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Intelligent feedback stabilization of MIMO systems based on additively decomposable property

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Abstract. Recently, stability of nonlinear multi-input, multi-output (MIMO) system has attracted a significant number of researchers specially, due to the large development in practical implementation of nonlinear system with uncertainty. In this paper, using additively decomposable property based on the structure of a LUR'E problem, an intelligent Takagi-Sugeno fuzzy block is added in the system feedback. The new proposed system includes all the system nonlinearities as result it grantees system absolute stability. A simulation result of the new algorithm for MIMO system is introduced.

1. Introduction

Recently with the large developments in modern control systems, they become more complex, multivariable and even more nonlinear. The stability of these systems is a challenging problem [1-3]. Many methods have been used in the literature to tackle this problem [4-14], the most common methods are Lyapunov, Popov, Circle criteria, describing function, etc. These methods use qualitative analysis to test the global stability by studying the system behaviour. However, many of these nonlinear systems are approximately linear in the operating region, the saturation outside this region causes system nonlinearity. This includes linear systems when input saturation is considered. Fuzzy systems are considered as universal approximators, which use the approximate reasoning method [15, 16] . Using additively decomposable property [17] based on the structure of LUR'E problem of a T–S fuzzy system [2]. The MIMO systems then can be split into a linear and a nonlinear part. In this paper, an intelligent fuzzy feedback controller is imposed in the feedback loop of the system which tunes using circle criteria theory [16] to include all the linearity of the system as result it grantees the system absolute stability.

This paper is organized in four sections. Section one introduces the problem discussed. Section two; defines the problem and the analysis which leads to the proposed algorithm.
An example to proof the idea of the algorithm and its simulation result are shown in section three. Finally section four concludes the proposed algorithm.

2. Problem Definition and the proposed method analysis

This paper discusses the complicity of MIMO systems that have complex behavior phenomena, such as multiple equilibrium points which cannot be detected using conventional methods. The proposed method is based on LUR'E problem [1]. The new method decomposes the T–S fuzzy system to a linear and a feedback nonlinear part as shown in Figure 1.

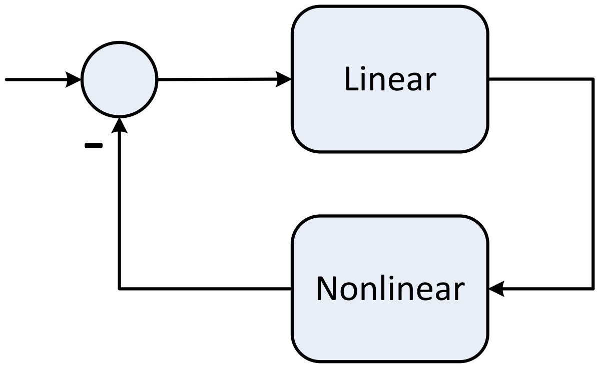

Figure 1. The system Decomposition

This split is based on additively decomposition technique which allows the use of the multivariable circle criterion. The idea is that the T-S Fuzzy system is added in the feedback loop of the system as shown in Figure 2. It has been tuned to ensure that the system nonlinearity lays in a bounded sector according to circle criterion theory, and then the system is absolute stable. Consider a T-S fuzzy system has a two-input one-output system. The inputs are the error signal *e(t)* and the system output signal *y(t)* while the output is the feedback signal *φ(t)*.

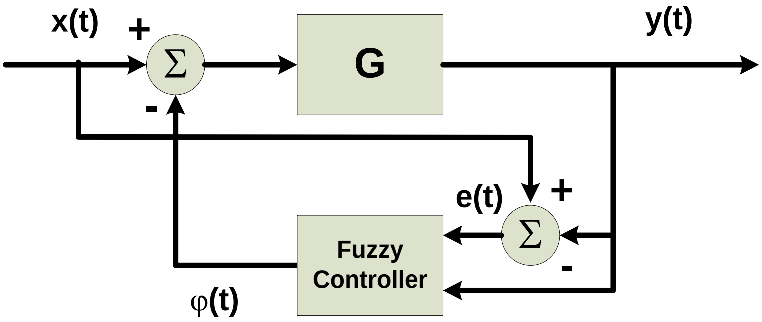

Figure 2. Adding a fuzzy controller in the feedback loop

The consequent part of the i^{th} fuzzy rule φ^c_i is a linear analytic function of the input:

$$\varphi_i^c(t) = \delta_i M y(t) \tag{1}$$

where δ_i takes the values of -1, 0, or 1
$y(t)$ the system input
M fuzzy parameter to be tuned.

The fuzzy controller is tunes by adjusting the universe of discourse limits which affect the system nonlinearity.
As a result for a given T-S fuzzy system the output consequent part can be expressed as follow [2]:

$$\dot{y} = \sum_{i=0}^{N} \delta_i(y) M_i y \tag{2}$$

where N the rules number
M_i a tuned parameter for the i^{th} rule

In order to test and ensure the system stability, the circle criterion theory [3, 4] will be considered.
Let's rewrite Equation (2) so that:

$$\dot{y} = \sum_{i=1}^{N} \{M_i y - [(1-\delta_i(y))M_i y]\} \tag{3}$$

Consider a closed loop system shown in Figure 3. According LUR'E formulation problem and by comparing Equations (2) and (3) then one can separates linear part and nonlinear part. The system process to be controlled G as a linear time-invariant part with a fuzzy block in the feedback path as a nonlinear part φ(t) which can be expressed as:

$$\varphi = \sum_{i=1}^{N} [(1-\delta_i(y))M_i y] \tag{4}$$

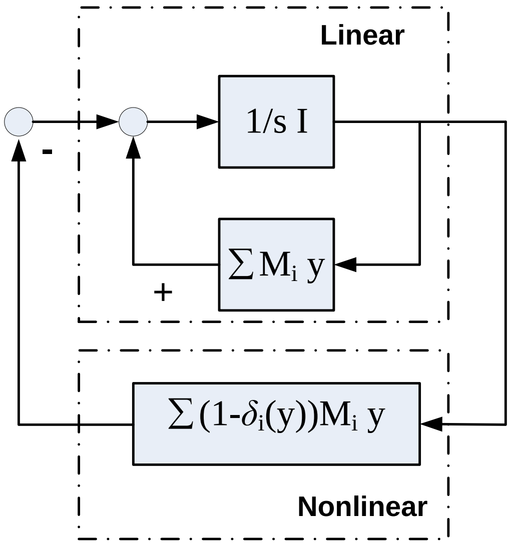

Figure 3. The considered problem according to LUR'E formulation.

Using circle criterion theory [5] as shown in Figure 4, if the function *φ(t)* can be bounded within a certain region such that:

α, β, a, b, (β>α, a<0<b) for which:

$$\alpha y \le \varphi(y) \le \beta y \tag{5}$$

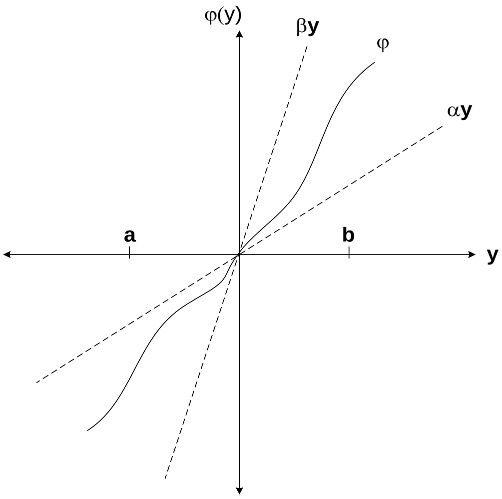

Figure 4. Circle criterion Bounded region

for all $t \geq 0$ and all $y \in [a, b]$ then: If $\alpha y \leq \varphi(y) \leq \beta y$ is true for all $y \in (-\infty,\infty)$ then the sector condition holds globally and the system is "absolutely stable". The idea is that no detailed information about nonlinearity is assumed, all that known it is that φ satisfies this condition [1]. As a result, given the Nyquist plot of the proposed system transfer function, then SISO system is absolute stable it does not enter the forbidden region left to the line passing through $\frac{-1}{\beta}$ in an anticlockwise direction as shown in Figure 5.

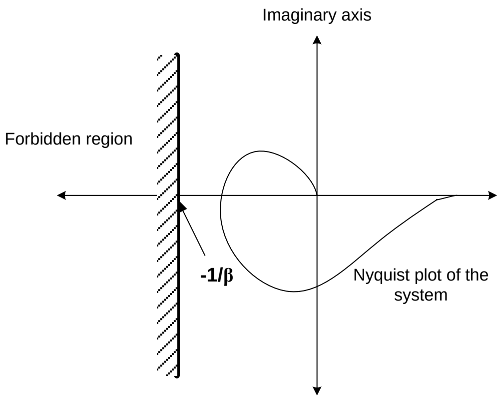

Figure 5. Nyquist plot of the proposed system.

Otherwise, the system can be tuned to lie in the bounded sector. This is done by tuning the fuzzy parameters of feedback block so that the system nonlinearity is bounded.

3. Extension of the proposed algorithm to MIMO systems

Given the structure of the LUR'E problem [6], and using additively decomposition technique [6, 7] , the proposed method can be valid for MIMO systems [8]
For $M_{ij} \in R_{2x2}$, is the fuzzy characteristic matrices, the system state can be written as follow:

$$\dot{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \delta_{ij}(z) M_{ij} z \tag{6}$$

$$\delta_{ij}(z) = \frac{w_{ij}(z)}{\sum_{k=1}^{m} \sum_{p=1}^{n} w_{kp}(z)}$$

where

and $w_{ij}(z)$ is the weight of the implication R_{ij} at time t
Then:

$$\dot{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ M_{ij} z - \left[\left(1 - \delta_{ij}(z)\right) M_{ij} z \right] \right\} \tag{7}$$

which shows that the first column of *Mij* depends on *i* while the second column depends on *j*
As a result, one can write the nonlinear part φ(z) such that:

$$\varphi(z) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - \delta_{ij}(z)\right) M_{ij} z \quad , \tag{8}$$

is additively decomposable, that is:

$$\varphi(z) = \varphi(z1, z2) = \varphi(z1, 0) + \varphi(0, z2) \tag{9}$$

4. The Proof using Additively Decomposable T-S Systems

Consider the state space representation of a T-S fuzzy system, with two dimension state vector. There are three different regions can be considered [6, 7]:

1. Operating regions with one active rule
2. Interpolation regions with two rules fire
3. Interpolation regions with four active rules.

The proof will be discus these three regions.

4.1. Operation region with one active rule

Let z = (z1, z2) ∈ Z such that two indices, k and l, exist so that

$$\mu_{F_k^1}(z_1)=1, \quad \mu_{F_i^1}(z_1)=0 \quad \forall\, i\neq k$$
$$\mu_{F_l^2}(z_2)=1, \quad \mu_{F_j^2}(z_2)=0 \quad \forall\, j\neq l$$

From above section the nonlinear part is given by:

$$\varphi(z)=\mu_{F_k^1}(z_1)\mu_{F_l^2}(z_2)[-\gamma_{kl}-M_{kl}z]+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z=[-\gamma_{kl}-M_{kl}z]+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z \tag{10}$$

It is also satisfy that:

$$\varphi(z_1,0)=\mu_{F_k^1}(z_1)\mu_{F_q^2}(0)\left(-\gamma_{kq}-M_{kq}\begin{bmatrix}z_1\\0\end{bmatrix}\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}\begin{bmatrix}z_1\\0\end{bmatrix}$$
$$\varphi(0,z_2)=\mu_{F_p^1}(0)\mu_{F_l^2}(z_2)\left(-\gamma_{pl}-M_{pl}\begin{bmatrix}0\\z_2\end{bmatrix}\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}\begin{bmatrix}0\\z_2\end{bmatrix}$$

Adding both equations:

$$\varphi(z_1,0)+\varphi(0,z_1)=-\gamma_{kq}-\begin{bmatrix}a_k & b_q\\c_k & d_q\end{bmatrix}\begin{bmatrix}z_1\\0\end{bmatrix}-\gamma_{pl}-\begin{bmatrix}a_p & b_l\\c_p & d_l\end{bmatrix}\begin{bmatrix}0\\z_2\end{bmatrix}+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z$$

$$=-\gamma_{kl}-M_{kl}z+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z$$
$$=\varphi(z).$$

Thus, the system is additively decomposable in the operating regions.

4.2. Interpolation regions with two active rules

The system output is calculated from two fired rules, that implies to:

$$\mu_{F_k^1}(z_1)+\mu_{F_{k+1}^1}(z_1)=1.$$

Similar to region one the nonlinear part is:

$$\varphi(z)=\mu_{F_k^1}(z_1)\mu_{F_l^2}(z_2)[-\gamma_{kl}-M_{kl}z]+\mu_{F_{k+1}^1}(z_1)\mu_{F_l^2}(z_2)[-\gamma_{k+1,l}-M_{k+1,l}z]+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z$$

and

$$\varphi(z_1,0)=\mu_{F_k^1}(z_1)\mu_{F_q^2}(0)\left(-\gamma_{kq}-M_{kq}\begin{bmatrix}z_1\\0\end{bmatrix}\right)+\mu_{F_{k+1}^1}(z_1)\mu_{F_q^2}(0)\left(-\gamma_{k+1,q}-M_{k+1,q}\begin{bmatrix}z_1\\0\end{bmatrix}\right)$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}\begin{bmatrix}z_1\\0\end{bmatrix}$$

$$\varphi(0,z_2)=\mu_{F_p^1}(0)\mu_{F_l^2}(z_2)\left(-\gamma_{pl}-M_{pl}\begin{bmatrix}0\\z_2\end{bmatrix}\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}\begin{bmatrix}0\\z_2\end{bmatrix}$$

Adding both equations:

$$\varphi(z_1,0)+\varphi(0,z_1)=\mu_{F_k^1}(z_1)\left(-\gamma_{kq}-\begin{bmatrix}a_k & b_q\\c_k & d_q\end{bmatrix}\begin{bmatrix}z_1\\0\end{bmatrix}\right)+\mu_{F_{k+1}^1}(z_1)\left(-\gamma_{k+1,q}-\begin{bmatrix}a_{k+1} & b_q\\c_{k+1} & d_q\end{bmatrix}\begin{bmatrix}z_1\\0\end{bmatrix}\right)$$
$$+\left(-\gamma_{pl}-\begin{bmatrix}a_p & b_l\\c_p & d_l\end{bmatrix}\begin{bmatrix}0\\z_2\end{bmatrix}\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z$$

$$=-\mu_{F_k^1}(z_1)\gamma_{kq}-\mu_{F_{k+1}^1}(z_1)\gamma_{k+1,q}-\gamma_{pl}-\begin{bmatrix}\mu_{F_k^1}(z_1)a_k+\mu_{F_{k+1}^1}(z_1)a_{k+1} & b_l\\\mu_{F_k^1}(z_1)c_k+\mu_{F_{k+1}^1}(z_1)c_{k+1} & d_l\end{bmatrix}z$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z$$

$$=-\mu_{F_k^1}(z_1)\gamma_{kq}-\mu_{F_{k+1}^1}(z_1)\gamma_{k+1,q}-\left(\mu_{F_k^1}(z_1)+\mu_{F_{k+1}^1}(z_1)\right)\gamma_{pl}$$
$$-\begin{bmatrix}\mu_{F_k^1}(z_1)a_k+\mu_{F_{k+1}^1}(z_1)a_{k+1} & \left(\mu_{F_k^1}(z_1)+\mu_{F_{k+1}^1}(z_1)\right)b_l\\\mu_{F_k^1}(z_1)c_k+\mu_{F_{k+1}^1}(z_1)c_{k+1} & \left(\mu_{F_k^1}(z_1)+\mu_{F_{k+1}^1}(z_1)\right)d_l\end{bmatrix}z$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z$$
$$=\mu_{F_k^1}(z_1)[-\gamma_{kq}-\gamma_{pl}-M_{kl}z]+\mu_{F_{k+1}^1}(z_1)[-\gamma_{k+1,q}-\gamma_{pl}-M_{k+1,l}z]$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z$$

$$=\mu_{F_k^1}(z_1)\mu_{F_l^2}(z_2)[-\gamma_{kl}-M_{kl}z]+\mu_{F_{k+1}^1}(z_1)\mu_{F_l^2}(x_2)[-\gamma_{k+1,l}-M_{k+1,l}z]$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z$$

= φ(z).

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4.3. Interpolation regions with four active rules:

The system output comes from four rules fired at the same time, which implies

$$\mu_{F_k^1}(z_1)+\mu_{F_{k+1}^1}(z_1)=1$$
$$\mu_{F_l^2}(z_2)+\mu_{F_{l+1}^2}(z_2)=1$$

Similar, the nonlinear part in this region is shown:

$$\begin{aligned}\varphi(z)=&\ \mu_{F_k^1}(z_1)\mu_{F_l^2}(z_2)[-\gamma_{kl}-M_{kl}z]\\&+\mu_{F_{k+1}^1}(z_1)\mu_{F_l^2}(z_2)[-\gamma_{k+1,l}-M_{k+1,l}z]\\&+\mu_{F_k^1}(z_1)\mu_{F_{l+1}^2}(z_2)[-\gamma_{k,l+1}-M_{k,l+1}z]\\&+\mu_{F_{k+1}^1}(z_1)\mu_{F_{l+1}^2}(z_2)[-\gamma_{k+1,l+1}-M_{k+1,l+1}z]\\&+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z\end{aligned}$$

and

$$\begin{aligned}\varphi(z_1,0)=&\ \mu_{F_k^1}(z_1)\left(-\gamma_{kq}-M_{kq}\begin{bmatrix}z_1\\0\end{bmatrix}\right)\\&+\mu_{F_{k+1}^1}(z_1)\left(-\gamma_{k+1,q}-M_{k+1,q}\begin{bmatrix}z_1\\0\end{bmatrix}\right)\\&+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}\begin{bmatrix}z_1\\0\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\varphi(0,z_2)=&\ \mu_{F_l^2}(z_2)\left(-\gamma_{p,l}-M_{pl}\begin{bmatrix}0\\z_2\end{bmatrix}\right)\\&+\mu_{F_{l+1}^2}(z_2)\left(-\gamma_{p,l+1}-M_{pl+1}\begin{bmatrix}0\\z_2\end{bmatrix}\right)\\&+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}\begin{bmatrix}0\\z_2\end{bmatrix}\end{aligned}$$

Thus adding both equations, it becomes:

$$
\begin{aligned}
&\varphi(z_1,0)+\varphi(0,z_2)\\
&\quad=\mu_{F_k^1}(z_1)\left(\mu_{F_l^2}(z_2)+\mu_{F_{l+1}^2}(z_2)\right)\cdot\left(-\gamma_{kq}-M_{kq}\begin{bmatrix}z_1\\0\end{bmatrix}\right)\\
&\quad+\mu_{F_{k+1}^1}(z_1)\left(\mu_{F_l^2}(z_2)+\mu_{F_{l+1}^2}(z_2)\right)\cdot\left(-\gamma_{k+1,q}-M_{k+1,q}\begin{bmatrix}z_1\\0\end{bmatrix}\right)\\
&\quad+\mu_{F_l^2}(z_2)\left(\mu_{F_k^1}(z_1)+\mu_{F_{k+1}^1}(z_1)\right)\cdot\left(-\gamma_{pl}-M_{pl}\begin{bmatrix}0\\z_2\end{bmatrix}\right)\\
&\quad+\mu_{F_{l+1}^2}(z_2)\left(\mu_{F_k^1}(z_1)+\mu_{F_{k+1}^1}(z_1)\right)\cdot\left(-\gamma_{p,l+1}-M_{p,l+1}\begin{bmatrix}0\\z_2\end{bmatrix}\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z\\
&\quad=\mu_{F_k^1}(z_1)\mu_{F_l^2}(z_2)\cdot\left(-\gamma_{kq}-M_{kq}\begin{bmatrix}z_1\\0\end{bmatrix}-\gamma_{pl}-M_{pl}\begin{bmatrix}0\\z_2\end{bmatrix}\right)\\
&\quad+\mu_{F_k^1}(z_1)\mu_{F_{l+1}^2}(z_2)\cdot\left(-\gamma_{k,q}-M_{kq}\begin{bmatrix}z_1\\0\end{bmatrix}-\gamma_{p,l+1}-M_{p,l+1}\begin{bmatrix}0\\z_2\end{bmatrix}\right)\\
&\quad+\mu_{F_{k+1}^1}(z_1)\mu_{F_l^2}(z_2)\cdot\left(-\gamma_{k+1,q}-M_{k+1,q}\begin{bmatrix}z_1\\0\end{bmatrix}-\gamma_{pl}-M_{pl}\begin{bmatrix}0\\z_2\end{bmatrix}\right)\\
&\quad+\mu_{F_{k+1}^1}(z_1)\mu_{F_{l+1}^2}(z_2)\cdot\left(-\gamma_{k+1,q}-M_{k+1,q}\begin{bmatrix}z_1\\0\end{bmatrix}-\gamma_{p,l+1}-M_{p,l+1}\begin{bmatrix}0\\z_2\end{bmatrix}\right)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z\\
&\quad=\mu_{F_k^1}(z_1)\mu_{F_l^2}(z_2)\cdot(-\gamma_{kl}-M_{kl}x)\\
&\quad+\mu_{F_{k+1}^1}(z_1)\mu_{F_l^2}(z_2)\cdot(-\gamma_{k+1,l}-M_{k+1,l}z)\\
&\quad+\mu_{F_k^1}(z_1)\mu_{F_{l+1}^2}(z_2)\cdot(-\gamma_{k,l+1}-M_{k,l+1}z)\\
&\quad+\mu_{F_{k+1}^1}(z_1)\mu_{F_{l+1}^2}(z_2)\cdot(-\gamma_{k+1,l+1}-M_{k+1,l+1}z)\\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{n}M_{ij}z\\
&\quad=\varphi(z)
\end{aligned}
$$

From above analysis, the fuzzy system is also additively decomposable in this region and, therefore, it is proved that the system is additively decomposable in all the state space. However, that the superposition principle does not apply in general:

$$\varphi(z_1+a,z_2+b)\neq\varphi(z_1,z_2)+\varphi(a,b)$$

From the above discussion, Similar analysis method to the one used on SISO system ac be extended to MIMO systems. For each feedback path, a fuzzy block will be added in the MIMO system as shown in Figure 6. The system then is globally stable, and all the nonlinearities can be included within a bounded sector.

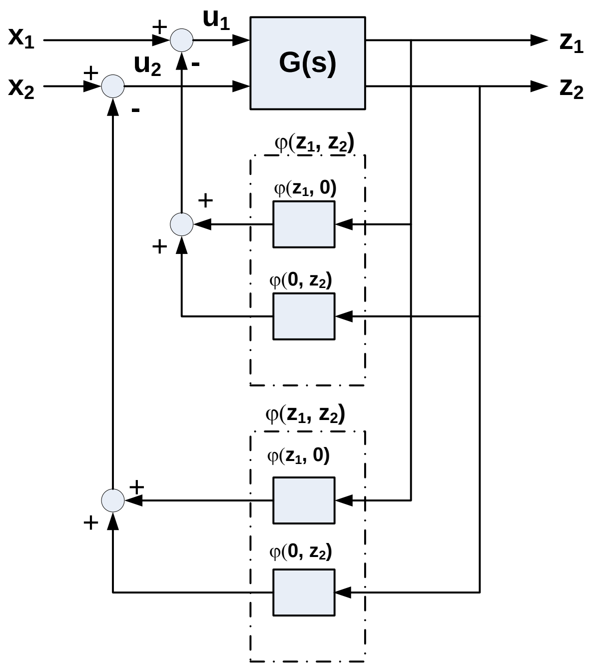

Figure 6. The proposed method applied to MIMO system

For closed loop system, consider the classical system shown in Figure7 (a).

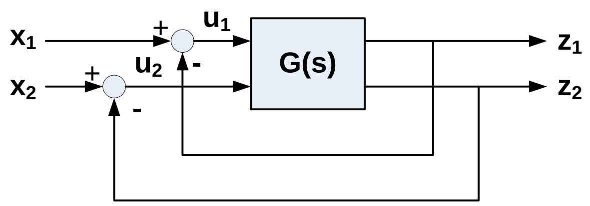

Figure 7(a). Classical MIMO system

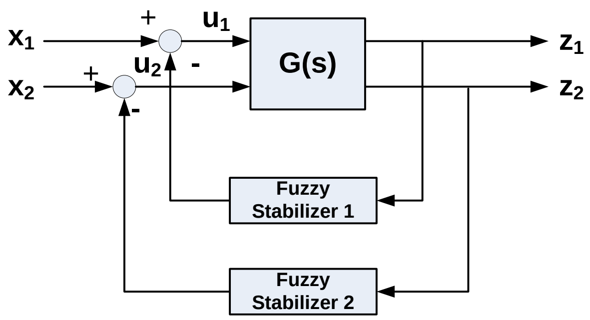

Figure 7(b). MIMO system with fuzzy stabilizers

5. Simulation Results

In order to test the proposed method, consider the state space representation of the given MIMO (two inputs, two outputs) system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -7 & -7 & -50 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Using MATLAB™ software, the transfer functions relates the inputs to the outputs have been calculated as follow:
For G11 relates first input to the first output

$$G_{11} = \frac{s^2}{s^3 + 7s^2 + 7s + 50}$$

G12 relates the first input to the second output

$$G_{12} = \frac{7s + 50}{s^3 + 7s^2 + 7s + 50}$$

Similar, G21 relates the second input to the first output

$$G_{21} = \frac{-s}{s^3 + 7s^2 + 7s + 50}$$

Finally, G22 relates the second input to the second output

$$G_{22} = \frac{s^2 + 7s}{s^3 + 7s^2 + 7s + 50}$$

The simulation of system output is shown in Figure 8.

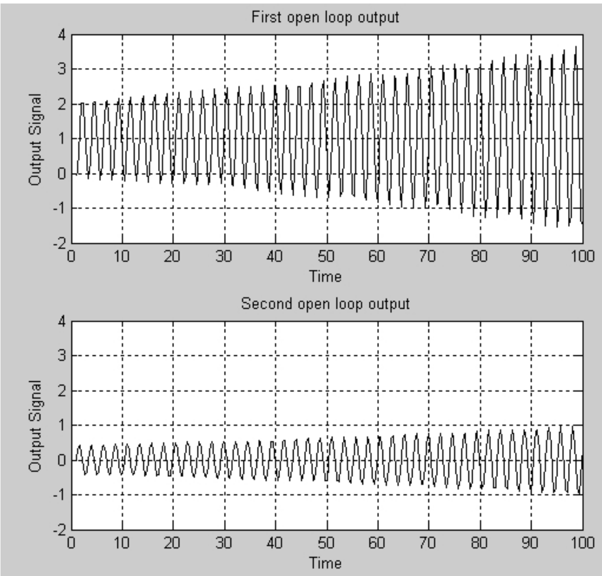

Figure 8. The proposed system output simulation

Using the above method and with aid of Nyquist plot, the forbidden region for each transfer function and consequently the bounded sector for each fuzzy block have been calculated using Figure 9.

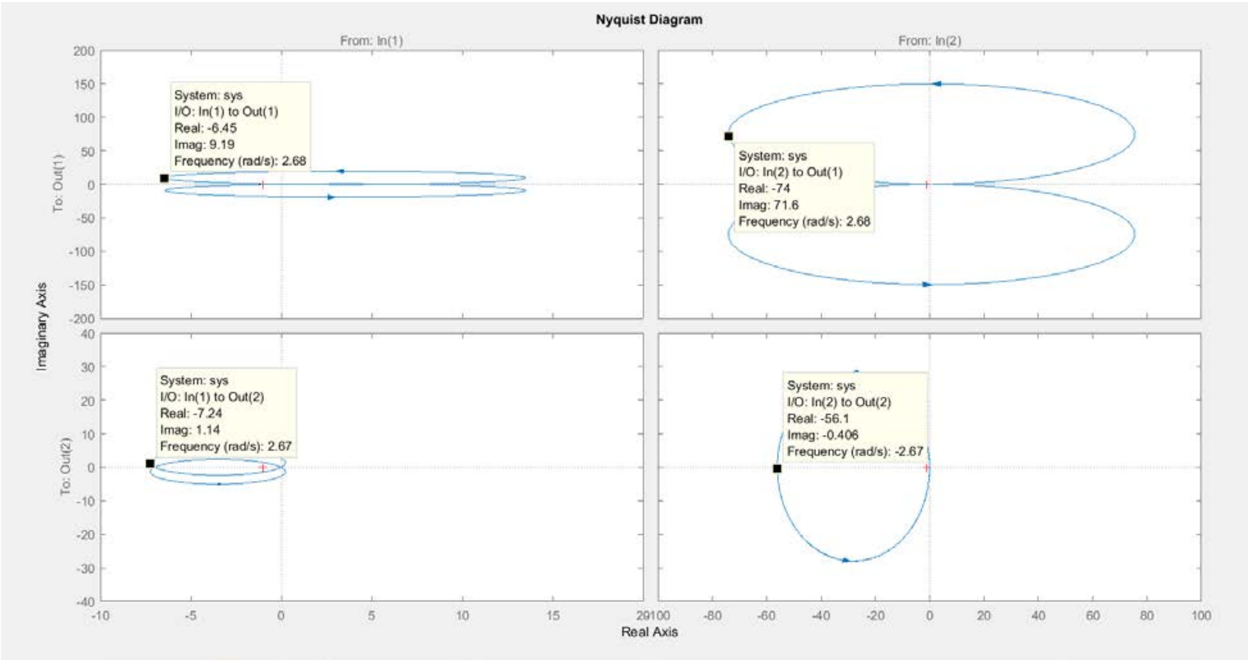

Figure 9. The Nyquist diagram for the MIMO proposed system

As a result, the forbidden regions are as follow:

For G11, $-\frac{1}{\beta_{11}}$ = - 6.45, then β_{11}= 0.155
For G12, $-\frac{1}{\beta_{12}}$ = - 7.24, then β_{12}= 0.138
For G21, $-\frac{1}{\beta_{21}}$ = - 74, then β_{21}= 0.0135
For G22, $-\frac{1}{\beta_{22}}$ = - 56.1, then β_{22}= 0.0178

Each fuzzy block has been tuned such that the boundary of the input and output universe of discourse meet the calculated parameters.

The simulation results introduced in Figure 10 show the feedback intelligent blocks enhance the system stability

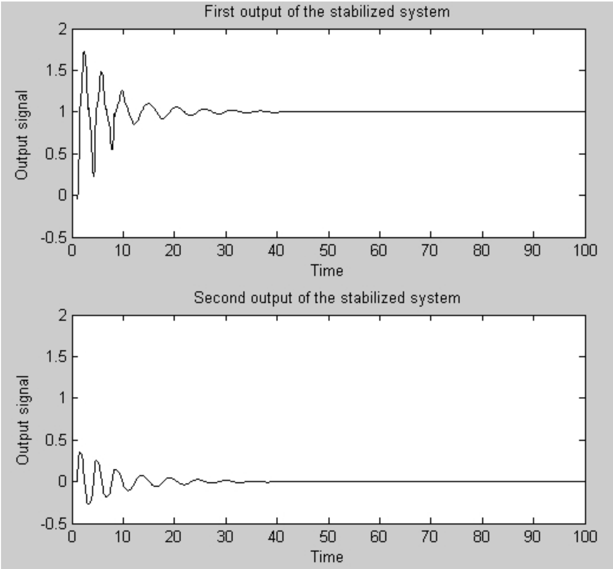

Figure 10. The simulation of the system output after stabilization

The simulation shows that using the proposed method by keeping the system nonlinearities in bounded sector can grantee the system global stability.

6. Conclusion

In this paper, using additively decomposable property based on the structure of a LUR'E problem, a proposed method of check global stability of nonlinear systems has been expanded to grantee stability of MIMO nonlinear systems. The proposed method adds an intelligent Takagi-Sugeno fuzzy block to every feedback path. The new proposed system includes all the system nonlinearities as result it grantees system absolute stability. A simulation result of the new algorithm for MIMO system is introduced.

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