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SUPPRESS THE VIBRATION OF A NONLINEAR DYNAMICAL SYSTEM SUBJECTED TO EXTERNAL FORCE USING A POSITIVE POSITION FEEDBACK CONTROL

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ABSTRACT

Vibration suppression using a positive position feedback (PPF) control for a nonlinear dynamical system which subjected to an external force is studied. The proposed model is the vertical dynamic excitation of structures induced by a single pedestrian walking along a straight path on flat and relatively stiff surfaces. The multiple scale perturbation technique was applied to derive the first order approximate solution of the system. The response equation and the stability criteria for the system were derived near the simultaneous primary and internal resonance cases. MATLAB 14.0 have been used for the numerical studying to show the time history of the main system with and without PPF controller. Also, the effect of the system parameters on the response system have been studied. A comparison between the approximate and numerical solutions is illustrated and it show a good agreement between them. It is found that (PPF) controller is very suitable for small natural frequency dynamical systems subjected to primary resonance excitations.

Keywords: Positive position feedback controller; multiple scale perturbation method; response equation; primary resonance case; external force.

1. Introduction

Studying the dynamical systems and suppressing the high amplitude of the resulted vibration attractive many researchers. There exists more one strategy to suppress the vibration of the dynamical systems, one of them is the active linear absorber based on positive position feedback (PPF) control. The PPF control technique is applied by Jun [1]. It is demonstrated that this strategy is effective in suppressing the high amplitude response of a flexible beam subjected to a primary external excitation. Wang and Inman [2] used four conventional controller and their versions of hybrid bang_bang control (on_off control) respectively. PPF controller was one of the used controllers for vibration suppression and compared in terms of their energy consumption.

Cazzulani et al. [3] presented a technique of an active modal tuned mass damper (AMTMD). The technique is compared with the PPF control which already presented in the literature. They resulted that AMTMD achieves the same performances of PPF around the resonances, without increasing the low frequency response. An experimental study is conducted by Orszulik and Shan [4] to show that a PPF controller can be used to suppress the vibrations of a structure with unknown natural frequencies. The PPF controller is applied on the structure which of a flexible manipulator with a collocated piezoelectric sensor/actuator pair.

The active vibration control of clamp beams is investigated by Shin et al. [5]. They used multiple of PPF controllers with a sensor/moment pair actuator to overcome the problems of the instability. It is illustrated numerically and experimentally that the vibration levels are reduced at the tuned modes of the PPF controllers. Mitura et al. [6] presented nonlinear horizontal and vertical beam models. Numerical results show that the PPF controller is a very good absorber for reducing the vibration of the vertical beam for low level of excitation amplitudes. El-Ganaini et al. [7] studied a nonlinear dynamic model which subjected to external primary resonance excitation. They used the PPF controller to suppress the vibration amplitude of the system. An approximate solution is obtained by applying the multiple scales perturbation technique (MSPT).

Jung et al. applied the PPF control [8] to reduce the vibration of the proposed model. Experimentally, the PPF controller have shown that more effective than the proportional integral derivative (PID) controller. The basic procedures for the modeling and simulation of a smart beam were presented by Ghareeb and Schmidt [9]. More than one controller were applied, one of them is the PPF controller. They illustrated the effect of the PPF controller on the amplitude of the peak displacement of the smart beam and its magnitude. Omidi and Mahmoodi [10] introduced a method of a nonlinear integral positive position feedback (NIPPF) that benefits from the advantages of both integral resonant control (IRC) and PPF control. The results of the study investigated that the NIPPF controller is effectiveness in suppression the nonlinear vibration. They used the multiple scales method to get the approximate solution for the system.

A cantilever beam subjected to random base excitation is studied by Kaushik et al. [11]. The piezoelectric patches and actuation are used as smart materials. The PPF controller is applied to control the vibrations at all the resonant modes using piezoelectric sensor voltage feedback. The vibrating system with cubic nonlinearities and external excitations is presented by EL-Sayed and Bauomy [12]. They used two PPF controller to reduce the vertical vibration of the system. MSPT was applied to the equations of the system to find approximate analytical solutions. Amer et al. [13] investigated the effect of the PPF controller in suppressing the vibration of a micro-electromechanical (MEMS) resonator. They used the multiple scales method to obtain the first order approximate solution.

The techniques of PPF and negative derivative feedback (NDF) controllers was applied by Syed [14] on a single link flexible manipulator featuring piezoelectric actuator. Based on the particular studied system, the comparison between the two controllers concluded that NDF controller is overall more effective in suppressing vibration than PPF controller. A nonlinear magnetic levitation system was studied by EL-Ganaini [15]. The horizontal vibration of the system is suppressed by applying the PPF controller. MSPT was applied to derive an approximate solution of the system which described by a four first order differential equations. A strategy of a compensated positive position feedback (CPPF) for active control of flexible structures with piezoelectric actuators was presented by Wu et al. [16]. The advantages of the proposed CPPF strategy confirmed by the simulation results compared with the conventional PPF methodology.

Enríquez-Zárate et al. [17] investigated that it could be using the PPF control to reduce the vibrations in the building-like structure. The numerical results illustrated that the optimized PPF control was effective in reducing the vibrations and lateral displacements of the building-like structure by around 97%. Yaghoub and Jamalabadi [18] studied suppression of the mechanical oscillations of the galloping system using the PPF control. The results show that the PPF controller is a powerful method to decrease the galloping amplitude of the D-shaped prism. Kumar et al. [19] presented a model of the vertical dynamic excitation of structures induced by a single pedestrian walking along a straight path on flat and relatively stiff surfaces. Amer et al. [20] studied a Duffing oscillator system subjected to harmonic force. They applied three different control methods; Positive Position Feedback (PPF), Integral Resonance Control (IRC) and Nonlinear Integrated Positive Position Feedback (NIPPF).The numerical comparison between the three different control methods illustrated that NIPPF controller is the best for reducing vibration at a high rate and after a short time. The nonlinear dynamic vibrations of a composite plate with square and cubic nonlinear terms subjected to external and parametric excitations are studied by Bauomy and EL-Sayed [21]. They applied Galerkin procedure to convert the nonlinear partial differential equation of motion into a nonlinear ordinary differential equation. PPF control is used to reduce the amount of vibration produced by the system. It is found that PPF control is better than Nonlinear Saturation Controller (NSC) as presented in a comparison which has been made between them. Also, there are many numerical methods for solving differential equations [22, 23].

In this study the effect of applying the PPF control is illustrated for suppressing the vibration of the system which described by (Eqn. 1) and reported in [19].

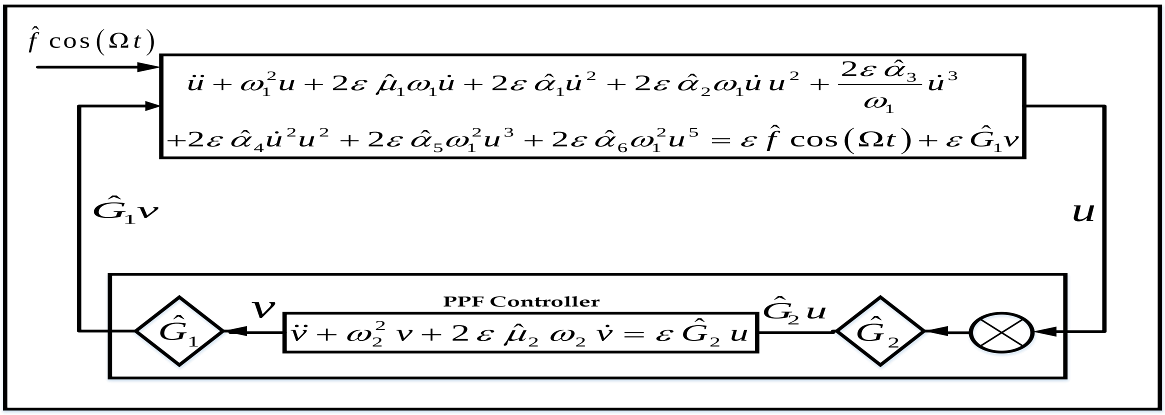


Fig. 1. Block diagram of the system

2. Formulation of the problem

The system under studying is proposed to be subjected to an external force and is expressed by the following equation:

$$\ddot{u}+\omega_1^2 u+2\mu_1\omega_1\dot{u}+2\alpha_1\dot{u}^2+2\alpha_2\omega_1\dot{u}u^2+\frac{2\alpha_3}{\omega_1}\dot{u}^3+2\alpha_4\dot{u}^2u^2+2\alpha_5\omega_1^2u^3+2\alpha_6\omega_1^2u^5=f\cos(\Omega t) \quad (1)$$

The system is effected by a PPF controller. The system is introduced by:

$$\ddot{u}+\omega_1^2 u+2\mu_1\omega_1\dot{u}+2\alpha_1\dot{u}^2+2\alpha_2\omega_1\dot{u}u^2+\frac{2\alpha_3}{\omega_1}\dot{u}^3+2\alpha_4\dot{u}^2u^2+2\alpha_5\omega_1^2u^3+2\alpha_6\omega_1^2u^5$$

$$=f\cos(\Omega t)+G_1 v \quad (2)$$

$$\ddot{v}+\omega_2^2 v+2\mu_2\omega_2\dot{v}=G_2 u \quad (3)$$

The values of the system is considered to be:

$$\mu_i=\varepsilon\hat{\mu}_i,\ G_i=\varepsilon\hat{G}_i,\ \alpha_n=\varepsilon\hat{\alpha}_n,\ f=\varepsilon\hat{f},\ (i=1,2;\ n=1,2,3,4,5,6) \quad (4)$$

Equation of the system becomes:

$$\ddot{u}+\omega_1^2 u+2\varepsilon\hat{\mu}_1\omega_1\dot{u}+2\varepsilon\hat{\alpha}_1\dot{u}^2+2\varepsilon\hat{\alpha}_2\omega_1\dot{u}u^2+\frac{2\varepsilon\hat{\alpha}_3}{\omega_1}\dot{u}^3+2\varepsilon\hat{\alpha}_4\dot{u}^2u^2+2\varepsilon\hat{\alpha}_5\omega_1^2u^3+2\varepsilon\hat{\alpha}_6\omega_1^2u^5=$$

$$\varepsilon\hat{f}\cos(\Omega t)+\varepsilon\hat{G}_1 v \quad (5)$$

$$\ddot{v}+\omega_2^2 v+2\varepsilon\hat{\mu}_2\omega_2\dot{v}=\varepsilon\hat{G}_2 u \quad (6)$$

3. Multiple scale perturbation technique

Applying the multiple scale perturbation method:

$$u(t,\varepsilon)=u_0(T_0,T_1)+\varepsilon u_1(T_0,T_1)+O(\varepsilon^2) \quad (7)$$

$$v(t,\varepsilon)=v_0(T_0,T_1)+\varepsilon v_1(T_0,T_1)+O(\varepsilon^2) \quad (8)$$

where $T_0 = t$, $T_1 = \varepsilon t$ (9)

The relations between the differential operators defined as follows:

$$\frac{d}{dt} = D_0 + \varepsilon D_1, \quad \frac{d^2}{dt^2} = D_0^2 + \varepsilon\left(2D_0 D_1\right) + \varepsilon^2 D_1^2 \tag{10}$$

where $D_0 = \frac{\partial}{\partial T_0}, \quad D_1 = \frac{\partial}{\partial T_1}$ (11)

Substituting equations (7) and (8) into equations (5) and (6) with using the relations (10) and equating the like order of ε , the following equations are obtained:

ε^0 :

$$\left(D_0^2 + \omega_1^2\right)u_0 = 0 \tag{12}$$

$$\left(D_0^2 + \omega_2^2\right)v_0 = 0 \tag{13}$$

$$\varepsilon^1 : \left(D_0^2 + \omega_1^2\right)u_1 = -2D_0 D_1 u_0 - 2\hat{\mu}_1\omega_1\left(D_0 u_0\right) - 2\hat{\alpha}_1\left(D_0 u_0\right)^2 - 2\hat{\alpha}_2\omega_1 u_0^2\left(D_0 u_0\right) - \frac{2\hat{\alpha}_3}{\omega_1}\left(D_0 u_0\right)^3$$

$$-2\hat{\alpha}_4 u_0^2\left(D_0 u_0\right)^2 - 2\hat{\alpha}_5\omega_1^2 u_0^3 - 2\hat{\alpha}_6\omega_1^2 u_0^5 + \hat{f}\cos\left(\Omega t\right) + \hat{G}_1 v_0 \tag{14}$$

$$\left(D_0^2 + \omega_2^2\right)v_1 = -2D_0 D_1 v_0 - 2\hat{\mu}_2\omega_2\left(D_0 v_0\right) + \hat{G}_2 u_0 \tag{15}$$

The general solution of equations (12) and (13) can be expressed in the form:

$$u_0 = A\left(T_1\right)e^{i\omega_1 T_0} + \bar{A}\left(T_1\right)e^{-i\omega_1 T_0} \tag{16}$$

$$v_0 = B\left(T_1\right)e^{i\omega_2 T_0} + \bar{B}\left(T_1\right)e^{-i\omega_2 T_0} \tag{17}$$

Substituting equations (16) and (17) into equations (14) and (15):

$$\begin{aligned}\left(D_0^2 + \omega_1^2\right)u_1 = &-\left[2i\omega_1\left(D_1 A\right) + 2i\hat{\mu}_1\omega_1^2 A + 2i\left(\hat{\alpha}_2 + 3\hat{\alpha}_3 - 3i\hat{\alpha}_5\right)\omega_1^2 A^2\bar{A} + 20\hat{\alpha}_6\omega_1^2 A^3\bar{A}^2\right]e^{i\omega_1 T_0}\\ &+\left[2\hat{\alpha}_1\omega_1^2 A^2\right]e^{2i\omega_1 T_0} - \left[2i\left(\hat{\alpha}_2 - \hat{\alpha}_3 - i\hat{\alpha}_5\right)\omega_1^2 A^3 + 10\hat{\alpha}_6\omega_1^2 A^4\bar{A}\right]e^{3i\omega_1 T_0} + \left[2\hat{\alpha}_4\omega_1^2 A^4\right]e^{4i\omega_1 T_0}\\ &-\left[2\hat{\alpha}_6\omega_1^2 A^5\right]e^{5i\omega_1 T_0} + \left[\hat{G}_1 B\right]e^{i\omega_2 T_0} + \left[\frac{\hat{f}}{2}\right]e^{i\Omega T_0} - 2\omega_1^2\left[\hat{\alpha}_1 A\bar{A} + \hat{\alpha}_4 A^2\bar{A}^2\right] + cc.\end{aligned} \tag{18}$$

$$\left(D_0^2 + \omega_2^2\right)v_1 = -\left[2i\omega_2\left(D_1 B\right) + 2i\hat{\mu}_2\omega_2^2 B\right]e^{i\omega_2 T_0} + \left[\hat{G}_2 A\right]e^{i\omega_1 T_0} \tag{19}$$

The general solution of eqns. (18) and (19) can be expressed as follows:

$$\begin{aligned}u_1 = &-\left[\frac{2\hat{\alpha}_1 A^2}{3}\right]e^{2i\omega_1 T_0} + \left[\frac{i\left(\hat{\alpha}_2 - \hat{\alpha}_3 - i\hat{\alpha}_5\right)A^3 + 5\hat{\alpha}_6 A^4\bar{A}}{4}\right]e^{3i\omega_1 T_0} - \left[\frac{2\hat{\alpha}_4 A^4}{15}\right]e^{4i\omega_1 T_0} + \left[\frac{\hat{\alpha}_6 A^5}{12}\right]e^{5i\omega_1 T_0}\\ &+\left[\frac{\hat{G}_1 B}{\left(\omega_1^2 - \omega_2^2\right)}\right]e^{i\omega_2 T_0} + \left[\frac{\hat{f}}{2\left(\omega_1^2 - \Omega^2\right)}\right]e^{i\Omega T_0} - 2\left[\hat{\alpha}_1 A\bar{A} + \hat{\alpha}_4 A^2\bar{A}^2\right] + cc.\end{aligned} \tag{20}$$

$$v_1 = -\left[\frac{\hat{G}_2 A}{\left(\omega_1^2 - \omega_2^2\right)}\right] e^{i\omega_1 T_0} + cc. \tag{21}$$

Studying the system at the simultaneous resonance case of the primary resonance case: $\Omega \cong \omega_1$ and the internal resonance case: $\omega_1 \cong \omega_2$.

Define the detuning parameters $\hat{\sigma}_1, \hat{\sigma}_2$ as follows:

$$\Omega = \omega_1 + \sigma_1 = \omega_1 + \varepsilon\hat{\sigma}_1 \ , \ \omega_2 = \omega_1 + \sigma_2 = \omega_1 + \varepsilon\hat{\sigma}_2 \tag{22}$$

4. Periodic Solution

4.1 simultaneous resonance

Eliminating the secular term in eqns. (20) and (21):

$$\left(D_1 A\right) = -\hat{\mu}_1\omega_1 A - \left(\hat{\alpha}_2 + 3\hat{\alpha}_3 - 3i\hat{\alpha}_5\right)\omega_1 A^2\bar{A} + 10i\hat{\alpha}_6\omega_1 A^3\bar{A}^2 - \frac{i\hat{f}}{4\omega_1} e^{i\hat{\sigma}_1 T_1} - \frac{i\hat{G}_1}{2\omega_1} B e^{i\hat{\sigma}_2 T_1} \tag{23}$$

$$\left(D_1 B\right) = -\hat{\mu}_2\omega_2 B - \frac{i\hat{G}_2}{2\omega_2} A e^{-i\hat{\sigma}_2 T_1} \tag{24}$$

It is convenient to express A, B in the polar form:

$$A = \frac{1}{2}\hat{a}_1\left(T_1\right)e^{i\hat{\varphi}_1(T_1)}, \ \ B = \frac{1}{2}\hat{a}_2\left(T_1\right)e^{i\hat{\varphi}_2(T_1)} \tag{25}$$

Substituting equations (25) into equations (23) and (24)

$$\left(\hat{a}_1' + i\hat{a}_1\hat{\varphi}_1'\right) = -\hat{\mu}_1\omega_1\hat{a}_1 - \frac{\left(\hat{\alpha}_2 + 3\hat{\alpha}_3 - 3i\hat{\alpha}_5\right)\omega_1}{4}\hat{a}_1^3 + \frac{5i\hat{\alpha}_6\omega_1}{8}\hat{a}_1^5 - \frac{i\hat{f}}{2\omega_1}e^{-i\left(\hat{\varphi}_1 - \hat{\sigma}_1 T_1\right)} - \frac{i\hat{G}_1}{2\omega_1}\hat{a}_2 e^{-i\left(\hat{\varphi}_1 - \hat{\varphi}_2 - \hat{\sigma}_2 T_1\right)} \tag{26}$$

$$\left(\hat{a}_2' + i\hat{a}_2\hat{\varphi}_2'\right) = -\hat{\mu}_2\omega_2\hat{a}_2 - \frac{i\hat{G}_2}{2\omega_2}\hat{a}_1 e^{i\left(\hat{\varphi}_1 - \hat{\varphi}_2 - \hat{\sigma}_2 T_1\right)} \tag{27}$$

where $\hat{a}_i'$ and $\hat{\varphi}_i', \left(i = 1, 2\right)$ represent the derivatives of $\hat{a}_i$ and $\hat{\varphi}_i, \left(i = 1, 2\right)$ with respect to T_1.

Return back every scaled parameter into its original form:

$$\hat{\mu}_i = \frac{\mu_i}{\varepsilon}, \ \hat{\alpha}_n = \frac{\alpha_n}{\varepsilon}, \hat{\sigma}_i = \frac{\sigma_i}{\varepsilon}, \ \hat{f} = \frac{f}{\varepsilon}, \ \hat{G}_i = \frac{G_i}{\varepsilon}, \ \hat{a}_i = a_i, \ \hat{\varphi}_i = \varphi_i; \quad \left(i = 1, 2; n = 1, 2, \ldots, 6\right) \tag{28}$$

Substituting equations (28) into equations (26) and (27):

$$\dot{a}_1 + ia_1\dot{\varphi}_1 = \mu_1\omega_1 a_1 - \frac{\left(\alpha_2 + 3\alpha_3 - 3i\alpha_5\right)\omega_1}{4}a_1^3 + \frac{5i\alpha_6\omega_1}{8}a_1^5 - \frac{if}{2\omega_1}e^{-i\left(\varphi_1 - \sigma_1 t\right)} - \frac{iG_1}{2\omega_1}a_2 e^{-i\left(\varphi_1 - \varphi_2 - \sigma_2 t\right)} \tag{29}$$

$$\dot{a}_2 + ia_2\dot{\varphi}_2 = \mu_2\omega_2 a_2 - \frac{iG_2}{2\omega_2}a_1 e^{i\left(\varphi_1 - \varphi_2 - \sigma_2 t\right)} \tag{30}$$

where $\dot{a}_i$ and $\dot{\varphi}_i, \left(i = 1, 2\right)$ represent the derivatives of a_i and $\varphi_i, \left(i = 1, 2\right)$ with respect to t.

Consider that: $\theta_1 = \varphi_1 - \sigma_1 t, \ \ \theta_2 = \varphi_1 - \varphi_2 - \sigma_2 t$ (31)

Substituting equation (31) into equations (29) and (30):

$$\begin{aligned}\dot{a}_1 + ia_1\left(\dot{\theta}_1 + \sigma_1\right) = -\mu_1\omega_1 a_1 - \frac{\left(\alpha_2 + 3\alpha_3\right)\omega_1}{4}a_1^3 + i\left(\frac{3\alpha_5\omega_1}{4}a_1^3 + \frac{5\alpha_6\omega_1}{8}a_1^5\right) \\ - \frac{if}{2\omega_1}\left[\cos\left(\theta_1\right) - i\sin\left(\theta_1\right)\right] - \frac{iG_1}{2\omega_1}a_2\left[\cos\left(\theta_2\right) - i\sin\left(\theta_2\right)\right]\end{aligned} \tag{32}$$

$$\dot{a}_2 + ia_2\left(\dot{\theta}_1 - \dot{\theta}_2 + \sigma_1 - \sigma_2\right) = -\mu_2\omega_2 a_2 - \frac{iG_2}{2\omega_2} a_1\left[\cos(\theta_2) + i\sin(\theta_2)\right] \tag{33}$$

Equating real and imaginary parts in eqns. (32) and (33), the following equations obtained:

$$\dot{a}_1 = \left(-\mu_1\omega_1 a_1 - \frac{(\alpha_2 + 3\alpha_3)\omega_1}{4} a_1^3\right) - \left(\frac{f}{2\omega_1}\right)\sin(\theta_1) - \left(\frac{G_1 a_2}{2\omega_1}\right)\sin(\theta_2) \tag{34}$$

$$\dot{\theta}_1 = \left(-\sigma_1 + \frac{3\alpha_5\omega_1}{4} a_1^2 + \frac{5\alpha_6\omega_1}{8} a_1^4\right) - \left(\frac{f}{2\omega_1 a_1}\right)\cos(\theta_1) - \left(\frac{G_1 a_2}{2\omega_1 a_1}\right)\cos(\theta_2) \tag{35}$$

$$\dot{a}_2 = \left(-\mu_2\omega_2 a_2\right) + \left(\frac{G_2 a_1}{2\omega_2}\right)\sin(\theta_2) \tag{36}$$

$$\dot{\theta}_2 = \left(-\sigma_2 + \frac{3\alpha_5\omega_1}{4} a_1^2 + \frac{5\alpha_6\omega_1}{8} a_1^4\right) - \left(\frac{f}{2\omega_1 a_1}\right)\cos(\theta_1) + \left(\frac{G_2 a_1}{2\omega_2 a_2} - \frac{G_1 a_2}{2\omega_1 a_1}\right)\cos(\theta_2) \tag{37}$$

4.2 Fixed Point

Applying the steady state condition $\dot{a}_1 = \dot{a}_2 = \dot{\theta}_1 = \dot{\theta}_2 = 0$; the response equation can be expressed in the following form.

$$d_1 a_1^{10} + d_2 a_1^8 + d_3 a_1^6 + d_4 a_1^4 + d_5 a_1^2 + d_6 = 0 \tag{38}$$

where $d_i\,(i = 1, 2, \ldots, 6)$ are constants which have been determined **(see Appendix)**

5. Nonlinear Solution

To study the stability of the nonlinear solution of the obtained fixed points, let:

$$a_1 = a_{10} + a_{11},\ \theta_1 = \theta_{10} + \theta_{11},\ a_2 = a_{20} + a_{21},\ \theta_2 = \theta_{20} + \theta_{21} \tag{39}$$

where $a_{10}, \theta_{10}, a_{20}, \theta_{20}$ are the solutions of eqns. (34), (35), (36) and (37), while $a_{11}, \theta_{11}, a_{21}, \theta_{21}$ are perturbations which are assumed to be small compared with $a_{10}, \theta_{10}, a_{20}, \theta_{20}$.

Substituting equation (39) into equations (34 – 37) and keeping only the linear terms in $a_{11}, \theta_{11}, a_{21}, \theta_{21}$ gives:

$$\dot{a}_{11} = -\left[\mu_1\omega_1 + \frac{3(\alpha_2 + 3\alpha_3)\omega_1}{4} a_{10}^2\right] a_{11} - \left[\left(\frac{f}{2\omega_1}\right)\cos(\theta_{10})\right]\theta_{11} - \left[\left(\frac{G_1}{2\omega_1}\right)\sin(\theta_{20})\right] a_{21} - \left[\left(\frac{G_1 a_{20}}{2\omega_1}\right)\cos(\theta_{20})\right]\theta_{21} \tag{40}$$

$$\dot{\theta}_{11} = \left[-\frac{\sigma_1}{a_{10}} + \frac{9\alpha_5\omega_1}{4} a_{10} + \frac{25\alpha_6\omega_1}{8} a_{10}^3\right] a_{11} + \left[\left(\frac{f}{2\omega_1 a_{10}}\right)\sin(\theta_{10})\right]\theta_{11} - \left[\left(\frac{G_1}{2\omega_1 a_{10}}\right)\cos(\theta_{20})\right] a_{21} + \left[\left(\frac{G_1 a_{20}}{2\omega_1 a_{10}}\right)\sin(\theta_{20})\right]\theta_{21} \tag{41}$$

$$\dot{a}_{21} = \left[\left(\frac{G_2}{2\omega_2}\right)\sin(\theta_{20})\right] a_{11} + [0]\theta_{21} - [\mu_2\omega_2] a_{21} + \left[\left(\frac{G_2 a_{10}}{2\omega_2}\right)\cos(\theta_{20})\right]\theta_{21} \tag{42}$$

$$\dot{\theta}_{21} = \left[-\frac{\sigma_1}{a_{10}} + \frac{9\alpha_5\omega_1}{4} a_{10} + \frac{25\alpha_6\omega_1}{8} a_{10}^3 + \left(\frac{G_2}{2\omega_2 a_{20}} \right) \cos(\theta_{20}) \right] a_{11} + \left[\left(\frac{f}{2\omega_1 a_{10}} \right) \sin(\theta_{10}) \right] \theta_{11}$$
$$+ \left[\frac{(\sigma_1 - \sigma_2)}{a_{20}} - \left(\frac{G_1}{2\omega_1 a_{10}} \right) \cos(\theta_{20}) \right] a_{21} + \left[\left(\frac{G_1 a_{20}}{2\omega_1 a_{10}} - \frac{G_2 a_{10}}{2\omega_2 a_{20}} \right) \sin(\theta_{20}) \right] \theta_{21} \quad (43)$$

6. Numerical Solution

The numerical solution of the system which introduced by the equations (5&6) is determined by applying the Runge-Kutta fourth-order method. The selected values for the system parameters are given by:

$$\Omega = 1,\ \omega_1 = 1,\ \omega_2 = 1,\ \varepsilon = 0.5,\ \hat{\mu}_1 = 0.1,\ \hat{\mu}_2 = 0.001,\ \hat{\alpha}_1 = 0.15,\ \hat{\alpha}_2 = 0.1,\ \hat{\alpha}_3 = 1/30,\ \hat{\alpha}_4 = -0.3,$$

$$\hat{\alpha}_5 = -0.3,\ \hat{\alpha}_6 = 0.2,\ \hat{f} = 0.5,\ \hat{G}_1 = 1.5,\ \hat{G}_2 = 1.3.$$

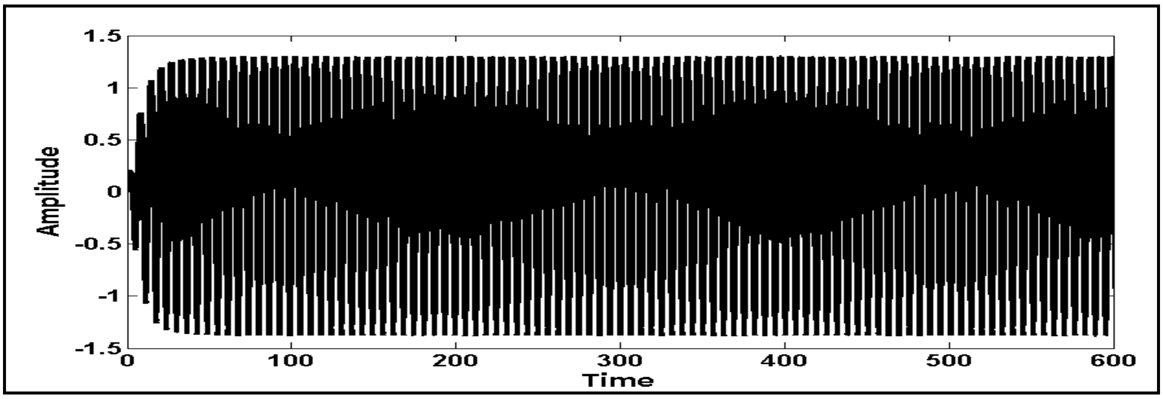


Fig.Fig. 2. Time history of the system without control at the resonance case $\Omega = \omega_1$.

The vibration of the system under studying before using PPF control when $\Omega = \omega_1$ is illustrated by Fig. (1). The amplitude of the system vibration is about 1.3.

The effect of using a positive position feedback control for reducing the system vibration at the simultaneous resonance case of the primary resonance case $\Omega = \omega_1$ and the internal resonance case $\omega_1 = \omega_2$ is illustrated by Fig.(2). As shown in Fig.(3), the system steady state amplitude under the effect of the PPF control is about 0.0005, and the controller effectiveness E_a is about 2600.The frequency response curves for the main system with PPF control is demonstrated in Fig. (4), which gives closed loop case. It is found two peaks are produced at the values $\sigma_1 = -0.44$ and $\sigma_1 = 1.84$, so they are creating a bandwidth in-between about 2.28.

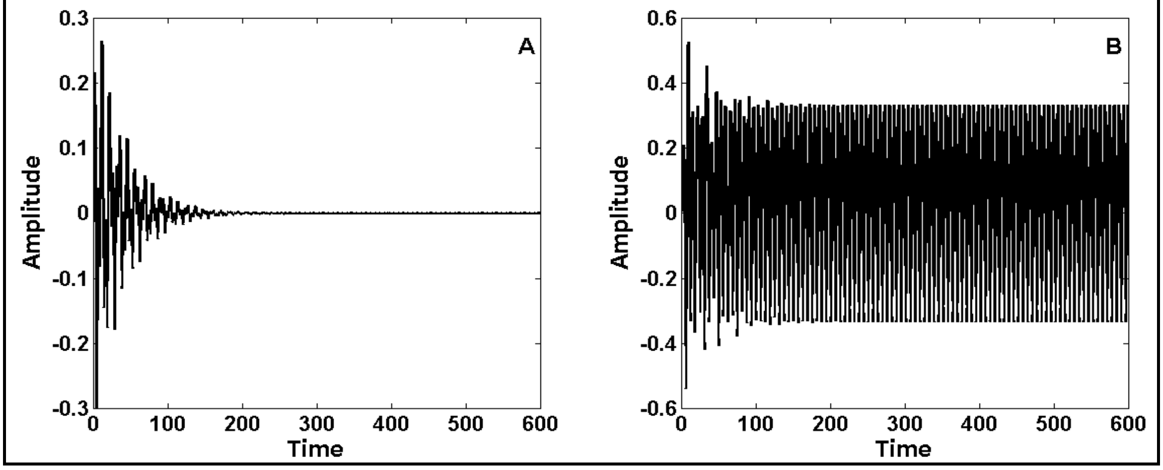


Fig. 3. A) Main system time history and B) Controller time history with PPF control at the simultaneous resonance case $\Omega = \omega_1$ and $\omega_1 = \omega_2$.

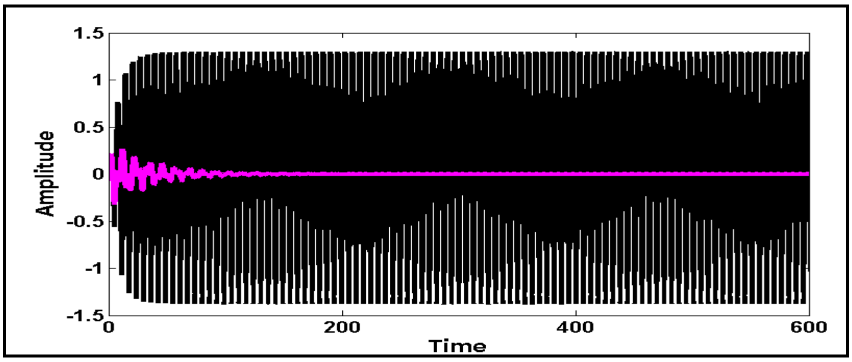


Fig. 4. The effectiveness of PPF control for reduction the vibration of the system

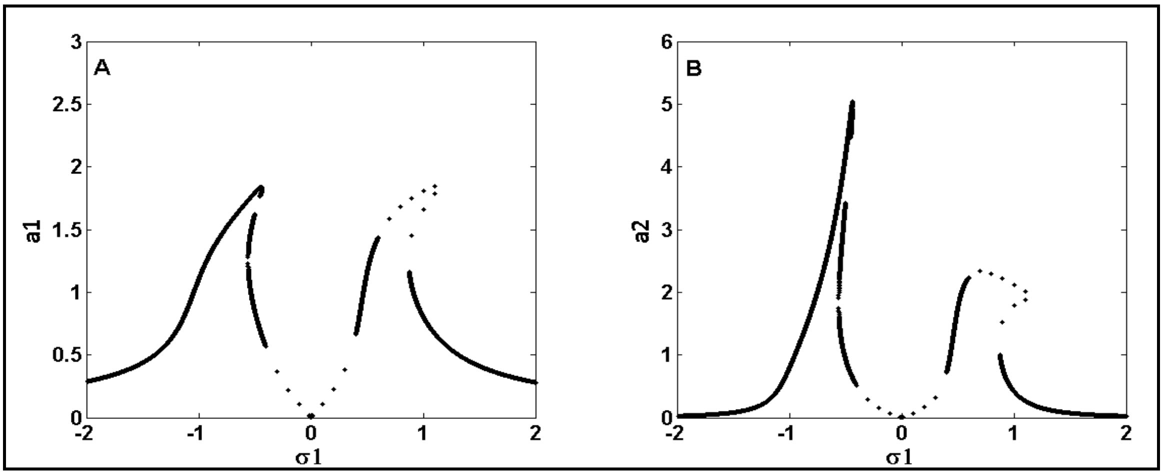
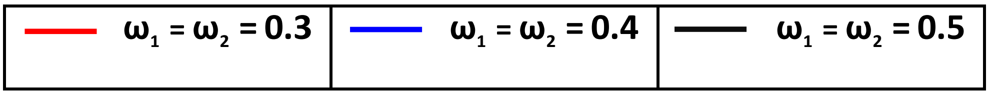


Fig. 5. Frequency response curve for the system with PPF control at the selected values for the system parameters.Solid line: stable solution & Dot line: unstable solution

7. Effect of the system parameters on the response curve

In this section, Fig.s (6 – 15) show the effects of different parameters on the behavior of the vibrating system: A) on the main system and B) on the controller. The selected values of the parameters in this study are as mentioned before.

Fig. (6) show that the steady-state amplitude of both the main system and the controller is monotonic decreasing for increasing the frequencies ω_1 and ω_2 in the case of the internal primary resonance case $\omega_1 = \omega_2$. So this type of controllers (PPF) is very suitable. The bandwidth of the two peaks of both the main system and the controller reduced with increasing of the frequencies.



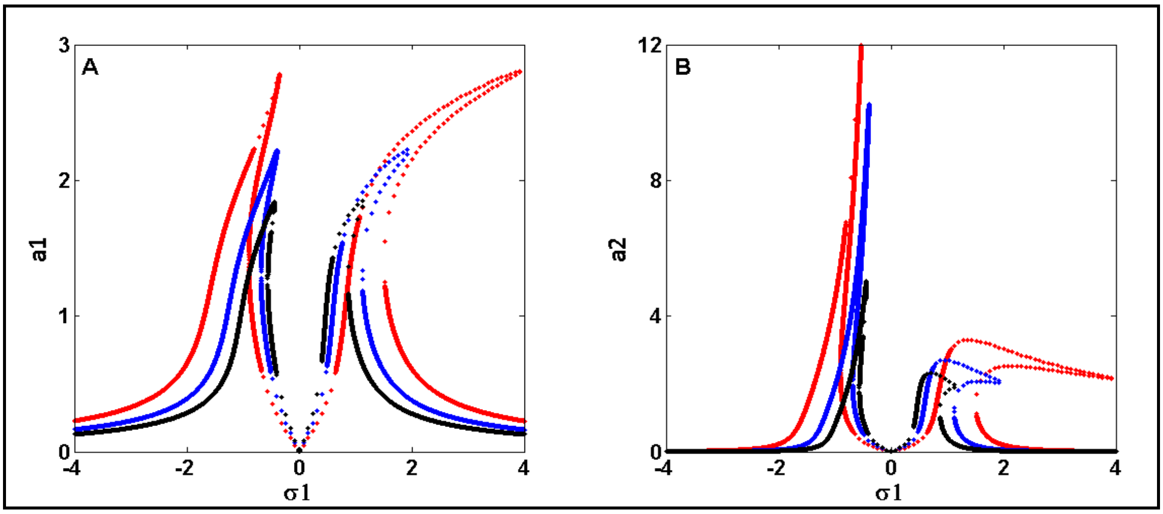


Fig. 6 Effect of the internal resonance on the response curve.

Effect of the damping coefficients μ_1 and μ_2 is illustrated in Fig.s (7&8) respectively. Fig.s (7) and (8) appear that within creasing of μ_1 and μ_2, the steady-state amplitude of the two peaks for both the main system and the controller is decreasing. The bandwidth of the two peaks is to be smaller with increasing of μ_1. The amplitude is slightly affected by μ_2 and the bandwidth is not affected.

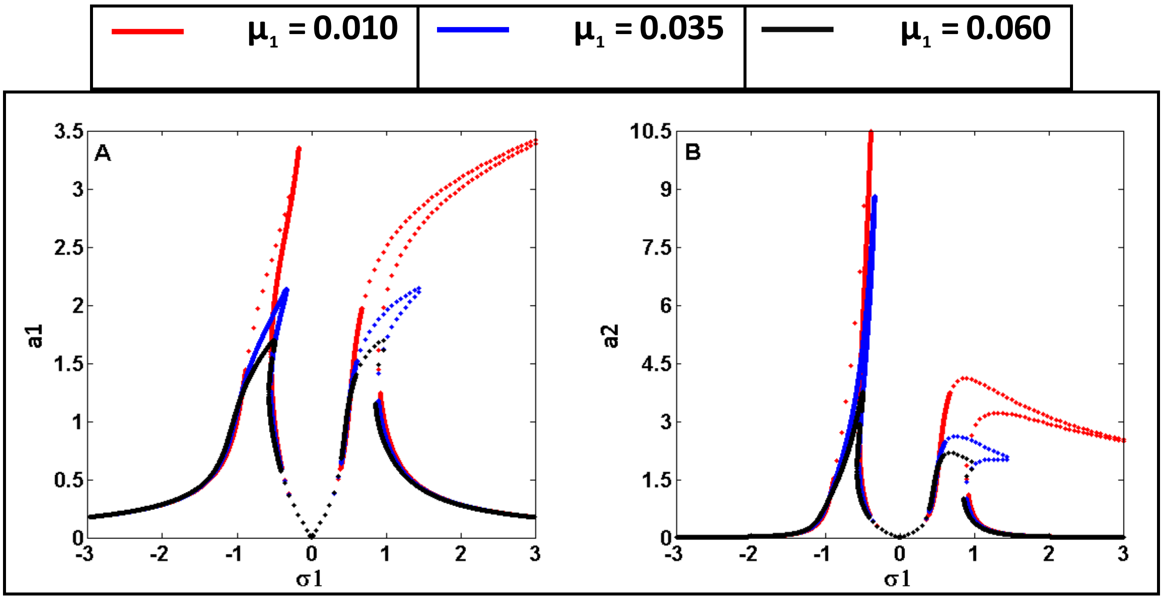
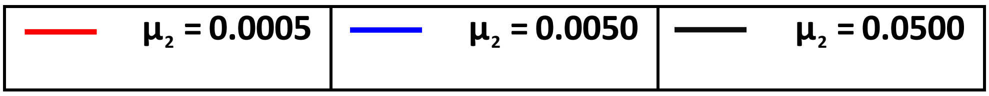


Fig. 7. Effect of μ_1 on the response curve.



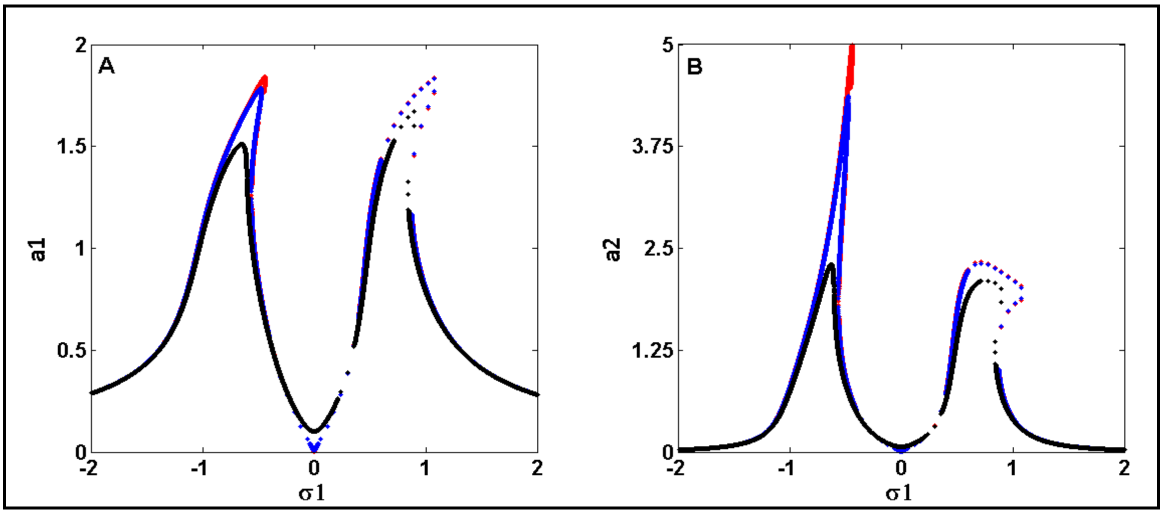


Fig. 8. Effect of μ_2 on the response curve.

Effect of the control signal gain G_1 and the feedback signal gain G_2 is illustrated in Fig.s (9&10) respectively. Fig.s (9) and (10) appear that the bandwidth between the two peaks increases with increasing of G_1 and G_2 which gives more flexibility for the controller job. This can increase the safety factor because the value of σ_1 may deviate from 0 and go towards one of the values where the peaks are located. The peaks values for the main system are not affected by variation of either G_1 or G_2 .

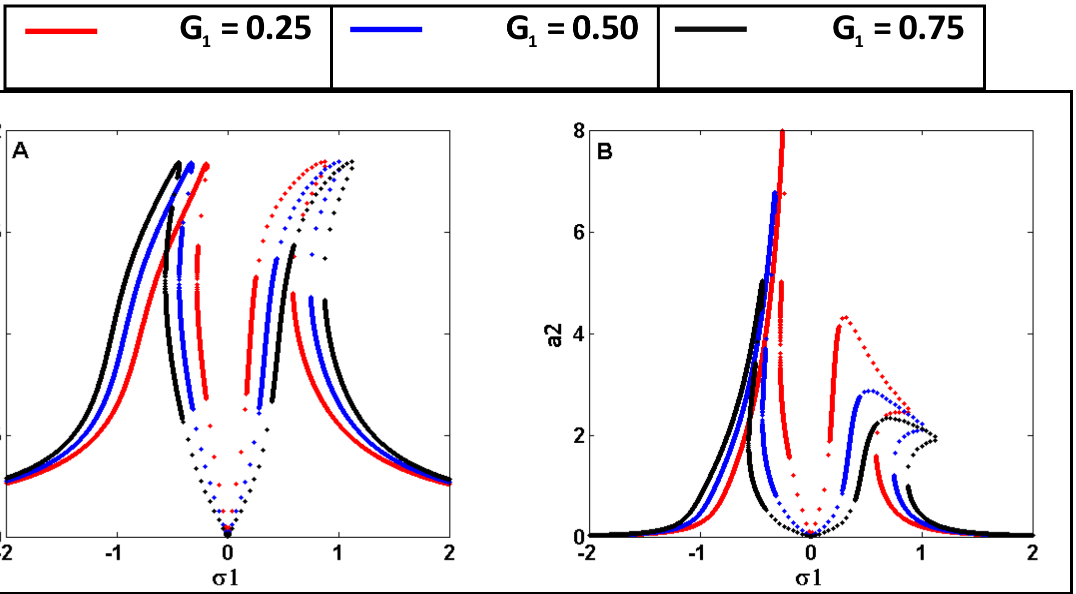
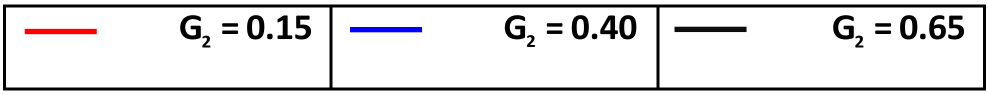


Fig. 9. Effect of G_1 on the response curve.



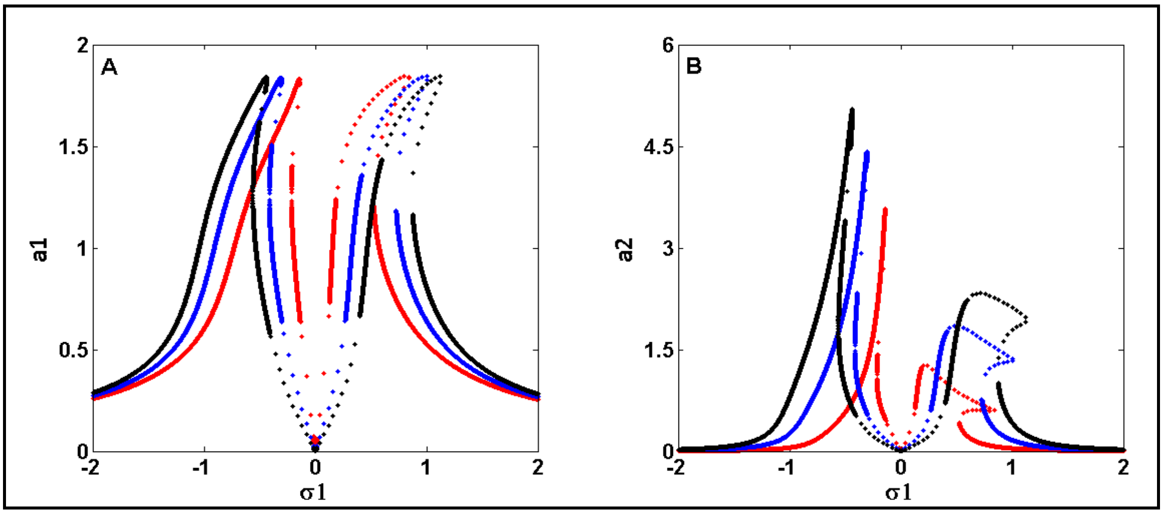


Fig. 10. Effect of G_2 on the response curve.

Fig. (11) show that for increasing external excitation force f, the amplitude of the main system and the controller is monotonic increasing and the frequency response curves bend away from the linear curves.

Fig. (12) show effect of the detuning parameter on the response curve. The minimum value of both the main system and the controller is observed that it occur when $\sigma_1 = \sigma_2$.

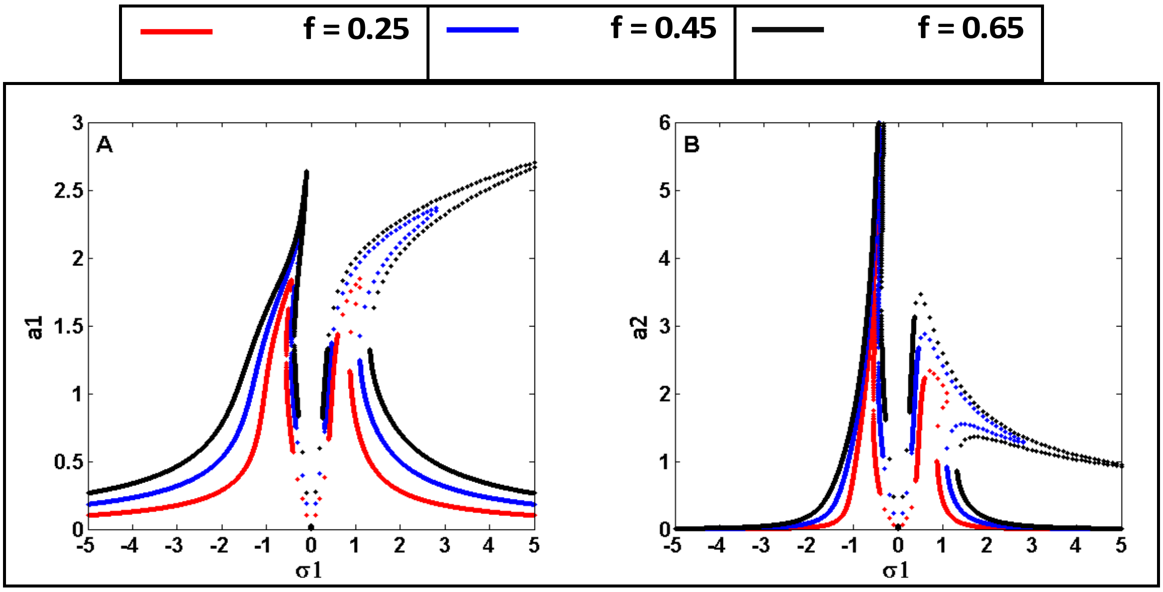
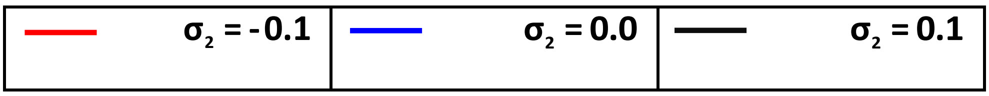


Fig. 11. Effect of f on the response curve.



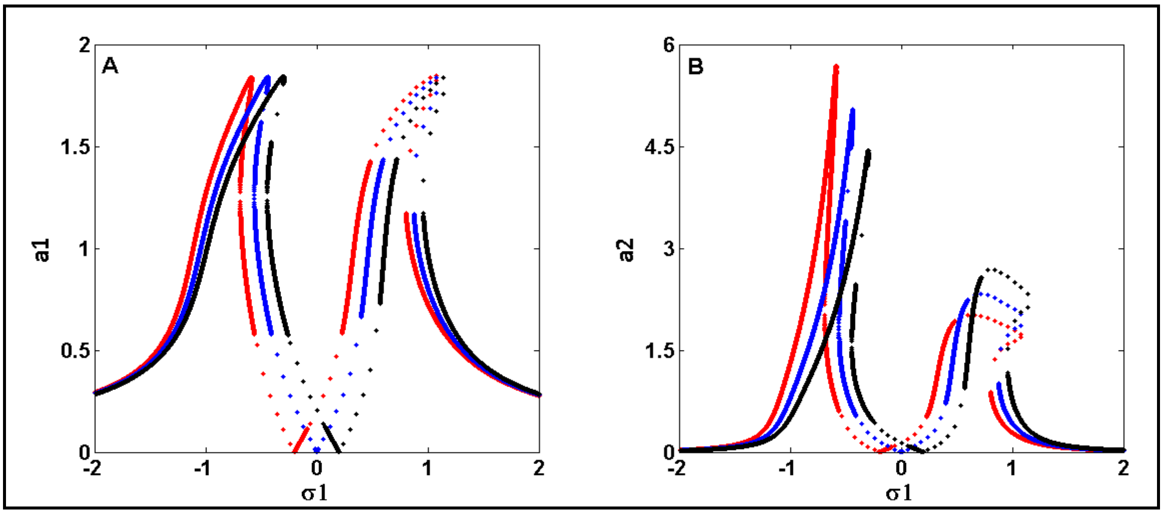


Fig. 12. Effect of σ_2 on the response curve.

Effect of the nonlinear parameter α_6 "the coefficient of the fifth order term" is illustrated by Fig. (13) which appears no effect in the main system amplitude and slightly effect in the bandwidth of the two peaks that is slightly increasing with increasing α_6. On the other hand the controller amplitude is increasing with increasing α_6 in the negative interval of σ_1; while the controller amplitude is decreasing with increasing α_6 in the positive interval of σ_1 and the bandwidth also slightly increasing with increasing α_6.

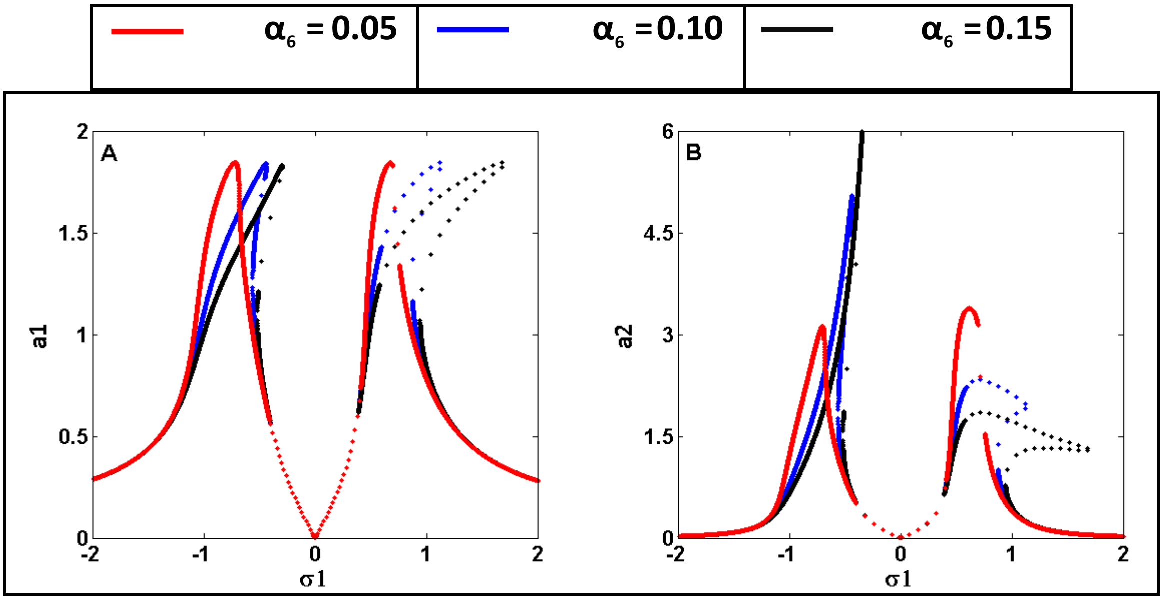


Fig. 13. Effect of α_6 on the response.

The relation between the system amplitude and the excitation force without and with the control is demonstrated in Fig. (14). In the absence of control the system amplitude is observed to increasing nonlinearly for a slight increase in the excitation force. After applying PPF controller, the system amplitude leads to a saturation case that the relation became horizontal, so the system amplitude is slightly change for largely increasing the excitation force.

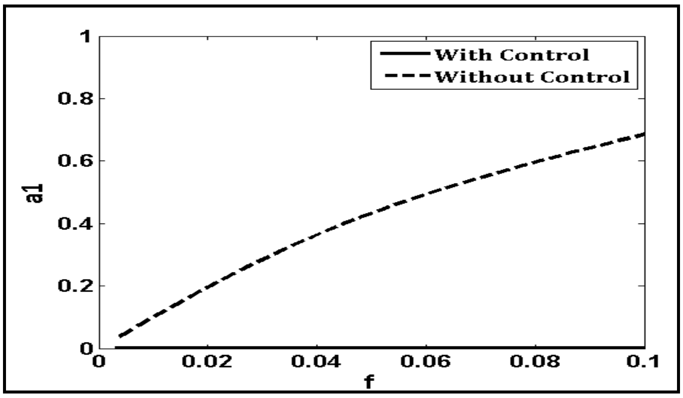


Fig. 14. Frequency response curve for the system at $\sigma_1 = \sigma_2 = 0$

Fig. (15) show the system with PPF control and illustrated a good agreement between the perturbation solution which obtained by MSPT and the numerical solution which determined by applying Runge-Kutta fourth-order method.

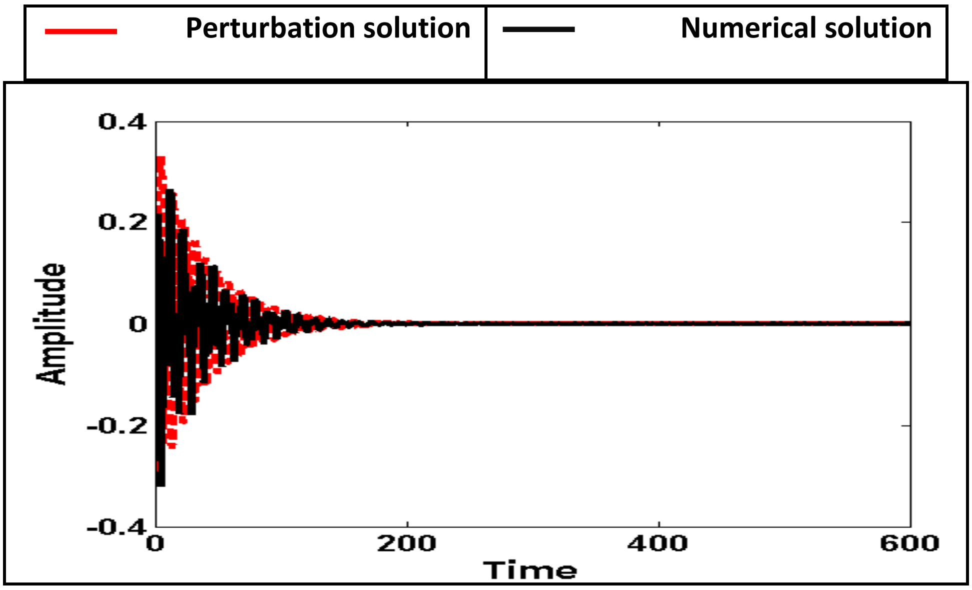


Fig. 15. the agreement between the perturbation solution and the numerical solution with PPF control

8. Comparison with previously published work

In a previous work [19], the authors present a model of the vertical dynamic excitation of structures induced by a single pedestrian walking along straight path on flat and relatively stiff surfaces. The main goal of the study was to create a model which could describe reliably time and frequency domain of continuously measured vertical walking force records. A linear least square identification technique was used in conjunction with the Fourier representation to identify values of the modelling parameters. A damping is added to the structure by acting as a negative damper during walking.

In this study, the presented modified system in [19] is controlled using PPF controller to reduce the vibration produced of the main system. A damping is proposed to be positive. MSPT is applied to get a solution of the studied system and examined the stability of this system. PPF controller succeeded for reducing the produced vibration with ratio 99.5 %.

9. Conclusions

In this paper PPF controller is used to reduce the vibration produced from a modified hybrid Rayleigh -Van der Pol-Duffing oscillator. The proposed oscillator is considered to be acted by a positive damper and subjected to an external excitation. MSPT technique was applied to derive an approximate solution of the system. Oscillator equation is coupled to an active positive position feedback (PPF) controller and they have been studied near the simultaneous primary and internal resonance case. The results of this paper can be summarized as follows:

1) The worst resonance cases of the system are the simultaneous resonance case is the primary resonance case $\Omega = \omega_1$ and the internal resonance case $\omega_1 = \omega_2$.
2) PPF controller is effective for reducing the produced vibration that the controller effectiveness E_a is about 2600.
3) This type of controllers (PPF) is very suitable for small natural frequency dynamical systems subjected to primary resonance excitations.
4) The vibration reduction controller frequency bandwidth may be controlled by controlling the control signal gain G_1 and the feedback signal gain G_2 that gives more flexibility for the controller job.
5) The steady-state amplitudes of the main system is decreasing with increasing of the damping coefficients and with decreasing of the excitation force.
6) The minimum steady-state amplitude of both the main system and the controller is observed that it occur when $\sigma_1 = \sigma_2$.
7) After control, the relation between the main system amplitude and the excitation force became horizontal that represents to a saturation case; while the relation between the controller and the excitation force is directed nonlinearly relation.
8) Even nonlinear terms have absolutely no effect on the response of the system; while the coefficient of the fifth order term has an effect on the response system. It is observed that its effect on the negative interval of σ_1 is different about its effect on the positive interval.

Appendix

$$\mathrm{K}_1 = \frac{G_1 G_2 (\sigma_1 - \sigma_2)}{\omega_1 \omega_2 \left[(\sigma_1 - \sigma_2)^2 + (\mu_2 \omega_2)^2 \right]} - 4\sigma_1;\ \mathrm{K}_2 = \frac{\mu_2 G_1 G_2}{\omega_1 \left[(\sigma_1 - \sigma_2)^2 + (\mu_2 \omega_2)^2 \right]} - 4\mu_1 \omega_1$$

$$\mathrm{d}_1 = \frac{25}{64} \alpha_6^2 \omega_1^2;\ \mathrm{d}_2 = \frac{15}{16} \alpha_5 \alpha_6 \omega_1^2;\ \mathrm{d}_3 = \frac{\omega_1^2}{16} \left[(\alpha_2 + 3\alpha_3)^2 + (3\alpha_5)^2 + \left(\frac{5\alpha_6 \mathrm{K}_1}{\omega_1} \right) \right]$$

$$\mathrm{d}_4 = \frac{1}{8} \left[3\alpha_5 \omega_1 \mathrm{K}_1 - (\alpha_2 + 3\alpha_3) \omega_1 \mathrm{K}_2 \right];\ \mathrm{d}_5 = \mathrm{K}_1^2 + \mathrm{K}_2^2;\ \mathrm{d}_6 = \frac{-f^2}{4\omega_1^2}$$

References

[1] Jun L. Positive Position Feedback Control for High-Amplitude Vibration of a Flexible Beam to a Principal Resonance Excitation. Shock and Vibration. 2010; 17:187-203.

[2] Wang Y, Inman DJ. Comparison of Control Laws for Vibration Suppression Based on Energy Consumption. Journal of Intelligent Material Systems and Structures. 2011; 22:795-809.

[3] Cazzulani G, Resta F, Ripamonti F. The Active Modal Tuned Mass Damper (AMTMD) for Vibration Suppression in Flexible Structures.

[4] Orszulik R, Shan J. Multi-mode adaptive positive position feedback: An experimental study. In: Proceedings of the 2011 American Control Conference.; 2011:3315-3319. doi:10.1109/ACC.2011.5991454

[5] Shin C, Hong C, Jeong WB. Active vibration control of clamped beams using positive position feedback controllers with moment pair. Journal of Mechanical Science and Technology. 2012;26(3):731-740. doi:10.1007/s12206-011-1233-y

[6] Mitura A, Warminski J, Bochenski M. Optimal control methods for vertical and horizontal beam dynamics. In: ; 2012.

[7] El-Ganaini WAA, Saeed NA, Eissa M. Positive position feedback (PPF) controller for suppression of nonlinear system vibration. Nonlinear Dynamics. 2013; 72:517-537.

[8] Jung W, Noh I, Kang D. The vibration controller design using positive position feedback control. In: 2013 13th International Conference on Control, Automation and Systems (ICCAS 2013). ; 2013:1721-1724. doi:10.1109/ICCAS.2013.6704213

[9] Ghareeb N, Schmidt R. Active vibration control of a super element model of a thin-walled structure. 2014 11th International Conference on Informatics in Control, Automation and Robotics (ICINCO). 2014; 01:657-664.

[10] Omidi E, Mahmoodi SN. Sensitivity analysis of the Nonlinear Integral Positive Position Feedback and Integral Resonant controllers on vibration suppression of nonlinear oscillatory systems. Commun Nonlinear Sci Numer Simul. 2015; 22:149-166.

[11] Kaushik AK, Krishna Y and Bangarubabu P. Active vibration control of smart beam under random vibrations. International Journal of Innovations in Engineering and Technology. 2015; 5(1):8 – 16.

[12] El-Sayed AT, Bauomy HS. Nonlinear analysis of vertical conveyor with positive position feedback (PPF) controllers. Nonlinear Dynamics. 2016; 83:919-939.

[13] Amer Y, EL-Sayed A, Salem AM. Vibration Control in MEMS Resonator Using Positive Position Feedback (PPF) Controller. Advances in Mathematics. 2016; 12:6821-6834. doi:10.24297/jam.v12i11.1114

[14] Syed HH. Comparative study between positive position feedback and negative derivative feedback for vibration control of a flexible arm featuring piezoelectric actuator. International Journal of Advanced Robotic Systems. 2017;14.

[15] EL-Ganaini WA. Vibration control of a nonlinear dynamical system excited at simultaneous resonance case. Electronic Journal of Mathematical Analysis and Applications. 2018; 6(2) :86 – 100.

[16] Wu Y, Zhang W, Meng X, Su Y. Compensated positive position feedback for active control of piezoelectric structures. Journal of Intelligent Material Systems and Structures. 2017; 29:397-410.

[17] Enríquez-Zárate J, Trujillo L, Toledo-Ramirez G, Ramos-Cirilo ÁJ, Hernández C. Optimization of PPF Control of a Building-like Structure for Vibration Control. Computación y Sistemas. 2018;22.

[18] Abdollahzadeh Jamalabadi MY. Positive Position Feedback Control of a Galloping Structure. Acoustics. 2019;1(1):47-58. doi:10.3390/acoustics1010005

[19] Kumar P, Kumar A, Racic V, Erlicher S. Modelling vertical human walking forces using self-sustained oscillator. Mechanical Systems and Signal Processing. 2018; 99:345-363.

[20] Amer YA, El-Sayed AT, Abdel-Wahab AM, Salman HF. The effectiveness of nonlinear integral positive position feedback control on a duffing oscillator system based on primary and super harmonic resonances. Journal of Vibroengineering. 2019; 21:133-153.

[21] Bauomy HS, EL-Sayed AT. A new six-degrees of freedom model designed for a composite plate through PPF controllers. Applied Mathematical Modelling. 2020; 88:604-630. doi:https://doi.org/10.1016/j.apm.2020.06.067

[22] Ali KK, Salam MAA el, Mohamed EMH. Chebyshev operational matrix for solving fractional order delay-differential equations using spectral collocation method. Arab Journal of Basic and Applied Sciences. 2019; 26:342-353.

[23] Raslan KR, Salam MAA el, Ali KK, Mohamed EMH. Spectral Tau method for solving general fractional order differential equations with linear functional argument. Journal of the Egyptian Mathematical Society. 2019; 27:1-16.

إخماد إهتزاز نظام ديناميكى غير خطى تحت تأثير قوة خارجية باستخدام PPF كنترول

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الملخص العربى :

فى هذا البحث تم استخدام Positive Position Feedback (PPF) Control وذلك للحد من الإهتزاز الناتج من النظام الديناميكى غير الخطى قيد الدراسة والذى يقع تحت تأثير قوة خارجية. النموذج المقترح يصف الإهتزازات الرأسية للمنشآت والتى تنتج عن سير شخص واحد فى مسار مستقيم على أسطح مستوية وصلبة نسبيا. تم تطبيق MSPT للحصول على الحلول التقريبية للنظام من الرتبة الأولى. تم اشتقاق ودراسة الشروط اللازمة للإستقرار فى جوار حالة الرنين الأسوأ وهى Simultaneous Primary and Internal Resonance Cases. تم استخدام برنامج MATLAB 14.0وذلك لتوضيح تأثير PPF Control على إخماد الإهتزاز وكذلك تم دراسة تأثير تغير بارامترات النظام المختلفة على Response System كما تم عرض مقارنة بين الحل التقريبى والحل العددى وتوضيح الإتفاق فيما بينهم. تم التحقق بأن تطبيق PPF Control مناسب جدا على الأنظمة الديناميكية غير الخطية ذات الترددات الصغيرة والتى تخضع لقوى خارجية ذات ترددات أولية.