ABSOLUTE DISTANCE MEASUREMENT USING FREQUENCY SCANNING INTERFEROMETRY

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Abstract

Absolute distance measurement is necessary for many applications where the distance to a fixed-target is required. In this paper, frequency scanning interferometry is implemented with an external cavity tunable diode laser, which has a wavelength scanning range of (665-675 nm), to measure distances up to 5 m absolutely (achieved by multi-reflection system). An ultra-low expansion (ULE) Fabry-Pérot (FP) cavity has been used as a reference to measure the tunable-laser scanning range. Absolute distance is determined by counting the synthetic fringes and FP peaks simultaneously while scanning the laser frequency. A fringe processing technique is developed to perform online distance measurement even though the tunable laser is not mode-hop free. This fringe processing method enables simple, fast and accurate distance measurements with repeatability $\pm 3.9\times10^{-6}$ L and combined uncertainty $\pm 38.9\times10^{-6}$ L which is limited by the calibration system used.

Keywords: Absolute distance, optical interferometers, mode hop, frequency scanning interferometry, Fabry-Pérot cavity

1. Introduction

A numerous applications in the field of length metrology have been developed by means of laser invention. One of the most important implementations of lasers in this field is absolute distance measurement employing the frequency scanning interferometry. Conventional laser interferometers basically measure a distance by accumulating the incremental movements of the target mirror continuously using the wavelength of light as a ruler. Although such interferometers are very accurate (accuracy in the nanometer range), it cannot be used to measure the distance to a fixed target which is needed for several applications such as range finder in military applications, total stations which is used in the field of surveying and building construction, and fiber length measurements in telecommunications.

Frequency scanning interferometry (FSI) aims at measuring the distance absolutely between an observer and an object without target movement. It is based on counting interference fringes produced from a measuring interferometer and a reference interferometer as the wavelength of the external cavity diode laser is changed in a continuous manner **(Xiaoli and Katuo (1998), Bechstein and Fuchs (1998), Stone et al. (1999), Meiners-Hagen et al. (2009), Terra et al. (2014)).**

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Semiconductor laser's output spectrum depends strongly on laser temperature and injection current. At some combined values of laser injection current and temperature, wavelength can make discrete jumps. These jumps occur when the laser switches from one longitudinal mode to another which is known as mode hopping.

Thiel et al. (1995), Edwards et al. (2000), Meiners-Hagen et al. (2009), and Dutta et al. (2011) stated that mode-hope free tuneable laser is a perquisite for measuring distance absolutely employing the frequency scanning interferometry. In this work, an external cavity tunable diode laser with one mode-hop occurs through the wavelength scanning range (665-675 nm) is used to measure distances up to 5 m absolutely. An ultra-low expansion (ULE) Fabry-Pérot (FP) cavity has been used as a reference to measure the tunable laser scanning range. Absolute distance is determined by counting the measuring interferometer fringes and FP peaks simultaneously while scanning the laser frequency. A fringe processing technique is developed to perform online distance measurement even though the tunable laser is not mode-hop free.

Theory

In an ordinary interferometer, the phase difference between the interferometer arms is given by,

$$\Delta\Phi = \frac{4\pi}{\lambda_0} n\Delta L = \frac{4\pi\nu}{\mathrm{c}} n\Delta L \qquad (1)$$

where λ_0 is the wavelength in vacuum, ν is the laser frequency and ΔL is the path difference between the interferometer arms, n is the refractive index of the medium and c is the velocity of light in vacuum.

For absolute distance measurement, the path difference between the two fixed interferometer arms can be determined from the phase change due to the frequency tuning according to

$$L = \frac{c}{4\pi n} \frac{\Delta\Phi}{\Delta\nu} \qquad (2)$$

The phase difference $\Delta\Phi$ is obtained by counting the number of interference orders N (integer part and fractional part) produced during the wavelength tuning.

$$\Delta\Phi = 2\pi N \qquad (3)$$

A reference Fabry-Pérot cavity can be used to measure the swept frequency ($\Delta\nu$) **(Edwards (2000), Meiners-Hagen et al. (2009) and Cabral et al. (2010))**. It could be obtained by multiplying the free spectral range (FSR) of a Fabry-Pérot cavity by the number of cavity transmission peaks (r) as follows:

$$\Delta\nu = r \cdot FSR \qquad (4)$$

From equations (2), (3) and (4), absolute distance is given by

$$L = \frac{c}{2 \cdot n \cdot FSR} \frac{N}{r} \qquad (5)$$

The refractive index (n) of the ambient air is dependent on air parameters, temperature (T), pressure (P) and relative humidity (RH). **Bönsch and E. Potulski (1998)** made a modification on Edlen equation to calculate air refractive index as following

$$n = 1 + \frac{7.86 \times 10^{-4} P}{(T + 273)} - 1.5 \times 10^{-11} RH(T^2 + 160) \qquad (6)$$

2. Materials and Methods

2.1. System configuration

Fig. 1 shows the system used to measure the absolute distance. A tunable laser from New Focus (Velocity 6308) with 10 nm tuning range from 665 nm to 675 nm and around 3 mW output power is the main element of the measuring system. The laser output is directed to a Faraday isolator to prevent back reflections from the system into the laser cavity. After the isolator, the beam is divided between a measuring interferometer and a reference interferometer using a beam splitter. In order to keep the diode laser beam collimated along the whole round trip of up to 10 m, a simple collimation system, which consists of two lenses L1 and L2 with focal lengths 25.4 mm and 100 mm respectively, is used before the measuring arm. The measuring interferometer is a typical Michelson interferometer with a 50:50 beam splitter and two mirrors. The mirror M1 is installed at the reference arm and the mirror M2 is installed at the measuring arm. The reference interferometer is a typical Fabry-Pérot resonator with cavity length of around 126 mm. In order to avoid the effect of the cavity thermal expansion on length measurement, the cavity spacer is made from ultra-low-expansion glass (ULE-glass) of a thermal expansion of 0 ± 30 ppb/°C from 5°C to 35°C (lower 10000x from normal glass). FP interferometer is used as a frequency reference to determine the laser scanning range. Cavity reflectors are dielectric mirror with a reflectivity of R = 50%, which produces a reflection limited Finesse of around 4.44 ($F = \pi\sqrt{R}/(1 - R)$). The low Finesse is intentionally chosen to increase the width of the detected peaks which increases the number of the sampling points in each peak **(Meiners-Hagen et al. (2009))**. Photo-detectors PD1 and PD2 are used to detect the interference fringes from the measuring and the reference interferometers, respectively, and to convert it to an electrical signal. The two electrical signals are directed to two channels of an oscilloscope (Yokogawa DLM2022). The air parameters (temperature, pressure and relative humidity), which are used in the compensation of air refractive index, are monitored using a data logger (HUMLOG20 THIP).

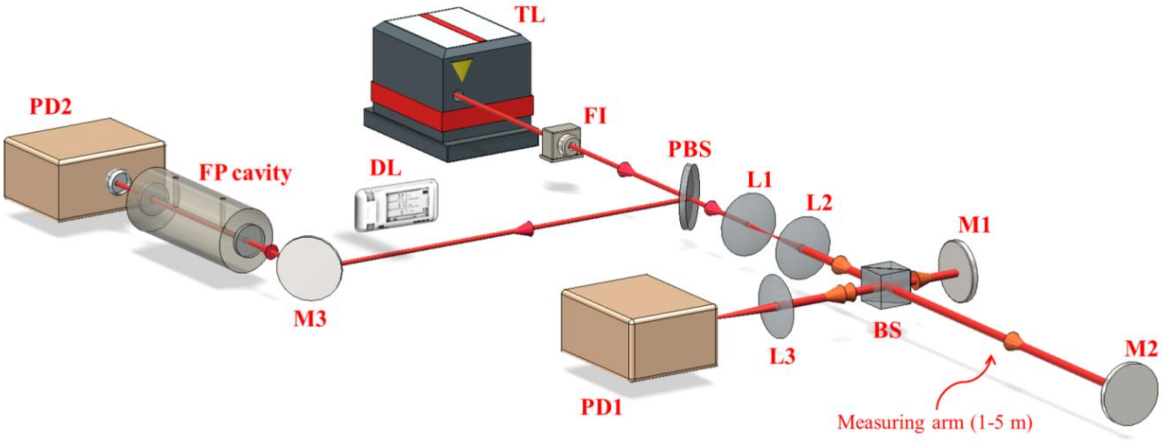

Fig. 1: Absolute distance measuring system setup. **TL:** Tunable diode laser, **FI:** Faraday isolator, **PBS:** Plate beam splitter, **BS:** Beam splitter, **L1, L2:** Collimating lenses, **L3:** Focusing lens, **M1, M2, M3:** Highly reflecting mirrors, **FP:** Ultra low expansion Fabry-Pérot cavity, **PD1, PD2:** Photo-detectors, **DL**: Data logger for air parameters

2.2. Fringe counting

One of the main sources of uncertainty in FSI is the fringe counting error. This error comes mainly from counting the fraction of a fringe. In order to avoid counting the fringe fraction, a fringe counting algorithm is used to count only the integer fringes from the measuring and reference interferometers. It is based on the facts that, the oscilloscope acquire signals from both channels simultaneously, and both signals are correlated since they come from the same laser. Consequently, there will be frequent positions on both reference and measuring signals where phases are matched. Therefore, the algorithm searches for the peaks that matches on both signals and count the integer number of peaks at each signal as shown in Fig. 2. The ratio between both numbers of peaks is *N/r*.

A Matlab program is developed based on this algorithm to count the number of peaks and calculate the ratio and then calculates the distance from the ratio and the air parameters as will be shown in section 3. Moreover, the program smoothes the signal using Savitzky–Golay filter to remove any ripples that could affect the process of signal maxima detection.

Fig. 2: Signals. **High frequency:** Interferometer signal, **Low frequency:** FP resonator signal.

Ideally, the laser should be mode-hop free, as reported by **Thiel et al. (1995), Edwards et al. (2000) Meiners-Hagen et al. (2009), and Dutta et al. (2011)**. Consequently the signals should show clean sinusoidal waves; however, when a mode-hop occurs while tuning the laser, it will appear on the measuring and the reference interferometers (see Fig. 3). Therefore, a counting error will appear. In order to avoid the miscounting due to these mode-hops, they should be removed from the signals in such a way that it doesn't affect the count. The program is modified to allow the removal of such mode-hops at the points of peaks match around the mode-hop. Therefore, the program counts the peaks after and before the mode-hop and adds them together. By following this method, mode-hops can be removed without affecting the counting; however, the uncertainty increases due to the peaks matching process.

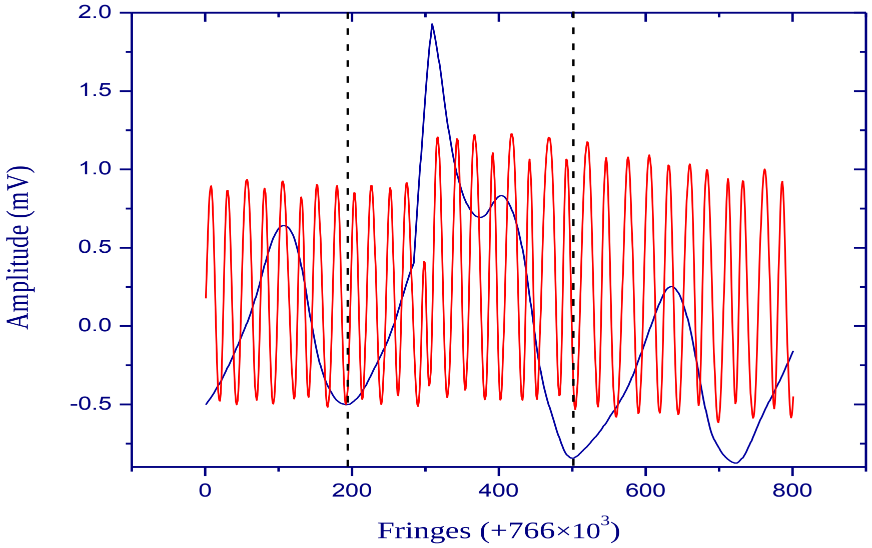

Fig. 3: A mode-hop of the tunable laser is observed on the measuring and the reference interferometers fringes.

It is assumed that the fringe counting process has no uncertainty since it is a digital process. Moreover, if a counting error occurred, it will be easily noticed. However, the cutting process involves some errors due to the search for matched peaks on both measuring and reference interferometers.

The uncertainty in the fringe counting caused by the cutting technique is evaluated by simulating the cutting process several times on different portions of the 1 m, 3 m and 5 m signals. Ideally, the fringe ratio (measuring/reference) should be the same whatever the number of signal splitting is. However, the ratio changes with changing the number of splitting. The deviation of the count from unperturbed signal count is calculated and found to be length dependent ($\pm 2.5 \times 10^{-6}$ L).

2.3 FP cavity calibration

As reported by **Cabral and J. Rebordão (2006)**, a calibrated translation stage is used to find the FP cavity free spectral range. This stage has a movement resolution of 10 nm and the accuracy of less than 1 µm in 2.5 cm movement. A preliminary FSR based on the length of the cavity is assumed for the cavity used in the absolute distance setup. The mirror M2 is fixed on the stage and moved a displacement of 24 mm in steps of 4 mm while measuring the distance absolutely with the interferometer. By partially differentiating equation (5), the values measured by the interferometer are compared with the values given by the stage. The corrected FSR is stated as follows:

$$FSR_{\mathrm{corr}} = FSR\left(\frac{E}{D} + 1\right) \qquad (7)$$

where E is the difference between the measured distances and the stage displacement, and D is the displacement of the stage.

3. Results and Discussion

The setup described in Fig. 1 is used to measure the distance from 1 m to 5 m absolutely. The tunable laser is scanned with a constant power mode from 665 nm to 675 nm and with scanning rate of 0.55 nm/s. To enable laser propagation with minimum divergence over long distances, the beam waist of the laser of around 1.5 mm is collimated and expanded to be 4 mm. The FSR of the FP interferometer is initially measured, and then the distances from 1 m to 5 m are measured.

3.1. FSR determination of the FP cavity

The FSR of the cavity is determined as described in section 2.3. The error in the measured distance by the absolute distance setup is found to be (14.6 ± 1) µm along the calibration distance range of 24 mm. Therefore, the correction factor $(E/D+1)$ is found to be 1.003658.

3.2. Air refractive index

Air refractive index is one of the most effective parameters in measuring long distances. Air refractive index can be calculated from the modified Edlén formula (equation (6)). The uncertainty in determining the refractive index will be limited by the uncertainty in temperature, pressure and relative humidity measurements. The uncertainty contribution to the distance measurement will be demonstrated in Table 2 based on the uncertainty in the measured air parameters, which is provided by the manufacturer.

Because the cavity has a hollow core, the change in the refractive index of the air inside the cavity affects its optical path and hence the FSR is as following

$$FSR = \frac{c}{2\ n_c\ d} \qquad (8)$$

where n_c is the refractive index of air inside the cavity and d is the cavity length. As both of the measuring and reference interferometers are placed close together in the same enclosed and controlled environment, air refractive index effect will be almost the same $(n_c = n)$ **(Meiners-Hagen et al. (2009))**. By substituting from equation (8) in (5), it is clear that the measured distance is independent of air refractive index.

$$L = \frac{N \times d}{r} \qquad (9)$$

However, since it is impossible to reach a perfect match between both refractive indices, additional uncertainty contribution should be considered. This contribution is based on the air fluctuations inside the measuring area.

Table 1 shows the measured variation in temperature, pressure and relative humidity during the experiment and its effect on the ratio between refractive index of the ambient air and air inside the cavity.

Table 1: Air parameters effect on refractive indices

Air parameter	Measured value	Variation	Effect on L (m)
Temperature	20.7 °C	±0.03 °C	$\pm 2.85\times10^{-7}$ L
Pressure	101.00 kPa	±0.05 kPa	$\pm 1.34\times10^{-7}$ L
Relative humidity	58.6 %	±2 %	$\pm 1.77\times10^{-8}$ L

3.3. Absolute distance measurement

After measuring the FSR of the FP cavity, distances from 1 m to 5 m can be accurately measured by placing the mirror M2 at the intended positions. A linear fitting is performed on the repeatability of the measured distances as shown in Fig. 4 Measured distance repeatability is found to be $\pm 3.9\times10^{-6}$ L.

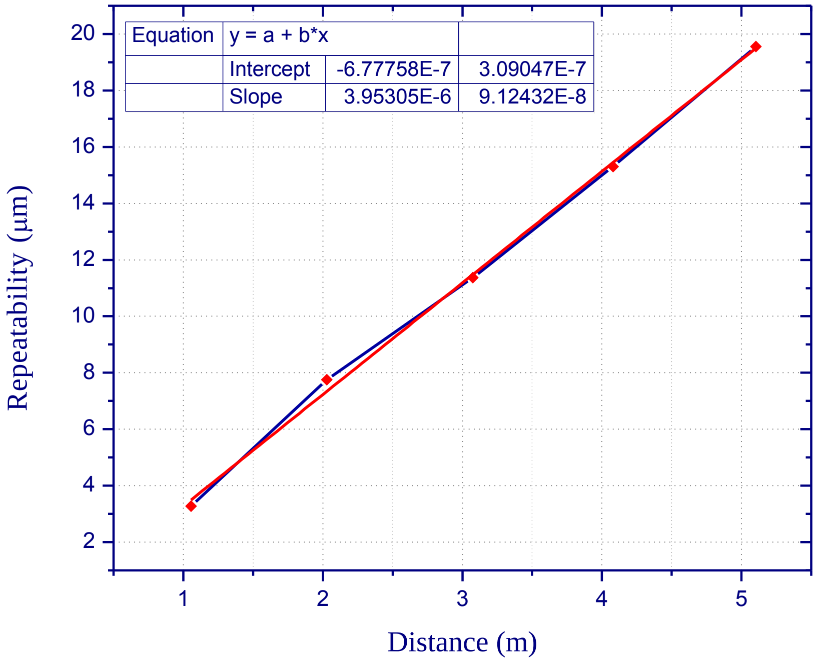

Fig. 4: Measured Repeatability of the measured distances

3.4. Uncertainty Analysis

In order to report the results of absolute distance measurement, the measurement uncertainty must be stated. The contributions from different sources of uncertainty are found by partially differentiating equation (5). The ration N/r is a constant ratio, so it is replaced by R.

$$\delta L = \sqrt{\left(\frac{c}{2 \cdot n \cdot FSR}\delta R\right)^2 + \left(\frac{c \cdot R}{2 \cdot n^2 \cdot FSR}\delta n\right)^2 + \left(\frac{c \cdot R}{2 \cdot n \cdot FSR^2}\delta FSR\right)^2} \qquad (10)$$

From equation (10), it is clear that the main sources of uncertainty are: 1- Counting uncertainty 2- Air refractive index and 3- The FSR of the FP cavity.

The uncertainty in the fringe counting is demonstrated in section 2.2. The uncertainty contribution of the air refractive index to the distance measurement was previously discussed in section 3.2 and its values are demonstrated in Table 2.

Using an ultra-low expansion (ULE) glass as a spacer for the FP cavity of thermal expansion coefficient of 30 ppb/ºC **(Berthold et al. (1977))**, allows the cancelation of the effect of temperature on cavity length.

From equation (7), the uncertainty in determining FSR will be limited by the accuracy of the translation stage and the repeatability of the measured stage displacements. The total uncertainty in measuring FSR is found to be 45.8 kHz. The contribution of uncertainty in FSR in measuring the absolute distance is $\pm 38.6 \times 10^{-6}$ L.

The evaluation of measurement uncertainty is made according to the Guide to the Expression of Uncertainty in Measurement GUM **(BIPM (2008))**. The total uncertainty budget for measured distances is shown in Table 2. Measured distance repeatability is found to be $\pm 3.9\times10^{-6}$ L. The combined uncertainty u_c is $\pm 38.9\times10^{-6}$ L. The results obtained for an expanded uncertainty of the measurement corresponding to a coverage probability of 95% (k = 2) is $\pm 77.75\times10^{-6}$ L. The uncertainty is limited mainly by the calibration system. If another calibration system is used with similar accuracy but wider range than the stage used, the uncertainty could be much less.

Table 2: Uncertainty budget for measured distances

Source of uncertainty	Value	Probability distribution	divisor	Uncertainty	Sensitivity coefficient	Standard uncertainty (m)
Temperature	±0.03 °C	Rectangular	$\sqrt{3}$	±0.02 °C	$\pm 9.5\times10^{-7}$L	$\pm 1.9\times10^{-8}$L

Pressure	±0.02 kPa	Rectangular	$\sqrt{3}$	±0.01 *kPa*	$\pm 2.68\times10^{-6}$L	$\pm 2.68\times10^{-8}$L
Relative humidity	±0.15%	Rectangular	$\sqrt{3}$	±0.09 %	$\pm 8.8\times10^{-9}$L	$\pm 7.9\times10^{-10}$L
FSR	±45.8 kHz	Normal	1	±45.8 kHz	1	$\pm 38.6\times10^{-6}$L
Fringe counting cutting	$\pm 2.5\times10^{-6}$L	Normal	1	$\pm 2.5\times10^{-6}$L	1	$\pm 2.5\times10^{-6}$L
Statistical uncertainty	$\pm 3.9\times10^{-6}$L	Normal	1	$\pm 3.9\times10^{-6}$L	1	$\pm 3.9\times10^{-6}$L
Combined Uncertainty						**$\pm 38.9\times10^{-6}$L**

4. Conclusion

This work shows that the frequency scanning interferometry could be applied to measure distances up to 5 meters absolutely with high accuracy even though the tuneable laser is not mode-hop free. Air refractive index effect on distance measurement could be partially neglected in case that both of reference interferometer and measuring interferometer are placed closely together. A new technique is used to count interference fringes produced from both the measuring and the reference interferometers. This technique allows counting interference fringes with superior accuracy even if a mode-hop occurs while scanning the tuneable laser. This fringe processing method enables simple, fast and accurate distance measurements repeatability $\pm 3.9\times10^{-6}$ L and combined uncertainty $\pm 38.9\times10^{-6}$ L which is limited by the calibration system used.

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الملخص باللغة العربية

"قياس المسافات المطلقة بإستخدام تقنية مسح ترددات التداخل الضوئي"

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تتطلب العديد من التطبيقات في مختلف المجالات قياس المسافات المطلقة (إلى هدف ثابت). يتم في هذا البحث تطبيق تقنية مسح ترددات التداخل الضوئي بإستخدام ليزر متغير الطول الموجي في المدي من 665 نانومتر و حتي 675 نانومتر لقياس مسافات مطلقة تصل إلي 5 أمتار. يستخدم مقياس تداخل فابري بيرو مصنوع من زجاج ذو معامل تمدد حراري منخض جدا كمرجع لقياس التغير في الطول الموجي لليزر المستخدم. لقياس المسافة المطلقة يتم إحصاء هدب التداخل الناشئة من مقياس تداخل مايكلسون و مقياس تداخل فابري بيرو أثناء مسح ترددات الليزر متغير الطول الموجي. يتم تطوير طريقة إحصاء هدب التداخل الضوئي لقياس المسافات المطلقة علي الرغم من أن الليزر المستخدم يحتوي قفزات عشوائية بين أطواله الموجية مما قد يعوق تحديد المسافة بالدقة المطلوبة في حالة عدم معالجتها. تمكن الطريقة المذكورة في هذا البحث قياس مسافات تصل إلي 5 أمتار بدقة ± 38.9 ×10^{-6}× الطول المقاس.