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Modeling and control of unstable mechanical systems using control moment gyro (CMG)

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Abstract. Stabilizing of statically unstable systems such as monorail trains and the Two-wheel bike is a wide area for research. In this work bike has been used as an unstable mechanical system**.** Control Moment Gyro (CMG) has been utilized as a control tool to a achieve stability of the proposed system. The proposed gyroscopic system consists of single degree of freedom gimbal and flywheel placed horizontally inside the bike frame. The gimbal angular velocity controlled by DC servo motor lay on a new control approach "*Linear Quadratic Regulator Plus Integral Action (LQR+I)* ". Matlab model was developed for the system using mathematical equations derived from nonlinear dynamics by using Lagrange's method to obtain simulated system response and behaviour. Proportional Integral Derivative (PID) controller has been applied to simulated model and actual test rig to ensure the simulation validity, the study compares the three techniques of control (PID ,LQR and LQR+I) in terms of settling time, overshoot and steady state error. "The simulation results are illustrated with the help of graphs and tables which significantly show superior performance and robustness of LQR + I over PID and LQR.

1. Introduction

Usage of bike increases as transportation in last year's. Bike is inherent unstable it can be considered as a simple inverted pendulum (IP). Over the last two decades, the inverted pendulum has been considered the most common system illustrating the concepts of multi-variable, high order, nonlinear, strong coupling, and instability. Consequently, it's considered as an optimum research area for many control theory [1] .Researchers have been exploring different solutions for a dynamically balance the inverted pendulum such as moving carts [2]. System described as vertical rod pivoted on rotation axis and attached to a moving cart .the cart is motorized and moves along a horizontal line. The cart moves forward and backward to keep the pendulum in vertical position.

Conventional PID control scheme has been applied to moving cart system [3], Although PID control is simple and efficient control scheme it requires a good knowledge of the system and accurate tuning to obtain good performance. PID controllers are very bad choices when dealing with multivariable systems. PID shows a better steady-state error in the linearized model, however, a very poor steady-state error in the non-linear model [4].anther configuration is inverted pendulum with Control Moment Gyroscope Stabilizer[5].CMG relies on change the direction of the angular momentum of a rotating object it's called gyroscopic effect phenomenon .A gyroscope consists essentially of a rotor which can spin freely about its geometric axis and free to rotate about one or more perpendicular axes. According to Newton's 1st law of motion, an object in motion will stay in motion unless acted on by an external

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force. This statement holds true for rotating objects as well. Therefore, a spinning flywheel (constant spin rate) with no applied external forces or torques will continue to spin about its spin axis and in its current orientation, with no other rotational or translational motion relative to a stationary frame of reference. However, a spinning flywheel with summation of external forces or torques (about an axis other than its spin axis) not equal to zero exhibits an interesting phenomenon called precession which dynamically balance the inverted pendulum [6] .

Control moment gyroscopes (CMGs) are torque amplification devices with a very high storage capacity. The control moment gyro has a wide range of application, gyroscopic anti-roll devices for ships[7], Local controllability and stabilization of spacecraft attitude by control moment gyros [8]not only provide a fail-operation mode, but also have a significant meaning in improving reliability of attitude control systems,also used for dynamic attitude control of small satellites, Control for Ballistic Missile Longitudinal Autopilot [9]. CMG used for stabilization of an unmanned vehicle sinc1905 Brennan's Duo gyro Monorail Car [7].After revolution at motor speed, solid state electronics, and computer capabilities. Researches introduce innovative controllers for an unmanned vehicle. Beznos [10] used the CMG for stabilization and apply simple simple servo control to his dual gyro bicycle which also had steering functionality.

Bui Trung proposes paper [11] "structure-specified mixed H2/H∞ controllers by using PSO algorithm". It is an advanced technique for designing robust and optimal controllers for systems associated with sources of uncertainties but, it is not commonly used like PID and lead-lag controllers due to complicated design procedures. Pom Yuan Lam [12] proposed a proportional plus derivative (PD) controller which was implemented to bike. The paper states that the PID contrller response is superior when compared to a PD controller. The phase margin decreases and a pair of poles had been shifted to the right-half plane. The system becomes stable and able to balance the bicycle. But the compensator rely on pole zero cancelation method to achive the system stability , which is not efficient when system exact model is unavailable.

In this paper, the system can be described as self-balancing bicycle equipped with sensors to detect the roll angle of the bicycle and gyro gimbal to bring it back to the balancing position whenever it tilts from its vertical position. Control Moment Gyro (CMG) simply is symmetrical rotor spinning rapidly about its axis and frees to rotate about one perpendicular axis. The CMG is a torque amplification device because small gimbal torque input produces large control torque on the bicycle. DC motor used to control the angular speed of the gimbal. is drove through H-bridge. Three different controllers was applied to the system (PID, LQR, LQR+I) and performance was compared for each one by simulations. The (LQR + I) controller shows better performance than conventional PID and LQR. The (LQR+I) have less over shoot and less steady state error, that’s led to better system performance during use of two wheel bicycle.

2. Hardware configuration

The proposed mechanical system shown in Figure 1. And Figure 2.was developed as a test rig for the new control approach (LQR+I), the mainframe for the bicycle is wooden frame (360mm*325mm*50mm), inner gimbal was mounted to the wooden frame using a ball bearing.

A Dc motor (12V-36W-250 rpm) was connected with the timing belt to rotate gimbal at ratio 2:1, flywheel 140mm diameter and .55 kg weight directly connected to DC motor (12V-120W-4200rpm) rotates with angular velocity 4200 rpm**.** Gimbal motor control gimbal angle and gimbal angular velocity through high-resolution magnetic encoder (1440pulse per revolution).the encoder connects directly to Arduino mega microcontroller running at 8MHz as a central controller. An MPU 6050 gyro sensor for measuring the lean angle of bike connected to the central control unit through the I2C protocol.

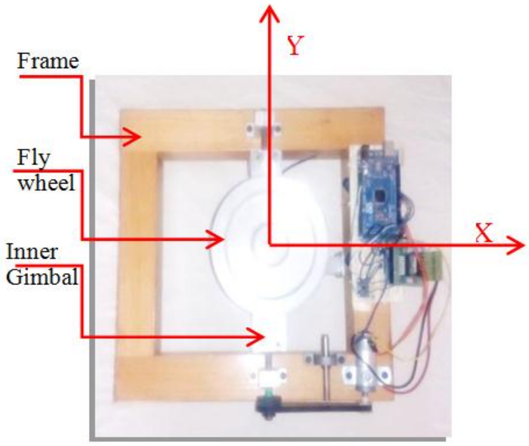

Figure 1. Plan view for mechanical system configuration

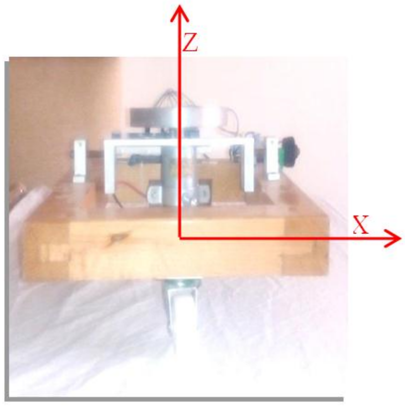

Figure 2. Side view for mechanical system configuration

3. Mathematical model of bicycle

3.1. Nonlinear mathematical model

As shown in Figure 3 the direction of rotation for each part.

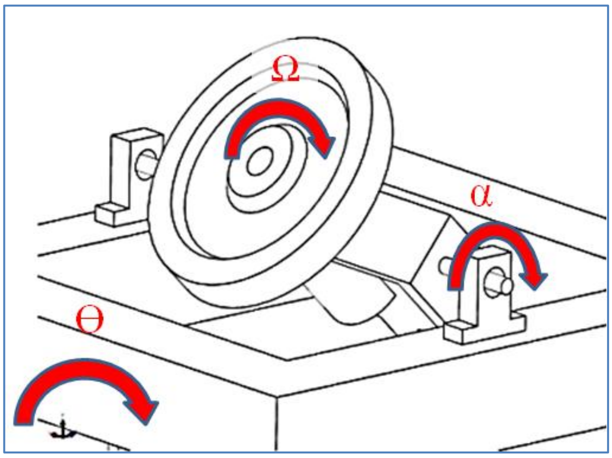

Figure 3. Direction of rotation for each part

To derive the dynamics model of the system, Lagrange equation is used:

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{q}_i}\right)-\left(\frac{\partial KE}{\partial q_i}\right)+\left(\frac{\partial PE}{\partial q_i}\right)=Q_i$$

where

- *KE, PE is the* total kinetic and potential energies of the system.
- *Qi* is external forces, and *qi* is generalized coordinate.
- *PE* and *KE* are determined, and represented by the following equations.

Total kinetic energy expressed as,

$$(K.E)=\frac{1}{2}l_1^2 m_O\theta^{\cdot 2}+\frac{1}{2}I_{xo}I\theta^{\cdot 2}+\frac{1}{2}L_2^2 m_i\alpha^{\cdot 2}+\frac{1}{2}m_i\theta^{\cdot 2}(\mathrm{L}_1+L_2\cos\alpha)^2+\frac{1}{2}I_{x2}\cos\alpha^2\theta^{\cdot 2}+\frac{1}{2}I_{y2}\alpha^{\cdot 2}$$
$$+\frac{1}{2}I_{z2}\sin\alpha^2\theta^{\cdot 2}+\frac{1}{2}m_D L^2\alpha^{\bullet 2}+\frac{1}{2}m_D(L_1+L\ \cos\alpha)^2\theta^{\cdot 2}+\frac{1}{2}I_{x3}(\cos\alpha)^2\theta^{\cdot 2}+\frac{1}{2}I_{y3}\alpha^{\bullet 2}+\frac{1}{2}I_{z3}(\theta^{\cdot}\sin\alpha+\Omega)^2 \quad (1)$$

Total potential energy expressed as,

$$\left(\mathrm{P.E}\right)_o=m_D g(\mathrm{L}_1+\mathrm{L}\ \cos\alpha\ )\cos\theta\ +m_i g(\mathrm{L}_1+\mathrm{L}_2\cos\alpha\ )\cos\theta\ +m_o g l_1\cos\theta \quad (2)$$

Apply Lagrange Equation,

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial q_i^{\bullet}}\right)-\left(\frac{\partial KE}{\partial q_i}\right)+\left(\frac{\partial PE}{\partial q_i}\right)=\mathrm{Q}_i \quad \text{when,} \quad q_i=\left[\theta \quad \alpha\right]^T \qquad q_i^{\bullet}=\left[\theta^{\bullet} \quad \alpha^{\bullet}\right]^T$$

For $q_i=\Theta$

$$\frac{\partial KE}{\partial\theta^{\bullet}}=l_1^2 m_O\theta^{\cdot}+I_{xo}I\theta^{\cdot}+m_i\theta^{\cdot}(\mathrm{L}_1+L_2\cos\alpha)^2+I_{x2}\cos\alpha^2\theta^{\cdot}$$
$$+I_{z2}\sin\alpha^2\theta^{\cdot}+\frac{1}{2}m_D L^2\alpha^{\bullet 2}+m_D(L_1+L\ \cos\alpha)^2\theta^{\cdot}+I_{x3}(\cos\alpha)^2\theta^{\cdot}+I_{z3}(\theta^{\cdot}\sin\alpha+\Omega)\sin\alpha \quad (3)$$

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial\theta^{\bullet}}\right)=[l_1^2 m_O+I_{xo}+m_i(\mathrm{L}_1+L_2\cos\alpha)^2+I_{x2}\cos\alpha^2+I_{z2}\sin\alpha^2+m_D(L_1+L\ \cos\alpha)^2+I_{x3}(\cos\alpha)^2$$
$$+I_{z3}(\sin\alpha^2)]\theta^{\bullet\bullet}+2I_{z2}\sin\alpha\ \cos\alpha\ \theta^{\cdot}\alpha^{\bullet}-2I_{x2}\sin\alpha\ \cos\alpha\ \theta^{\cdot}\alpha^{\bullet}+2I_{z3}\sin\alpha\ \cos\alpha\ \theta^{\cdot}\alpha^{\bullet}$$
$$-2I_{x3}\sin\alpha\ \cos\alpha\ \theta^{\cdot}\alpha^{\bullet}-2m_i\theta^{\cdot}\alpha^{\cdot}(\mathrm{L}_1+L_2\cos\alpha)L_2\sin\alpha-2m_D(L_1+L\ \cos\alpha)\theta^{\cdot}\alpha^{\cdot}L\ \sin\alpha+I_{z3}\alpha^{\cdot}\Omega\cos\alpha \quad (4)$$

$$\frac{\partial KE}{\partial\theta}=zero$$

$$\left(\mathrm{P.E}\right)_o=-m_D g(\mathrm{L}_1+\mathrm{L}\cos\alpha\ )\sin\theta\ -m_i g(\mathrm{L}_1+\mathrm{L}_2\cos\alpha\ )\sin\theta\ -m_o g l_1\sin\theta \quad (5)$$

For $q_i=\alpha$

$$\frac{\partial KE}{\partial\alpha^{\cdot}}=L_2^2 m_i\alpha^{\cdot}+I_{y2}\alpha^{\cdot}+m_D L^2\alpha^{\bullet}+I_{y3}\alpha^{\bullet} \quad (6)$$

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial\alpha^{\cdot}}\right)=[L_2^2 m_i+I_{y2}+m_D L^2+I_{y3}]\alpha^{\cdot\cdot} \quad (7)$$

$$\frac{\partial KE}{\partial\alpha}=m_i\theta^{\cdot 2}(\mathrm{L}_1+L_2\cos\alpha)L_2\sin\alpha\ +I_{x2}\cos\alpha\sin\alpha\theta^{\cdot 2}$$
$$+I_{z2}\cos\alpha\sin\alpha\theta^{\cdot 2}+m_D(L_1+L\ \cos\alpha)L_2\sin\alpha\theta^{\bullet 2}+\frac{1}{2}I_{x3}(\cos\alpha)^2\theta^{\bullet 2}+I_{z3}\theta^{\cdot}\cos\alpha\ (\theta^{\cdot}\sin\alpha+\Omega) \quad (8)$$

$$\left(\mathrm{P.E}\right)_i=-[m_D g\,\mathrm{L}+m_i g\,\mathrm{L}_2]\sin\alpha\ \cos\theta \quad (9)$$

Hence,

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$$\ddot{\theta} = \frac{-2((I_{z3}+I_{z2})-(I_{x2}+I_{x3}))\sin\alpha\cos\alpha\dot{\theta}\dot{\alpha} + 2\dot{\theta}\dot{\alpha}\sin\alpha(m_i L_2(L_1+L_2\cos\alpha) - m_D L(L_1+L\cos\alpha)) + }{[l_1^2 m_O + I_{xo} + m_i(L_1+L_2\cos\alpha)^2 + ((I_{x2}+I_{x3})\cos\alpha^2) + m_D(L_1+L\cos\alpha)^2 + ((I_{z3}+I_{z2})\sin\alpha^2)]}\ldots$$

$$\ldots\frac{+\sin\theta(m_D(L_1+L\cos\alpha) + m_i(L_1+L_2\cos\alpha) + m_o l_1) - I_{z3}\dot{\alpha}\Omega\cos\alpha + T_D}{[l_1^2 m_O + I_{xo} + m_i(L_1+L_2\cos\alpha)^2 + ((I_{x2}+I_{x3})\cos\alpha^2) + m_D(L_1+L\cos\alpha)^2 + ((I_{z3}+I_{z2})\sin\alpha^2)]} \quad (10)$$

And,

$$\ddot{\alpha} = \frac{((I_{z3}+I_{z2})-(I_{x2}+I_{x3}))\sin\alpha\cos\alpha + \dot{\theta}^2\sin\alpha(m_i L_2(L_1+L_2\cos\alpha) - m_D L(L_1+L\cos\alpha)) + }{[m_i(L_2)^2 + m_D(L)^2 + I_{y3} + I_{y2}]}\ldots$$

$$\ldots\frac{+((m_D L) + (m_i L_2)) g\sin\alpha\cos\theta) + I_{z3}\dot{\alpha}\Omega\cos\alpha + [2K_m i - B_m\dot{\alpha}]}{[m_i(L_2)^2 + m_D(L)^2 + I_{y3} + I_{y2}]} \quad (11)$$

3.2. Linearized Model For The System

By linearization for equation 2.10 and 2.11

$$\ddot{\theta} = \frac{\overbrace{g(m_D L_1 + m_i L_1 + m_o L_1)}^{K1}\theta - I_{z3}\dot{\alpha}\Omega + T_D}{\underbrace{[l_1^2 m_O + I_{xo} + m_i(L_1)^2 + (I_{x2}+I_{x3}) + m_D(L_1)^2]}_{K2}}$$

$$\ddot{\alpha} = \frac{I_{z3}\dot{\theta}\Omega + [2K_m i - B_m\dot{\alpha}]}{\underbrace{[I_{y3}+I_{y2}]}_{K3}}$$

$$\dot{i} = \frac{V_a - K_e\dot{\alpha} - R_a i}{L_a}$$

where K1 = 3.234 K2 =0.027225 K3=0.00154 TD=0

3.2.1. System in State Space

$$\dot{x} = Ax + Bu$$

$$\dot{y} = Cx + Du \qquad when \rightarrow x = \begin{bmatrix}\theta \\ \dot{\theta} \\ \dot{\alpha} \\ i\end{bmatrix}$$

$$then,,, \begin{bmatrix} \theta^{\bullet} \\ \theta^{\bullet\bullet} \\ \alpha^{\bullet\bullet} \\ i^{\bullet} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{K1}{K2} & 0 & \frac{-0.624}{K2} & 0 \\ 0 & \frac{0.624}{K3} & \frac{-Bm}{K3} & \frac{2Km}{K3} \\ 0 & 0 & \frac{-Ke}{La} & \frac{-Ra}{La} \end{bmatrix} * \begin{bmatrix} \theta \\ \theta^{\bullet} \\ \alpha^{\bullet} \\ i \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \frac{1}{La} \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 0 \\ Va \end{bmatrix}$$

$$[\theta] = [1 \quad 0 \quad 0 \quad 0] * \begin{bmatrix} \theta \\ \theta^{\bullet} \\ \alpha^{\bullet} \\ i \end{bmatrix} + [0 \quad 0 \quad 0 \quad 0] * \begin{bmatrix} 0 \\ 0 \\ 0 \\ Va \end{bmatrix}$$

3.2.2. System in frequency domain

$$T.f = \frac{714187}{S^4 + 3315\ S^3 + 61063\ S^2 + 30246328\ S\ - 6164425}$$

4. Control algorithm

Bicycle is a non-linear system with disturbance. Conventional (PID), (LQR) and (LQR + I) are applied to the system and their performances are compared.

4.1. (PID) controller

The proportional –integral -derivative (PID) controller is the most common form of feedback controllers .The control signal in PID is sum of P-term (proportional to the error), the I-term (proportional to the integral of the error) and the D-term (proportional to derivative of the error). Gains for PID controllers were obtained using trial and error method.

$$u(t) = k_p(\mathrm{t}) + k_i \int_0^t e(\tau)d\tau + k_d \frac{de}{dt} \qquad \text{When, } k_p = 15, k_i = 1, k_d = 5$$

Linearized system step response (roll angle -time) using PID controllers are shown in figure 5. And Table 1.The input voltage controlled by PID controller indicates the power consumption. Input voltage is shown in figure 6.

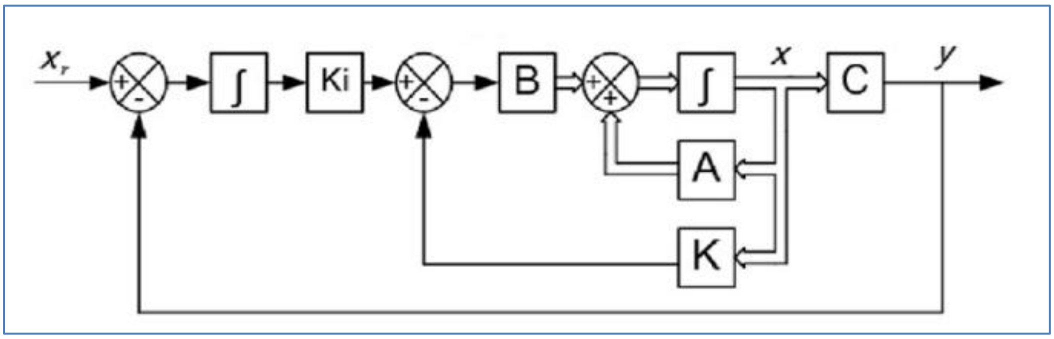
4.2. Linear quadratic regulator plus integral action

LQR+I is improved conventional LQR optimal controller by add integral action to eliminate the steady state error. LQR is optimally determines the gains by compromising the state and control-input cost. The LQR cost function is expressed by

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

The Q and R matrices are the state and control weighting matrices, and the input u=-$K\,x$. The basic approach in integral feedback is to create a state within the controller that computes the integral of the error signal figure.4, which is then used as a feedback term. This can be achieved by augmenting the matrix describe the system with integral state z:

$$\frac{d}{dt}\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} Ax+Bu \\ y-r \end{bmatrix} = \begin{bmatrix} Ax+Bu \\ Cx-r \end{bmatrix}$$



,

Figure 4. LQR optimal controller by add integral action

$$u = -K(x - x_e) - K_i z$$

$$z^{\bullet} = y - r$$

$$u = -K(x - x_e) - K_i \int y - r$$

The optimal gain matrix K is determined by solving the Algebraic Ricotta equation

$$A^T + PA + Q - PBR^{-1}B^T P = 0 \quad (12)$$

$$K = R^{-1}B^T P \quad (13)$$

The weighting matrices (Q and *R)* are selected based on the degree of importance. Since the roll angle is the most important state the matrixes are chosen as follow:

- For conventional LQR weighting matrices are:

  Q= [10 1 1 1] and R= [1] By substation in Eq.(12) and Eq.(13)

  (before implement integral term [$K_i \int$ y-r])

  Hence, K= [17.9 .028 .0753 .015]

- For LQR +I weighting matrices are:

  Q= [10 1 1 1 100] and R= [1] By substation in Eq.(12) and Eq.(13)

  (after implement integral term [$K_i \int$ y-r])

  Hence, K= [42.71 .0643 .38 .151 10]

Linearized System step response (roll angle - time) using (LQR and LQR+I) controllers are shown in Figure 5. And Table 1. Input voltage controlled by (LQR and LQR+I) controller indicates the power consumption. Input voltage is shown in figure 6.

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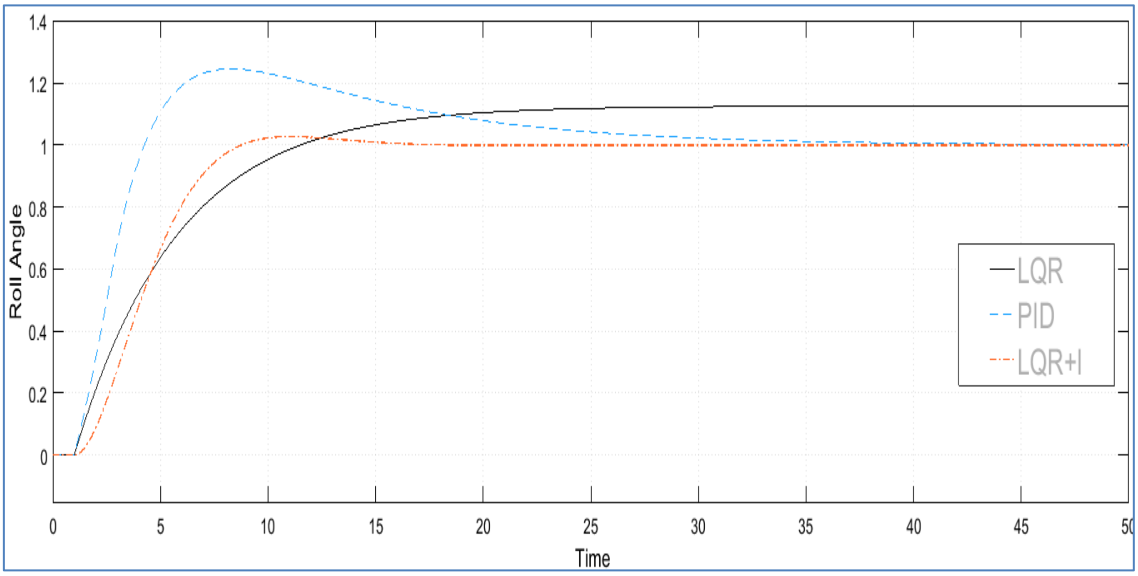

Figure 5. Step response for (LQR, PID and LQR+I)

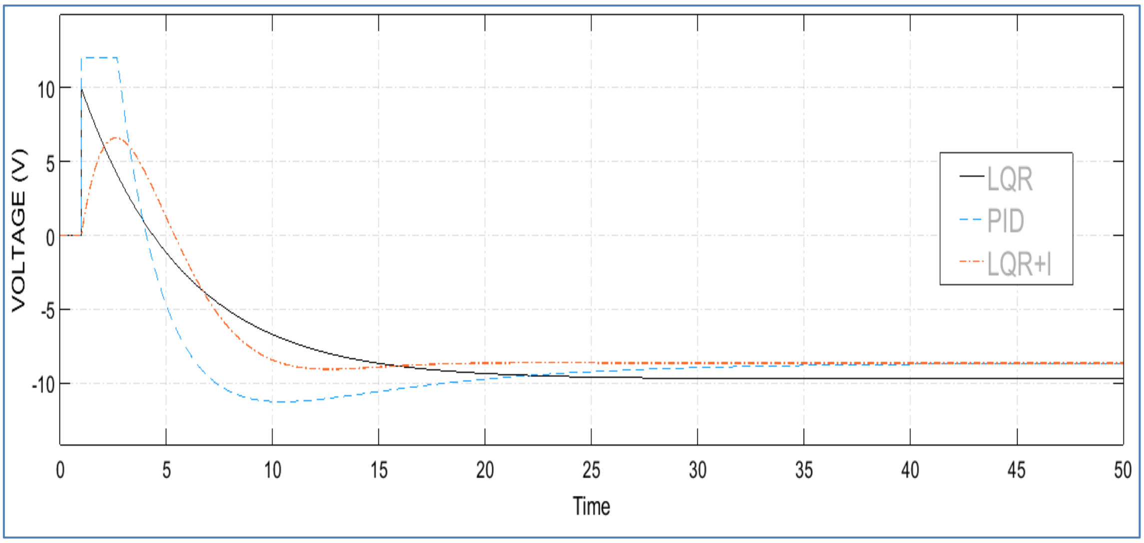

Figure 6. Input voltage applied by (LQR, PID and LQR+I))

4.3. Simulation results analysis

The Simulink model created was based on the simplified governing equation of motion derived from the summation of moments acting about the bicycle's x-axis. Figure 5 shows the step response for the system using three different controllers, Figure 6 shows the input voltage applied by different controllers .controller performance comparison is state in Table 1

Table 1. Simulation results of the system using PID,LQR and LQR+I .

	Overshoot	Rise Time	Settling Time	steady state error	max voltage
PID	24.39%	2.4 s	30.28 s	.002%	12 v
LQR	12.6%	7.24 s	18.9 s	15.6%	10.1 v
LQR+I	2.7%	4.8s	13.03 s	.0015%	6 v

4.4. Controller robustness verification

The effects of decreasing the gyro flywheel angular velocity was explored using simulation to shows the effect of decreasing the gyro flywheel angular velocity from a value of 400 rad/s to 200 rad/s while keeping all other parameter constant ,simulation for each controller shown in below figures(7,8,9)

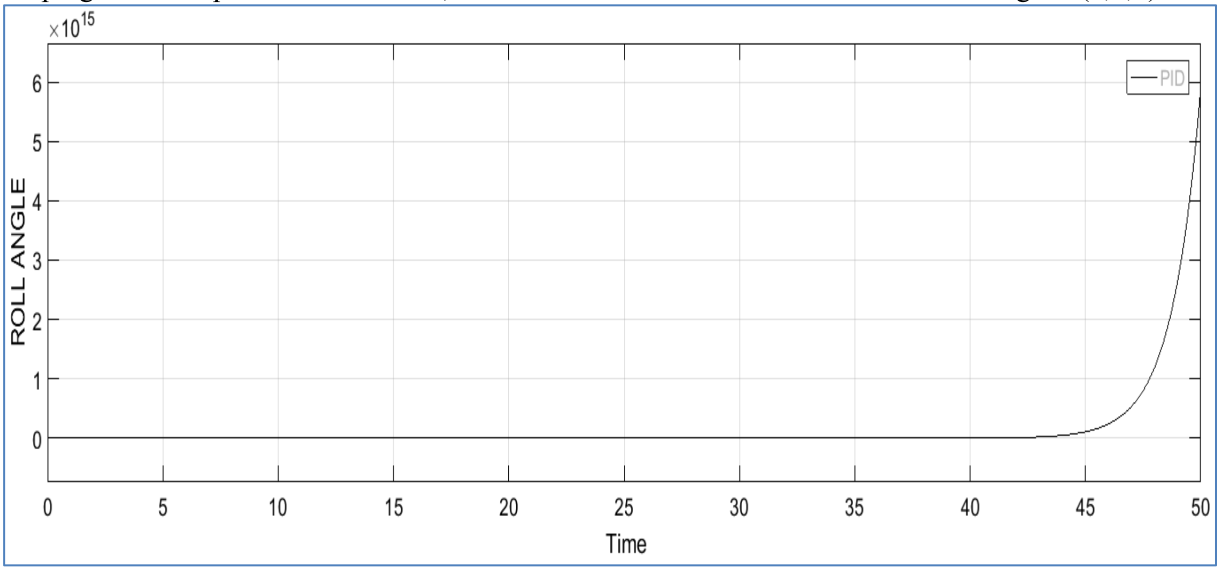

Figure 7. System response using PID controller (at w 200 rad/s)

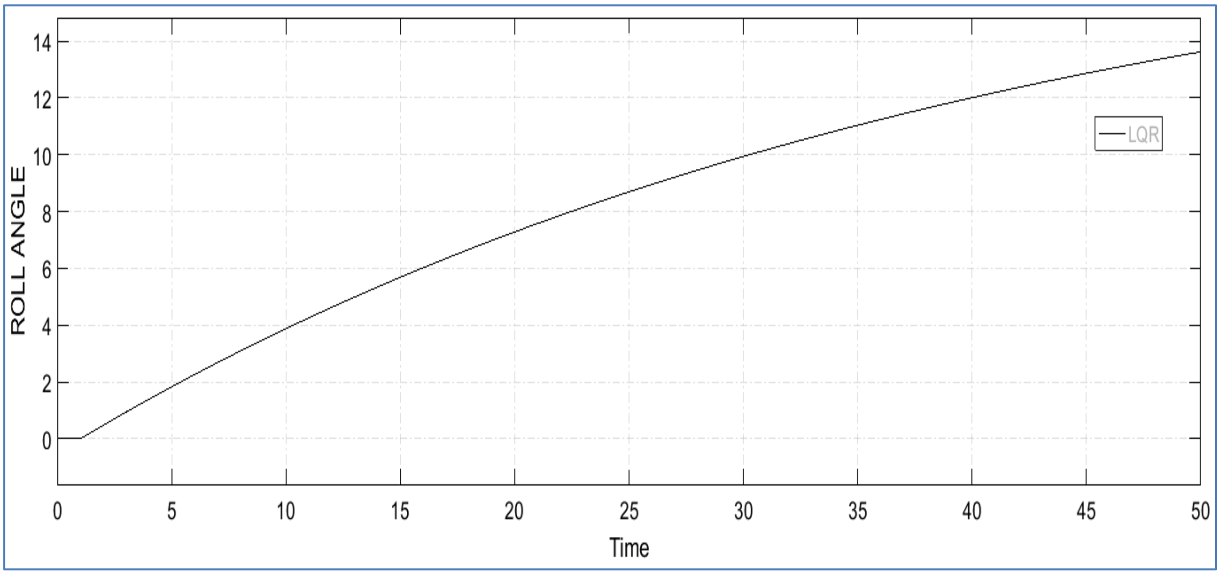

Figure 8. System response using LQR controller (at w 200 rad/s)

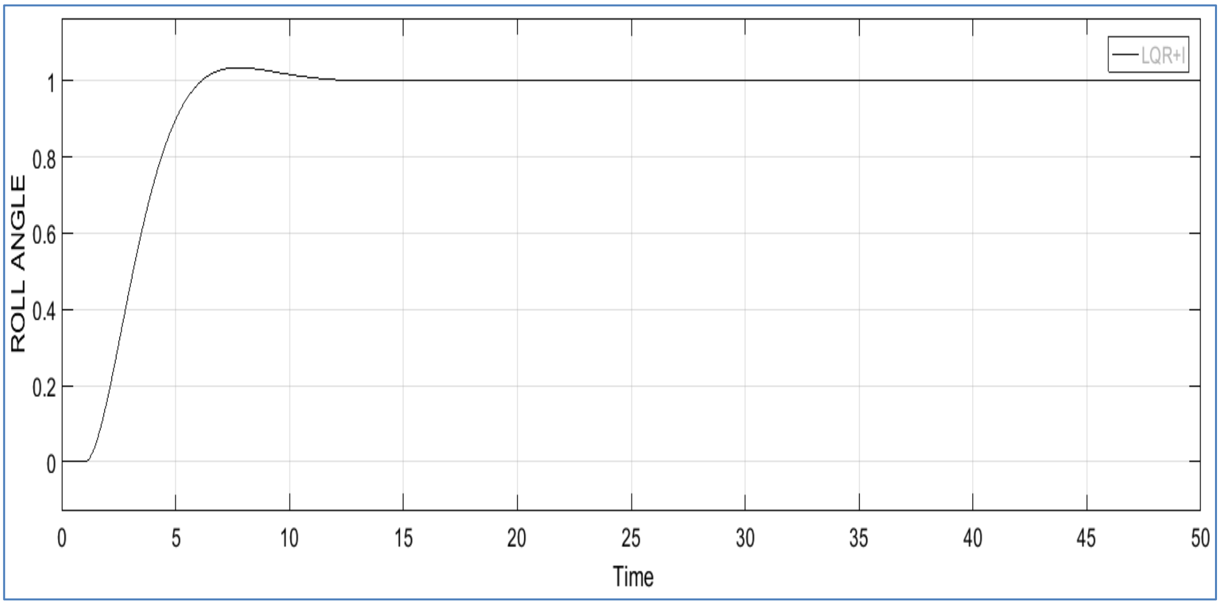

Figure 9. System response using LQR+I controller (at w 200 rad/s)

Figure 7 shows the step response for, the system using PID after of decreasing the gyro flywheel angular velocity from 400 rad/s to 200 rad/s .the result shows the system goes unstable consume high power, Figure 8 shows the step response for the system using LQR, the result shows the system goes unstable .Figure 9 shows the step response for the system using LQR+I, The result shows the system still stable which indicate controller robustness.

4.5. Experimental results

For sake of simulated system validation the simulation result same PID controller was applied to test rig and result was plotted to online arduino graphical plotter as shown in Figure.10. And Table 2

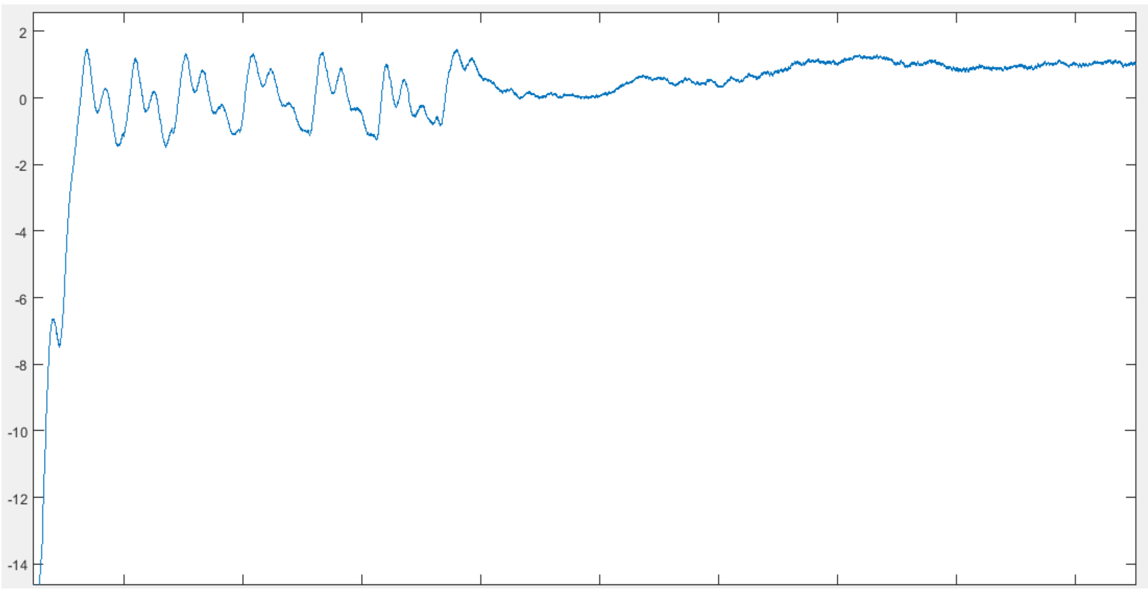

Figure 10. Experimental responses with PID controller

Table 2. Experimental data with PID controller

	PID system result
Max error	1.8 DEGREE
Over shoot	16 %
Steady state error	1 DEGREE
Rising time	5 S
Settling time	24 S

5. Conclusion

A bike is proposed in this paper as inverted pendulum model. Single control moment gyro designed for bicycle balancing. Dynamic model of the bicycle is derived by using Lagrangian's equation of motion. Three different controllers were applied to the system (PID, LQR, LQR+I) and performance was compared for each one by simulations. The (LQR + I) controller shows better performance than conventional PID and LQR. The (LQR+I) have less over shoot and less steady state error, that's led to better system performance during use of two-wheel bicycle.

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