



## PARAMETRIC MODELING DEVELOPEMENT THROUGH OUT ANALOGUE AND DIGITAL AGES

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### الملخص

أصبح التصميم المعماري في الفترة الأخيرة معقد جداً، حيث أنه يعتمد على تقنيات مختلفة ومعقدة عن الأساليب التقليدية، ومن أحد وسائل النمذجة الحديثة ما يعرف بالنمذجة باستخدام المتغيرات القيمية، حيث أن تغيير كل قيمة ينتج عنه نموذج تصميمي جديد. وترجع جذور نمذجة المتغيرات القيمية إلى عصر المصريين القدماء حيث أنهم استخدموا المعادلات الرياضية في تقسيم الأراضي ونسب أهرامتهم ومعابدهم وهو ما يمثل جنور هذا الاتجاه. وبعده استخدمت أيضاً المعادلات الرياضية والتي هي نواه لنمذجة المتغيرات القيمية في عصور مختلفه أبرزها عصور اليونان والرومان من خلال انسبة الذهبية وغيرها. وفي العصر الحديث وبعد بزوخ عصر تكنولوجيا المعلومات بدأ يظهر التصميم المعماري المعتمد على المتغيرات القيمية، وقد تدرج في تطوره في خلال ثلاثة مراحل معتمداً أساساً على تطور تكنولوجيا المعلومات في العصور الرقمية حيث أنه: في المرحلة الأولى كانت المتغيرات القيمية معتمده على المعادلات الرياضية وفي المرحلة الثانية اعتمدت على العلاقات الهندسية بين عناصر المشروع وفي المرحلة الحالية اعتمدت على الخوارزميات. يستعرض البحث مراحل النمذجة باستخدام المتغيرات القيمية في العمارة بمراحلها المختلفة ويستخلص تطورها وإمكاناتها وتأثيرها على التصميم في كل مرحلة.

### ABSTRACT

Architectural design methods are starting to be based on very complicated techniques different from the old ones. These methods of design are usually the black box of the famous architectural firms, which are usually hidden. One of these techniques of design is what is known by Parametric Modeling PM, its roots in architecture started ages ago. Ancient Egyptians<sup>1</sup>, Greek/Roman<sup>2</sup>, and Islamic architecture<sup>3</sup> used mathematical relations as an application for construction, dividing lands and designing Temples. Architectural components are first theoretically studied for various styles from bibliography and from existing monuments. Each component is then described with a minimal set of parameters.

However, the Greeks were the first to apply many influential theories which have sure applied mathematics, and consequently techniques similar to parametric architecture over the history. Applying PM in architecture redefines the use of information technology in architecture from only a presentation tool to a counterpart to the human imagination, and a portal to another world new to the human mind. The contemporary applications of PM in architecture within digital ages passed through three generations to reach the current form of the parametrical applications in contemporary design. These generations reflect clearly the information technology progression in hardware, software, and their applications in architecture throughout the previous century. The first generation shows pure applications on mathematical equations, second one is based on creating relations between geometrical objects, and the third is the contemporary application that represents the most powerful and

<sup>1</sup> Gillings, Richard J. (1982). Mathematics in the Time of the Pharaohs. Dover.

<sup>2</sup> Maor, Eli (2007). The Pythagorean Theorem: A 4,000-year History. Princeton University Press.

<sup>3</sup> Blair, Sheila; Bloom, Jonathan M. (1995). The Art and Architecture of Islam 1250-1800. Yale University Press.

mature through the use of algorithms, the role of each step in architectural design will be discussed and stated.

KEYWORDS: Parametric Modeling, Analogue, Digital, History.

1. INTRODUCTION








hyperboloid of 1sheet cone cylinder	$\begin{bmatrix} \alpha(\cos u \mp v \sin u) \\ \beta(\sin u \pm v \cos u) \\ \pm cv \end{bmatrix} = \begin{bmatrix} \alpha \cos u \\ \beta \sin u \\ 0 \end{bmatrix} + v \begin{bmatrix} -\alpha \sin u \\ \beta \cos u \\ c \end{bmatrix}$	ALL CYLINDRICAL SURFACES  When a straight line moves in the space without changing its direction, the ruled surface it sweeps is called a cylindrical surface.	CONICAL SURFACES  When a straight line moves passing constantly through a certain point O the ruled surface it sweeps is called a conical surface or a generalised cone.
hyperbolic paraboloid	$\begin{bmatrix} \alpha(u+v) \\ \pm \beta v \\ u^2 + 2uv \end{bmatrix} = \begin{bmatrix} \alpha u \\ 0 \\ u^2 \end{bmatrix} + v \begin{bmatrix} \alpha \\ \pm \beta \\ 2u \end{bmatrix}$	HYPERBOLIC PARABOLOID or SADDLE (a doubly ruled surface)  Its vertical cross sections are parabolas, while the horizontal cross sections are hyperbolas.	HELICOID  MOEBIUS STRIP 
moebius strip	$\alpha \begin{bmatrix} \cos u + v \cos(\frac{1}{2}u) \cos u \\ \sin u + v \cos(\frac{1}{2}u) \sin u \\ v \sin(\frac{1}{2}u) \end{bmatrix} = \alpha \begin{bmatrix} \cos u \\ \sin u \\ 0 \end{bmatrix} + \alpha v \begin{bmatrix} \cos(\frac{1}{2}u) \cos u \\ \cos(\frac{1}{2}u) \sin u \\ \sin(\frac{1}{2}u) \end{bmatrix}$	PLUCKER'S CONOID (or CYLINDROID) 	HYPERBOLOID OF 1 SHEET a doubly ruled surface 
plucker's conoid	$\begin{bmatrix} r \cos \theta \\ r \sin \theta \\ 2r \cos \theta \sin \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2r \cos \theta \sin \theta \end{bmatrix} + r \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix}$		

Fig.1 Some surfaces with equivalent parametric equations

Parametric Modeling is turning design into a set of principles encoded as a sequence of parametric equations. The equations are used to express certain quantities as explicit functions of a number of variables. By changing any parameter in the equation new forms and new shapes could be generated (Fig.1).

Parametric design is a subcategory of algorithmic design, and is strictly based on an algorithmic construct.

Computationally speaking, there is no difference between algorithmic and parametric systems; algorithms by default operate on parameters, and a parametric system's fundamental component is the algorithm itself, called the schema or definition.

However, different than algorithmic design, parametric systems emphasize the explicit and direct manipulation of the parameter values in order to induce a change on the design artifact. This simple difference between a purely algorithmic versus parametric design manifests itself only during the design process, where the parameter values are changed by the designer in order to manipulate the design geometry in search of the optimal design solution. Parametric modeling PM is based mainly on using certain input variables to generate new designs based on these variables. This concept application differs according to the period and the technologies available at each period, but usually the parametric modeling is related to mathematics.

Parametric Modeling is interferred in many related terminologies for a modeling technique.

(Fig.2) shows It's existence in a hierarchy with those related terms: digital, computational, algorithmic and generative modeling.

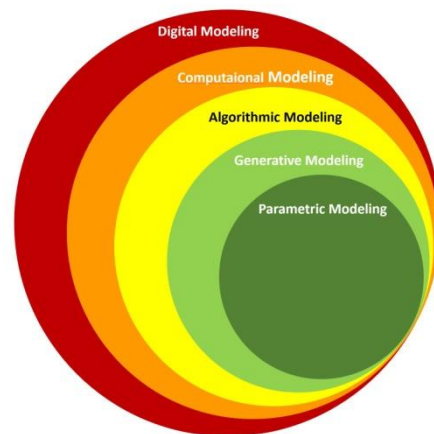


Fig.2 Digital Modeling hierarchy

Digital Modeling	It's modeling using digital software without any computation restrictions. It's a digital form-making process rather than a form finding-process.
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Computational Modeling	It is modeling using digital software using computational rules and simulation.
Algorithmic Modeling	Is a finite set of instructions that aim to fulfill a clearly defined purpose in a finite number of steps? An algorithm takes one value or a set of values as input, executes a series of computational steps that transform the input, and finally produces one value or a set of values as output.
Generative Modeling	Two important and distinguishing points that make a computational model generative; the rule or logic and the feedback loop, where the model takes its own output for input, to relatively generate complex results using iteration.
Parametric Modeling	The simple difference between a purely algorithmic versus parametric design manifests itself during the design process, where the parameter values are changed by the designer in direct manipulation in order to induce a change on the output.

Historically many buildings or facades were designed based on certain proportional mathematical rules that maintain the relation between certain proportions, and if the main length side is changed the whole façade or design should be changed based on the rule, which represents a parametrical approach in design. In our contemporary architecture, the parametric design progressed through many generations which are based on computers applications.

The journey of PM in architecture from ancient world to our contemporary architecture will be discussed and analyzed.

## 2. PARAMETRIC MODELING IN ANALOGUE AGES.

The applications of parametrical rules could go back in the history to the ancient Egyptian period; according to researches ancient Egyptians architects were the first to create their Facades based on mathematical relations which represents the simplest form of parametric design<sup>4</sup>.

### 2.1 PM in Ancient Ages

#### 2.1.1 PM in Ancient Egyptian

In 1863, Viollet-le-Duc suggested that the ancient Egyptians used three main triangles in their designs: the 3-4-5 Triangle, the equilateral triangle, and what he called Egyptian triangle. The Egyptian triangle is connected to equilateral triangle and in fact derived from it, and this is clear in figure 1. In the Egyptian triangle, the ratio between the base and the height can be approximated by means of the ratio 8:5 .

Also in 1899, French architectural historian Auguste Choisy Stated that the Egyptians used also three other triangles similar to these of Viollet but he discussed that the third triangle is generated by the 'mean and extreme ratio' which is an introduction to the golden section (Fig. 3)<sup>5</sup>.

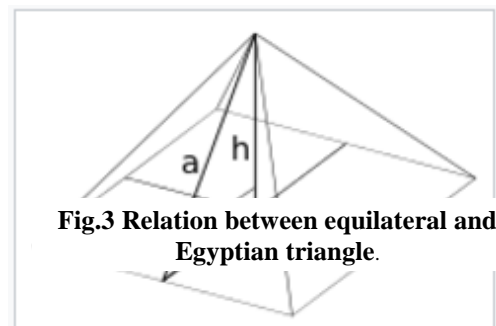


Fig.3 Relation between equilateral and Egyptian triangle.

A proportion may be defined as a relation among four values, or as an equality of two ratios:  $a : b = c : d$ . A proportion is called 'continuous' if there is a common term between the ratios:  $a : b = b : c$ . If  $c = a + b$ , it is possible to establish a proportion which involves two terms only, instead of the four of the first example and the three of the second:  $a : b = b : (a + b)$  that is, the

<sup>4</sup> Gillings, Richard J. (1982). Mathematics in the Time of the Pharaohs. Dover.

<sup>5</sup> Archibald, R. C. "Notes on the Logarithmic Spiral, Golden Section and the Fibonacci Series". Retrieved 1 October 2015.

ratio between two terms is equal to the ratio between the larger term and the sum of the two. (Fig. 4).

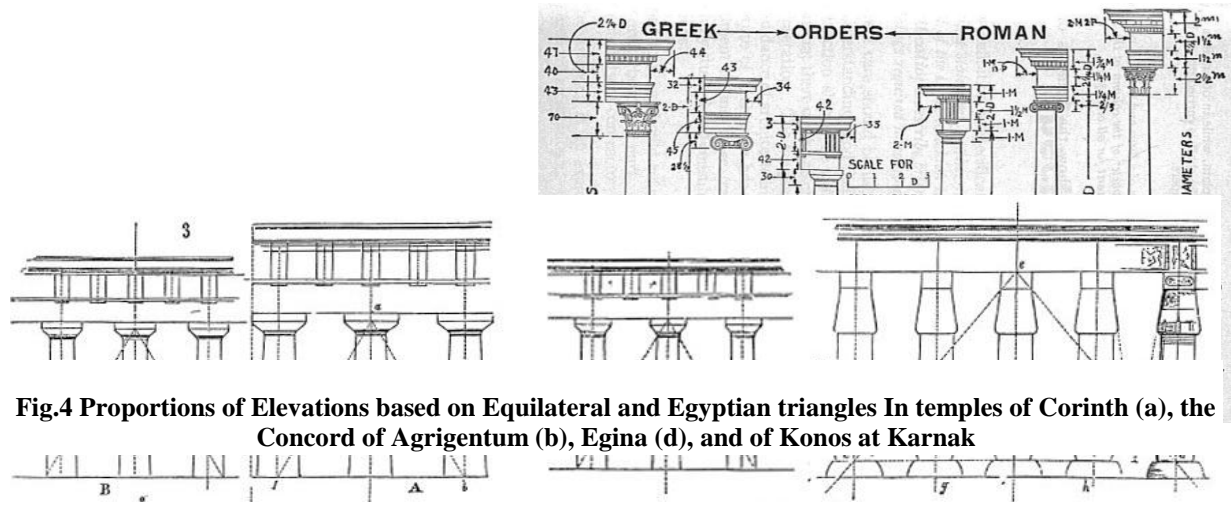
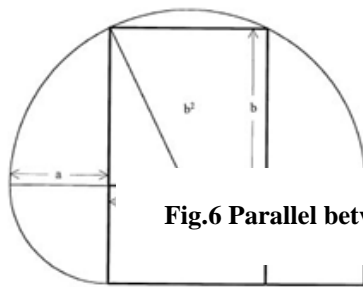


Fig.4 Proportions of Elevations based on Equilateral and Egyptian triangles In temples of Corinth (a), the Concord of Agrigentum (b), Egina (d), and of Konos at Karnak

Fig.5 Visualization of the relationship among elements of a continuous proportion (golden ratio).

And consequently the golden section rule can be generated based



$$ab = b(a+b) \rightarrow b^2 = ab + a^2$$

on the same rule "In  $a : b = b : c$ , then  $b^2 = ac$ , whereas in

$a : b = b : (a + b)$  then  $b^2 = ab + a^2$ . These conditions are visualized in figure 5, where the scheme represents the geometrical properties of the Golden Section: if we assume that  $b = 1$ , the semi-diagonal of the square can be easily calculated as  $\sqrt{5}/2$ . Therefore, the smaller term is equal to  $\sqrt{5}/2 - 1/2$ , while the sum is equal to  $\sqrt{5}/2 + 1/2 = 1.618033989 \dots$ : this irrational number, usually defined by the Greek letter  $\Phi$ , is the numerical value of the Golden Section." (Fig. 5).

According to other researches the golden section was clear in many of the ancient Egyptian buildings, many of statues and reliefs and claimed to have found that they were all designed according to the Golden Section. One of the discussions was the created section by Matila Ghyka for the sketch of pyramid Khufu (Fig. 6).

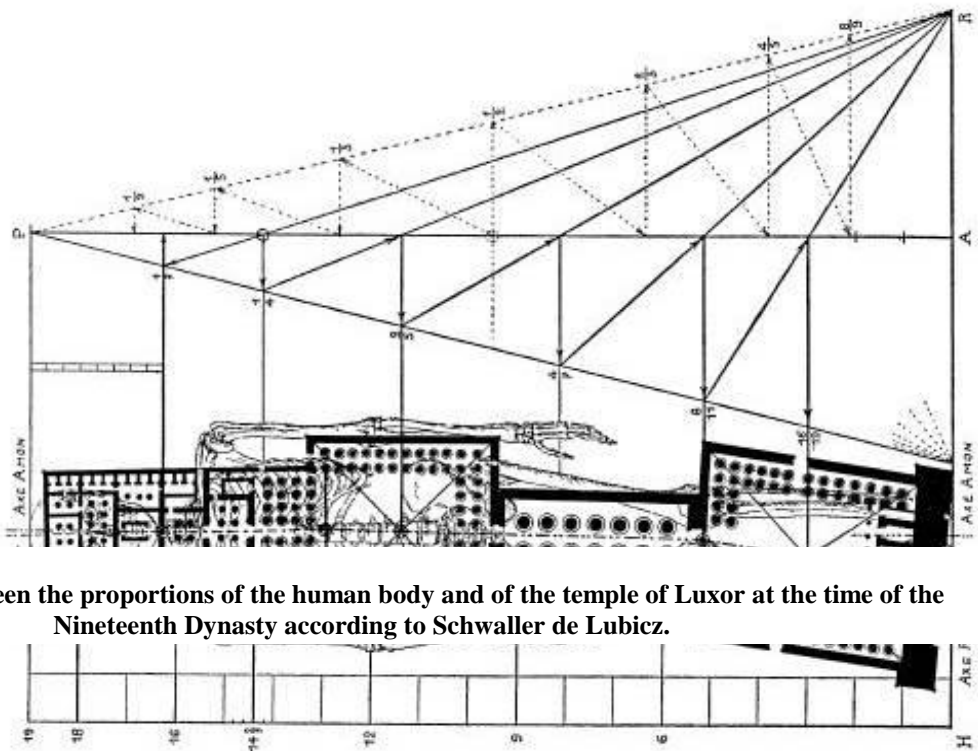


Fig.6 Parallel between the proportions of the human body and of the temple of Luxor at the time of the Nineteenth Dynasty according to Schwaller de Lubicz.

**2.1.2 PM in Greek and Roman Architecture**

The main features of Greek and Roman architecture was based on ordered components, geometric proportion. Gottfried Semper outlines concerning the history of Greek and Roman architecture a scheme of proportions, which he is careful to state is mere means of comparison it's the following: "if we take three average distances from columns-axis as the basis of rectangle, whose vertical sides are equal to the height of the order, measured from the level of the top step of the level of the top of the stylobate, to the upper level of cornice, we thus construct the normal rectangle, whose measure of length or modulus is the lower radius of the column"<sup>6</sup>  
 (Fig. 7).

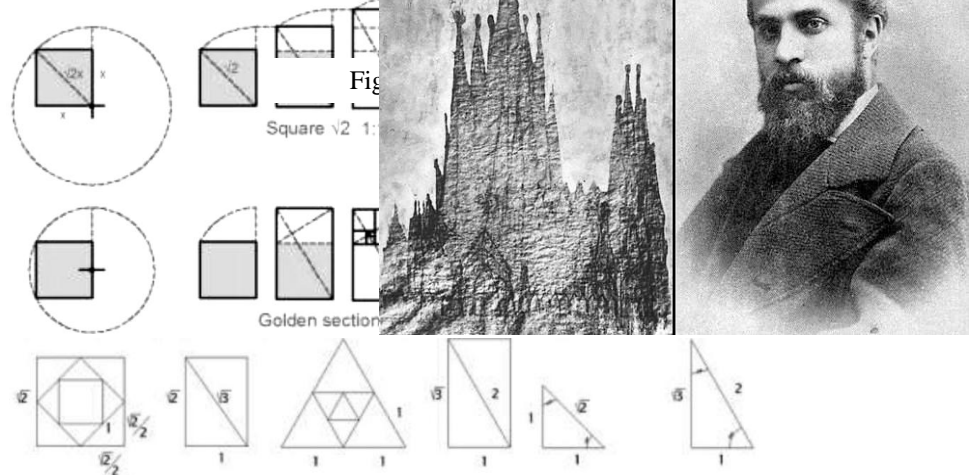


Fig.8 geometrical relationships used mostly in Islamic architecture.

**2.1.3 PM in Islamic Architecture**

In the Islamic designs, geometry represents the order, harmony and beauty in calculations, scale and proportion. It clearly exists in the design of plans, façade, ornaments and patterns. It expresses many concepts of Islam such as the unity and the oneness of Allah, the perfection and the infinity of creation in the universe, the containments and the continuity.<sup>7</sup>

The main proportions of the Islamic style depend on the square proportions (ex: Islamic proportion 1: √2) in which the square is the basic module shape that generates the other geometric forms such as the famous Islamic star, octagonal Islamic rose. In addition, square gives the basic axes and symmetry in the main internal spaces such as the Courtyard in the mosques. So far the role of geometry has been fundamentally dependent in which it contains, regulates and supports the module of the elements. (Fig. 8).

**2.2 PM in the Nineteenth Century**

**2.2.1 Antoni Gaudí (1852-1926)**

Architect Antoni (Fig. 9) Gaudí was the first who began designing with parametric catenary curves and parametric hyperbolic paraboloids at the end of the ninetieth century. It is impossible to know whether Gaudí

was directly influenced by the various scientists and mathematicians who had earlier used parametric equations to define geometry.

Gaudí's deep understanding of mathematics underlies his architecture, especially his later architecture, which almost exclusively consists of mathematical ruled surfaces – helicoids, paraboloids, and hyperboloids – parametrically associated together with ruled lines, booleans, ratios, and catenary arches. (Fig.10).

Whether or not Gaudí knew of the earlier work defining geometry with parametric equations, Gaudí certainly employed models underpinned by parametric equations when designing architecture.

The use of parametric equations can be seen in many



<sup>6</sup>M. Marzocchi, E. McGovern, S. Pavia, HISTORIC BUILDING INFORMATION MODELLING, March 2011, Trento, Italy.  
<sup>7</sup>Mai M. AbElsalam, The Use of Smart Geometry in Islamic Patterns, Alexandria University, Alexandria, Manama, Bahrain.

Fig.10 Free falling catenary curves

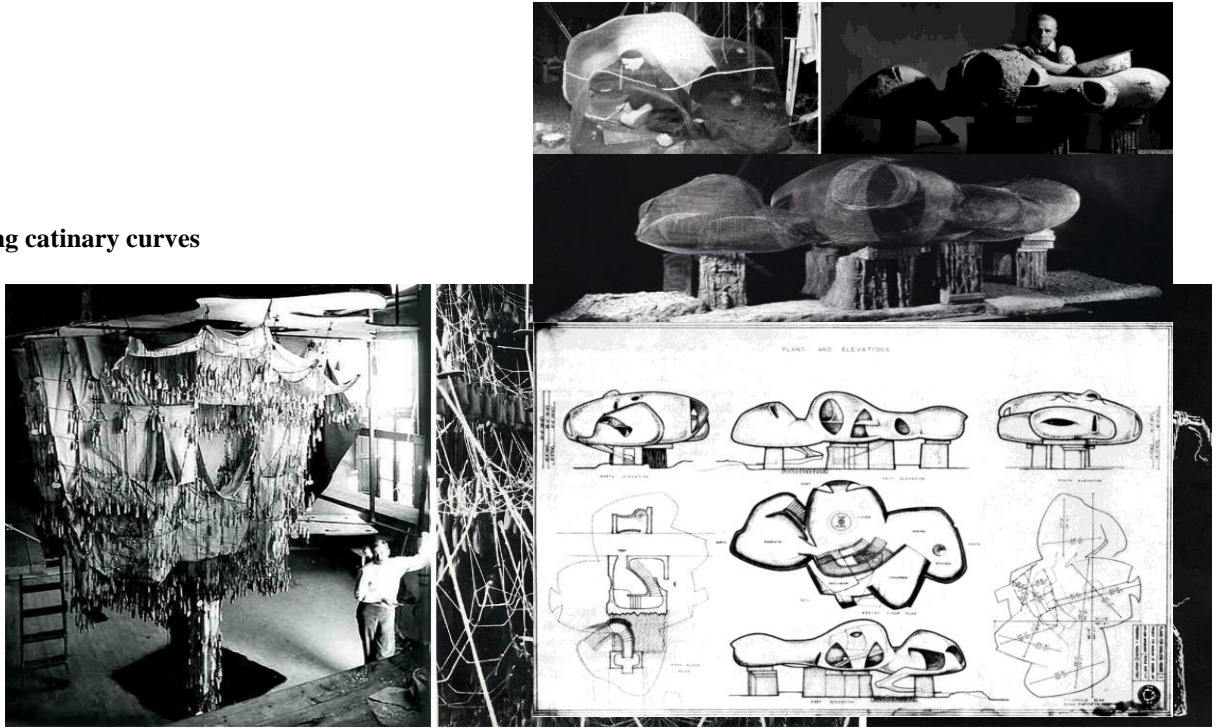


Fig.11 The upside down working force model of weights.

aspects of Gaudí's architecture but is perhaps best illustrated by his use of the hanging chain model. The hanging chain model originates from Robert Hooke's anagram, which unscrambled and translated from Latin reads "as hangs the flexible line, so but inverted will stand the rigid arch".<sup>8</sup>

Gaudí used this principle to design the Colònia Güell Chapel by creating an inverted model of the chapel using strings weighed down with birdshot.

Because of Hooke's principle, the strings would always settle into a shape that, when inverted, would stand in pure compression. The hanging chain model has all the components of a parametric equation. There are a set of independent parameters (string length, anchor point location, birdshot weight) and there are a set of outcomes (the various vertex locations of points on the strings) which derive from the parameters using explicit functions (in this case Newton's laws of motion).

By modifying the independent parameters of this parametric model Gaudí could generate versions of the Colònia Güell Chapel and be assured the resulting structure would stand in pure compression.

### 2.2.2 Kiesler (1890- 1965)

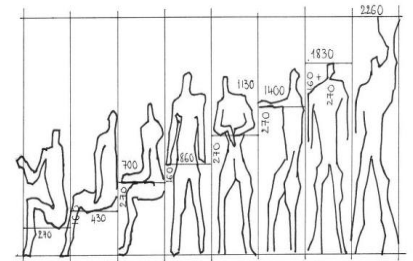
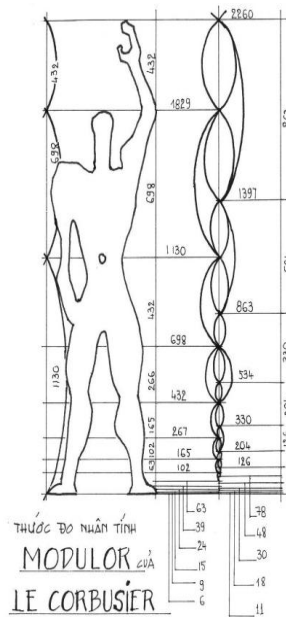
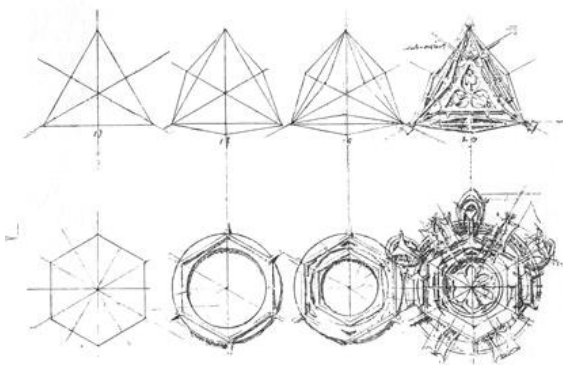
Kiesler built large 1:1 prototypes of parts of the Endless House design using suspended netted structures to simulate tension of multiple- directional forces and used his bodily experience to shape the space from inside the model. It is not just a sculptural

<sup>8</sup> <http://www.danieldavis.com/a-history-of-parametric>.

form. It is a co-ordinate of strictly dimensional areas, all different in height, width, shape, and textures.”. He used splines, borrowing techniques established in the shipbuilding industry, to draw horizontal and vertical sections to map the complex external curvature of the surface of the Endless House. (Fig. 12).

**2.2.3 Louis Sullivan (1856- 1924)**

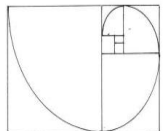
Louis Sullivan (1856- 1924) was an early American modernist architect, known for his pioneering work with the skyscraper form and for his intricate and integrated ornamentation. His ornamentation has to a large extent become the defining



“MODULOR KHÔNG TIEM LẠI TÀI NĂNG, LẠI CÀNG KHÔNG BEM LẠI THIÊN TÀI, NỮ KHÔNG GIÚP CÁI THỦ TỤC TRỞ THÀNH CÁI TÌNH TẾ, NỮ CHỈ CÒNG HIỆN SỰ THOẢI MÁI TIỀN LỢI DO VIỆC SỬ DỤNG THƯỜNG SỞ BÙ CHẮC CHẮN. TRƯỚC KHO TÀI NG VỐ TÂM CẢ HỀ THƯỜNG MODULOR CHỈ CÒN VIỆC LỰA CHỌN CHO MINH THƯỜNG TỶ LỆ THÍCH HỢP NHẤT.”  
LE CORBUSIER

HÌNH CHỮ THẬT TỶ LỆ VÀNG

$$\frac{a}{b} = \frac{b}{a+b}$$



characteristic of his work – in part because it is so unique and in part because it is so incredibly complex. While at first glance his ornamentation appears to be fluid and organic – plastically and intuitively manipulated – his processes of design

generation and production were actually a complex, rigorous system with very specific rules. Just before his death in 1924, Sullivan produced a series of sketches in plates titled A System of Architectural Ornament. A process that he followed to generate ornamentation. (Fig.13)<sup>9</sup>

**2.2.4 Le Corbusier in (1887, 1965)**

One of the most common theories in the early of the twentieth century was that stated by Le Corbusier in 1925 which is “geometry and gods sit side by side” he held geometry to be one of mankind’s greatest achievements, and his belief in transferring the laws found in nature, mathematics, and music into art and architecture produced the modular. The modular represents the revivalism of other old theories. (Fig. 14)

Le Corbusier generated a system of harmony and with relation to the human body. The Modular system is based on using two heights 113 cm and 226 cm where he scaled these up or down according to golden ratio. Mathematically, he multiplied these heights based on golden ratio to obtain height. The concept is similar also to the Fibonacci theory. Many of his works reflect clearly the modular theory.

**Fig.14 The Modular theory of Le Corbusier; Le Corbusier described it as a range of harmonious measurements to suit the human scale, universally applicable to architecture and to mechanical**

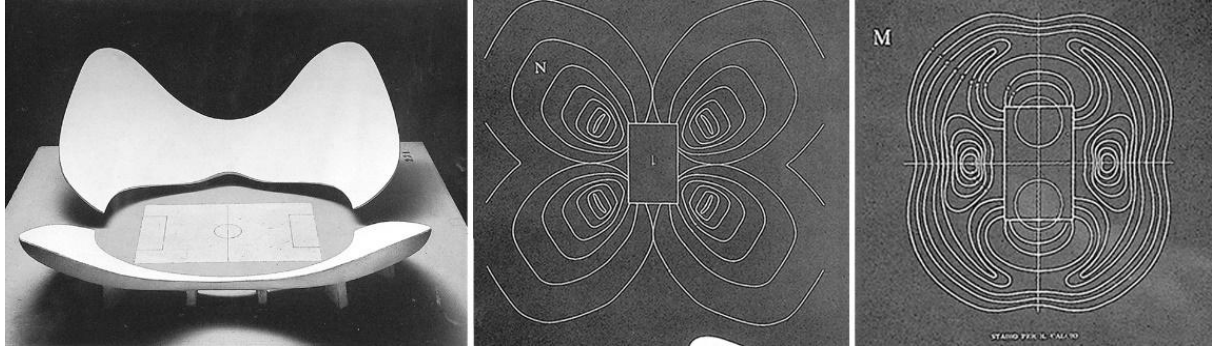
**2.3 PM in the Twentieth Century**

**2.3.1 Luigi Moretti (1907, 1973)**

Moretti wrote extensively about “parametric architecture,” which he defines as the study of architecture systems with the goal of “defining

<sup>9</sup> Selçuk Köse, sources of the exoticism in the architecture of Louis Sullivan: the primitive, the oriental, the natural, the middle east technical university, 2004.

the relationships between the dimensions' dependent upon the various parameters.”



**Fig.15** A model of stadium N by Luigi Moretti. Exhibited at the 1960 Parametric Architecture exhibition at the Twelfth Milan Triennial. The stadium derives from a parametric model consisting of nineteen



Moretti uses the design of a stadium as an example (Fig.15), explaining how the stadium's form can derive from nineteen parameters concerning things like viewing angles and the economic cost of concrete. Versions of a parametric stadium designed by Moretti were presented as part of his Parametric Architecture exhibition at the Twelfth Milan Triennial in 1960.<sup>10</sup>

### 2.3.2 Frei Otto 1967

Otto (Fig.16) calls designing with these models form finding. A phrase that foregrounds the exploratory nature of parametric modelling. In Gaudí's case, the hanging chain model facilitates exploration of form both by constraining Gaudí to structurally sound shapes, and by automatically deriving these shapes whenever Gaudí modifies the parameters of the model.<sup>11</sup>

Frei Otto regarded form-finding as a method of research and design of the emergent properties of natural systems, through which buildings will become less unnatural than in the past. This is a large step in lessening the impact mankind has on the planet. The work of Otto's practice, The Institute for Lightweight Structures, is based around process of seeking form for large engineering constructions through careful construction and documentation of experimental physical models.

Every experiment conducted by the group serves as a model for the future explanation of forces and force transpositions of the particular form in question. Forces of surface tension of soap bubbles where the trigger for some of his designs. (Fig. 17-18).

Otto deems it absolutely necessary that Form-finding Architecture be an interdisciplinary pursuit, made up of architects, engineers, biologists, philosophers, physicists, and synergists. It is the ethical task of the group to fit every unavoidable new structure into its environment with a minimum of materials and energy consumption, eventually in such a way that it becomes part of an ecological system.

### 3.PARAMETRIC MODELING IN THE DIGITAL AGE.

Computer graphics technology was developed in the early 1950s; this was the start for the new technology that helped architects to discover a new world of designs based on complicated computations. This technology starts with a very primary capabilities and starts to progress till it reached the contemporary form. The progression of this technology is reflected clearly on architectural design and consequently, on the contemporary architectural language. The progression of the computer graphics technology is reflected clearly on architectural design and consequently on the ideology and methodology of applying

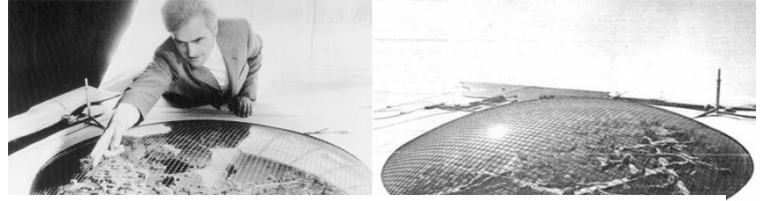


Fig.16 Frei Otto Working on Arctic City Model.

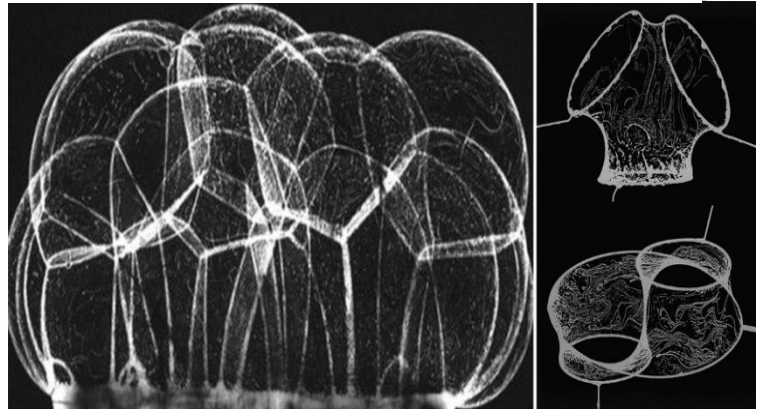


Fig.17 The surface tension of the soap solution formed by the bubble tends to minimize its surface area for the given boundary.

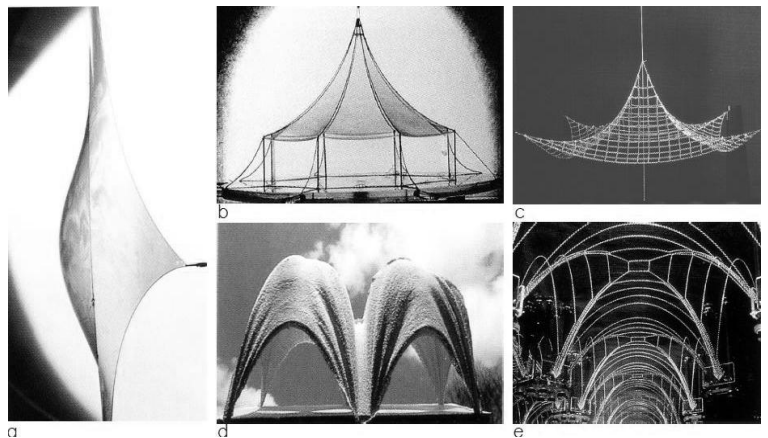
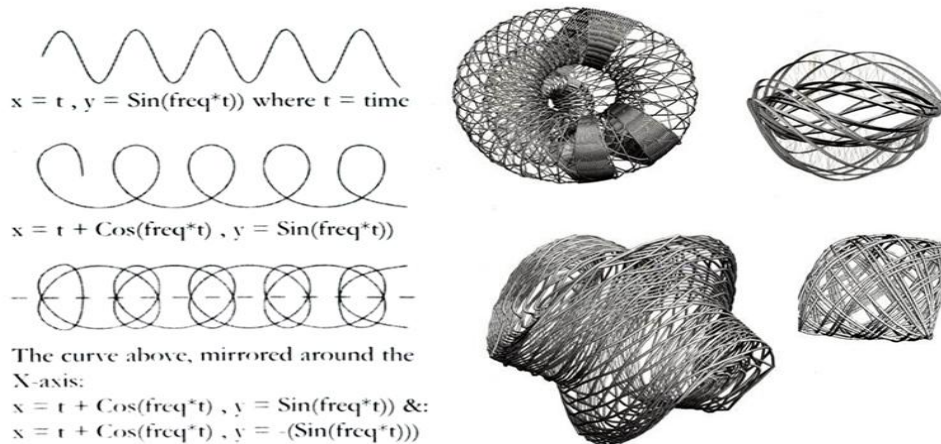


Fig.18 Form-Finding Experiments Conducted by Frei Otto and Bodo Rasch at the Institute for Lightweight Structures.

<sup>10</sup> <http://www.danieldavis.com/a-history-of-parametric>.

<sup>11</sup> Ibid.



**Fig.19 Various forms generated by Mathematical equations.**

parametrical design. 12

The main role of computer graphics technology in architecture was different through the whole journey of this technology progression. In the 1960s the computers' role in architecture was trying to imitate human. through many concepts such as graph theory and machine learning. In the 1970s, the role was to create intelligent systems to assist designers in their decision making by reducing their efforts in taking many design decisions, by many systems such as automated design, expert systems, and other formalistic based concepts. Later on, by the 1980s and the spread of the personal computers architects start to use their personal computers to present their work, and discover new forms.<sup>13</sup>

Today, computers are gradually involved in the architectural design process. Their roles differ from drafting, and modeling to intelligent knowledge based processing of architectural information. The contemporary parametrical design in architecture is moving side to side with the progression of computer technology, and their applications in architecture. Its progression depends mainly on the available technology at each time. Through three main generations the progression of this method can be discussed, from the first generation of only using mathematical relations based on primary graphics applications to the next generation as a reflection for the spread of the personal computers, till the last generation which reflects the mature application for the computational power.

### **3.1 First generation (1960 -): Pure application for the parametric equations.**

This phase is the primary application for the concept of parametric design. It is based on mathematical equations that generate forms based on changing the equations or the variables (Fig. 19). By changing one of the variables or the equations, new forms will be generated. Architects like Felix Candela started using this method in his projects based on parametric equations that generate forms, and consequently, these equations helped him in designing the form structurally. It reflects also the progression of the historical mathematical rules that constraints the proportions of the elevations, since the designer here generates the form based on his mathematical equations.

<sup>12</sup> Zellner, Peter, "Hybrid Space: New forms in digital architecture" New York: Rizzoli,1999.

<sup>13</sup> Negroponte, N. "The Architecture Machine, Cambridge: MIT Press, 1970.

This generation reflects clearly the basic and primary applications for the computer technologies at the sixties period, which is based mainly on pure programming and mathematics, and also on very primary display technologies. It shows the classical ideology of using computers in generating architectural forms which was influenced by other engineering applications based mainly on mathematical equations.

No software was available in this phase (as the computing technology was still primary). Despite that the appearance of this concept was at 1960s many of our architects are still applying this concept but with our contemporary technologies.<sup>14</sup>

Examples; -

Restaurant Los Manantiales in Mexico, by Felix Candela at 1958.

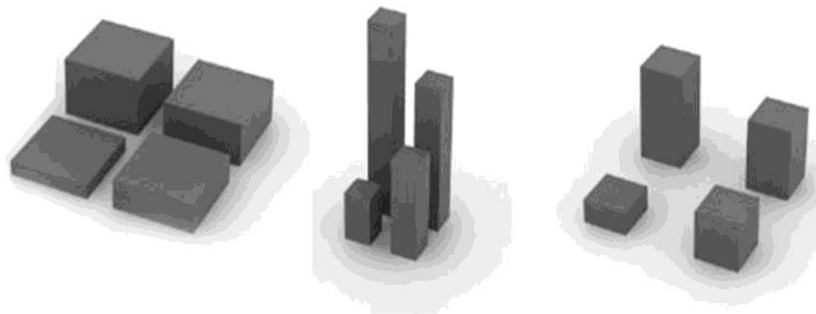
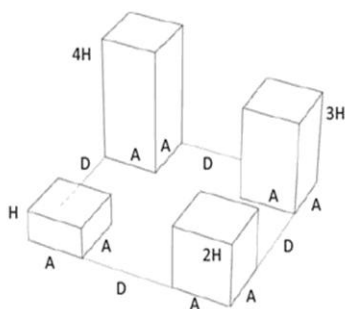
This example was designed by Felix Candela who generated the form of the restaurant, based on a matrix of mathematical equations that generated Hyperbolic paraboloid, this matrix contained all the equations that generated all the parts of the form, and by changing the main variables which were the height and the width, new forms were generated based on these equations. (Fig. 20)

### 3.2 Second generation (1960 -): Formalistic description of relations.

It is based on creating relationships between geometrical objects of a certain project, and by changing any dimension of one of the objects the whole design will be modified (Fig. 21). Parametric design in this generation suggests the use of parameters to generate a form but the greater importance here, are the underlying relations between elements of the form. Set of relationships between objects are maintained while the elements can be independently modified. When interdependencies between objects are determined, the objects' behavior under transformations is efficiently defined. Relationships can also be revisited and revised during the design process.

This concept comprises perfectly with new common software available at that time, where the designer mainly starts creating these relationships based on parametrical sliders in this software. The applications of 3d software start to be clear here; usually most of these forms are generated based on 3d software, which is different from the previous phase, where the forms were created based on mathematical equations. It reflects the ideology of digital architecture that is based on the spread of personal computers, which is only discovering new forms without any targets. It can be simply stated that it is only the experimental era of the powerful potentials of the personal computers, and the use of the available software to generate new forms that may not solve any design problems.

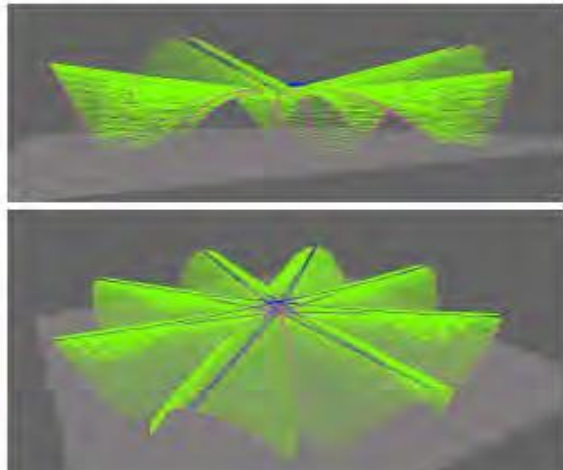
15



**Fig.21 The geometry of the four boxes is parametrically related to each other and to the platform.**

<sup>14</sup> Kolarevic, Branko, "Architecture in the digital age: Design and manufacturing", Published by Tylor & Francis group,2003.

<sup>15</sup> Kotonik, Toni, "Algorithmic extension of architecture" master degree at ETHARCH/CAAD, Zurich, 2006.

$$\begin{bmatrix} 0 & 0 & 50 & -40 & 180 & 0 & 0 & -100 & 10 & 0 & -350 & 0 & \pi/8 & 0 & 0 \\ 0 & 0 & 50 & -40 & 180 & 0 & 0 & -100 & 10 & 0 & -350 & 0 & \pi/8 & 0 & \pi/4 \\ 0 & 0 & 50 & -40 & 180 & 0 & 0 & -100 & 10 & 0 & -350 & 0 & \pi/8 & 0 & 2*\pi/4 \\ 0 & 0 & 50 & -40 & 180 & 0 & 0 & -100 & 10 & 0 & -350 & 0 & \pi/8 & 0 & 3*\pi/4 \\ 0 & 0 & 50 & -40 & 180 & 0 & 0 & -100 & 10 & 0 & -350 & 0 & \pi/8 & 0 & 4*\pi/4 \\ 0 & 0 & 50 & -40 & 180 & 0 & 0 & -100 & 10 & 0 & -350 & 0 & \pi/8 & 0 & 5*\pi/4 \\ 0 & 0 & 50 & -40 & 180 & 0 & 0 & -100 & 10 & 0 & -350 & 0 & \pi/8 & 0 & 6*\pi/4 \\ 0 & 0 & 50 & -40 & 180 & 0 & 0 & -100 & 10 & 0 & -350 & 0 & \pi/8 & 0 & 7*\pi/4 \end{bmatrix}$$


**Fig.20 The Matrices of equations used in generating the parametrical form of the restaurant.**

Available software was used with their primary capabilities, with only showy results more than using the software with their embedded languages or scripts full potentials.

Examples; -

Programmable housing, By Kas Oosterhuis (1999).

The house is a resultant of a collaborative design process between the designer and the client. The concept involves the future client, who can design his residential unit based on changing certain variables (sliders in a software), and since all the parameters of the form are related, the change of main parameters can generate another designs.

It reflects the concept of the second generation clearly through the unrealistic form, and the relationships between the parameters of the house. (Fig. 22).

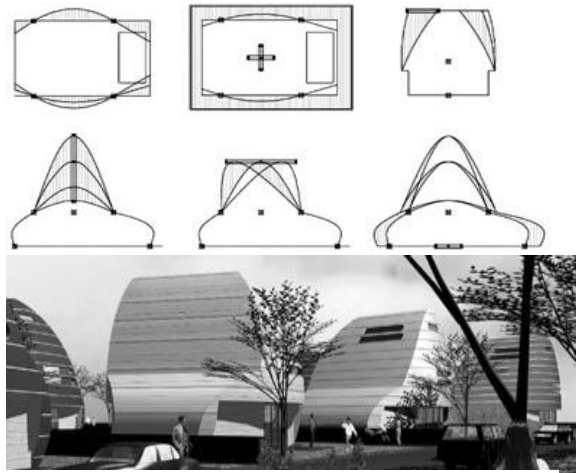


Fig.22 Variometric house as generated from parametrical sliders by the users. By changing the sliders values, new houses are generated.

Embryological house, Greg Lynn (1997-2002),

The “spline” proved most relevant for its simple and concise parametric capacity. It could be pushed, pulled, stretched in a relation with rotation around another spline curve to produce blobby surfaces. (Fig. 23).

**3.3 Third generation (2000 -): Contemporary parametric architecture.**

The contemporary parametric design is based on designing an algorithm to generate the project. The parameters here will be the inputs of this algorithm, and by modifying the parameters new designs will be generated. It is similar to the previous phases but the different here is that the parameters are inputs in algorithms not in pure algebraic equations, which represents a wider spectrum for the idea of parametric design. The designer here starts to create his algorithm by determining certain procedures that at the end generates the final form which fulfills the designer rules (Fig. 24). The algorithmic steps could be used to create relations between geometrical entities similar to the previous parametric relations, or it may be based on mathematical equations. And by changing any of the input parameters new design variables will be generated.

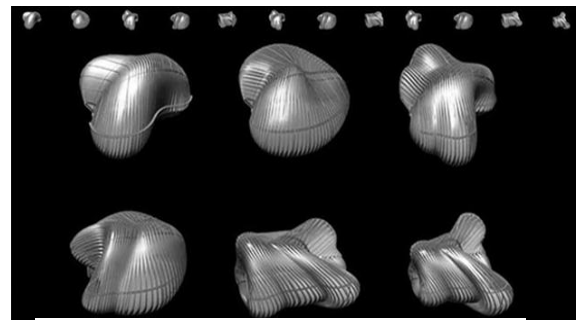


Fig.23 Greg Lynn, Embryologic House, 1998.

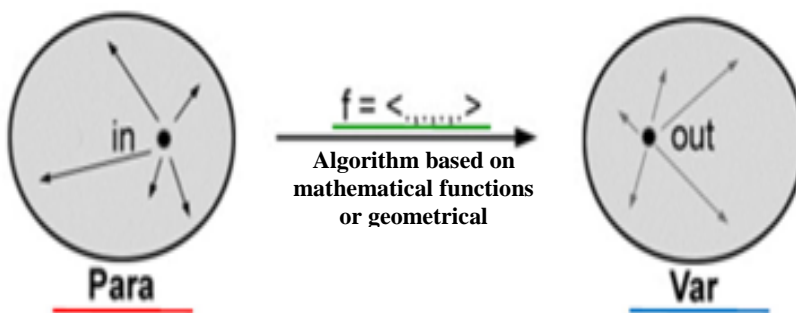


Fig.24 The contemporary parametric design is based on input parameters in an algorithm to generate variables.

This generation represents clearly matured applications for using computers in the architectural designs. It reflects the full computational power of computers with their software and hardware, which is different from the previous generations where most of the designs are not-realistic. Most of the designed projects of the first two

generations, show forms reflecting information technology potentials without any concerns about solving real design problems.

Recently, architects started to use common algorithmic steps such as Voronoi, A-star, Stochastic search, Tabu Search, Swarm intelligence in their designs which consequently start to take part in their parametric designs. These common algorithms could be the main or part of the algorithmic process done by the architect.

This phase reflects the current ideology of using the full computational power to try to solve certain architectural design problems, or to generate a project that meets certain design criteria.

Examples; -

Mixed use tower in Seattle, USA, by Hollander at 2010.

It is a multi-functional building, commercial and residential tower, and through the use of parametric design the architect was able to generate a tower that fulfills the client's aim, which is to reach the maximum profit.

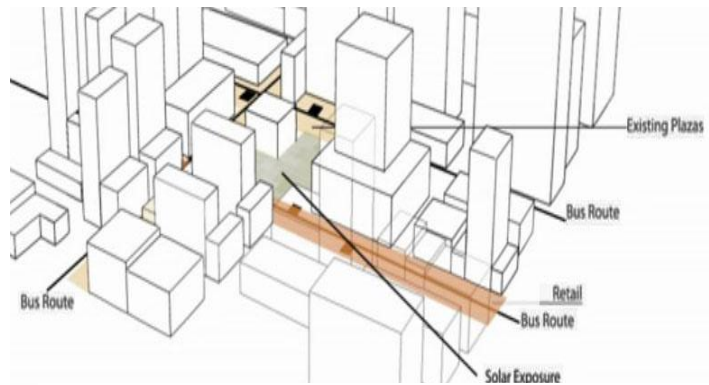
The concept of increasing the revenue was based on improving the design of residential units, through improving their orientation, their sight to the sea, and designing the commercial podium with the best function. Based on the previous concept the architect starts designing his algorithm.

The main concept is the use of parametric design to study all the alternatives to reach the optimum solution through the input of variables in an algorithm. It is fulfilled by using a parametrical design which is based on creating inputs in a Genetic algorithm. A genetic algorithm is a subset of heuristic search methods for solving optimization problems, simulating biological evolution. Genetic Algorithm has unique terminology due to its origins in biological evolutionary science. The process of problem solving in GA is fairly simple, despite its ability to solve highly complex problems. Initially; a population is randomly generated with a sufficient amount of characters (called chromosomes) allowing for a wealth of genetic information to be processed and therefore, statistically provides a better solution, although the bigger the population; the more computing power is needed.

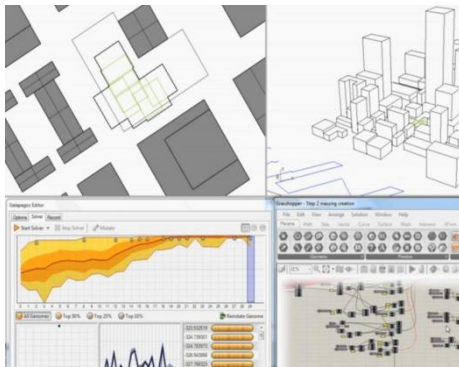
Individuals are then selected on a basis of performance rules defined by the architect (in our example is the sight) prior to the process. At this stage, a number of genetic operators (stochastic operators) become in effect, such as crossovers. Crossovers are swapping of parts of two characteristics (chromosomes) selected for their fitness, producing a new offspring of individuals containing some genetic material from each parent individual. This process is repeated and reiterated to produce more refined generations gradually through selection.

#### Step 1: Generating the Podium (commercial)

The first step done by the architect was to determine the place of the main plaza to create the design of the retails zone. And to determine the position of this plaza the architect studied the important nodes around the site like the distance to transportation, existing plazas, surrounding retail, and directions of solar exposure (Fig. 25-26). These nodes were used in the algorithm as numerical functions to show the best position for the plaza according to the surrounding nodes. Using the evolutionary algorithm many alternatives were generated after thousands of iterations, based on the evolutionary function stated by the architect. By the end of this step, the final position for the plaza was determined.



**Fig.25 The Main important nodes in the atmosphere of the site that will impact on the design of the main podium such that bus route, existing plazas, bus routes, solar exposure, etc.**



Step 2:

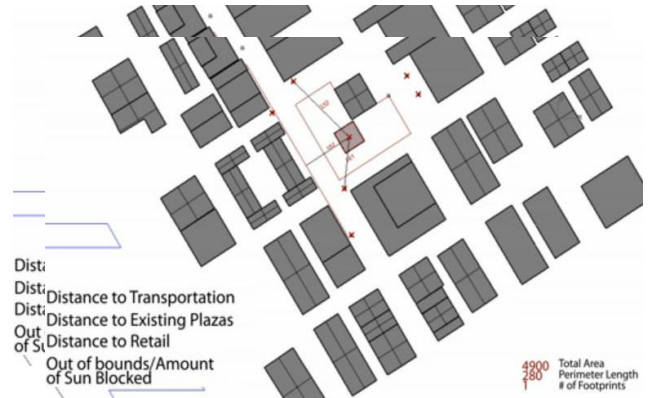
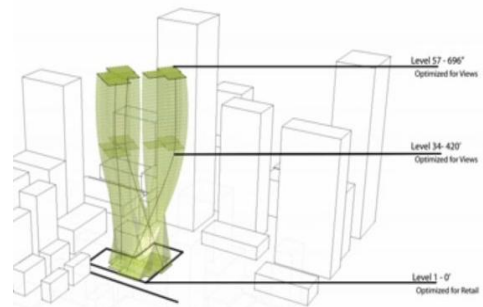
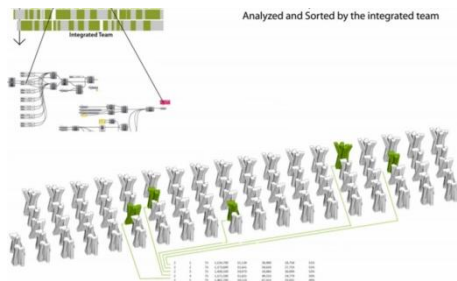


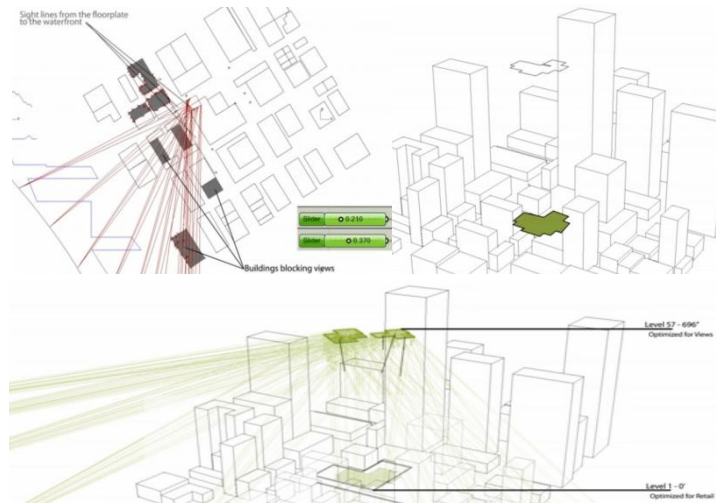
Fig.26 The algorithm generates the alternatives to fulfill the designers aim.



Generating the residential towers.

Fig.28 various forms are generated based on the optimization algorithm.

This step represents the crucial step for generating the main form to reach the optimum sight for the residential units. The architect here used the evolutionary algorithm which is based on generating the final form in such a way that the circumference 's facing the sight will be increased to the maximum and at the same time avoiding the obstacles (buildings in the lower floors). In This phase, the input is a proposed circumference for a plan and many variables are generated using the algorithm to optimize the sight for some floors till last floor. The algorithm now generates the optimum plans for the two towers. (Fig. 27).



Step 3: Generating the final form.

In this step, the algorithm starts generating many alternatives for the final form according to the previous steps. At the same time these alternatives are compared. Then the final form is selected, according to performance and cost. (Fig. 28-29).

Fig.27 Circumference of the residential tower is being generated to improve the sea sight for the units.

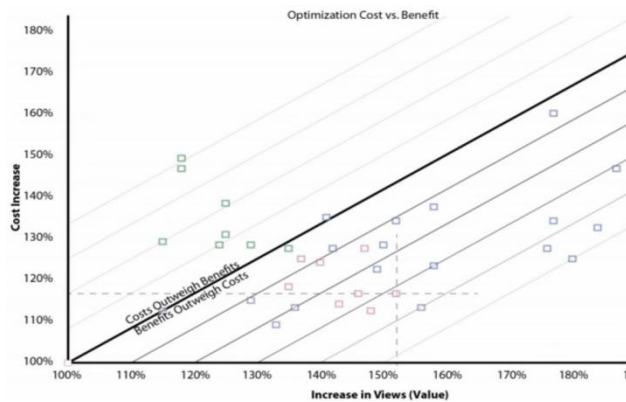


Fig.29 on the left: The group of the generated towers are compared according to their performance, on the right; the final form.

#### 4.

### CONCLUSIONS

Parametrical Modeling role in architecture varies from primary to complicated applications. The method of applying parametrical Modeling, its role, and its potentials in architecture, are reflection to the technology and culture available at each time. Parametrical Modeling role during analogue ages was a primary application that concerned only with designing facades or sections with certain geometrical proportions, to assure the aesthetics aspects in design. It also reflects the primary knowledge with its simple mathematics, and lack of technology. Parametrical applications during digital ages started to develop regarding the progression of information technology. Their role in architecture can be summarized in three generations; in first generation, their role was developed from only creating geometrical proportions to generating forms based on mathematical equations. In second generation, their role progressed from only mathematical equations generating forms, (depending on progressed software) to relations between geometrical forms. The third generation reflects the use of algorithms in parametric design in a way of regarding and discovering the unknown at its very best, it is programming that goes beyond developing commercial software applications. It becomes a technique of exploring and mapping our own way of thinking. The role of parametric design in this generation is solving architectural problems, experimenting design, generating new forms, and sometimes generating plans. Parametric design becomes the technique by which one can extend and experiment with rules, principles, and outcomes of traditionally defined architectural processes.

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تطور نمذجة المتغيرات القيمة من العصور التناظرية وحتى الرقمية