

WATER CONING CORRELATIONS IN VERTICAL WELLS

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ABSTRACT

One of the major problems that reduces significantly the crude oil production is water coning. When the well is produced, water moves up from the bottom water toward the wellbore in a cone shape. At certain conditions, water breaks through into the well and concurrent oil and water production starts.

Many methods have been presented to overcome this problem, whatever before or after the taking place of water coning. The critical rate is the widely method used to overcome this problem before its occurring.

The aim of this paper is to introduce a method to predict the critical rate, breakthrough time, and WOR (water oil ratio) after breakthrough in vertical wells. A 3-D numerical simulator model was used to perform a comprehensive sensitivity analysis of water coning process. From this analysis, an empirical coning correlation was developed based on the basic flow equations and regression analysis. The format of the correlation is similar to Addington's gas-coning and Yang's water coning correlations.

The correlations presented in this paper provide a hand calculation fashion of coning prediction for vertical wells. The correlations were tested and found to be reliable and accurate in predicting the critical rate, breakthrough time and WOR, within the correlation parameter range.

Keywords: Water coning; Critical rate; Water production.

INTRODUCTION

Water coning is the expression describing the upward movement of water into the perforations of producing wells that are produced from oil layers underlain by bottom water. Several problems arise due to the excessive production of water from oil wells such as decrease in oil flow rate, decrease efficiency in the depletion mechanism and hence loss of field total overall recovery. Produced water is often corrosive and thereby increase in water disposal cost.

During production from an oil well underlain by bottom water, there are two forces that control the movements of oil-water contact, the pressure gradients and gravity forces. When the pressure gradients are dominant, the oil-water contact can be raised near the well and the coning of water will break into the well to produce water along with the oil. The gravity forces that arise from fluid density differences counterbalance the flowing pressure gradients and tend to keep the water out of the oil zone. Numerous authors have dealt with the coning problem in terms of critical rate, water

breakthrough time, and water-oil ratio (WOR) after water breakthrough. Many approaches have been developed for predicting these behaviors.

Several correlations were developed to predict the critical rate. In general, these correlations can be divided into two groups. The first group determines the critical rate analytically based on the equilibrium conditions of the pressure gradients and gravity forces. Muskat and Wyckoff¹ developed an approximate solution of the critical flow rate in isotropic formations. They solved Laplace's equation for single phase, steady-state, incompressible flow. Meyer and Garder² simplified the analytical solution of Muskat and Wyckoff work for radial-flow, whereas Chaney et al.³ and Chierici et al.⁴ used potentiometric models to obtain the critical flow rate. Chaperon⁵ investigated the water coning in vertical wells in anisotropic formations in a closed system and presented a solution for the critical rate and reported that the critical rate increases slightly when the vertical permeability decreases, but the elevation of the critical cone does not change appreciably. Wheatly⁶ took into account the influence of cone shape on the oil potential, which others had not done before. Tabatabaei et al.⁷ presented an analytical model to calculate the optimum completion interval and the critical flow rate for a steady state flow from a vertical well that is partially completed in an anisotropic reservoir with a low pressure gradient.

The second group is empirical correlations. Schols⁸ presented an expression from experiments conducted in his lab, while Hoyland et al.⁹ developed their correlation from computer simulation runs. Addington¹⁰ presented generalized expressions for the critical coning rate and the gas-liquid ratio (GLR) after gas breakthrough employing a three-dimensional simulation study of the Prudhoe Bay field. The concept of Addington for calculating the critical rate was different from others. Addington was solving a closed outer boundary problem that never reaches steady-state conditions, while others were dealing with open outer boundary problems at steady-state conditions. Moreover, Addington's critical rate is decreasing with time, while others had a constant critical rate.

Methods are also available to predict the water breakthrough time. Sobocinski and Cornelius¹¹ presented dimensionless correlating groups in forms of dimensionless plot based on their experimental and computer simulation runs for a homogeneous, incompressible system with no gas cap, producing at a constant rate to calculate the cone breakthrough time and critical rate. Bournazel and Jeanson¹² conducted laboratory experiments and developed a correlation for the water breakthrough time in vertical wells similar to the correlation of Sobocinski and Cornelius. Also they developed a method assuming that water is separated from oil, the oil-water interface rises and stays at some point of perforation interval. By determining the length of the perforation interval in the water, WOR can be calculated. Byrne and Morse¹³, Mungan¹⁴, Blades and Strightll¹⁵ searched the effects of different reservoir and well variables on WOR performance using numerical simulation. However, they had not reached to a general predictive model. Kuo and DesBrisay¹⁶ presented an expression for predicting water cut performance based on a sensitivity analysis of water coning performance for different reservoir parameters using numerical simulation. Yang and Wattenbarger¹⁷ presented correlations to predict the critical rate, breakthrough time and WOR after breakthrough for water coning in vertical and horizontal wells. This work presents water-coning correlations for predicting the critical rate, water breakthrough time and WOR after breakthrough for vertical wells.

Method

Yang and Wattenbarger¹⁷ noticed that the relationship between the WOR plus a constant (c) and the average oil column height below perforations after water breakthrough (h_{bp}) on a semi-log scale is a straight line as shown in Fig.1. They described this diagram mathematically as follows:

$$\begin{aligned} \text{WOR} = 0 & & h_{bp} > h_{wb} \\ \text{Log}(\text{WOR} + C) = S(h_{bp} - h_{wb}) + \text{Log}(C) & & h_{bp} \leq h_{wb} \end{aligned} \quad (1)$$

Where, h_{wb} is the average oil column height below perforation at breakthrough, S is the slope of the straight line and C is a constant.

In the presented work, a method for determining h_{wb} , S and C was developed from a stepwise procedure. First, a number of simulation runs was made to analyze the performance of coning at different reservoir and fluid properties. Then, for each run, $(WOR + C)$ was graphed against h_{bp} on a semi-log scale, from which S and h_{wb} were determined using regression analysis. Once the h_{wb} and S data was obtained for all the simulation runs, regression analysis was then used to define the relationship between S , h_{wb} and different reservoir and fluid properties, respectively.

Correlations Development

In this paper Eclipse, a black-oil, three-dimensional, commercial simulator was used to simulate the water coning in a vertical well. The formation is considered to be homogeneous and anisotropic with capillary forces. The vertical well is modeled with a 3-D, r-z model as shown in Fig.2.

To develop correlations to calculate the water breakthrough height and slope of the straight line after breakthrough, the parameters sensitivity analysis was made to supply the required data. The relative permeability data is illustrated in Table 1.

A base case was installed to start the parameters sensitivity analysis. Afterwards each parameter was varied from the lower value to the upper value of its range in each simulation run. The parameters used in the sensitivity analysis are oil flow rate, horizontal and vertical permeabilities, drainage radius, net pay thickness, perforation and above perforation thicknesses, oil and water viscosities, porosity and water-oil gravity difference.

In Table 2, the simulation data and outcomes are illustrated. The base case parameters values are shown in the top line. For the rest of the cases reported, the parameters are varied independently over the range presented in the table while preserving the values of the remainder of the parameters in the base case. A blank space displays that the base case value is carried forward. The average oil column height below perforation at breakthrough h_{wb} and slope of the straight line S are recorded in the last two column. For a particular variable under investigation, a semi-log plot of $(WOR+C)$ vs. h_{bp} was made. From the plot, h_{wb} , S and C are obtained. It was found that the constant, C , is 0.25. Then, the WOR changes can be described by the following equation.

$$WOR = 0 \quad h_{bp} > h_{wb}$$

$$\text{Log}(WOR + 0.25) = S(h_{bp} - h_{wb}) + \text{Log}(0.25) \quad h_{bp} \leq h_{wb} \quad (2)$$

After investigating the effect of the various reservoir and fluid properties on h_{wp} and S , the following equations were defined as follows.

$$h_{wp} = a_0 \frac{q_0^{a_1} k v^{a_2} h^{a_3} \mu_o^{a_4} r e^{a_5}}{k h^{a_6} h_{ap}^{a_7} h_p^{a_8} \mu_w^{a_9} \Delta \bar{\sigma}^{a_{10}}} \quad (3)$$

Where

$a_0 = 0.307116$	$a_1 = 0.036326$	$a_2 = 0.05255$
$a_3 = 1.181052$	$a_4 = 0.03772$	$a_5 = 0.0765$
$a_6 = 0.046041$	$a_7 = 0.132478$	$a_8 = 0.183205$
$a_9 = 0.036$	$a_{10} = 0.01$	

$$S = b_0 \frac{k h^{b_1} h^{b_2} \mu_w^{b_3} \Delta \bar{\sigma}^{b_4} q_0^{b_5}}{k v^{b_6} h_{ap}^{b_7} h_p^{b_8} \mu_o^{b_9} r e^{b_{10}}} - 1 \quad (4)$$

Where

$b_0 = 1.106253$	$b_1 = 0.050788$	$b_2 = 0.036605$
$b_3 = 0.016241$	$b_4 = 0.000746$	$b_5 = 0.0016$
$b_6 = 0.018312$	$b_7 = 0.000639$	$b_8 = 0.004854$
$b_9 = 0.0306$	$b_{10} = 0.07$	

Correlations Validation

In order to validate the accuracy of the derived correlations, a statistical analysis has been used to evaluate their performance. The statistical indicators are presented in the appendix. The obtained outcomes include an average relative error (ARE) of 0.013, 0.006 an average absolute relative error (AARE) of 0.74, 0.39 and regression coefficient (R^2) of 0.99, 0.9 for h_{wp} and S correlations, respectively.

Calculation example

An oil well with the following data, calculate the critical rate, time at breakthrough and WOR performance after breakthrough.

h_{wp} , ft =	161.8	μ_o , cp =	1.11
h_p , ft =	20	μ_w , cp =	0.3
k_h , md =	200	ρ_o , lb/ft ³ =	50
k_v , md =	20	ρ_w , lb/ft ³ =	62.4
h , ft =	200	Φ =	0.2
h_{ap} , ft =	10	β_o , bbl/STB =	1.364
r_e , ft =	2000	r_w , ft =	0.5

Obtained results

Equation 3 for h_{wb} can be used as a critical rate correlation. At the height h_{wb} water breaks into the well. Then the oil flow rate in this correlation is the critical coning rate. The following equation is used to calculate the breakthrough time.

$$t_{bt} = \frac{(N_p)_{bt}}{q_o} \quad (5)$$

Where $(N_p)_{bt}$ is the cumulative oil production at breakthrough. From Fig. 3, the average oil column height below perforations h_{bp} is linearly related to the cumulative oil production N_p . Then, the cumulative oil production at breakthrough can be calculated from the breakthrough height h_{wb} :

$$(N_p)_{bt} = A\Phi(1 - S_{wc} - S_{or} - S_{gc})(h - h_{wp} - h_{ap} - h_p) \quad (6)$$

Table (3) compares the results obtained from the present research with those obtained from some other correlations and simulation. The present correlations show a good match of the critical rate and breakthrough time with the simulation results.

After calculation of S from Eq. 4, use Eq. 2 to calculate WOR for a vertical well. The results were compared with the simulation results. The comparison is shown in Fig. 4. The figures show that the present correlation shows a good match with the simulation results.

CONCLUSIONS

As presented in this study, the following items were achieved.

- Numerical method was used to study the water coning phenomenon in vertical wells.
- A sensitivity analysis was conducted to estimate the effects of the various reservoir rock and fluid properties on the average oil column height below perforations and slope.
- The developed empirical water coning correlations were derived based on three-dimensional simulation results to predict the critical rate, breakthrough time and WOR after breakthrough for vertical wells.

- The correlations were developed based on the regression analysis using the data from numerical simulations.
- The developed correlations show a good match of the critical rate, breakthrough time and WOR after breakthrough with the simulation results.

NOMENCLATURE

A	cross sectional area, ft ²
a ₀ - a ₁₀	correlation coefficients
b ₀ - b ₁₀	correlation coefficients
B _o	oil formation volume factor, bbl/STB
C	constant
h	initial oil formation thickness, ft
h _{ap}	oil column height above perforations, ft
h _p	perforation thickness, ft
h _{bp}	average oil column height below perforation, ft
h _{av}	height increase of oil water contact due to production, ft
r _w	wellbore radius, ft
h _{wb}	average oil column height below perforations at breakthrough, ft
k _h	horizontal permeability, md
K _v	vertical permeability, md
k _{rg}	gas relative permeability
k _{rw}	water relative permeability
k _{row}	oil relative permeability in oil-water system
k _{rog}	oil relative permeability in gas-oil-irreducible water system
N _p	cumulative oil production, STB
q _o	oil production rate, STB/d
P _{cog}	capillary pressure of gas-oil system, psi
P _{cow}	capillary pressure of water-oil system, psi
r _e	drainage radius, ft
S	slope of the after breakthrough straight line
S _o	oil saturation, fraction
S _g	gas saturation, fraction
S _w	water saturation, fraction
S _{wc}	connate water saturation, fraction
S _{or}	residual oil saturation, fraction
S _{gc}	Critical gas saturation, fraction
t _{bt}	breakthrough time, days

WOR	water-oil ratio
μ_o	oil viscosity, cp
μ_w	water viscosity, cp
ρ_o	oil density, lb/ft ³
ρ_w	water density, lb/ft ³
ϕ	porosity, fraction
$\Delta\gamma$	water-oil gravity difference, psi/ft

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Statistical Error Analysis

The following three statistical parameters were used in this study to evaluate the accuracy of the correlations.

- 1- Average percent relative error (ARE)

$$E_r = \frac{1}{n_d} \sum_1^{n_d} E_i$$

Where

$$E_i = \left(\frac{x_{measured} - x_{estimated}}{x_{measured}} \right)_i * 100 (i = 1, 2, \dots, n_d)$$

- 2- Average absolute percent relative error (AARE)

$$E_a = \frac{1}{n_d} \sum_1^{n_d} E_i$$

$$r^2 = 1 - \frac{\sum_1^{n_d} (x_{measured} - x_{estimated})^2}{\sum_1^{n_d} (x_{measured} - x_{average})^2}$$

- 3- Coefficient of correlation

The lower the value of E_r the more equally distributed are the errors between positive and negative values.

The lower value of E_a the better in the correlation.

The correlation coefficient describes the range of connection between two variables namely experimental and estimated values obtained from the correlation.

The value of r^2 varies from -1 to +1. As the value of correlation coefficient approaches +1, it means there is a strong positive relationship between these two variables.

Table 1. Relative permeability data

Sw	Krw	Pcow		Sg	Krg	Pcog		So	Krow	Krog
0.22	0	1		0	0	0		0	0	0
0.3	0.051	0.5		0.04	0	0		0.2	0	0
0.4	0.12	0.3		0.1	0.022	0		0.35	0	0.02
0.5	0.218	0.16		0.2	0.1	0		0.4	0.0048	0.038
0.6	0.352	0.1		0.3	0.195	0		0.45	0.029	0.058
0.7	0.5	0.05		0.4	0.289	0		0.5	0.0649	0.102
0.8	0.65	0.03		0.5	0.42	0		0.55	0.11298	0.163
0.9	0.83	0.01		0.6	0.58	0		0.6	0.197	0.234
1	1	0		0.7	0.8125	0		0.65	0.287	0.33
				0.78	1	0		0.7	0.4	0.454
								0.75	0.637	0.67
								0.78	1	1

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Table 2. Simulation input data and results

Parameters	qo	ϕ	Kh	Kv	h	hap	hp	μ_o	μ_w	$\Delta\gamma$	re	S	hwp
Base Case	3000	0.2	200	20	200	10	20	1.11	0.3	0.086	2000	-0.0403	161.8
1	2000											-0.0328	160.40
2	2500											-0.0398	160.90
3	3000											-0.0403	161.80
4	4000											-0.0405	163.20
5	5000											-0.0406	165.60
6		0.15										-0.0411	161.69
7		0.2										-0.0403	161.80
8		0.25										-0.0405	161.79
9		0.35										-0.0373	162.06
10		0.4										-0.0318	162.49
11			50									-0.1121	168.76
12			150									-0.0425	165.02
13			200									-0.0403	161.80
14			300									-0.0261	156.59
15			400									-0.0162	151.91
16				10								-0.0243	155.82
17				15								-0.0358	159.19
18				20								-0.0403	161.80
19				30								-0.0498	164.62
20				40								-0.0498	167.78
21					110							-0.04	79.70
22					200							-0.0403	161.80
23					290							-0.0267	253.30
24					380							-0.00176	343.00
25						10						-0.0403	161.80
26						20						-0.0398	152.71
27						30						-0.0406	143.53
28						40						-0.0401	134.86
29						50						-0.0419	125.87
30							10					-0.0283	179.48
31							20					-0.0403	161.80
32							30					-0.0411	152.15
33							40					-0.0404	142.78
34							50					-0.0421	133.00

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Table 2. Continued

35								1.11					-0.0403	161.80
36								1.61					-0.0505	164.56
37								2.11					-0.0491	166.63
38								3.11					-0.0689	168.00
39								4.11					-0.0833	168.47
40									0.2				-0.0481	163.75
41									0.3				-0.0403	161.80
42									0.4				-0.0356	160.19
43									0.5				-0.0330	158.49
44									0.7				-0.0264	156.34
45										0.017			-0.0406	164.13
46										0.052			-0.0405	162.80
47										0.086			-0.0403	161.80
48										0.121			-0.0395	161.00
49										0.156			-0.0385	160.30
50											1000		-0.0107	151.22
51											1500		-0.0331	156.02
52											2000		-0.0403	161.80
53											3000		-0.0708	166.01
54											4000		-0.1081	167.61

Table 3. Comparison with correlations

Correlation	qc, STB/D	Breakthrough time, day
Meyer and Garder	192.4	
Schols	298	
Chaperon	140	
Hoyland et al.	548	
Sobocinski and Cornelius		1596
Bournazel and Jeanson		697
Yang and Wattenbarger	1556	674
This Study	3231	343
Simulation	3000	350

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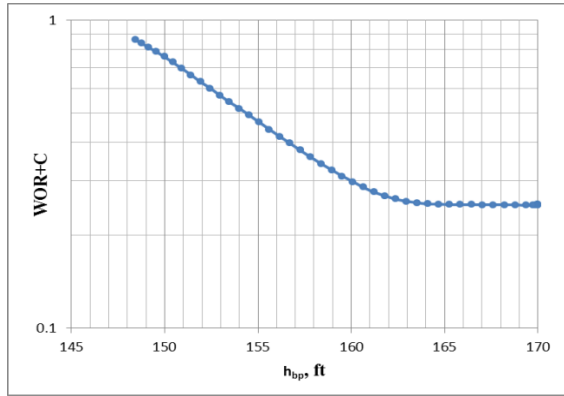


Fig. 1. WOR+C vs. oil column height below perforations from a simulation run

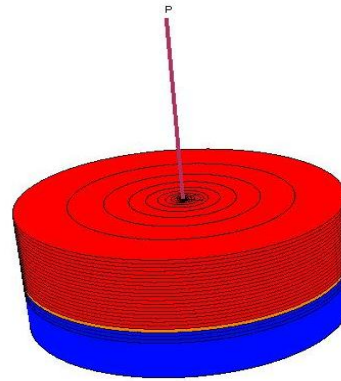


Fig. 2. Simulation grid for a vertical well

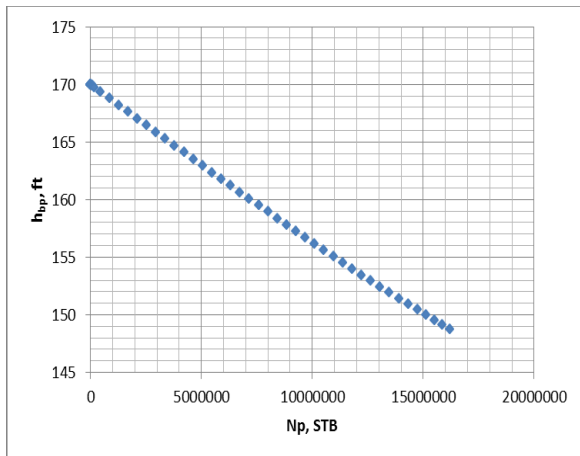


Fig. 3. The relationship between the average oil column height below perforations and the cumulative oil production

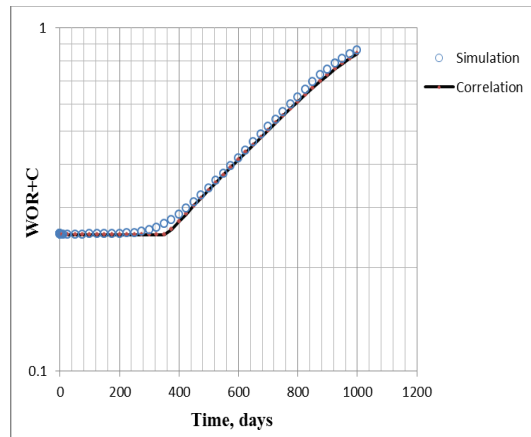


Fig. 4 - WOR+C comparison between simulation and the present correlation