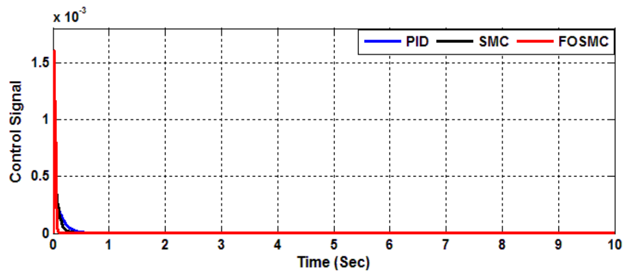

Figure 3: Control signal for normal case.

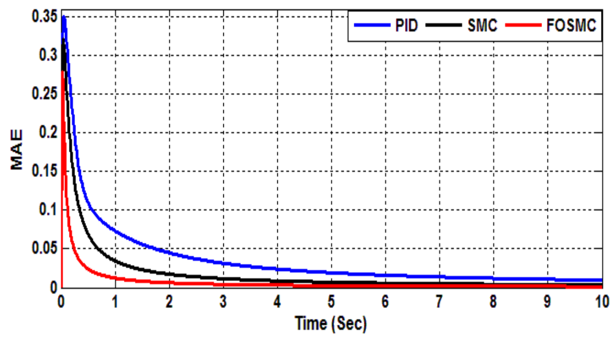

Figure 4: MAE for normal case.

B. *Task 2: External disturbance*

This simulation task shows the response of the coupled tank system under the effect of the external disturbance. Figs. (5-7) shows the system response for this task. An external disturbance; $d(t)$ equals 0.5 is added at 5 sec. The simulation results in these figures prove that the proposed FOSMC exposes better control performance and eliminates the disturbances.

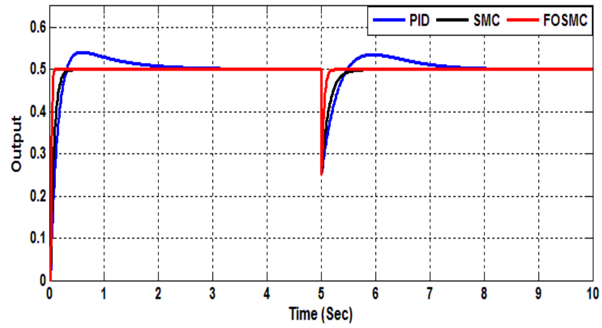

Figure 5: Output response of coupled tank system for external disturbance.

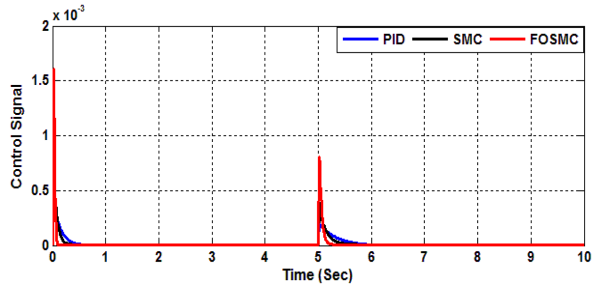

Figure 6: Control signal for external disturbance.

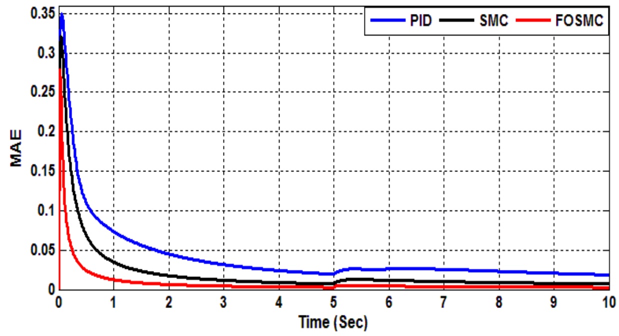

Figure 7: MAE Response for external disturbance.

C. *Task 3: Variations of the desired output*

Figs. (8-10) indicate the system response when the set-point is changed. The desired output is changed from 0.5 to 1 at 3 sec and then changed into 1.5 at 6 sec. The proposed FOSMC has smaller rise time and settling time for tracking the reference trajectory than other controllers. It is noted that, the proposed controller have lower values of MAE as indicated in Fig. 10.

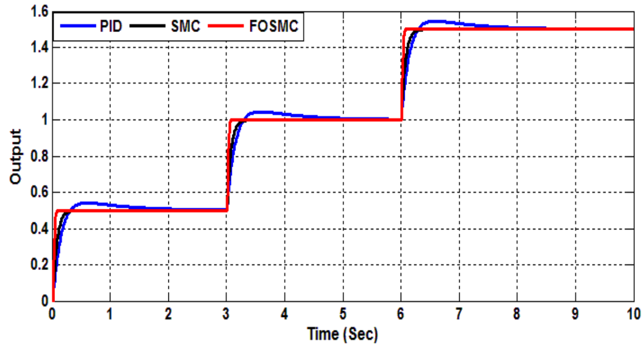

Figure 8: Output response of coupled tank system for step change.

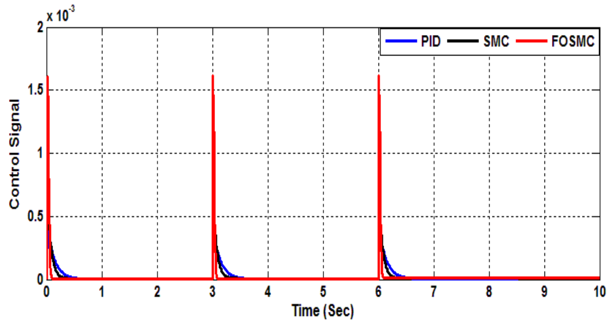

Figure 9: Control signal for step change.

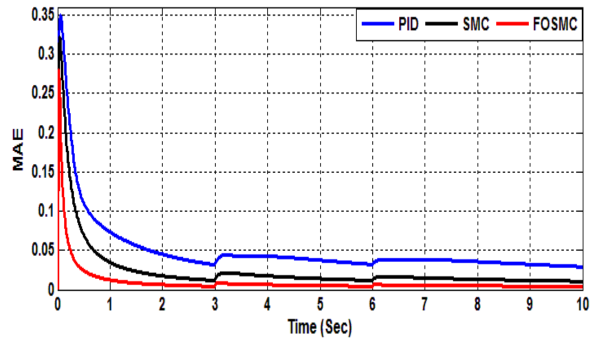

Figure 10: MAE for step change.

Tables (2-4) list the RMSE, the MAE and the MAPE values, respectively, for the proposed FOSMC, SMC, and PID controller for all the above simulation tasks. As shown in Tables (2-4) the values of the RMSE, the MAE and the MAPE for the proposed FOSMC are a lower than that obtained for the other controllers. So, the proposed FOSMC is superior to respond the uncertainties in the system compared to the other controllers.

Table 2: RMSE values for the coupledtank system.

Controller	**Task 1**	**Task 2**	**Task 3**
PID [41]	0.0367	0.0462	0.6159
SMC [40]	0.0262	0.0337	0.6214
FOSMC	0.0132	0.0188	0.6148

Table 3: MAE values for the coupled tank system.

Controller	Task 1	Task 2	Task 3
PID [41]	0.0092	0.0181	0.4598
SMC [40]	0.0031	0.0066	0.4594
FOSMC	0.0009	0.0022	0.4528

Table 4: MAPE values for the coupled tank system.

Controller	Task 1	Task 2	Task 3
PID [41]	1.8438	3.6185	3.5290
SMC [40]	0.6147	1.3141	1.2807
FOSMC	0.1744	0.4301	0.4677

V. CONCLUSION

This paper proposeda fractional order sliding mode control for coupled tank system with disturbances, and variation of desired output. The proposed FOSMC makes the output quickly tracking the reference without any affection from the external disturbances. The proposed FOSMC can decrease the tracking error, compared with the SMC. The results show that FOSMC achieves better system performance compared with SMC and PID controller with the shortest rising time and settling time and the least overshoot and verify that the proposed FOSMC could obtain better robustness and more superior adaptability to variable control systems. A comparative study has been carried out and the obtained results are quite acceptable for FOSMC, SMC and PID controller but the FOSMC seems to be more robust.

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