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Application of the level set method in solid propellant grain burnback with dogbone grain as a case study

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Abstract. Solid propellant grain burnback is a crucial step internal ballistic module. There are different methods to predict grain burnback; the oldest method is the drafting technique where the burning surface is tracked using manual drawing applying surface regression rules. Then there is the analytical method, where a geometric analysis of the surface is performed and analytical expressions for the burning surface segments are derived. These expressions are evaluated at different values of burnt distance to find the evolution of burning surface and port areas with time. A level set method is a different approach which, in some aspects, may be considered a drafting technique where the burning surface segments are tracked. From another point of view, it is a numerical method that tracks the burning surface with no need of trying to derive analytical expressions for burning segments. Dogbone grain is known for its structural superiority; however, researches dealing with dogbone grain design parameters are scarce. In this paper, the level set method is introduced and the main steps for its implementation are elaborated. Validation of the level set method is performed against the drafting technique using CAD and analytical expressions for the well-known star grain. Finally, dogbone grain geometry with different configurations is introduced and a parametric study for the governing parameters that affect dogbone grain burnback is performed.

1. Introduction

Burnback analysis is an important step in predicting the ballistic performance of a solid-propellant rocket motor. There are several methods to perform the burnback analysis; the oldest method is the drafting technique where the simulation of the burning is done by combining basic shapes to describe the grain initial geometry and the burning surface is assumed to propagate normal to itself [1]. The famous Solid Performance Program (SPP™) is based on the drafting technique [2]. The main drawbacks of this method are that it is discrete, requires high human-computer interaction which may lead to errors, and it is also a time-consuming method.

Another way to perform the burnback analysis is the analytical method where the main idea is to describe the burning surface or interface through mathematical relations. Unlike the drafting technique, the analytical method is fast and continuous in representing the burning progression [3]. The disadvantage of the analytical method is that the relations are dedicated to a single grain configuration, so for some complex configurations, it might be difficult to deduce the proper mathematical relations.

The other method which is the scope of this research is one of the numerical interface tracking methods or namely the level set method. The level set method is first introduced in 1988 by Sethian and Osher [4] and offers a numerical technique for tracking the motion of a curve with time.

The level set method has so many applications in different fields from physics, chemistry, fluid mechanics, image processing, materials sciences, and fabrication of microelectronic components [4]. Also, it has many applications in rockets such as burning rate modeling [5], sloshing in liquid propellant tanks [6], droplet collision in dense spray applications [7], and G-equation for flame front modeling [8].

Unlike the analytical method, the level set method affords a high ability in dealing with complex shapes as it can easily work with the sharp edges, cusps, and corners; as well as the change in the topology of the interface. This enables to perform the burnback analysis for any grain configuration such as dogbone grain; which is an internal burning grain known for its structural superiority [9]. Its geometry can be represented typically as a special case of the wagon wheel grain. This configuration is commonly used when high initial burning area or chamber pressures are desired [10]. However, the current work aims to implement a procedure to perform a level-set-based burnback analysis and apply this procedure to different grain geometries to validate it. Finally, a parametric study for a dogbone grain is performed. As to the author's knowledge, the researches dealing with dogbone grain design parameters are scarce.

2. Level Set Methodology

2.1 Level set basics

There are two approaches in studying moving interfaces or fluids, the Eulerian approach and the Lagrangian approach, which are completely different. “*Lagrangian approach follows the path of each particle of fluid as it moves, Eulerian approach sees which particles pass through each point it sits*”. The Lagrangian approach is more direct as it tracks the flow, so if we mark a set of equally spaced points on it at the beginning, some difficulties will appear later in the mesh such as the spacing could be too narrow or too wide, the initial curve can intersect itself or can split apart. The level set method cures all these difficulties by using the Eulerian approach where the coordinates are fixed and it only captures the interface implicitly, when expanding or tangling the interface it is translated into changes in the level set function ‘$\emptyset$’ not in the mesh [11].

This remarkable feature made it able to deal with different types of cusps in the propellant grain configurations. For example, as shown in figure 1; the burning progression does not affect the convex cusp and it remains as it was; while the concave one becomes an arc of a circle and its center is the cusp vertex. This can cause some difficulties in using the drafting technique for the grain burnback analysis, since one has to create a new surface for the arc, unlike the level set method which deals with this easily as this topological change can be handled by converting the 2D interface into a 3D surface called the level set function ‘$\emptyset$’ as shown in figure 2. This function is constructed numerically and on intersecting it at zero level one gets the initial interface which is star grain.

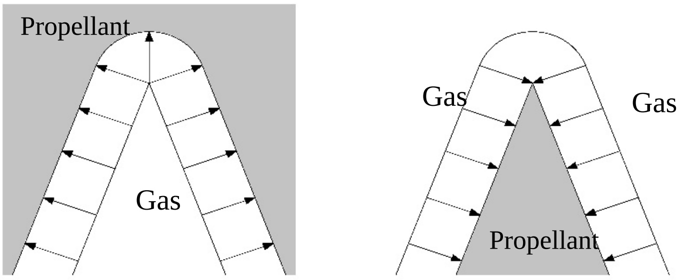


Figure 1 Effect of burning on a convex cusp (right) and concave cusp (left).

For further explanation, consider a simple 2D circular interface shown in figure (3) such that:

$$\emptyset(x,y) = x^2 + y^2 = c \quad (1)$$

When ‘c’ is positive the level sets are circles and if it is negative there are no level sets. To create the function ‘$\emptyset$’ one has to introduce a third dimension ‘z’ such that:

$$z = \emptyset(x, y) = x^2 + y^2 = c \tag{2}$$

This will result in a 3D surface shown also in figure 3. That is exactly what happens in any other interface.

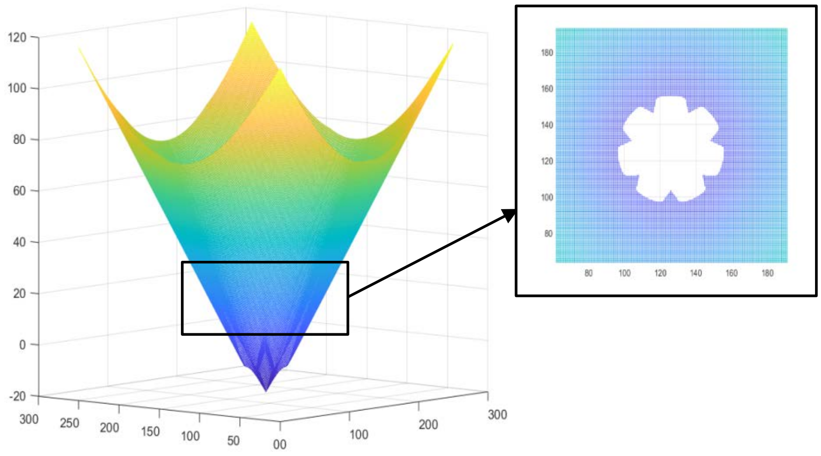


Figure 2 Representation of the function ‘∅’ for star grain configuration.

The characteristic feature of the level set method consists of representing the interface implicitly, by using the zero-level set of a time-dependent function ‘∅’ [12]. During the formation of the level set method a principle of *“once a particle burns, it remains burnt”*. This means that when two interfaces pass by the same point whatever one arrives first; the particle located there will be ignited. Thus, the function ∅ is created as an indicator function of the propagating interface, from which the boundary can be determined.

The first interface is represented by Eqn. 3:

$$\emptyset[\vec{x}(t), t] = 0 \tag{3}$$

which expresses that at any time the interface points are defined by ∅ function for all x not just for the points of the boundary. Figure (3) is a schematic representation of the level set method for an interface (circle) propagating with a velocity F (which in grain burnback analysis is equivalent to burning rate) normal to the boundary and to the outward direction:

The velocity F (burning rate) can be represented as:

$$\frac{d\vec{x}}{dt} \cdot \vec{n} = F \tag{4}$$

Differentiating $\emptyset[\vec{x}(t), t] = 0$ with respect to time:

$$\frac{\partial\emptyset}{\partial t} + \nabla\emptyset \cdot \frac{\partial\vec{x}}{\partial t} = 0 \tag{5}$$

Because ∅ is constant at a certain level set, ∇∅, is normal to the interface. So the normal direction is calculated using Eqn. 4:

$$\vec{n} = \frac{\nabla\emptyset}{|\nabla\emptyset|} \tag{6}$$

Substituting, the level set equation is obtained:

$$\frac{\partial\emptyset}{\partial t} + \mathrm{F} \cdot |\nabla\emptyset| = 0 \tag{7}$$

with the initial condition given by

$$\emptyset[\vec{x}, t = 0] \tag{8}$$

The initial condition involves knowing all the values of ∅ for all x at t=0. So, Starting from this initial condition, the interface can be propagated by solving the level set equation for a given velocity F, the previous equations are implemented in a MATLAB code, with an explanation of the procedure as discussed in the next section

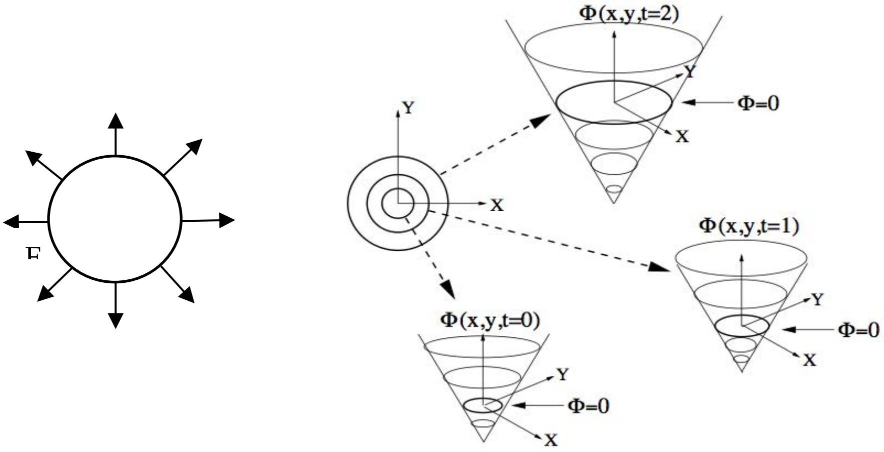


Figure 3 Level set representation for a circular interface

2.2 *Level set procedure*

The procedure starts with drawing the 2D grain configuration using CAD software (in this case SOLIDWORKS), then this 2D grain configuration –which is basically a line describing grain initial burning area- is discretized into XYZ points and exported into a text file. This text file is received by a MATLAB code where the basic equations of the level set method are implemented (as introduced in the previous section), allowing the calculation to start. This MATLAB code initializes values of Ø and adds the background mesh where the calculation will be performed. A block diagram for the procedures is shown in figure 4.

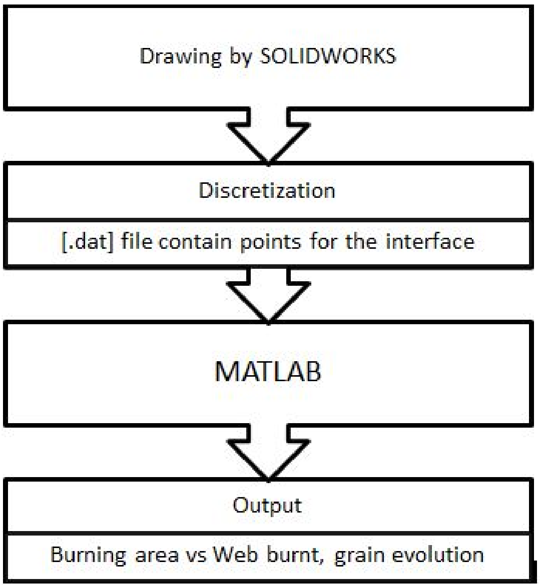
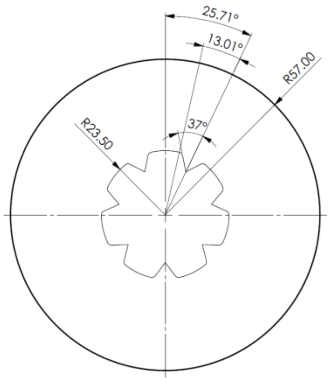


Figure 4 Block diagram for the procedures of level set code

3. Validation and mesh size effect

To validate the method, a standard 7-point star grain, figure 5, with zero cusp radius was selected with a total length of 1.6 m and a web thickness of 33.5 mm [13]. The progress of the burning area against the web burnt was represented by the 3 aforementioned techniques.



No of star points [N]	7
Star point angle [θ]	74°
Angle fraction [ε]	0.5058
Grain inner radius [R_{in}]	23.5 mm
Fillet radius [f]	1.6 mm
Web thickness [w]	33.5 mm
Length of star grain L_g	1604.1 mm

Figure 5 Geometric parameters for the used star grain

First, the drafting technique is applied to the star grain with data as in Table 1. The grain is drawn using CAD software with several parallel offsets as sketched in figure 6.

Figure 6 Drafting technique for the selected star grain

Second, the analytical method is applied to the same grain using star grain equations for different phases of burning into a MATLAB code written by Hashish et al. [13]. The inputs are the star grain parameters (N, f, θ, w, L, R), and the burning area vs. the web burnt is calculated. Finally, the level set method is applied using the procedure illustrated in figure 4. For this case (star grain); it was first drawn and represented by SOLIDWORKS and then it was imported as an IGS file format into a module which is used for discretization of the model by meshing its outlines for a 2D section into an XYZ file that contains model meshing points to be recognized by the MATLAB code. The code creates the initial burning surface and then propagates it normal to itself to represent the grain regression. The comparison between the three methods is shown in figure 7.

As perceived from these figures, the results from drafting and analytical methods are nearly the same with a very narrow error that could be minimized by increasing the number of the offsets in the drafting method at the expense of consuming more time and effort. However, for the level set method, there is an error in comparison with the other methods. This error is a characteristic of any numerical method due to discretization and local truncation error in solving the level-set equation (Eqn. 7). This error can be minimized by decreasing the cell size of the background mesh at which the level-set equations are solved (figure 8 a-c). A comparison between calculation times for solving this case using different mesh sizes is shown in Table 1, with the root mean square error (RMSE) in each case.

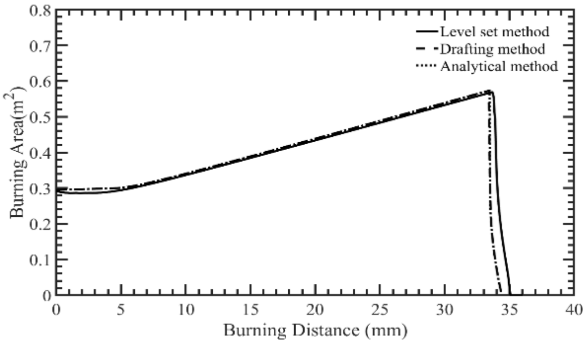
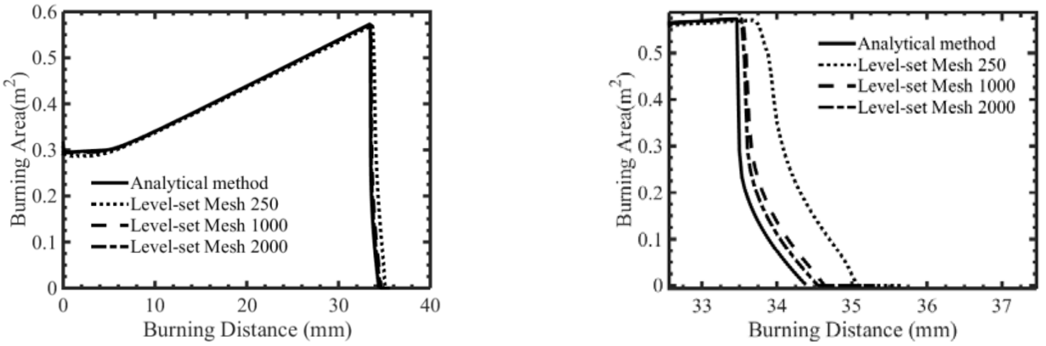


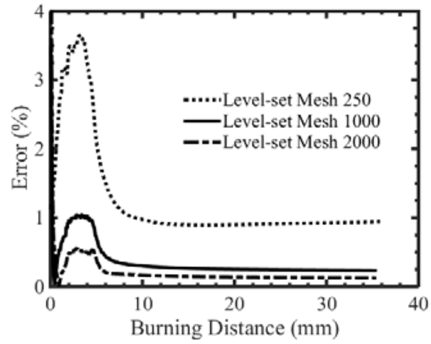
Figure 7 Validation results for the star grain

From figure 8c and Table 1, the maximum error with using mesh size of 250x250 is less than 4 % and RMSE is ~1.5 % compared to the analytical method, which may be considered as an accepted error margin without consuming much more time especially in cases of studying parameters effects (as in this paper)-, while denser mesh size (1000x1000 or 2000x2000) may be used in cases of experimental validation to minimize the discrepancy between prediction and experimental results.



a. Effect of mesh size

b. Effect of mesh size (tail-off region zoomed)



c. Error % for different mesh size-discarding error for the tail-off region-

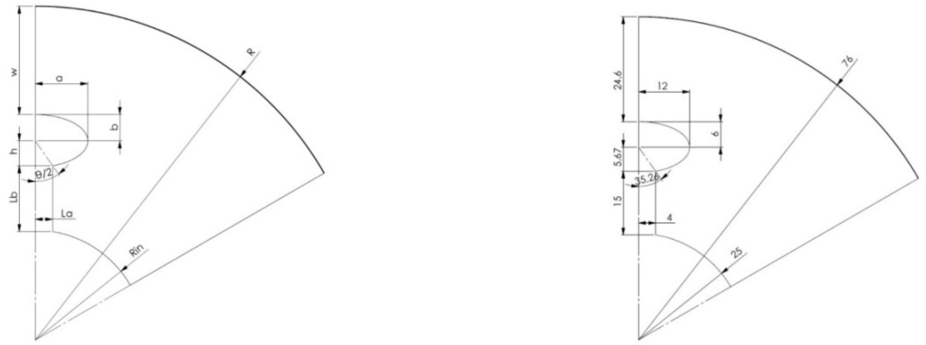
Figure 8 Effect of mesh size on level-set accuracy

Table 1 Calculation time for different mesh sizes

Mesh size	Time (sec)	RMSE (%)
250x250	20	1.5
500x500	84	0.8
1000x1000	280	0.46
2000x2000	1148	0.36

4. Dogbone grain

Dogbone configuration is selected to test the applicability of the level-set method as actually -to the author's knowledge- published sources have neither studied the dogbone grain nor provided the analytical relations that can be used to find the effect of different design parameters. The dogbone is an internal burning grain that has been developed and it is known by its structural reliability, it can be defined by 8 parameters: (R_{in}, e, θ, h, L_b, L_a, N, R) as shown in figure 9a, with the baseline case, (shown in figure 9b).



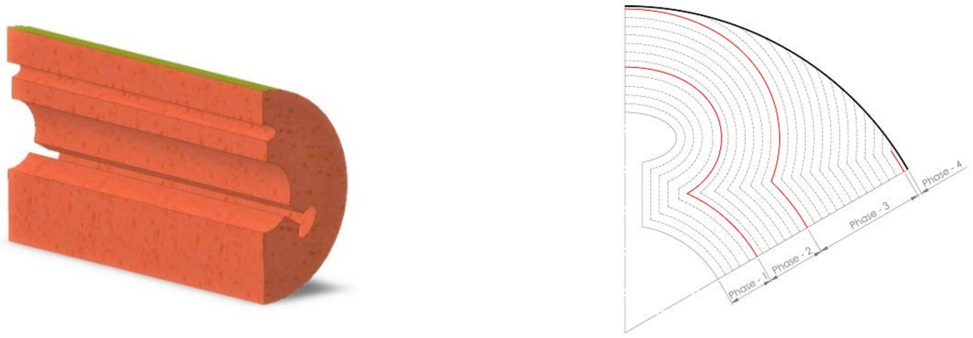
(a) Dogbone grain basic parameters (b) Baseline dogbone grain

Figure 9 Dogbone grain configuration

The baseline case study is a 3-arm dogbone grain (figure 10a), with all geometric data presented in Table 2. First, a drafting technique was executed for the baseline case, where different burning phases are shown in figure 10b. Phase one ends when the arm vanishes; this phase is slightly progressive as the decrease in the burning area due to diminishing of the arm is less than its increase due to the growth of the elliptical part size and the inner radius. Phase two is also progressive and it ends when the web disappears. In the third phase, the tail-off starts, with a regressive behavior as the area of burning decreases. The last phase is the sliver phase where the burning area decreases sharply near the burnout until the propellant grain is completely burned.

Table 2 Baseline case Parameters

Rin	e (a/b)	θ	h	Lb	La	N	R
25	2	70.53°	5.66 mm	15 mm	4 mm	3	76 mm



a- 3D Cross-section for baseline case b- Burning phases representation

Figure 10 Dogbone grain

The following is a comparison between the drafting technique and the LS method (figure 11). As one can see here the error (figure 12) is less than 3% which is also an acceptable error. This proves that the level set method is a reliable technique and can be used in this study.

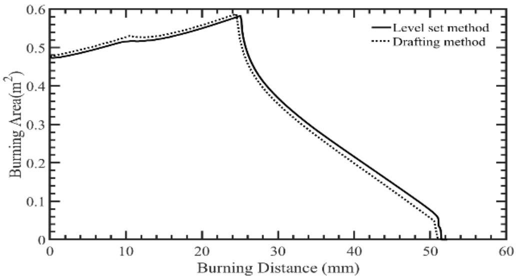


Figure 11 Results for baseline case using drafting technique and LS method.

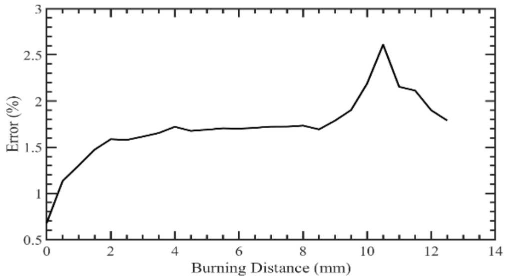


Figure 12 Error percentage for baseline grain.

5. Parametric study for dogbone grain

In order to conclude the effect of each geometric parameter on grain burnback, a parametric study is performed where a single parameter variation at a time method is applied for the following parameters: (R_{in}, e, θ, h, L_b, L_a, and N). Each parameter was changed 3 times to study its effect on the burning area. Table 3 presents a total of 16 studied cases. To compare between different cases three well-known parameters were introduced for each case. These parameters are the blocking factor (K_I) which is the ratio between the burning area to the port area, the clamping factor (J) which is the ratio between the critical area to the port area, and the filling coefficient ($K_{F.C.}$) which is the ratio between the propellant area to chamber cross-section area.

5.1 Effect of changing inner radius Rin':

The effect of changing the inner radius 'R_{in}' was studied by applying 2 different values for it (35 mm and 15 mm) and comparing it with the baseline case#1 (25 mm). For the 3 different configurations, the results of performance parameters and the grain evolution during burning are presented in (figures.13, 14) respectively. Increasing the inner radius obviously decreases the filling coefficient, increases the blocking factor (initial burning perimeter), and increases the sliver phase.

Table 3 Parametric Study Parameters

Case No.	Rin (mm)	L_b (mm)	L_a (mm)	a/b	θ °	h (mm)	N
Case#1	25	15	4	2	70.53	5.66	3
Case#2	35	15	4	2	70.53	5.66	3
Case#3	15	15	4	2	70.53	5.66	3
Case#4	25	10	4	2	70.53	5.66	3
Case#5	25	20	4	2	70.53	5.66	3
Case#6	25	15	6	2	70.53	5.66	3
Case#7	25	15	2	2	70.53	5.66	3
Case#8	25	15	4	2	70.53	5.66	5
Case#9	25	15	4	2	70.53	5.66	6
Case#10	25	15	4	1	56.14	7.50	3
Case#11	25	15	4	0.5	48.19	8.94	3
Case#12	25	15	4	0.5	180	0	3
Case#13	25	15	4	1	180	0	3
Case#14	25	15	4	2	180	0	3
Case#15	25	15	5.37	2	90	5.37	3
Case#16	25	15	5.37	2	270	-5.37	3

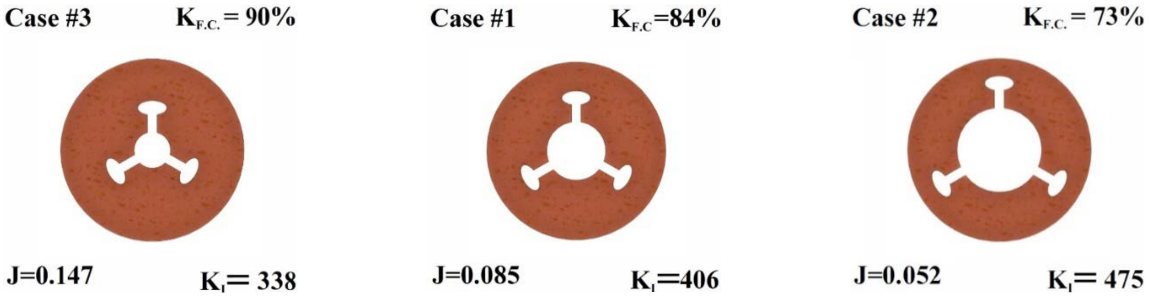


Figure 13 Effect of the inner radius on performance parameters.

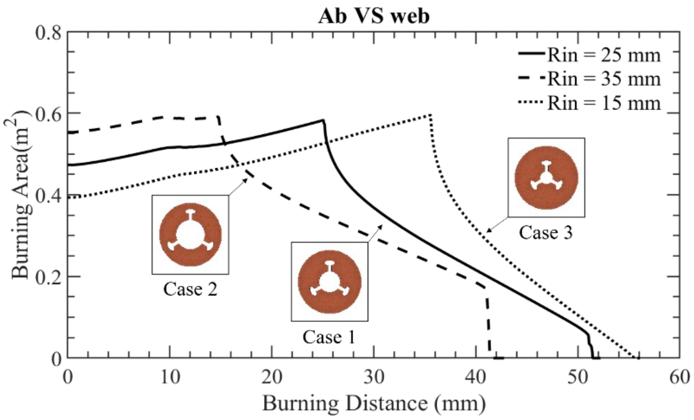


Figure 14 Effect of the inner radius on grain burnback

5.2 *Effect of changing arm length 'Lb'*

The effect of changing the thickness of the arm 'L_b' was studied by applying 2 different values for it, (20 mm, and 10mm) and comparing it with the baseline case#1 (15 mm). For the 3 different configurations, the results of performance parameters, and the grain evolution during burning are shown in (figures 15, 16) respectively. Increasing the arm length does not affect the filling coefficient, but increases the blocking factor (initial burning perimeter) and the sliver phase. From grain burnback, it is noticed that the start of the sliver phase is earlier as arm length is increased. The final burning phases are almost the same.

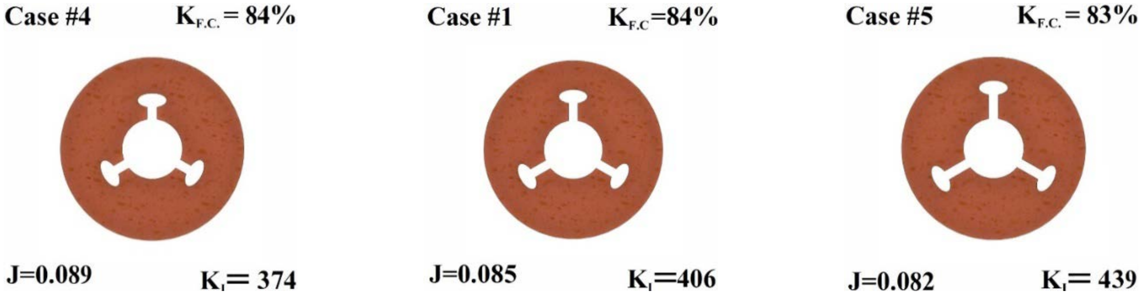


Figure 15 Effect of arm length on performance parameters.

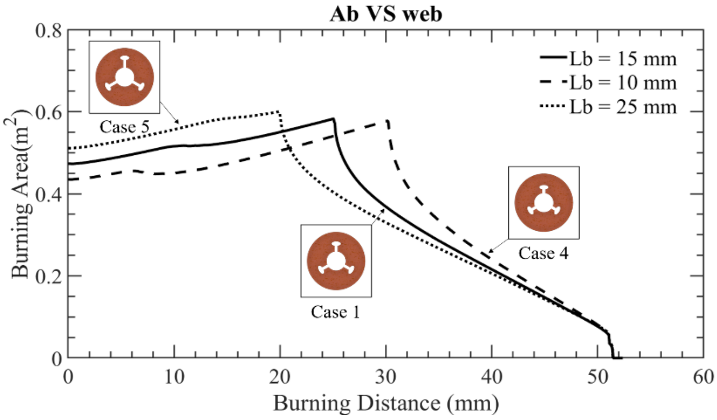


Figure 16 Effect of arm length on grain burnback

5.3 *Effect of changing arm thickness ‘La’*

The effect of changing the thickness of the arm ‘L_a’ was studied by applying 2 different values for it, (2 mm, and 6 mm) and comparing it with the baseline case#1, (4 mm). For the 3 different configurations, the results of performance parameters, and the grain evolution during burning are shown in (figures.17, 18) respectively. Increasing the arm thickness decreases the filling coefficient and the blocking factor, but the total change can be observed only in the initial burning area; the other changes are quite small.

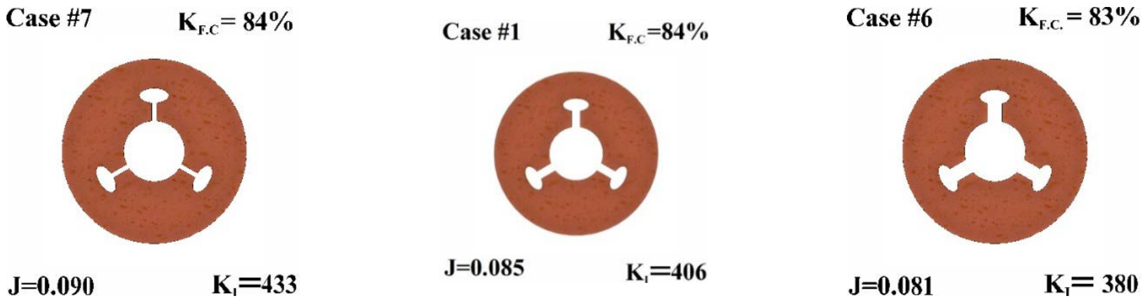


Figure 17 Effect of arm width on performance parameters.

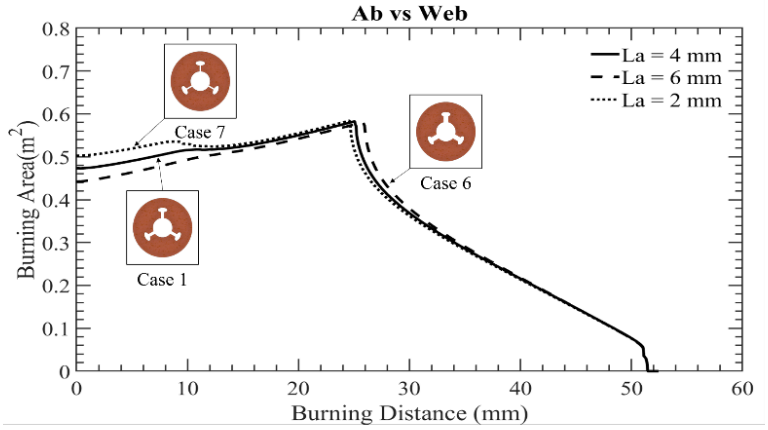


Figure 18 Effect of arm width on grain burnback

5.4 *Effect of changing the number of arms ‘N’*

The effect of changing the number of arms 'N' was studied by applying 2 different values for it (5 and 6) and comparing it with the baseline case#1. For the 3 different configurations, the results of performance parameters, and the grain evolution during burning are shown in (figures.19, 20) respectively. Increasing the number of arms made an obvious effect on the behavior of burning as shown. It decreases the filling coefficient and visibly increases the blocking factor but the major effect can be observed in the burning area evolution with burned distance; a dual thrust behavior is observed and the sliver area is reduced.

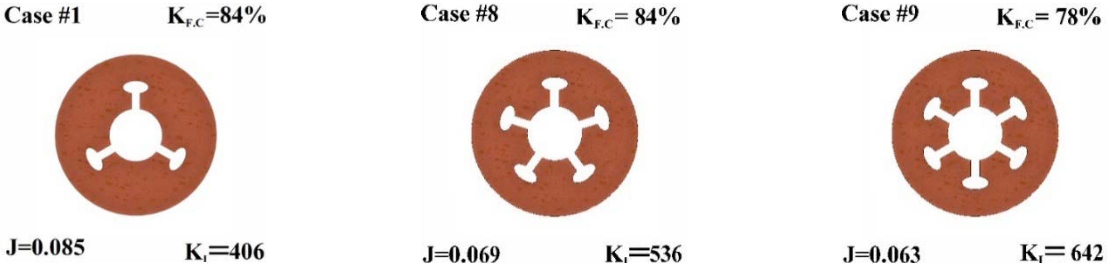


Figure 19 Effect of number arms on performance parameters.

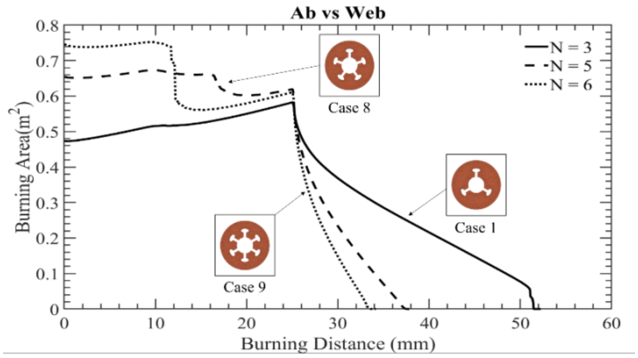
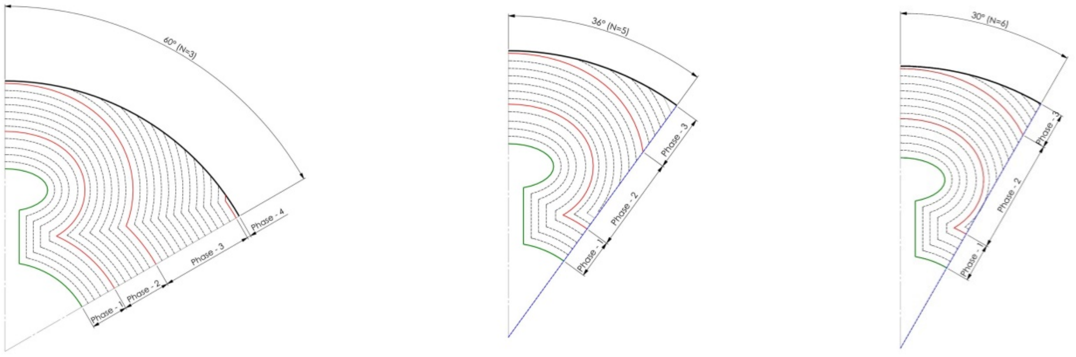


Figure 20 Effect of number of arms on grain burnback

For further understanding of the dual thrust behavior, the drafting technique for the 3 aforementioned cases was performed as shown in figures 21a-c. The blue dashed line is the sector border for each case; it was found that the surfaces became much closer and a detached sliver is formed as shown in figure 22. This indicates that the dual thrust behavior shown in figure 20 is caused by a sudden vanishing of the surface with the formation of a detached sliver, not a smooth decrease like star grains or sliver-less jump like wagon wheel grain.



a. Case #1 b. Case #8 c. Case #9

Figure 21 Drafting technique for cases #1, #8 and #9: Explanation of dual thrust behavior

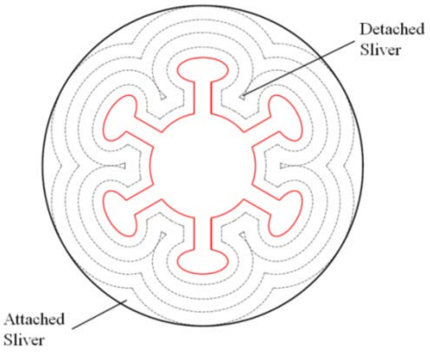


Figure 22 Dogbone grain sliver illustration

5.5 Effect of changing ratio (a/b)

The effect of changing the ellipse shape or (a/b) ratio was studied by applying 2 different values for it (2 and 0.5) and comparing it with the baseline case#1. For the 3 different configurations, the results of performance parameters, and the grain evolution during burning are shown in (figures. 23, 24) respectively.

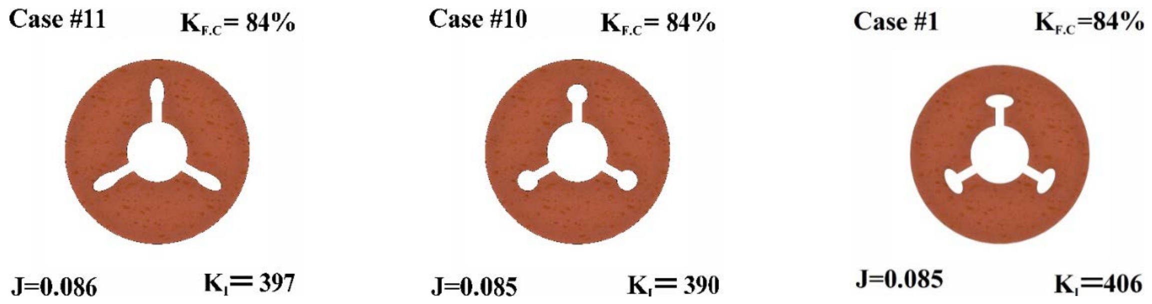


Figure 23 Effect of ellipse ratio a/b on performance parameters.

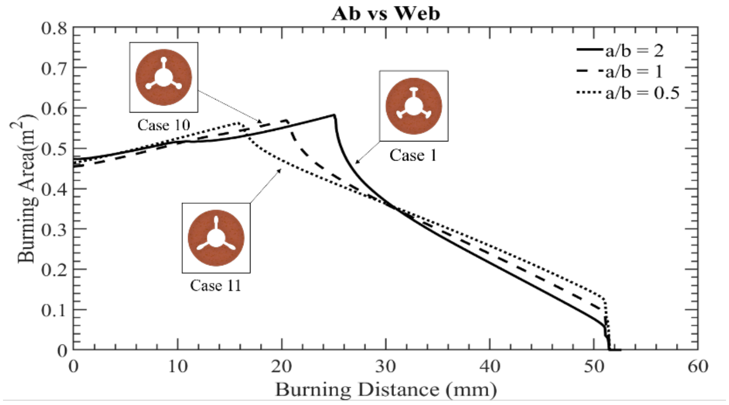


Figure 24 Effect of the ellipse ratio or (a/b) ratio on grain burnback

Changing the ratio (a/b) quietly affects the blocking factor and the filling coefficient. For a/b=0.5, the sliver area increases obviously, and the progressive phase decreases. For a/b=1 (circular tip) change can be observed in the sliver area too, and the progressive phase decreases a little.

5.6 Effect of tip center shape

The change in the tip center and shape was studied by assuming 3 cases as follows. The first one is case #12 as shown below. It is a special case of case #11 where a=La, b=2a, and the center h=0. When a=b=La and the center h=0, the grain configuration became finocyl. Finally, case#14 is a special case of case#1 where a=La=2b and radius h=0. For the 3 different configurations, the results of performance parameters, and the grain evolution during burning are shown in figures 25, 26 respectively.

As shown in the figure when a=La, b=2a, and the center h=0, the blocking factor is maximum and the filling is minimum, the sliver area is also the minimum. For the finocyl configuration, the sliver area increased but the progressive phase of burning is also increased and the blocking factor is decreased. Finally, when a=La=2b and radius h =0, the progressive phase is maximum and the sliver area is also increased, the blocking factor change is negligible.

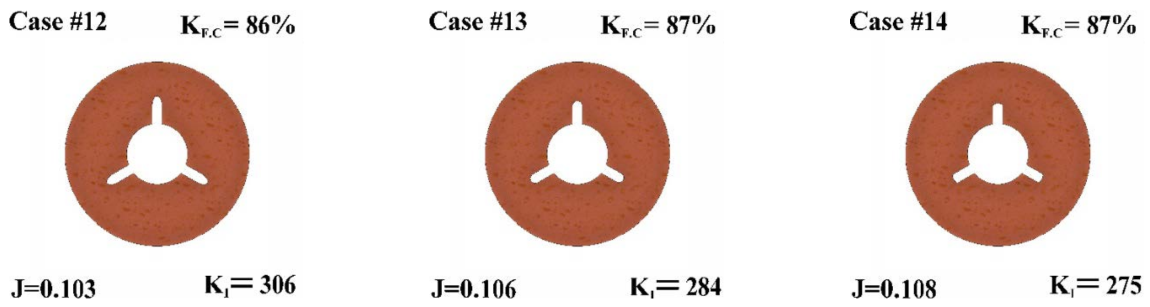


Figure 25 Effect of tip center shape on performance parameters.

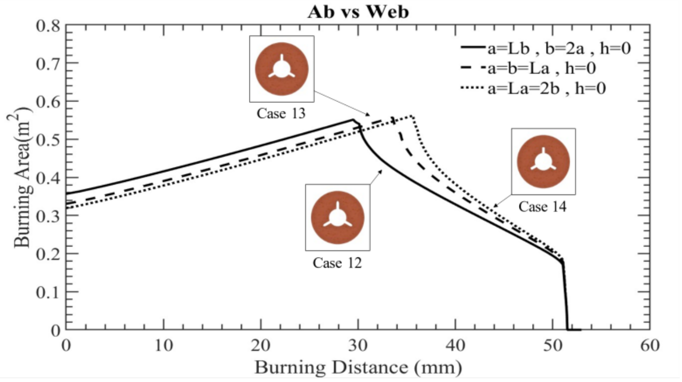


Figure 26 Effect of tip center shape on grain burnback.

5.7 Effect of ellipse angle ‘θ’

The ellipse angle ‘θ’ effect was studied by applying 2 different values for it (270°and 90°). The two cases were compared together and the results of performance parameters are shown in figure 27. The grain evolution during burning is shown in figure 28, which indicates that the ellipse center angle changes the entire configuration. When θ=270, the blocking factor decreases, the sliver area decreases obviously, the progressive phase increases, and the filling coefficient increases.

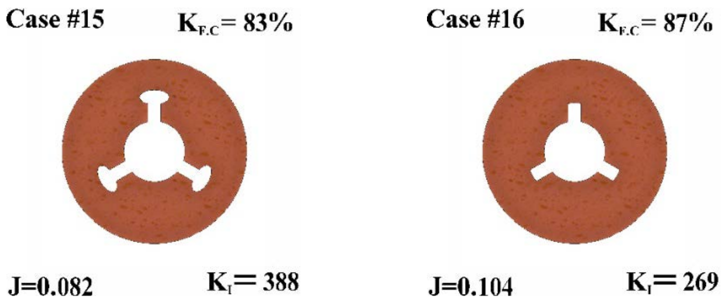


Figure 27 Effect of ellipse angle on performance parameters.

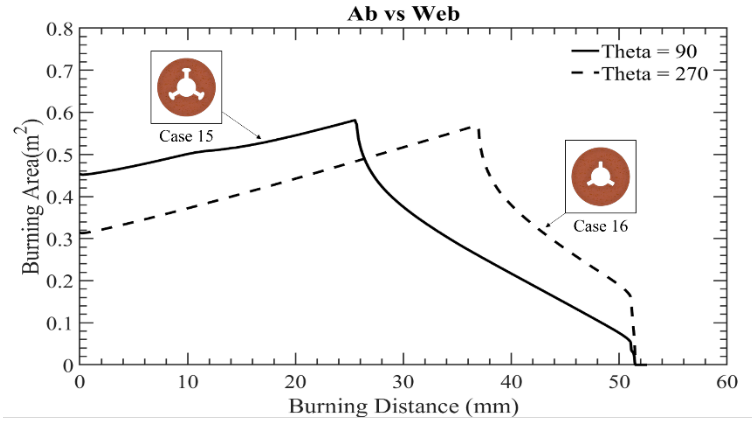


Figure 28 Effect of ellipse angle ‘θ’ on grain burnback

6. Conclusion and Future Work

The level set method and its application in evaluating grain burnback are introduced. Validation of the procedure is performed by comparing level set results for both star and dogbone grains with other validated techniques, namely drafting and analytical techniques, with an acceptable error. The level set method has the advantage of the ability to deal with complex geometry in a rapid way suitable throughout the preliminary design phase of a rocket motor. A parametric study for dogbone grain was performed, which showed that the dogbone grain is capable of providing a dual thrust profile. It is shown that the most important parameters are the number of arms, arm length, and inner radius, while the other parameters show minor effects on performance and grain burnback.

However, in order to get useful results from the proposed procedure, it needs to be automated to plug in an optimization module to get the most favorable design parameters according to the required mission. Furthermore, as the present methodology deals only with 2D grains, it could be modified to be usable in axisymmetric and 3D grains.

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