PAPER • OPEN ACCESS

Simulation and modelling of flight missile dynamics and autopilot analysis

To cite this article: Mohamed M Kamel *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **610** 012033

IOP Conf. Series: Materials Science and Engineering **610** (2019) 012033 doi:10.1088/1757-899X/610/1/012033

Simulation and modelling of flight missile dynamics and autopilot analysis

Mohamed M Kamel[1,3], Karim M Ali[1], M A H Abozied[2] and Yehia Z Elhalwagy[2]

[1]Egyptian Armed Forces, Egypt
[2]Military Technical College, Egypt

[3]Email: m_mamdouh77@hotmail.com

Abstract. In the current paper, nonlinear modelling and dynamical analysis of a surface to surface missile with both aerodynamic and thrust vector control is investigated. Aerodynamic force and moment modelling of the presented missile includes the challenging high angle of attack aerodynamics behaviour and the so-called hybrid control, which utilizes both tail fins for stabilization and rudder vanes as control surfaces. Then, concerning relative rotation between bodies, the missile aerodynamic model is built by Leabedev aerodynamic calculations software. After using the generated aerodynamic data obtained the required aerodynamic stability derivatives, the required transfer functions are resolute based on the equations of motion of the missile. The missile dynamic motion in pitch and roll channel are analysed. The guidance and control strategy are developed through an actuation system and autopilot design in the four channels based on the poles assignment technique. The roll autopilot is constructed to regulate the roll angle of the front body of the missile which is the controlled part, for the command signals generated by the guidance laws both pitch and yaw autopilots are designed to realize. The guidance command is in the form of rate of change of flight path angles of the missile with time.

1. Introduction

Classification of guided missile depends on range whatever Ballistic or Cruise Missiles to Tactical and Strategic missiles. The tactical missiles depend on seeker in guidance and are used in short and medium range scenarios. On the contrary the tactical missiles, Strategic ones are travelled longer distances and designed to fire on stationary targets, which accurately position [1]. According to missions, the missiles can be classified into four subsets; Surface-to-Air Missiles (SAM), Air-to-Surface Missiles (ASM), Air-to-Air Missiles (AAM), and Surface-to-Surface Missiles (SSM).

In guided missiles problems, the guidance and control equations are solved in some steps including, navigation states, external disturbances, missile equation of motion in 6DOF, which commonly uses Euler or Quaternion methodologies to carry out the overall algorithm [2] and to obtain guidance commands by using methods of guidance and sensors like seekers [3]. And then make our controller that achieve system stability during flight time and finally, estimate the target motion in modelling stage. Guidance commands are built starting in design steps of controller to obey the guidance commands depend on dynamic model and then calculate kinematics of target [4] and [5]. Control algorithm here depends on Kalman filter and fading memory filter to state estimation. That's requirement to obtain relations moments and forces on missile and kinematics states. Some external sources of force like forces due to aerodynamics motion on control surfaces.

These effects produce components in three directions (down, cross range and altitude) change as well as yaw, pitch and roll. It's easier to model inertial force than aerodynamics forces because aerodynamics forces depend on both present and past values of kinematics parameters of the missile. The Maple-Synge analysis is the most used method in modelling aerodynamics of the symmetric missiles [6].in this method, there were functions between aerodynamic force and moment coefficients related to the kinematic flight parameters. For the propose of design autopilot these parameters express as a linear function of angle of attack (AOA), side-slip angle (SSA) and some analytical methods developed to predict aerodynamics forces and moments effect on missile under steady and unsteady manoeuvres condition [7].

The guided missile motion can be investigated utilizing one of two approaches: simplified dynamic methods and Dynamic methods. First, simplified dynamic methods continuously determine the missile velocity along the concrete trajectory. The common feature here is the neglecting of the missile rotary motion around its c.g. (the missile is considered as a mass point). These methods have two variants according to what extent the constriction limiting the motion of the guided missile is satisfied. Second, in the dynamic method apply with full consideration of dynamic properties of the guided missile and control system [7]. In these methods the whole set of equations describing the missile motion including the equations representing the deviations of control fins, called law of control, are used. The law of control is a set of equations determining the deviations of control fins in dependence on signals produced by the control system and determining the dependence of these signals on motion parameters of missile and target.

2. Guidance and Control System Computations

The performance of ballistic missile systems is measured through destruction capability (warhead), maximum range, and accuracy at impact. The maximum range and the accuracy at impact besides some other constraints are called guidance constraints. Guidance constraints might be achieved through suitable guidance and control strategy. It is design to; pre-compute the reference trajectory using the selected guidance law, generates the guidance commands, compensates for the predicted hit error and generates the engine shut-off command [8]. The reference trajectory is computed satisfying the guidance constraint, according to launch position, target position, and environmental data [7], [4].

The choice of the optimal guidance law is one of the particular problems in ballistic design and does not has an analytical solution [4]. Then, the missile is guided along the reference trajectory in the powered flight phase, such that it arrives the shut-off point with the required attitude and velocity. The guidance law has specified one of the reference trajectories; the boost stage guidance has to guide the missile with appropriate performance along the pre-computed reference trajectory through a reference demand. This guidance is based on the difference between the real position of the missile in space determined by inertial measuring unit (IMU) and navigation algorithm, and reference position. The guidance commands are sending to the autopilot over the flight path control [9]. During the sustain phase, the guidance system has to guide the missile along the reference trajectory, and correct for the expected booster-impulse tolerance according to the flight path correction algorithm [10].

The motion of the missile around its center of gravity controlled via the so-called short period dynamics control (autopilot) to achieve adequate stability and reasonable rapid and well damped response to input control demand, with moderate insensitivity to external disturbances [11]. Steering the missile in a way such that the actual trajectory follows the required trajectory (long period dynamics control) and the two criteria's may be designed via conventional control theories or modern control theories too [11]. The steering of the missile carried out using different approaches depending on the environmental conditions:

IOP Conf. Series: Materials Science and Engineering **610** (2019) 012033 doi:10.1088/1757-899X/610/1/012033

In the atmospheric phase, where the aerodynamic effects are appreciable and where the nominal trajectory is zero lift, or gravity turn, there are two approaches [12]:
Attitude steering: In which the attitude is programmed as a function of time and used to maneuver the missile in a way to keep it as close as possible to the nominal trajectory without causing excessive structural loading and in this case a considerable angle of attack is necessary.
Velocity steering: In which the flight path is controlled such that a predetermined velocity function is satisfied. In this case, smaller angle of attack is required, and trajectory perturbations due to missile perturbations are smaller.
In vacuum phase**,** during which yaw attitude is commanded with some function of velocity gained (difference between required velocity and measured velocity), pitch attitude may be commanded as in the atmospheric phase or as some function of velocity gained. In this phase the steering may be either velocity to be gained steering or cross product steering [12], [13].

As a conclusion, guidance and control strategy works for minimizing the errors between actual trajectory carried out by the missile and required trajectory (guidance constraints) in the powered flight with high accuracy at impact position or shut-off velocity.

3. Ballistic Trajectory

The ballistic missile moves in space under the effect of three forces: thrust force, aerodynamic force, and gravitational force. The profile of the ballistic trajectory from launch to impact point can be divided into three dependent phases of flight [7], [4]:

- Boost phase/powered flight

This phase is the most complex phase because of the exit through the atmosphere. This phase of flight includes short vertical rise, transition turn and gravity turn. The missile is usually launched vertically to clear the launching site and allow the launching transients to die out before starting the pitch program [4]. Thereafter, the missile has to achieve (transition turn) between the vertical rise and gravity turn to obtain optimum gravity turn that satisfies the guidance constraints imposed by other systems.

- Free (ballistic) flight phase

It begins when the thrust terminates and during which gravity is the only acting force. The free flight trajectory lies completely within a plane, which contains the center of the earth and has the shape of a conic section. It may be an ellipse, a parabola, or above escape velocity; the parabola is limiting case.

- Re-entry flight phase

During the re-entry phase, the missile re-enters the atmosphere facing the unstable conditions such as winds or atmospheric density variations that contribute to hitting errors. If terminal guidance is used, then these effects can be minimized or eliminated. From re-entry point to the point of height (60km), the flight path is nearly a straight line and the direction of the missile velocity remains nearly the same. From (60km-15km) height the flight path remains a straight line and the missile losses most of its kinetic energy and undergoes intense heating. The terminal phase start from (15km) up to impact and the missile departs from a straight line and begins to cool off [7].

4. Mathematical Model Simulation

For investigating the performance of the system the model is structured as a series of modules. These modules can be individually developed. These modules are described relative to one or too many basic coordinate systems namely: missile axes, and earth axes. The possibility of vector transformation to and from each coordinate system is introduced using directional cosine method [13]. The proposed model is uniformly valid for analysis, design, and development of Endo/exo atmospheric flight problems [14], [15]. In addition, the equations provided are of sufficient fidelity to enable many major engineering decisions to be made by analysis of simulated system performance. the numerical integration method (Runge-Kutta) is used for the simulation of the underlying system on the PC using

IOP Conf. Series: Materials Science and Engineering **610** (2019) 012033 doi:10.1088/1757-899X/610/1/012033

the result of simulation is the flight parameters including trajectory shape, missile velocity profile and different angular displacements. These results shown in Fig.1 are validated against real data and show a reasonable profile such that it could be considered nominal for subsequent analysis.

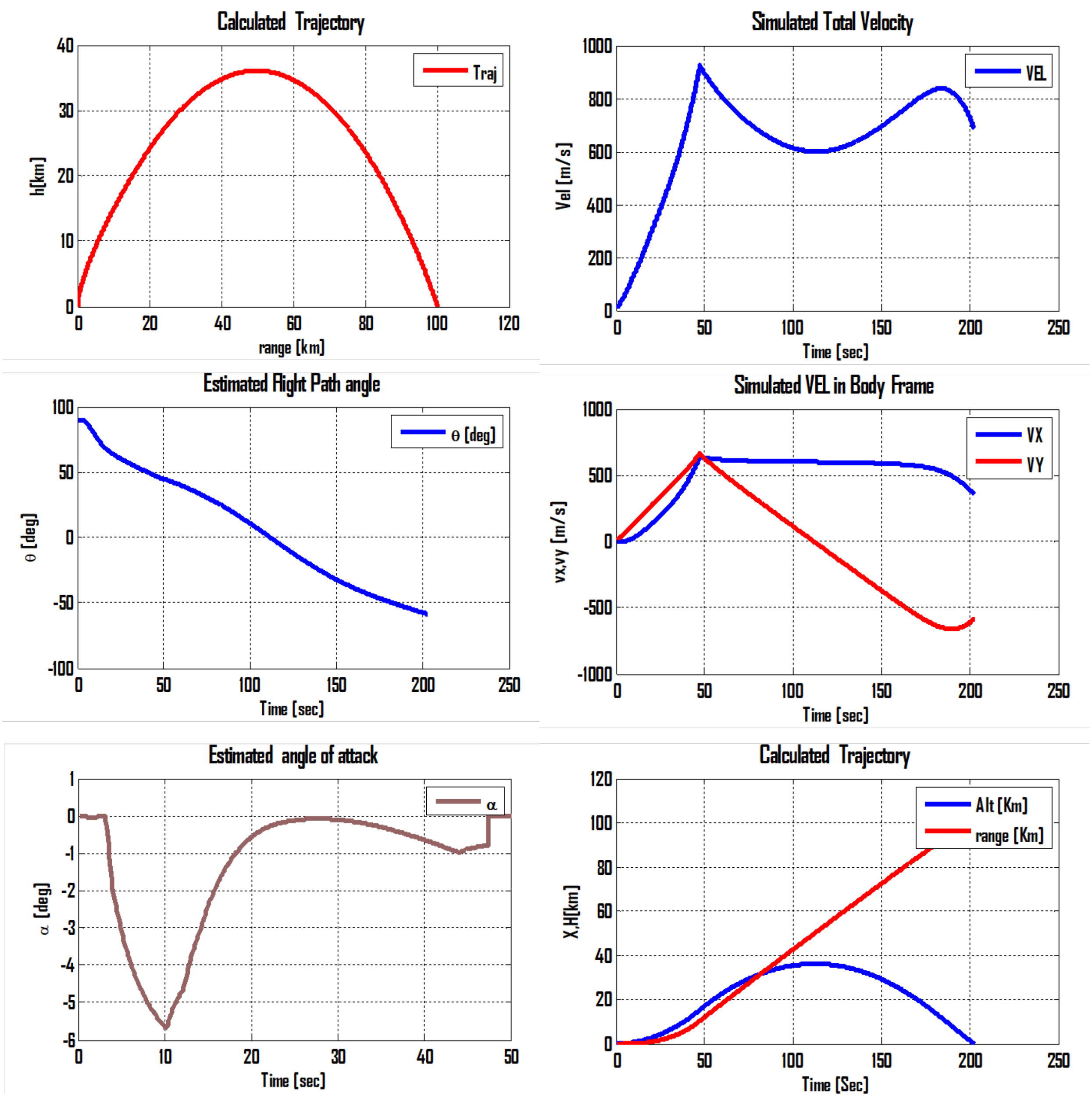

Figure 1. Missile Flight Parameters

5. Missile Pitch Dynamic Analysis

The missile maneuver is mainly carried out in the pitch plane, so the missile pitch dynamics developed for the analysis by using perturbation approach for the appropriate equation of motion of the missile [16]. The force along the longitudinal axis can be demonstrated in velocity coordinate system can be rewritten as follows [17] and [4]:

5.1 The axial force along x-axis in velocity coordinate

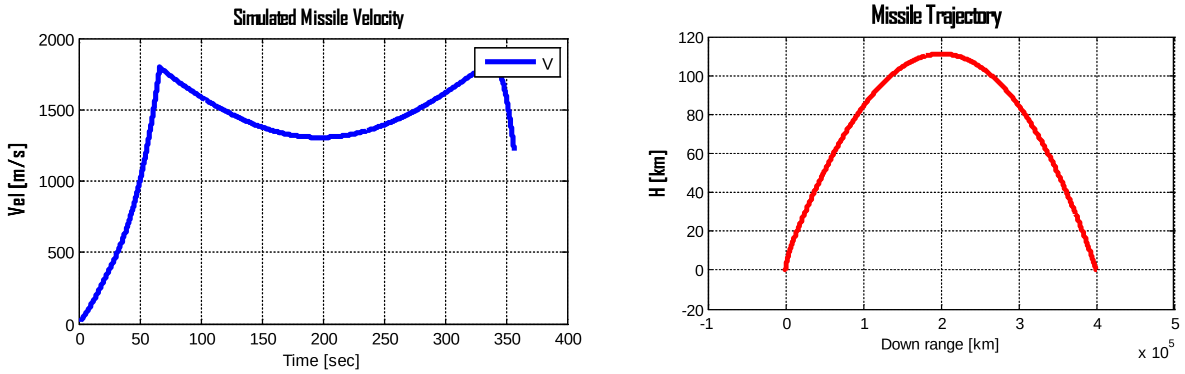

Figure 2 Missile Pitch Flight Parameters

$$F_x = P\cos\alpha\cos\beta + X_a - mg\sin\theta = m\dot{v}_M \tag{1}$$

$$X_a = -\frac{1}{2}\rho v_M^2 S_M (c_{xo} + c_c^{\alpha^2}\alpha^2) - c_{xru} q_{ru} S_{ru}\cos\alpha\cos\beta + 2c_{yru} q_{ru} S_{ru}\sin\alpha.\delta_p \tag{2}$$

Where,

$$\begin{aligned} \delta_p &= \delta_2 + \delta_4 \\ c_x &= c_{xo} + c_x^{\alpha^2}\alpha^2 \\ c_{xru} &= 4c_{xoru} + c_{xru}^{\delta^2}(\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2) \\ c_{yru} &= c_{yru}^{\delta} - c_{xoru} \end{aligned}$$

The perturbation of axial equation is given by:

$$m\frac{\partial \Delta v_M}{\partial t} = (P^{v_M} + X_a^{v_M})\Delta v_M + (X_a^{\alpha} - P.\alpha)\Delta\alpha - (mg\cos\theta)\Delta\theta + X_a^{\delta_p} + x_B \tag{3}$$

Where: $P^{v_M} = (\frac{\partial P}{\partial v_M})$; $X_a^{v_M} = (\frac{\partial X_a}{\partial v_M})$; $X_a^{\alpha} = (\frac{\partial X_a}{\partial \alpha})$; $X_a^{\delta_p} = (\frac{\partial X_a}{\partial \delta_p})$

Then, the force equation can be arranged as follows:

$$\Delta\dot{v}_M + a_{00}\Delta v_M + a_{02}\Delta\alpha + a_{04}\Delta\theta = a_{03}\Delta\delta_p + a_{05}x_B \tag{4}$$

Where:

$$a_{00} = \frac{1}{m}[(c_{xo} + \frac{\partial c_{xo}}{\partial M}.\frac{M}{2})\rho v_M S_M]$$
$$a_{02} = \frac{1}{m}[P\alpha + \rho v_M^2 S_M c_x^{\alpha^2}.\alpha - 2c_{yru} q_{ru} S_{ru}.\delta_p$$
$$a_{03} = \frac{1}{m}[2c_{yru} q_{ru} S_{ru}.\alpha]$$
$$a_{04} = g.\cos\theta$$
$$a_{05} = \frac{1}{m}$$

5.2 The lift force equation

$$F_y = P(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma) + Y_a\cos\gamma - Z_a\sin\gamma$$
$$- X_{ru}\sin\alpha + Y_{ru}\cos\alpha - mg\cos\theta = mv_M\dot{\theta} \quad (5)$$

Where γ is the roll angle in the velocity coordinate system and it has approximately zero value. Therefore, considering also, small values for the angle of attack and the side slips angle we can simplify the above equation as follows [7], [4]:

$$mv_M\dot{\theta} = P\alpha + Y_a - X_{ru}\alpha + Y_{ru} - mg\cos\theta \quad (6)$$

Applying the perturbation to the lift force equation yields:

$$mv_M\frac{\partial\Delta\dot{\theta}}{\partial t} = (P^{v_M}\alpha + Y_a^{v_M})\Delta v_M + (P - X_{ru} + Y_a^{\alpha})\Delta\alpha + mg\sin\theta\Delta\theta + Y_a^{\delta_p}\Delta\delta_p + y_B \quad (7)$$

Where,

$$X_{ru} = c_{xru}.q_{ru}$$
$$Y_a = c_y S_M q = \frac{1}{2}\rho v_M^2.c_y S_M$$
$$Z_a = c_z S_M q = \frac{1}{2}\rho v_M^2.c_z S_M$$

Then, equation (7) can be written as:

$$\Delta\dot{\theta} + a_{40}\Delta v_M + a_{42}\Delta\alpha + a_{44}\Delta\theta = a_{43}\Delta\delta_p + a_{45}y_B$$

(8)

Where,

$$a_{40} = -\frac{1}{mv_M}\rho S_M.\alpha(c_y^{\alpha} + \frac{\partial c_y^{\alpha}}{\partial M}\frac{M}{2}) \qquad a_{44} = \frac{-g\sin\theta}{v_M} \qquad a_{45} = \frac{1}{mv_M}$$

$$a_{42} = \frac{1}{mv_M}(c_{xru}q_{ru}S_{ru} - P - \frac{1}{2}\rho v_M^2 S_M c_y^{\alpha}) \qquad a_{43} = \frac{2c_{yru}^{\delta}}{mv_M}q_{ru}S_{ru}$$

IOP Conf. Series: Materials Science and Engineering **610** (2019) 012033 doi:10.1088/1757-899X/610/1/012033

5.3 The torque equation along lateral axis

$$\ddot{\theta}=[\frac{2c_{yru}^{\delta}.\phi_T q_{ru} S_{ru}(x_{cpru}-x_{cg})}{Iz}]\delta_p+\frac{\rho v_M^2 S_M D_M}{2}[c_y^{\alpha}\frac{x_{cg}-x_{cp}}{Iz.D_M}.\alpha+(\frac{A_z^{\omega z}x_{cg}^2+B_z^{\omega z}x_{cg}+C_z^{\omega z}}{Iz.v_M})] \tag{9}$$

The perturbation of torque equation can be obtained as follows [17]:

$$\Delta\ddot{\vartheta}=M_z^{v_M}\Delta v_M+M_z^{\alpha}\Delta\alpha+M_z^{\omega z}\Delta\dot{\vartheta}+M_z^{\delta_p}\Delta\delta_p+M_B \tag{10}$$
$$\Delta\ddot{\vartheta}=\Delta\ddot{\theta}+\Delta\alpha$$

$$\Delta\dot{\vartheta}=\Delta\dot{\theta}+\Delta\dot{\alpha} \tag{11}$$
$$\Delta\ddot{\vartheta}+\Delta\ddot{\alpha}+a_{10}\Delta v_M+a_{11}\Delta\dot{\alpha}+a_{12}\Delta\alpha+a_{11}\Delta\dot{\theta}=a_{13}\Delta\delta_p$$

Where,

$$a_{11}=\frac{-\rho v_M S_M}{I_z}(A_z^{\omega z}x_{cg}^2+B_z^{\omega z}x_{cg}+C_z^{\omega z})$$

$$a_{12}=\frac{-\rho v_M S_M c_y^{\alpha}}{I_z}(x_{cg}-x_{cp})\ ;\quad a_{13}=\frac{2c_{yru}^{\delta}q_{ru}S_{ru}}{I_z}(x_{cpru}-x_{cg})$$

$$a_{10}=\frac{-\rho v_M S_M}{I_z}[c_y^{\alpha}(x_{cg}-x_{cp})\alpha+(x_{cg}-x_{cp})\alpha\frac{\partial c_y^{\alpha}}{\partial M}\frac{M}{2}+\frac{\dot{\vartheta}D_M^2}{2v_M}(A_z^{\omega z}x_{cg}^2+B_z^{\omega z}x_{cg}+C_z^{\omega z})]$$

The missile pitch plane transfer function can be obtained by solving the following three differential equations [7]:

$$\begin{aligned}
&\Delta\dot{v}_M+a_{00}\Delta v_M+a_{02}\Delta\alpha+a_{04}\Delta\theta &&=a_{03}\Delta\delta_p\\
&\Delta\dot{\theta}+a_{40}\Delta v_M+a_{42}\Delta\alpha+a_{44}\Delta\theta &&=a_{43}\Delta\delta_p\\
&\Delta\ddot{\theta}+a_{10}\Delta v_M+\Delta\ddot{\alpha}+a_{11}\Delta\dot{\alpha}+a_{12}\Delta\alpha+a_{11}\Delta\dot{\theta} &&=a_{13}\Delta\delta_p
\end{aligned} \tag{12}$$

The Laplace transform of these equations yields the following matrix equation:

$$\begin{bmatrix} s+a_{00} & a_{02} & a_{04}\\ a_{40} & a_{42} & s+a_{44}\\ a_{10} & s^2+a_{11}+a_{12} & s^2+a_{11}s\end{bmatrix}\begin{bmatrix}\Delta v_M\\ \Delta\alpha\\ \Delta\theta\end{bmatrix}=\begin{bmatrix}a_{03}\\ a_{43}\\ a_{13}\end{bmatrix}\delta_p \tag{13}$$

The determinant of this matrix can be obtained as follows:

$$\Delta=s^4+A_1s^3+A_2s^2+A_3s+A_4 \tag{14}$$

A_1, A_2, A_3 and A_4 are functions of different coefficients. The transfer function can be simplified by considering the following relation:

$$\begin{aligned}\Delta\theta&=\Delta\vartheta-\Delta\alpha\\ \Delta\dot{\theta}&=\Delta\dot{\vartheta}-\Delta\dot{\alpha}\end{aligned} \tag{15}$$

Then equation (12) becomes:

$$\begin{aligned} &\Delta\dot{\vartheta} - \Delta\dot{\alpha} + a_{40}\Delta v_M + a_{42}\Delta\alpha - a_{44}\Delta\vartheta - a_{44}\Delta\alpha = a_{43}\Delta\delta_p \\ &\Delta\ddot{\vartheta} + a_{10}\Delta v_M + a_{11}\Delta\dot{\vartheta} + a_{12}\Delta\alpha = \mathrm{a}_{13}\Delta\delta_p \end{aligned} \tag{16}$$

The simulation of missile dynamics in pitch plane at different times during the guided phase results in different parameters as shown in Table 1:

$$\begin{bmatrix} -s + a_{42} - a_{44} & s + a_{44} \\ a_{12} & s^2 + a_{11}s \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta\vartheta \end{bmatrix} = \begin{bmatrix} a_{43} \\ a_{13} \end{bmatrix} \Delta\delta_p \tag{17}$$

The solution of this matrix equation is given by:

$$\begin{bmatrix} \Delta\alpha \\ \Delta\vartheta \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} s^2 + a_{11}s & -s - a_{44} \\ -a_{12} & -s + a_{42} - a_{44} \end{bmatrix} \begin{bmatrix} a_{43} \\ a_{13} \end{bmatrix} \Delta\delta_p \tag{18}$$

The transfer function of the missile dynamics in pitch can be written as [14]:

$$w_\delta^\vartheta(s) = \frac{k_\delta^\vartheta(1 + T_1 s)}{s^3 + A_2 s^2 + A_3 s + A_4} = \frac{k_\delta^\vartheta(1 + T_1 s)}{(T_2^2 s^2 + 2\xi T_2 s + 1)(\tau s + 1)} \tag{19}$$

Where,

$$T_2 = \frac{1}{\sqrt{a_{12} + a_{11}(a_{42} - a_{44})}} \quad ; \quad \xi = \frac{\mathrm{a}_{42} + a_{11} - a_{44}}{2}.T_2 \quad ; \quad \tau = \frac{\mathrm{a}_{12} + a_{11}(a_{42} - a_{44})}{-a_{12}a_{44}}$$

6. Missile roll dynamics

The roll autopilot functions with actuation systems to control the roll dynamics and system stabilization of the missile. So, it is important to analyze the missile roll dynamics.
The missile roll dynamics transfer function can be calculated as follows:

$$\mathrm{I}_x\, \ddot{\gamma} = M_x \tag{20}$$

$$\omega_x = \dot{\gamma} \tag{21}$$

$$M_x = m_x^{\omega_x}.\omega_x + m_x^\delta.\delta \tag{22}$$

Table 1. Missile Dynamic Flight Coefficients

Time[sec]	A_4	A_3	A_2	$T_1 k_\delta^\vartheta$	τ	ξ	T_2
2.000	-0.02113	0.06992	0.57307	2.47998	-7.72523	0.81631	2.45429
20.000	-0.05013	8.10706	0.65946	2.55422	-34.23169	0.09977	0.35127
30.000	-0.02128	11.95629	0.69055	2.65816	-58.61643	0.09156	0.28918
40.000	-0.00265	6.33166	0.26757	2.86391	-93.6548	0.112676	0.51384
50.000	-0.00063	0.06738	0.26757	2.89262	-163.5631	0.08621	0.84312
60.000	-0.0200	1.1109	0.1217	2.49546	-218.9254	0.04174	1.49978

$$\ddot{\gamma} = \frac{m_x^{\omega_x}}{\mathrm{I}_x}\dot{\gamma} + \frac{m_x^{\delta}}{\mathrm{I}_x}.\delta = c_1\dot{\gamma} + c_2.\delta \tag{23}$$

$$\ddot{\gamma} - c_1\dot{\gamma} = c_2.\delta \tag{24}$$

$$T_\gamma\ddot{\gamma} + \dot{\gamma} = \frac{c_2}{T_\gamma}.\delta \tag{25}$$

where M_x is the total moment acting on the missile along x-axis, $m_x^{\omega_x}$ is the rolling moment coefficient due to aerodynamic force, m_x^{δ} is the rolling moment coefficient due to actuator aerodynamic force, and δ are the deflection angles from pitch and yaw actuators $(\delta_{2,4};\delta_{1,3})$.
From (25) the transfer function of missile roll channel dynamics can be written as:

$$w_\delta^\gamma = \frac{\gamma(s)}{\delta(s)} = \frac{k_\delta^\gamma}{s(T_\gamma s+1)} = \frac{k_{99}}{s(T_9 s+1)} \tag{26}$$

Where,

$$k_\delta^\gamma = k_{99} = \frac{c_2}{c_1} \ ; \ T_\gamma = T_9 = -\frac{1}{c_1}$$

An open-loop simulation is performed, which means no autopilot is considered [7], [18]. The flight condition selected is at the time of flight, velocity, altitude, Mach, angle of attack, and at the time of flight. The simulation analysis of missile dynamics in pitch and roll planes during the powered phase results in different parameters as shown in Table 2 and Figure 2.

Table 2. Missile Pitch & Roll dynamics parameters without autopilot

	Pitch Channel		Roll Channel	
Flight time [sec]	30	60	30	60
Damping ratio (ξ)	0.1000	.0417	1.0000	1.0000
Gain margin [dB]	-27.994	-47.981	65.866	35.000
Phase margin [deg]	65.886	35.000	4.6055	.67592
Natural frequency[r/s]	3.5925	1.4034	4.7785	6.2772

The parameters of system performance at different times can be obtained using MATLAB control toolbox as shown in Figure 3. From the result it is clear that the missile dynamic pitch and roll channels are unstable.

IOP Conf. Series: Materials Science and Engineering **610** (2019) 012033 doi:10.1088/1757-899X/610/1/012033

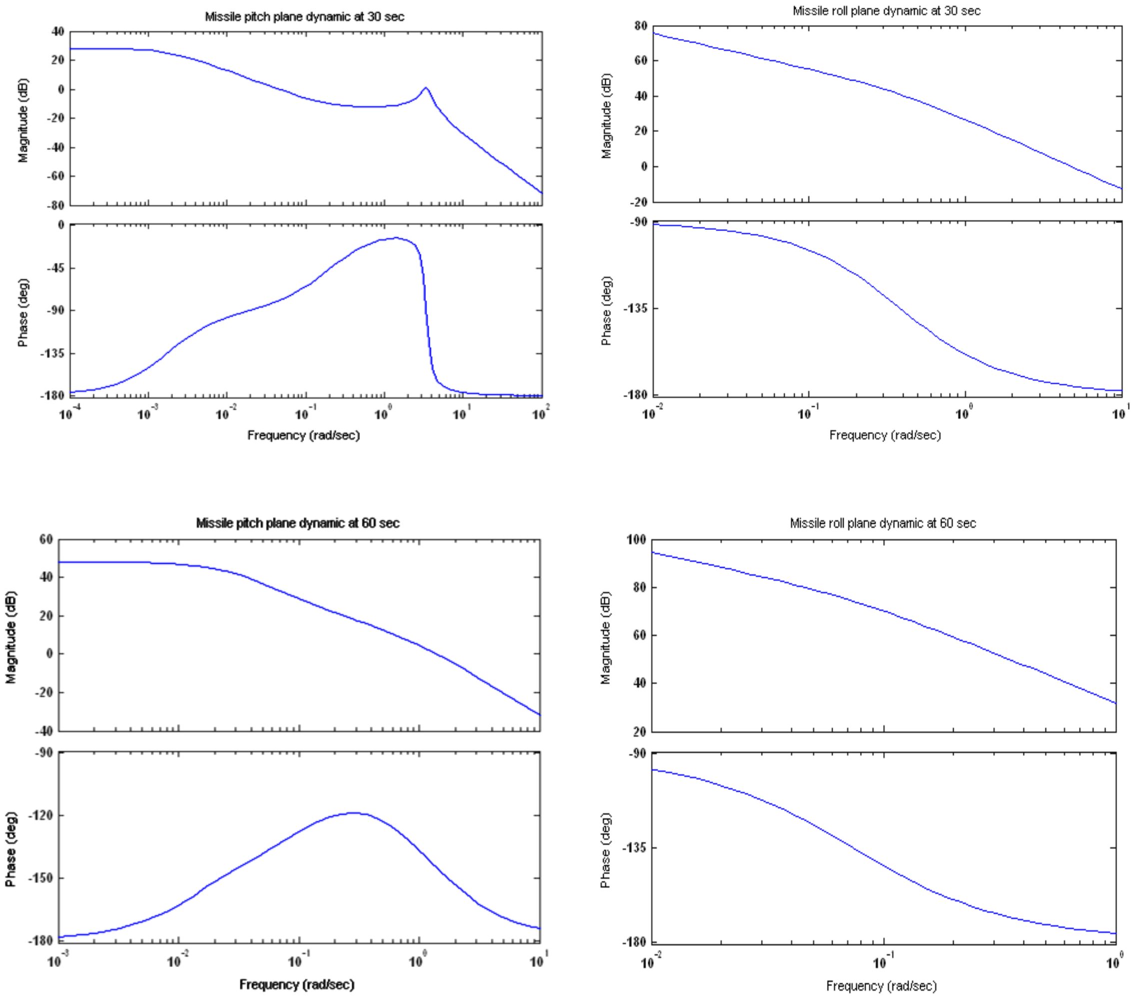

Figure 3. Missile Frequency Response for Pitch and Roll Dynamics

7. Conclusion

This paper presented the mathematical modelling of a particular missile system including, the reference frames and coordinate transformation to overcome great difficulty and avoid often occurring confusion and mistakes. These coordinate transformations and their characteristics appeared as very important tool in derivation of the equation of motion of flight vehicle and missiles, complete equations of missile were rigorously and systematically derived. In the establishment of mathematical model in this paper a variety of factors are taken into account, such as: Action of missile engine/motor's thrust, aerodynamic forces and moments acted on missile body, gravitational force of the earth, effect of surface and rotation of the earth, weight data, c.g and rotational motion of the missile.

The results obtained show the profile of different flight parameters. The analysis carried out in this paper, indicates that the missile manoeuvre is mainly restricted in the pitch plane and consequently the analysis is performed for the pitch and roll dynamics. The results show that, the missile dynamic motion in pitch and roll are unstable. Generally, the autopilot must be designed to control the dynamic motion of the missile around its centre of gravity and to achieve adequate stability and reasonable rapid and well damped response for the control demand.

IOP Conf. Series: Materials Science and Engineering **610** (2019) 012033 doi:10.1088/1757-899X/610/1/012033

8. References

[1] Shu He L 2012 *Ballistic Missile and Launch Vehicle Design* (China: Beihang University-School of space technology)
[2] Hendy H, Rui X and Khalil M 2013 *An Integrated GPS / INS Navigation System for land vehicle* Applied Mechanics and Materials ***336*** *221–226*
[3] Greensite A L 1970 *Analysis and Design of Space Vehicle Flight Control Systems* Spartan Book New York
[4] Arafa I I 2004 *Analysis and Design of Guidance Navigation and Control System for Missile Applications* (China: Beihang University) Ph.D Thesis
[5] Yanwang, X. 1999 *Design and Analysis on Control System of the Ballistic Missile and Launch Vehicle* **421** 63-65 Beijing China
[6] AKGÜL A and KARASOY S 2005 *Development of A Tactical Ballistic Missile Trajectory Prediction Tool* Istanbul
[7] Arafa I I 1998 *Guidance algorithms in present of uncertainties* M.sc Thesis, Egypt
[8] Berglund E 2000 *Guidance and Control Technology* RTO SCI Lecture Series on Technologies for Future Precision Strike Missile Systems Atlanta USA pp 1-10
[9] Sharma M and Ward D G 2002 *Flight-Path Angle Control via Neuro- Adaptive Backstepping* American Institute of Aeronautics and Astronautics USA
[10] Elbeltagy A M, Bayoumy Ali A M, Youssef A and Elhalwagy Y Z 2016 *Modeling and simulation of spacecraft pointing modes using quaternion-based nonlinear control laws* AIAA Modeling and Simulation Technologies Conference
[11] Friedland B 1987 *Control System Design, An Introduction to State-Space Methods* (New York: McGraw-Hill)
[12] Elleithy M A, Tantawy M O and Ghoniemy M E 1989 *Development of Six-Degrees of Freedom Model for Surface to Surface Strap Down Ballistic Missile* GC-5 ASAT-3 M.T.C Egypt
[13] Etkin B 1972 *Dynamics of Atmospheric Flight* (John Wiely and Sons Inc.)
[14] Tanrıkulu Ö 1999 *Non-Linear Flight Dynamics of Unguided Missiles* (Türkiye: Middle East Technical University) Ph.D Thesis
[15] Ahmed A H, Ouda A N, Kamel A M and Elhalwagy Y Z 2017 *Attitude stabilization and altitude control of Quadrotor* 12th International Computer Engineering Conference ICENCO Boundless Smart Societies
[16] Xiao Y 2005 *Rocket Ballistic And Dynamics* (China: Beijing)
[17] George M S 2004 *Missile Guidance and Control Systems* Springer Verlag New York
[18] Haytham A, Elhalwagy Y Z, Wassal A and Darwish N M 2015 *Modeling and simulation of four-wheel steering unmanned ground vehicles using a PID controller* 2nd International Conference on Engineering and Technology ICET