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Vibration analysis of composite wing with geometric and material coupling

Vibration analysis of composite wing with geometric and material coupling

A Magdy[1], M Kamel[1], M A Elshafei[1] and M Kassem[1]

[1]Department of Aircraft Mechanics, Military Technical College, Egypt

E-mail: a.magdy@mtc.edu.eg

Abstract. Composite wing design is complicated but inevitable to enlighten modern airplanes while maintaining the required performance. Using the dynamic transfer method, this paper discusses intensively the dynamic characteristics of a cantilever composite wing with both torsion and bending coupling to represent both material and geometric coupling. The governing differential equations are obtained based upon the principle of Hamilton and are solved analytically using a harmonic oscillation assumption. For this purpose, a MATLAB code is developed and results are validated in comparison with published work. Such a comparison shows a good agreement between both results. Finally, a parametric study is carried out to show the influence of the variation of both geometric coupling and torsion bending coupling rigidity on the free vibration analysis of the composite wing. The study shows the crucial effect of both factors on the dynamic behavior of the composite wing. The current research can be considered as a base for aeroelasticians while designing composite structures.

1. Introduction

Free vibration analysis of composite beams has many practical engineering applications in mechanical and aerospace structural designs such as spacecraft, rotor blades of helicopters, some parts of robots and aircraft wings. It's a very important prerequisite when carrying out a response and aeroelastic analysis. Controlling and improving the dynamic effects of a structural element is a very important desirable target for all designers, this can be done in composite materials by changing the stacking sequence and ply orientation of fibers which in turns control the coupling between different modes of deformation due to the anisotropic properties of composite materials. Due to this coupling, composite and metallic structures have two different free vibration analyses [1].

The vibration analysis of a composite cantilever beam is chosen because of its important applications in the idealization of the structural elements such as the composite wing of high aspect ratio. The characteristics of the free vibration analysis of composite beams can be controlled favorably by selecting a suitable stacking sequence and ply orientation [2].

The coupling of composite materials occurs due to two main sources, the geometric coupling is one of these sources which comes from the distance between the shear center and the cross-sectional centroid of the beam. This kind of coupling occurs in the asymmetric cross-sections, of composite beams and metallic beams. Because of the inertial nature of this coupling bending and torsion motions under static loads are uncoupled.

The other source of coupling occurs only in composite materials because its anisotropic properties arise from fiber orientations. This coupling depends only on material properties so, it is called material coupling, bending and torsion motions can occur under static and dynamic loads. For double symmetric cross sections, the coupling depends only on the material coupling and there is no effect for the geometric one [1]. Studying the free vibration analysis of composite beams has attracted the interest of many researchers due to their practical importance and potential advantages stated above. There are many analytical, numerical and experimental approaches are used for the vibration analysis of coupled bending and torsion in composite beams.

Hodges et al. [3] has proposed methods to predict composite beams' natural frequencies and mode shapes, two qualitatively different methods are used to evaluate the sectional elastic constants: simple analytical methods to calculate the stiffnesses that are given in a closed form and a detailed cross-sectional finite element method. They solved the equations of motion by an exact integration method and by a mixed finite element method. Eslimy-Isfany and Banerjee [4] developed a theory suitable for either open-section or closed-section composite beams of any cross-section, stacking sequence, and boundary conditions. They calculated the response of composite beams under the action of deterministic and random loads. Coupling between bending and torsion deformation, which comes from the anisotropic properties of the fibrous composite, investigated in their work is the material coupling.

RKaya and Ozgumus [5] used Hamilton’s principle to derive the governing equations of motion of composite Timoshenko beams and solved them by using the differential transformation method to investigate the effects on natural frequencies due to bending-torsion coupling, the axial force and the slenderness ratio are studied. Mirtalaie et.al [6] presented a numerical solution using the method of differential quadrature to solve the coupling of torsional and lateral vibrations by modeling it with coupling rigidity of the bending twisting material. They took into account the effect of material coupling, shear deformation and rotary inertia.

Lottati [7] made an investigation for a swept forward composite cantilever rectangular wing carrying a pylon at the wingtip and a fuselage at its semi span assuming that the case of an unrestrained vehicle. He analyzed the variation of the divergence and flutter velocities due to the warping effect. He deduced that the aeroelastic characteristic changes with the warping effect. Lottati [8] made a similar analysis for the same wing without any load to obtain its divergence behavior and the aeroelastic flutter. He analyzed the variation in the influence of the torsion-bending stiffness coupling of that wing on the critical dynamic pressure of the flutter and divergence. He indicated that increasing the flutter velocity tends to decrease the divergence speed.

Kashani and Hashemi [9] made a free vibrations study for single delamination composite beams undergoing bending-torsion coupling by using the finite element method to analyze the delaminated beams subjected to tip moment and axial compressive load. He obtained a linear eigenvalue problem by discretizing the beam along its span to find the natural frequencies and mode shapes from “free mode” and “constrained mode”.

In this paper, the dynamic transfer method is used to conduct free vibration analysis for a cantilever composite wing to calculate its natural frequencies and mode shapes and comparing them with a published work to validate this study then a parametric study is made to show the effect of geometric and material parameters on the natural frequency of the wing.

2. Theory

2.1. Formulation of the governing equations

In this part, the governing equations of a composite beam coupled in bending and torsional vibration are discussed as an idealized cross-section for the aircraft wing as shown in fig. 1. Y-axis represents the elastic axis of the wing, x_α is the distance between the elastic axis and the mass axis which represents the loci of the geometric mass centers of the wing cross-sections, EI is the bending rigidity, GJ is the torsion rigidity, K is the torsion bending coupling rigidity, m is the mass per unit length, I_α is mass moment of inertia per unit length about the Y-axis, L is the length of the wing, h(y , t) is the deflection out of the plane $\psi(y\,,t)$ is the angle of rotation about the Y-axis. The most important parameters are K and x_α because if x_α exists means there is no coincident between the shear center axis and the mass axis (geometric coupling) and K is an indication to the material coupling [1].

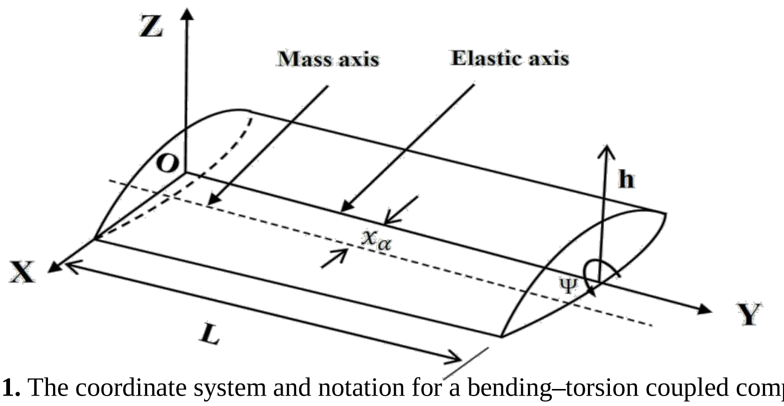

Figure 1. The coordinate system and notation for a bending–torsion coupled composite beam

It should be remembered that the presented theory does not involve the influence of rotatory inertia and shear deformation, it is applicable to analyze composite beams with high length to cross-sectional dimension ratio, for example, aircraft wing with high aspect ratio [10, 11]. The definition of kinetic energy and the potential energy for a composite beam subjected to torsion bending coupling are respectively given by [1].

$$T = \frac{1}{2}\int_0^L [m(\dot{h}) - 2mx_\alpha \dot{h}\dot{\psi} + I_\alpha\left(\dot{\psi}\right)^2]dy \quad (1)$$

$$V = \frac{1}{2}\int_0^L [EI(h'') + 2Kh''\psi' + GJ(\psi')^2]dy \quad (2)$$

where an over dot and a prime denote partial differentiation with respect to time t and location y respectively. Using the definition of Hamilton's principal

$$\delta\int_{t1}^{t2}(T - V)dt = 0 \quad (3)$$

where the integration is made between the time interval from t_1 to t_2 and δ is the variational operator. Using the definitions of kinetic energy and potential energy to substitute into eq. (3), The governing equations in the free vibration analysis are given by:

$$EIh'''' + K\psi''' + m\ddot{h} - mx_\alpha\ddot{\psi} = 0 \quad (4)$$

$$GJ\psi'' + Kh''' - I_\alpha\ddot{\psi} + mx_\alpha\ddot{h} = 0 \quad (5)$$

Due to the existence of coupling, shearing force (F), bending moment (M_b) and torque moment (M_T) can be expressed by:

$$F = EIh''' + k\psi'' \quad (6a)$$
$$M_b = -EIh'' - K\psi' \quad (6b)$$
$$M_T = -Kh'' - GJ\psi' \quad (6c)$$

Assuming harmonic oscillation for h and ψ.

$$\begin{cases} h\,(y\,,t) = H(y)e^{i\omega t} \\ \psi\,(y\,,t) = \Psi(y)e^{i\omega t} \end{cases} \quad (7)$$

where ω represents the angular frequency, H is the amplitude of h and Ψ is the amplitude of ψ. By substituting Eq. (7) into Eqs. (4) and (5) the governing equations become:

$$EIH'''' + K\Psi''' - m\omega^2 H + m\omega^2 x_\alpha \Psi = 0 \quad (8)$$

$$GJ\Psi'' + KH''' + I_\alpha \omega^2 \Psi - m\omega^2 x_\alpha H = 0 \tag{9}$$

Using the definition of the dimensionless length ζ and the differential operator D where

$$\zeta = \frac{y}{L} \qquad D = \frac{d}{d\zeta} \tag{10}$$

Putting Eqs. (8) and (9) into matrix form [12]

$$\begin{bmatrix} EI\frac{D^4}{L^4} - m\omega^2 & K\frac{D^3}{L^3} + m\omega^2 x_\alpha \\ K\frac{D^3}{L^3} - m\omega^2 x_\alpha & GJ\frac{D^2}{L^2} + I_\alpha \omega^2 \end{bmatrix} \begin{Bmatrix} H(\zeta) \\ \Psi(\zeta) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{11}$$

Then the differential equations of the bending displacement $H(\zeta)$and the torsion rotation $\Psi(\zeta)$ are given by:

$$\left[\left(EI\frac{D^4}{L^4} - m\omega^2\right)\left(GJ\frac{D^2}{L^2} + I_\alpha \omega^2\right) - \left(K^2\frac{D^6}{L^6} - m^2\omega^4 {x_\alpha}^2\right)\right] H(\zeta) = 0 \tag{12}$$

$$\left[\left(EI\frac{D^4}{L^4} - m\omega^2\right)\left(GJ\frac{D^2}{L^2} + I_\alpha \omega^2\right) - \left(K^2\frac{D^6}{L^6} - m^2\omega^4 {x_\alpha}^2\right)\right] \Psi(\zeta) = 0 \tag{13}$$

Equations (12) and (13) can be simplified into sixth order as stated below.

$$\left[(EIGJ - K^2)\frac{D^6}{L^6} + (EII_\alpha\omega^2)\frac{D^4}{L^4} - (m\omega^2 GJ)\frac{D^2}{L^2} - m\omega^4 I_\alpha + m^2\omega^4 {x_\alpha}^4\right] H(\zeta) = 0 \tag{14}$$

$$\left[(EIGJ - K^2)\frac{D^6}{L^6} + (EII_\alpha\omega^2)\frac{D^4}{L^4} - (m\omega^2 GJ)\frac{D^2}{L^2} - m\omega^4 I_\alpha + m^2\omega^4 {x_\alpha}^4\right] \Psi(\zeta) = 0 \tag{15}$$

Representing the characteristic equations of the two previous equations [12]

$$(D^6 + aD^4 - bD^2 - abc)W = 0 \tag{16}$$

Where W refer to H or Ψ and

$$a = \frac{\bar{a}}{1-k_m} \quad , \quad b = \frac{\bar{b}}{1-k_m} \quad , \quad c = (1-k_g)(1-k_m) \tag{17}$$

$$\bar{a} = \frac{I_\alpha \omega^2 L^2}{GJ} \quad , \quad \bar{b} = \frac{m\omega^2 L^4}{EI} \quad , \quad k_g = \frac{m{x_\alpha}^2}{I_\alpha} \quad , \quad k_m = \frac{K^2}{EI\,GJ} \tag{18}$$

Using trial solution to solve eq. (16) [13] , let $W = e^{n\zeta}$ then the auxiliary equation is

$$N^6 + aN^4 - bN^2 - abc = 0 \tag{19}$$

let $\lambda = P^2$ then eq. (19) becomes

$$\lambda^3 + a\lambda^2 - b\lambda - abc = 0 \tag{20}$$

the previous equation can be reduced to the standard form

$$x^3 - qx - r = 0 \tag{21}$$

where

$$x = \lambda + \frac{a}{3} \quad , \quad q = b + \frac{a^2}{3} \quad , \quad r = a\left(bc - \frac{b}{3} - \frac{2a^2}{27}\right) \tag{22}$$

Let $\delta = 27r^2 - 4q^3$ then there are three cases for the three roots of eq. (21)

- $\delta > 0$ there are two conjugate imaginaries and one is real.
- $\delta = 0$ they are all real roots at least two of them are equal.
- $\delta < 0$ they are all real and unequal, one is positive and the other two are negative. $\delta < 0$ for all the physical meaningful values of a, b and c then [14]

$$\begin{cases} x_1 = 2(q/3)^{1/2}\cos(\phi/3) \\ x_2 = -2(q/3)^{1/2}\cos\{(\pi-\phi)/3\} \\ x_2 = -2(q/3)^{1/2}\cos\{(\pi+\phi)/3\} \end{cases} \tag{23}$$

where

$$\phi = cos^{-1}\frac{27abc-9ab-2a^3}{2(a^2+3b)^{3/2}} \tag{24}$$

Substituting eq. (23) into the definition of x given in eq. (22), the sixth roots are the plus and the minus sign of the square roots of the three values of λ, so if the sixth roots are given by α, $-\alpha$, $i\beta$, $-i\beta$, $i\gamma$ and $-i\gamma$ with α, β and γ are real. They are defined by [2]:

$$\left.\begin{aligned} \alpha &= \left[2(q/3)^{1/2}cos(\phi/3) - a/3\right]^{1/2} \\ \beta &= \left[2(q/3)^{1/2}cos\{(\pi-\phi)/3\} + a/3\right]^{1/2} \\ \gamma &= \left[2(q/3)^{1/2}cos\{(\pi+\phi)/3\} + a/3\right]^{1/2} \end{aligned}\right\} \tag{25}$$

Then the solution of eq. (16) is

$$W(\zeta) = a_1 cosh\alpha\zeta + a_2 sinh\alpha\zeta + a_3 cos\beta\zeta + a_4 sin\beta\zeta + a_5 cos\gamma\zeta + a_6 sin\gamma\zeta \tag{26}$$

Where $a_1 - a_2$ are constants, defining the expressions of H and Ψ.

$$H(\zeta) = A_1 cosh\alpha\zeta + A_2 sinh\alpha\zeta + A_3 cos\beta\zeta + A_4 sin\beta\zeta + A_5 cos\gamma\zeta + A_6 sin\gamma\zeta \tag{27}$$

$$\Psi(\zeta) = B_1 cosh\alpha\zeta + B_2 sinh\alpha\zeta + B_3 cos\beta\zeta + B_4 sin\beta\zeta + B_5 cos\gamma\zeta + B_6 sin\gamma\zeta \tag{28}$$

by substitution with the definitions of bending displacement and torsion rotation in eq. (8)

$$\begin{aligned} B_1 &= \tfrac{1}{L}(-A_1 e_\alpha g_\alpha + A_2 e_\alpha) \quad B_2 = \tfrac{1}{L}(A_1 e_\alpha - A_2 e_\alpha g_\alpha) \quad B_3 = \tfrac{1}{L}\left(A_3 e_\beta g_\beta + A_4 e_\beta\right) \\ B_4 &= \tfrac{1}{L}\left(-A_3 e_\beta + A_4 e_\beta g_\beta\right) \quad B_5 = \tfrac{1}{L}\left(A_5 e_\gamma g_\gamma + A_6 e_\gamma\right) \quad B_6 = \tfrac{1}{L}\left(-A_5 e_\gamma + A_6 e_\gamma g_\gamma\right) \end{aligned} \tag{29}$$

where

$$\begin{aligned} g_\alpha &= \bar{b}k_\delta/\alpha^3 \qquad g_\beta = \bar{b}k_\delta/\beta^3 \qquad g_\gamma = \bar{b}k_\delta/\gamma^3 \\ e_\alpha &= k_\alpha/(1-g_\alpha{}^2) \quad e_\beta = k_\beta/(1+g_\beta{}^2) \quad e_\gamma = k_\gamma/(1+g_\gamma{}^2) \end{aligned} \tag{30}$$

$$k_\alpha = \frac{EI}{K}\left(\frac{\bar{b}-\alpha^4}{\alpha^3}\right), \quad k_\beta = \frac{EI}{K}\left(\frac{\bar{b}-\beta^4}{\beta^3}\right), \quad k_\gamma = \frac{EI}{K}\left(\frac{\bar{b}-\gamma^4}{\gamma^3}\right), \quad k_\delta = \frac{EI}{K}\frac{x_\alpha}{L} \tag{31}$$

Substituting Eq. (29) into Eq. (28), the expression of Ψ will be:

$$\Psi(\zeta) = \frac{1}{L}\left(A_1 u_\alpha + A_2 v_\alpha + A_3 u_\beta + A_4 v_\beta + A_5 u_\gamma + A_6 v_\gamma\right) \tag{32}$$

where

$$\begin{aligned} u_\alpha &= e_\alpha(sinh\alpha\zeta - g_\alpha cosh\alpha\zeta) \quad v_\alpha = e_\alpha(cosh\alpha\zeta - g_\alpha sinh\alpha\zeta) \\ u_\beta &= e_\beta\left(-sin\beta\zeta + g_\beta cos\beta\zeta\right) \quad v_\beta = e_\beta\left(cos\beta\zeta + g_\beta sin\beta\zeta\right) \\ u_\gamma &= e_\gamma\left(-sin\gamma\zeta + g_\gamma cos\gamma\zeta\right) \quad v_\alpha = e_\gamma\left(cos\gamma\zeta + g_\gamma sin\gamma\zeta\right) \end{aligned} \tag{33}$$

The bending rotation $\Theta(\zeta)$ can be expressed by $\Theta(\zeta) = \frac{dH}{dy} = \frac{1}{L}\frac{dH}{d\zeta}$ then

$$\Theta(\zeta) = \frac{1}{L}(A_1\alpha sinh\alpha\zeta + A_2\alpha cosh\alpha\zeta - A_3\beta sin\beta\zeta + A_4 cos\beta\zeta - A_5\gamma sin\gamma\zeta + A_6\gamma cos\gamma\zeta) \tag{34}$$

Representing the expressions of shear force $S(\zeta)$, bending moment $M(\zeta)$ and torque moment $T(\zeta)$ mentioned in Eq. (6)

$$F(\zeta) = \frac{EI}{L^3}\frac{d^3H}{d\zeta^3} + \frac{K}{L^2}\frac{d^2\Psi}{d\zeta^2} = W_3\big[A_1(\alpha P_\alpha sinh\alpha\zeta - \alpha^2 e_\alpha g_\alpha k_b cosh\alpha\zeta) + A_2(\alpha P_\alpha cosh\alpha\zeta - \alpha^2 e_\alpha g_\alpha k_b sinh\alpha\zeta) + A_3\big(\beta P_\beta sin\beta\zeta - \beta^2 e_\beta g_\beta k_b cos\beta\zeta\big) - A_4(\beta P_\beta cos\beta\zeta + \beta^2 e_\beta g_\beta k_b cos\beta\zeta) + A_5\big(\gamma P_\gamma sin\gamma\zeta - \gamma^2 e_\gamma g_\gamma k_b cos\gamma\zeta\big) - A_6\big(\gamma P_\gamma cos\gamma\zeta + \gamma^2 e_\gamma g_\gamma k_b sin\gamma\zeta\big)\big] \tag{35}$$

$$M_b(\zeta) = -\frac{EI}{L^2}\frac{d^2H}{d\zeta^2} - \frac{K}{L}\frac{d\Psi}{d\zeta} = -W_2\big[A_1(P_\alpha cosh\alpha\zeta - \alpha e_\alpha g_\alpha k_b sinh\alpha\zeta) + A_2(sinh\alpha\zeta - \alpha e_\alpha g_\alpha k_b cosh\alpha\zeta) - A_3\big(P_\beta cos\beta\zeta + \beta e_\beta g_\beta k_b sin\beta\zeta\big) - A_4\big(P_\beta sin\beta\zeta - \beta e_\beta g_\beta k_b cos\beta\zeta\big) - A_5\big(P_\gamma cos\gamma\zeta + \gamma e_\gamma g_\gamma k_b sin\gamma\zeta\big) - A_6\big(P_\gamma sin\gamma\zeta - \gamma e_\gamma g_\gamma k_b cos\gamma\zeta\big)\big] \tag{36}$$

$$M_T(\zeta) = -\frac{K}{L^2}\frac{d^2H}{d\zeta^2} - \frac{GJ}{L}\frac{d\Psi}{d\zeta} = -\frac{W_1}{L}\big[A_1(q_\alpha cosh\alpha\zeta - \alpha e_\alpha g_\alpha sinh\alpha\zeta) + A_2(q_\alpha sinh\alpha\zeta - \alpha e_\alpha g_\alpha cosh\alpha\zeta) - A_3\big(q_\beta cos\beta\zeta + \beta e_\beta g_\beta sin\beta\zeta\big) - A_4\big(q_\beta sin\beta\zeta - \beta e_\beta g_\beta cos\beta\zeta\big) - A_5\big(q_\gamma cos\gamma\zeta + \gamma e_\gamma g_\gamma sin\gamma\zeta\big) - A_6\big(P_\gamma sin\gamma\zeta - \gamma e_\gamma g_\gamma cos\gamma\zeta\big)\big] \tag{37}$$

where

$$W_3 = \frac{EI}{L^3} \quad , \quad W_2 = \frac{EI}{L^2} \quad , \quad W_1 = \frac{GJ}{L} \tag{38}$$

$$\begin{array}{lll} P_\alpha = \alpha(\alpha + e_\alpha k_b) & P_\beta = \beta(\beta + e_\beta k_b) & P_\gamma = \gamma(\gamma + e_\gamma k_b) \\ q_\alpha = \alpha(e_\alpha + \alpha k_t) & q_\beta = \beta(e_\beta + \beta k_t) & q_\gamma = \gamma(\gamma + \gamma k_t) \end{array} \tag{39}$$

$$k_b = \frac{x_\alpha}{L k_\delta} \quad , \quad k_t = \frac{k_m k_\delta L}{x_\alpha} \tag{40}$$

2.2. Application of Dynamic transfer matrix method

The boundary conditions as shown in Figs. (2) and (3)

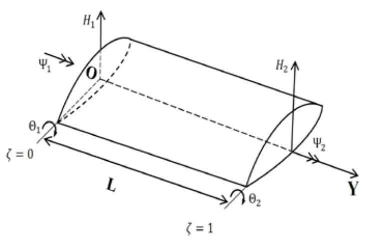

Figure 2. Boundary conditions for displacements.

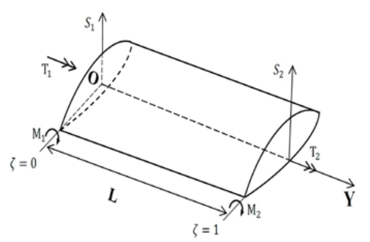

Figure 3. Boundary conditions for forces.

Displacements

$$\left.\begin{array}{lll} \text{at y} = 0\ (\xi = 0) & H = 0 \quad , \Theta = 0 & , \Psi = 0 \\ \text{at y} = \text{L}\ (\xi = 1) & H = H_2 \ , \ \Theta = \Theta_2 & , \ \Psi = \Psi_2 \end{array}\right\} \tag{41}$$

Forces

$$\left.\begin{array}{ll} \text{at y} = 0\ (\xi = 0) & F = F_1 , M_b = M_{b1} , M_T = M_{T1} \\ \text{at y} = \text{L}\ (\xi = 1) & F = 0 \quad , M_b = 0 \quad , M_T = 0 \end{array}\right\} \tag{42}$$

for a cantilever beam, fixed at the root ($\xi = 0$) and free at the tip ($\xi = 1$). Substituting with these boundary conditions into Eqs. (27), (32), (34), (35), (36) and (37)

$$\{Q\}_{F_x} = [R(0,\omega)]_{Fx}\{A\} \tag{43}$$

$$\{Q\}_{F_r} = [R(1,\omega)]_{Fr}\{A\} \tag{44}$$

where F_x and F_r refer to fixed end and free end, respectively and

$$\left.\begin{aligned} \{Q\}_{F_x} &= \{H_1\ \Theta_1\ \Psi_1\ S_1\ M_1\ T_1\}^T \\ \{Q\}_{F_r} &= \{H_2\ \Theta_2\ \Psi_2\ S_2\ M_2\ T_2\}^T \\ \{A\} &= \{A_1\ A_1\ A_1\ A_1\ A_1\ A_1\}^T \end{aligned}\right\} \tag{45}$$

From Eq. (43) and Eq. (44) the matrix relates the two ends with their boundary conditions will be

$$\{Q\}_{F_r} = [R]_{F_r}[R]_{F_x}{}^{-1}\{Q\}_{F_x} \tag{46}$$

Then the natural frequencies and mode shapes will be obtained by satisfying the boundary conditions and substituting in Eq. (46) with Eqs. (27), (32), (34), (35), (36) and (37), where $[R]_{F_r}[R]_{F_x}{}^{-1}$ is called the dynamic transfer matrix and the matrices $[R]_{F_x}$ and $[R]_{F_r}$ are defined as follows:

$$[R]_{F_x} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & \frac{\alpha}{L} & 0 & \frac{\beta}{L} & 0 & \frac{\gamma}{L} \\ \frac{-e_\alpha g_\alpha}{L} & \frac{e_\alpha}{L} & \frac{e_\beta g_\beta}{L} & \frac{e_\beta}{L} & \frac{e_\gamma g_\gamma}{L} & \frac{e_\gamma}{L} \\ \alpha^2 e_\alpha g_\alpha k_b W_3 & \alpha P_\alpha W_3 & -B^2 e_\beta g_\beta k_b W_3 & -\beta P_\beta W_3 & -\gamma^2 e_\gamma g_\gamma k_b W_3 & -\gamma P_\gamma W_3 \\ -P_\alpha W_2 & \alpha e_\alpha g_\alpha k_b W_2 & P_\beta W_2 & -\beta e_\beta g_\beta k_b W_2 & P_\gamma W_2 & -\gamma e_\gamma g_\gamma k_b W_2 \\ -q_\alpha W_1/L & \alpha e_\alpha g_\alpha W_1/L & q_\beta W_1/L & -\beta e_\beta g_\beta W_1/L & q_\gamma W_1/L & -\gamma e_\gamma g_\gamma W_1/L \end{bmatrix} \tag{47}$$

$$\left[[R]_{F_r}\right]_{11} = \begin{bmatrix} C_{h\alpha} & S_{h\alpha} & C_\beta \\ \frac{\alpha}{L}S_{h\alpha} & \frac{\alpha}{L}C_{h\alpha} & -\frac{\beta}{L}S_\beta \\ \frac{u_\alpha{}^*}{L} & \frac{v_\alpha{}^*}{L} & \frac{u_\beta{}^*}{L} \end{bmatrix} \qquad \left[[R]_{F_r}\right]_{12} = \begin{bmatrix} S_\beta & C_\gamma & S_\gamma \\ \frac{\beta}{L}C_\beta & -\frac{\gamma}{L}S_\gamma & -\frac{\gamma}{L}C_\gamma \\ \frac{v_\beta{}^*}{L} & \frac{u_\gamma{}^*}{L} & \frac{v_\gamma{}^*}{L} \end{bmatrix} \tag{48}$$

where

$$\left.\begin{aligned} C_{h\alpha} &= \cosh\alpha & C_\beta &= \cos\beta & C_\gamma &= \cos\gamma \\ S_{h\alpha} &= \sinh\alpha & S_\beta &= \sin\beta & S_\gamma &= \sin\gamma \\ u_\alpha{}^* &= e_\alpha(S_{h\alpha} - g_\alpha C_{h\alpha}) & & & v_\alpha{}^* &= e_\alpha(C_{h\alpha} - g_\alpha S_{h\alpha}) \\ u_\beta{}^* &= e_\beta\left(-S_\beta + g_\beta C_\beta\right) & & & v_\beta{}^* &= e_\beta\left(C_\beta + g_\beta S_\beta\right) \\ u_\gamma{}^* &= e_\gamma\left(-S_\gamma + g_\gamma C_\gamma\right) & & & v_\gamma{}^* &= e_\gamma\left(C_\gamma + g_\gamma S_\gamma\right) \end{aligned}\right\} \tag{49}$$

$$\left[[R]_{F_r}\right]_{21} = \begin{bmatrix} \alpha(\alpha e_\alpha g_\alpha k_b C_{h\alpha} - P_\alpha S_{h\alpha})W_3 & \alpha(\alpha e_\alpha g_\alpha k_b S_{h\alpha} - P_\alpha C_{h\alpha})W_3 & \beta(\beta e_\beta g_\beta k_b C_\beta - P_\beta S_\beta)W_3 \\ (P_\alpha C_{h\alpha} - \alpha e_\alpha g_\alpha k_b S_{h\alpha})W_2 & (P_\alpha S_{h\alpha} - \alpha e_\alpha g_\alpha k_b C_{h\alpha})W_2 & -(P_\beta C_\beta + \beta e_\beta g_\beta k_b S_\beta)W_2 \\ (q_\alpha C_{h\alpha} - \alpha e_\alpha g_\alpha S_{h\alpha})\frac{W_1}{L} & (q_\alpha S_{h\alpha} - \alpha e_\alpha g_\alpha C_{h\alpha})\frac{W_1}{L} & -(q_\beta C_\beta + \beta e_\beta g_\beta S_\beta)\frac{W_1}{L} \end{bmatrix} \tag{50}$$

$$\left[[R]_{F_r}\right]_{22} = \begin{bmatrix} \beta(P_\beta C_\beta + \beta e_\beta g_\beta k_b S_\beta)W_3 & \gamma(\gamma e_\gamma g_\gamma k_b C_\gamma - P_\gamma S_\gamma)W_3 & \gamma(P_\gamma C_\gamma + \gamma e_\gamma g_\gamma k_b S_\gamma)W_3 \\ (\beta e_\beta g_\beta k_b C_\beta - P_\beta S_\beta)W_2 & -(P_\gamma C_\gamma + \gamma e_\gamma g_\gamma k_b S_\gamma)W_2 & -(P_\gamma S_\gamma - \gamma e_\gamma g_\gamma k_b C_\gamma)W_2 \\ (\beta e_\beta g_\beta C_\beta - q_\beta S_\beta)\frac{W_1}{L} & -(q_\gamma C_\gamma + \gamma e_\gamma g_\gamma S_\gamma)\frac{W_1}{L} & -(q_\gamma S_\gamma - \gamma e_\gamma g_\gamma C_\gamma)\frac{W_1}{L} \end{bmatrix} \tag{51}$$

$$\left[[R]_{F_r}\right] = \begin{bmatrix} \left[[R]_{F_r}\right]_{11} & \left[[R]_{F_r}\right]_{12} \\ \left[[R]_{F_r}\right]_{21} & \left[[R]_{F_r}\right]_{22} \end{bmatrix} \tag{52}$$

2.3. Natural frequencies

substituting with the boundary conditions into Eq. (46), the following matrix equation is obtained

$$\begin{bmatrix} \bar{r}_{44} & \bar{r}_{45} & \bar{r}_{46} \\ \bar{r}_{54} & \bar{r}_{55} & \bar{r}_{56} \\ \bar{r}_{64} & \bar{r}_{65} & \bar{r}_{66} \end{bmatrix} \begin{Bmatrix} S_1 \\ M_1 \\ T_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{53}$$

For a non-trivial solution and in order to obtain the frequency equation for the composite beam in the free vibration analysis, the determinant of the coefficient 3 × 3 matrix must be equal to zero.

2.4. Mode shapes

Taking the first three equations from Eq. (43) and the last three equations from Eq. (44) then substituting with the natural frequency ω_n, the following system of equations is obtained.

$$\begin{bmatrix} R_{Fx_{11}} & \cdots & R_{Fx_{16}} \\ \vdots & \ddots & \vdots \\ R_{Fr_{61}} & \cdots & R_{Fr_{66}} \end{bmatrix} \begin{Bmatrix} A_1 \\ \vdots \\ A_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{54}$$

To get these constants and to avoid the nontrivial solution, one row from the coefficient matrix will be deleted arbitrarily and its corresponding constant will be chosen arbitrarily, then expressing the remaining constants in terms of the chosen one.

3. Results and discussion

To validate the current work, a MATLAB code was used to obtain the dynamic equation which is used to get the natural frequencies and mode shapes of the analyzed composite beam. The data of this cantilever composite wing was taken from Banerjee.

Table 1. The characteristics used for the composite beam analysis

item	Value	units
EI	$9.75 * 10^6$	N.m^2
GJ	$9.88 * 10^5$	N.m^2
L	6	m
m	35.75	Kg/m
I_α	8.65	Kg. m
K	$9{:}75 * 10^6$	N.m^2
x_α	0.2	m

To study the change in the vibration analysis of a composite beam due to coupling in both material and geometric, some values of x_α and K have been analyzed. In the vibratory motion, there will be no coupling when x_α and K are both zero. The values of x_α and K must be taken equal to 10^{-5} or 10^{-6} to get the results with acceptable accuracy rather than putting them exactly zero to prevent any numerical overflow.

Table 2. The variation of the first natural frequencies for different values of x_α and K

x_α (m)	K($*10^6$ Nm^2)		Natural frequencies (rad/s)		
			ω_1	ω_2	ω_3
0.1	0	Presented	50.539246	91.01975	258.42778
		Reference	50.539	91.02	258.43
	1.5	Presented	40251539	99.0718825	197.5697536
		Reference	40.252	99.072	197.57
	2	Presented	33.96224	100.4672	168.547189
		Reference	33.962	100.47	168.55
	2.5	Presented	25.44152	94.8172838	137.337642
		Reference	25.442	94.817	137.34
0.2	0	Presented	49.33140496	99.20276666	246.5961495
		Reference	49.331	99.202	246.6
	1.5	Presented	38.0708548	112.2243426	185.5736177
		Reference	38.071	112.22	185.57
	2	Presented	31.981323	114.7058	157.80644
		Reference	31.981	114.71	157.81
	2.5	Presented	23.88528092	104.3580622	133.8713782
		Reference	23.885	104.36	133.87

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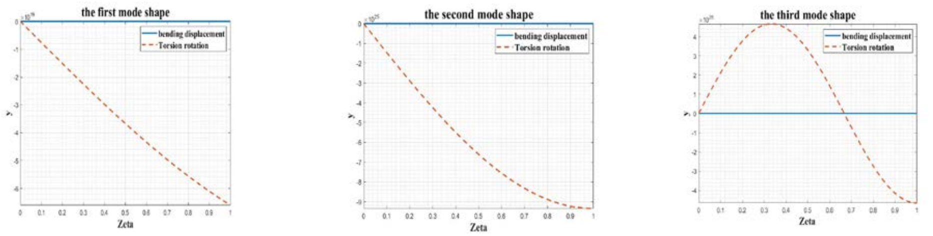
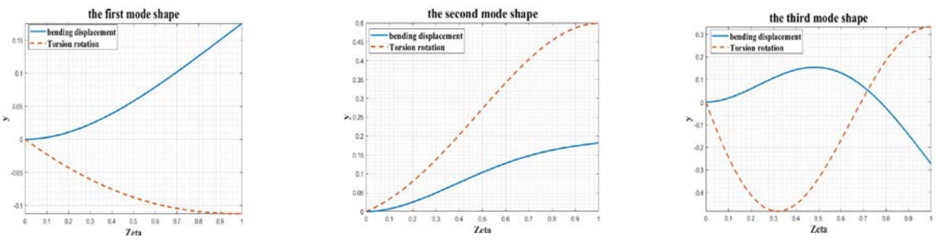
Banerjee made this analysis on the same cantilever beam using the assumption of harmonic oscillation and solved it with the dynamic stiffness method but in this paper with the same assumption of Banerjee, the dynamic transfer method has been used and a comparison between both methods has been made as shown in table (2)

The mode shapes of the two cases are shown in Figs. (4) and (5). In the first case as shown in fig. (4) the values of x_α and K are equal to zero to represent the uncoupled case, while the second case which represents the coupling between bending and torsion as shown in fig. (5) their values are 0.2m and $2.0*10^6$N.m^2 respectively.



(a) ω_1=51.005 (b) ω_2=88.478 (c) ω_3=265.44

Figure 4. The uncoupled case



(a) ω_1=31.981 (b) ω_2=114.71 (c) ω_3=157.81

Figure 5. The coupled case

Note that natural frequencies and mode shapes are very different in the coupled case than the uncoupled one and this is important from the aeroelastic point of view.

After validation of the developed code, a parametric study was performed to show the effect of variation of K and x_α on the natural frequencies corresponding to the first three modes.

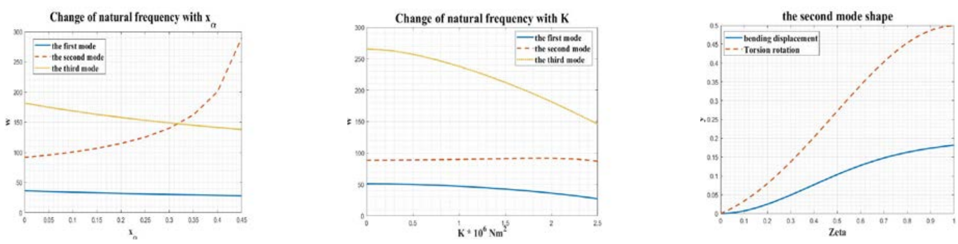


Figure 6. Change of natural frequency with x_α and K

Figure (6) shows the change in the first three mode shapes of the cantilever beam with the change in the values of K and x_α where as shown in Fig. 6 (a), ω_1 slightly decreases with x_α increasing, while ω_2 corresponding to the second mode is greatly increased. At $x_\alpha = 0.3169813$, the second and third modes intersect. Further increasing of x_α switches the order of the two modes (i.e., the natural frequency of the second mode becomes greater than that of the third mode. Figure 6 (b) represents the change of the natural frequency with the change in the value of the torsion bending coupling rigidity K when $x_\alpha = 0$ which means that the elastic axis and mass axis are coincided (i.e, the coupling depends only on the material coupling). As shown in this figure, ω_1 decreases with K increasing, while ω_2 increases very slowly with K increasing till approximately K = 2.0 *10^6 N. m^2 then decreases after that and ω_3 corresponding to the third mode is greatly decreased with K increasing. Figure 6 (c) represents the change of the natural frequency with the change of K at a constant value for x_α = 0.2 m (i.e., the coupling depending on both material and geometric coupling). In this figure with the increase in the value of K, ω_1 and ω_3 decrease but ω_2 increases till K = 2.0 *10^6 N. m^2 then decreases after that.

4. Conclusion

The present paper introduces an analytical solution for the free vibration analysis of composite beam with bending and torsional coupling representing material and geometric coupling for composite structures by a mathematical relation between both sides of the beam using the dynamic transfer method. This work was validated using a composite beam with fixed-free boundary conditions and the first three natural frequencies and mode shapes were calculated and compared with published results and a good agreement was found between them. The parametric study shows that x_α and K are very important parameters having a great effect on the free vibration analysis of composite structure, and this is an important benefit from the aeroelastic point of view that can be used to achieve desirable properties when design composite structures. In the future, this study will be conducted for composite wings under aeroelastic loading.

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