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Assessment of Uncertainties in Soil Parameters Using Finite Element Coupling Technique

M.Mourad[1], *M. ElGendy*[2], *E.Tolba*[3], *E.Galal*[4]

[1] *Civil Department, Faculty of Engineering, Port Said University, Port Said, Egypt, email:* mohamed.hamed@eng.psu.edu.eg
[2] *Civil Department, Faculty of Engineering, Port Said University, Port Said, Egypt, email:* melgendy@elpla.com
[3] *Civil Department, Faculty of Engineering, Port Said University, Port Said, Egypt, email:* prof.tolba@gmail.com
[4] *Civil Department, Faculty of Engineering, Port Said University, Port Said, Egypt, email:* elsayed.galal@eng.psu.edu.eg

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ABSTRACT

Uncertainties are unavoidable in geotechnical engineering design. Quantifying uncertainties and the related risks are very important in the overall design. For problems involving complex soil-structure interactions, using a fixed partial or global safety factor may lead to unrealistic failure or conservative design. The probabilistic analysis could be used as a basis for handling various types of uncertainties. The reliability analysis for complex structures requires a coupling between two models. A finite element model (FEM) to evaluate the limit state function (LSF) and a reliability model to deal with the random variables and reliability methods. Since the coupling between FEM and reliability methods is considered a new field, there is a need to use a hybrid technique for coupling. In the present paper, a developed technique for coupling between PLAXIS 2D v20 and Probabilistic tool-kit (PTK) is illustrated and checked throughout verification examples. The main goal is to present an “easy to use” reliability analysis technique to identify the probability of failure, the reliability index, and the influence factors for complex geotechnical structures. The results show that the developed coupling technique has a good reliability analysis result compared with the previous studies and it can be used to optimize the current design procedure for geotechnical structures.

Keywords: Soil structure interaction, Reliability analysis, Coupling technique, PTK, FORM, Uncertainty.

1.INTRODUCTION

Geotechnical engineering is primarily dealing with natural materials. Therefore, the uncertainty in soil properties is unavoidable. The uncertainty in the values of soil properties may be arises due to many sources of errors. These errors can be categorized into three main types according to F.H. 1992 [1] as:

1) Measurement errors.
2) Human errors.
3) Transformation errors.

The inaccuracy of the devices and equipment used in field and laboratory experiments may be the source of measurement errors. On the other hand, human errors may occur due to the lack of experience during laboratory experiments. Transformation errors may arise due to using the measured parameters through analytical equations.

One of the most important sources of uncertainty in geotechnical engineering is called aleatoric uncertainty. It can be characterized as the inherent randomness of natural processes; this variability can be defined by changes of the soil properties with time at a specific location, temporal variability, or over space at a single time, spatial variability, Baecher and Christian 2003 [2].

The conventional design of the geotechnical structures such as footings and retaining walls depends on the deterministic method. This method divides the ultimate resistance by a global safety factor to get the allowable resistance and reach a required safety level. This safety factor accounts for the uncertainties in soil, loads, the model used, etc.

However, due to the high uncertainties resulting from the variability of soils, measurement uncertainties, and transformation uncertainties, the deterministic method may not accurately reflect the real-world situations, and the effect of soil variability cannot be adequately represented. Recently, reliability and probability analysis started to take place in the engineering community. These methods account for the possible uncertainties. (Cederstrom 2014 [3]).

Although a deterministic design ensures a certain level of structural safety, it is worth considering a probabilistic approach to quantify safety and reliability that cannot be measured with a deterministic method (Russelli 2008 [4]). In some cases, the deterministic analysis is more conservative in determining stability and safety than the reliability analysis with the same safety level. (Muhammed 2019 [5]).

According to Chandrasekaran 2016 [6], the significant advantages of reliability methods can be summarized as:

- All sources of uncertainties in the project are considered.
- Offer decision-making support regarding the risk-cost analysis.
- Determine the probability of failure for each design method.
- The structure can be designed based on serviceability conditions.
- The overall risk which takes place in the project is figured out.

To overcome the previous sources of errors and to carry out a reliability analysis, the soil parameters should be regarded as random rather than deterministic variables. The reliability analysis requires a definition of the statistical characteristics for each soil parameter. These characteristics are at least the distribution type, the variance and the mean value of each random variable.

The most important factor related to reliability analysis is the limit state (LS) separating line, curve, or surface between the failure and non-failure domains. In reliability analysis, limit state function (LSF) is defined with a mathematical equation describing the failure mechanism. For simple structures, an equation describing the response of the structure can be used to evaluate the LSF for a certain failure mechanism. For complex structures, it is not possible to define an equation to describe the structure's response to assess the LSF. Therefore, in this case, the evaluation of LSF should be done using Finite Element Models (FEM). The problem here is to find a technique for coupling the FEM and the statistical model to carry out the reliability analysis.

Nowadays, reliability analysis is considered a new field, so that, in the literature, there are a number of studies that carried out a reliability analysis for different types of simple structures. Abdel-Fattah 2017 [7] presented a reliability-based analysis for slope stability problem since it is a representative application for calibrating the soil strength parameters. Ashraf et al., 2020 [8] performed a probabilistic analysis of strip footings resting on soil with uncertain properties to compute the probability of failure using Monte Carlo Simulation method. Muhammed 2019 [5] compared between the deterministic and probabilistic approaches in the analysis of the bearing capacity of a bridge foundation on undrained clay soil.

Limited studies for the reliability analysis for complex structures were carried out. Some of these studies are: Schweckendiek 2006 [9] studied the applicability of several probabilistic methods on the coupling with FEM-models of retaining structures using the prob2b reliability library. Wolters 2012 [10] contributed to the development of sheet-pile structures concerning quay wall design via a PLAXIS-FORM coupling using prob2b. Teixeira and Rippi 2016 [11] presented the benefits of a reliability-based analysis and employed the open-source open turns tool for the reliability study, combined with a finite element tool for limit state evaluation. Van der Wel 2018 [12] verified the design guidelines using partial factors and determined whether using a semi-probabilistic design strategy for quay walls achieved the needed level of safety while also being the most cost-effective, using PROBANA reliability library. Manoj 2017 [13] introduced probabilistic principles into finite element calculations using the point estimate approach by using PROBANA reliability library.

The statistical and reliability libraries which were used in the previous studies are either outdated, and their production has stopped, or its coupling technique with FEM is so complicated. In this paper, a developed technique for coupling finite element model (PLAXIS 2D V20) and the newest reliability model (PTK) from deltares will be illustrated and checked throughout verification examples. The main goal of this study is to present an “easy to use” reliability analysis technique for complex structures.

2.RELIABILITY METHODS

2.1 Reliability Analysis Principals.

Determination of failure probability is the most crucial factor in determining an element’s reliability. The limit state is the boundary between failure and non-failure. In contrast, the possibility that this threshold will not be exceeded is known as reliability. Limit State Functions (LSF) are used to interpret limit states, and their typical form (Baecher and Christian 2003 [2]) is:

$$Z = R - S \qquad (1)$$

Where (R) is the resistance or the component which contribute to non-failure. while (S) and sometimes (Q) is the load or the component which contribute to failure.

Resistance should be greater than load in structural design, or the performance function must be greater than zero (Z>0). The primary goal for the system is that this requirement remains relevant during the life time of the system. However, almost all of the parameters which forms resistance and load are uncertain. Therefore, the probability of non-failure for a given system is calculated and used to indicate the system reliability, P(Z>0). Probability of failure is:

$$Pf = P\ (Z \leq 0) = P\ (S \geq R) \qquad (2)$$

As a result, the reliability could be in the sense that:

$$P\ (Z > 0) = 1 - Pf \qquad (3)$$

Figure 1 shows the safe domain, failure domain and the LSF corresponds to R-S = 0 according to Chandrasekaran 2016 [6].

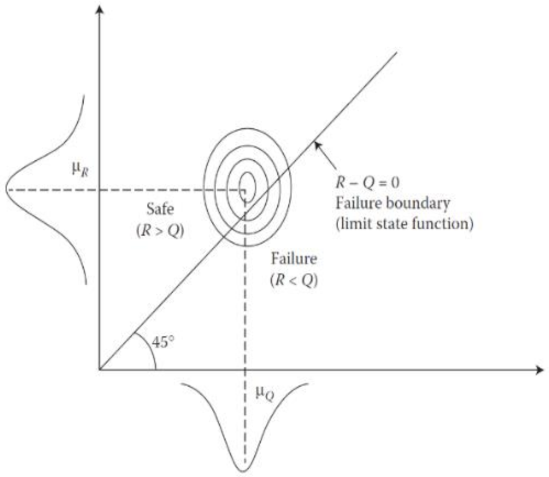

Figure 1: Safe domain and failure domain

The design concept in this scenario is based on determining the design point, that is located in failure space with highest probability density. This location is usually found on the dividing line between safe and unsafe areas. A level classification of the computation methods is created by Joint Committee on Structural Safety 1981 [14]. The three levels of this classification are:

- **Level I**: Semi Probabilistic, no failure probabilities evaluated at this level.
- **Level II**: Fully Probabilistic with approximations.
- **Level III**: Fully Probabilistic computations.

2.2 First Order Reliability Method (FORM)

First Order Reliability Method (FORM) is considered a level II method. The principal of this method is to linearize the LSF in U-space. Its accuracy decreases as the LSF becomes more non-linear in the areas of large probability density. FORM has significant deffect in which it cannot deal with numerous LSF. As a result, FORM should only be employed when there are no consedrables impact on outputs due to non-linearities or system effects. Figure 2 shows the linearization of the LSF carried out on the point in which (Z = 0), which is called the design point.

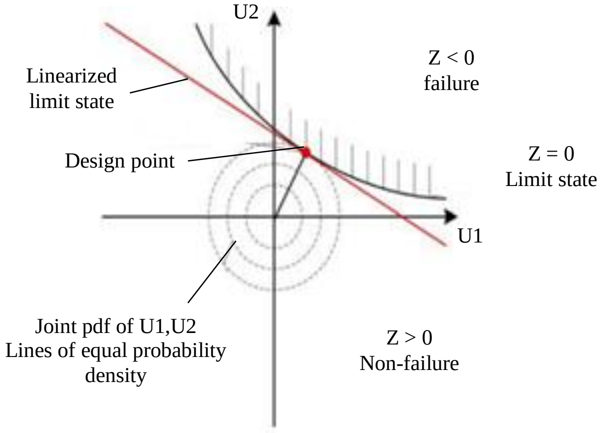

Figure 2: Design Point and Linearized Limit State in case of FORM.

The stochastic parameters are converted into typical standard normally distributed parameters and the overall process is done in U-space. According to Hasofer and Lind 1974 [15], for parameters with a normal distribution the conversion process may be expressed as shown:

$$ui = \frac{x_i - \mu_{xi}}{\sigma_{xi}} \qquad (4)$$

where; (ui) is the normalized variable, (x_i) is the variable value in x space, (μ_{xi}) is the normalized mean and (σ_{xi}) is the normalized standard deviation.

The FORM algorithms need derivatives of the limit state function concerning xi to be determined. Aiming this, the following relationship can be used:

$$dxi\ /\ dui = \sigma_{xi} \qquad (5)$$

$$\frac{\partial Z}{\partial ui} = \frac{\partial Z}{\partial xi} \cdot \frac{\partial xi}{\partial ui} = \frac{\partial Z}{\partial xi} \cdot \sigma_{xi} \qquad (6)$$

The limit state function, only expressed in terms of resistance and load becomes:

$$Z = R - S = \sigma_R U_R - \sigma_S U_S + \mu_R - \mu_S = 0 \qquad (7)$$

To determine the reliability index β and probability of failure Pf, this distance must be minimized. A Lagrangian Multiplier Approach or a Tylor Series Approach might be used to elaborate on this minimization problem. Both of these approaches lead to the same solution.

$$\beta = \frac{\mu_Z}{\sigma_Z} = -\frac{\sum u_i(\frac{\partial Z}{\partial u_i})}{\sqrt{\sum(\frac{\partial Z}{\partial x_i})^2}} \qquad (8)$$

Then the term influence factors may be defined as follows:

$$\alpha_i = \frac{(\frac{\partial Z}{\partial u_i})}{\sqrt{\sum(\frac{\partial Z}{\partial x_i})^2}} \qquad (9)$$

2.3 Directional Sampling Method (DS)

Directional Sampling Method (DS) is considered a level-III method. Directional sampling takes place in U-space. The n joint probability distribution is created then the random values for the variable's combination is calculated. The steps in the procedure according to Schweckendiek 2006 [9] are listed as follows:

1- A mean value computation is performed in U = 0.

2- A vector U is generated which is the vector connecting the origin of the variable space and a randomly produced point in the variable space.

3- The generated vector is converted to a certain length. At this stage the vector direction is kept as an information and the limit state function is evaluated. Figure (3) shows the procedure steps from 1 to 3.

4- An iterative procedure is used to determine the scale factor λ $(\lambda > 0)$ which agree with Z = 0 at this point the vector direction is considered constant. Figure (4) shows the procedure step number 4.

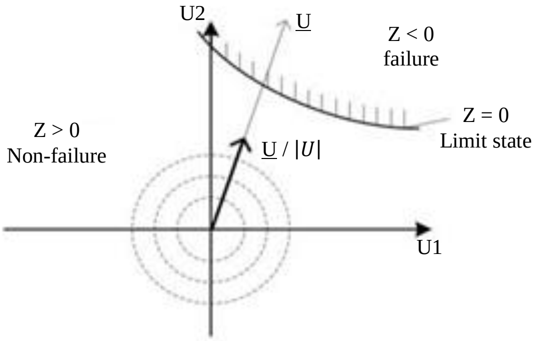

Figure 3: Directional sampling procedures (Steps 1 – 3)

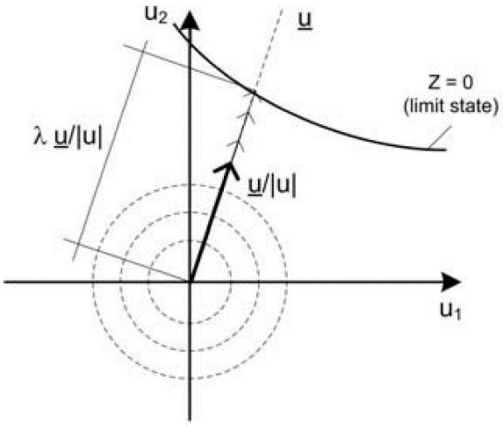

Figure 4: Directional Sampling procedures (Step 4)

5- $\sum_{i=1}^{n} \lambda_i^2$ is X^2 – distributed with degrees of freedom equals to stochastic parameters. If λ did not change in all directions then the failure probability may be calculated as follows:

$$Pf = 1 - X^2(\lambda^2, n) \qquad (10)$$

6- The previous process is repeated (step 1-5) as iteration process till the convergence criteria is reached. The convergence criteria may be the probability of failure variance or a specified number of iterations.

3.COUPLING RELIABILTY ANALYSIS WITH FEM

Calculating the probability of failure and the determination of influence coefficients is crucial when examining the reliability of an element or structure (reliability analysis). The limit state evaluation for complex systems requires a finite element program such as PLAXIS. While the reliability analysis requires a probabilistic library to deal with the reliability methods, such as the Probabilistic ToolKit (PTK).

A communication interface is required for coupling the PTK with PLAXIS. The interface should modify PLAXIS inputs and read PLAXIS outputs like soil and structural properties, water pressures, stress processing, and the displacements in soil. According to the chosen reliability analysis approach, PLAXIS must read the new values that PTK simulates for the selected random variables during the iterative process. Figure 5 shows the coupling scheme for PLAXIS and PTK.

In principle to run a reliability analysis using PTK, the following steps must be performed:

- Choosing the preferable reliability method.
- Defining the input random variables and their probability distributions.
- Defining the correlation matrix if any.
- Set the limit state function according to situation.

According to the probabilistic run in the PTK, PTK will calculate a new value for the chosen random variables, and now they are ready to be sent to PLAXIS. For this transmission, a python script (input interpreter) is required to send this input data to PLAXIS and perform the PLAXIS calculations. After the PLAXIS calculation is finished, a python script (output interpreter) extracts the desired results and sends them back to the PTK. PTK evaluates the new outcomes and prepares a new PLAXIS simulation to be sent again to PLAXIS. This loop will continue until specific convergence requirements are reached; at this point, the loop will be terminated.

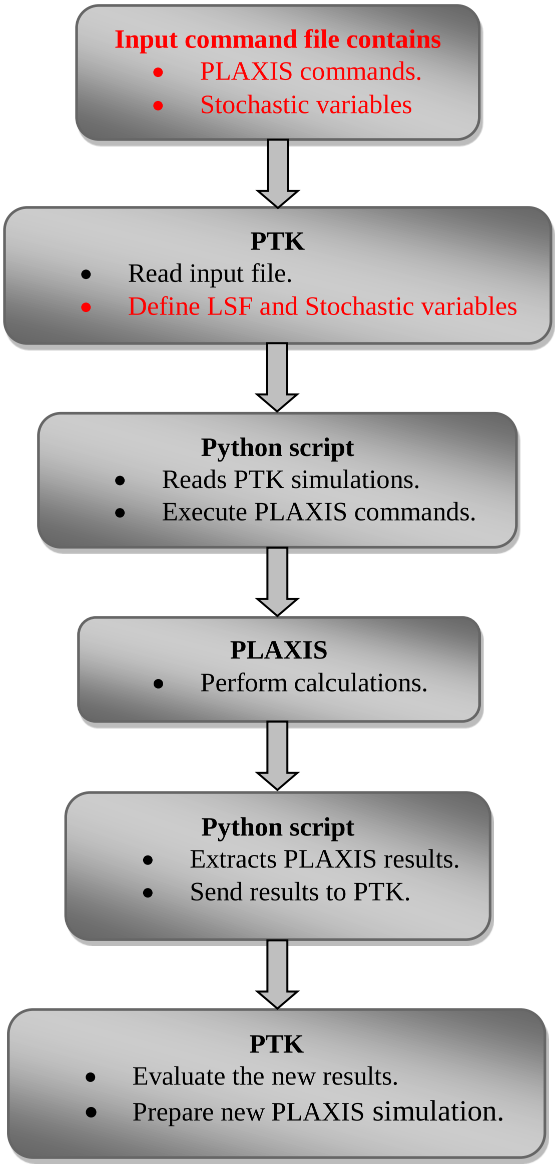

Figure 5: Coupling scheme: Probabilistic Toolkit-PLAXIS. In red all the required user input and in black the automated features.

Figure 6 shows the flow chart for the developed coupling between the PTK and PLAXIS 2D V20.

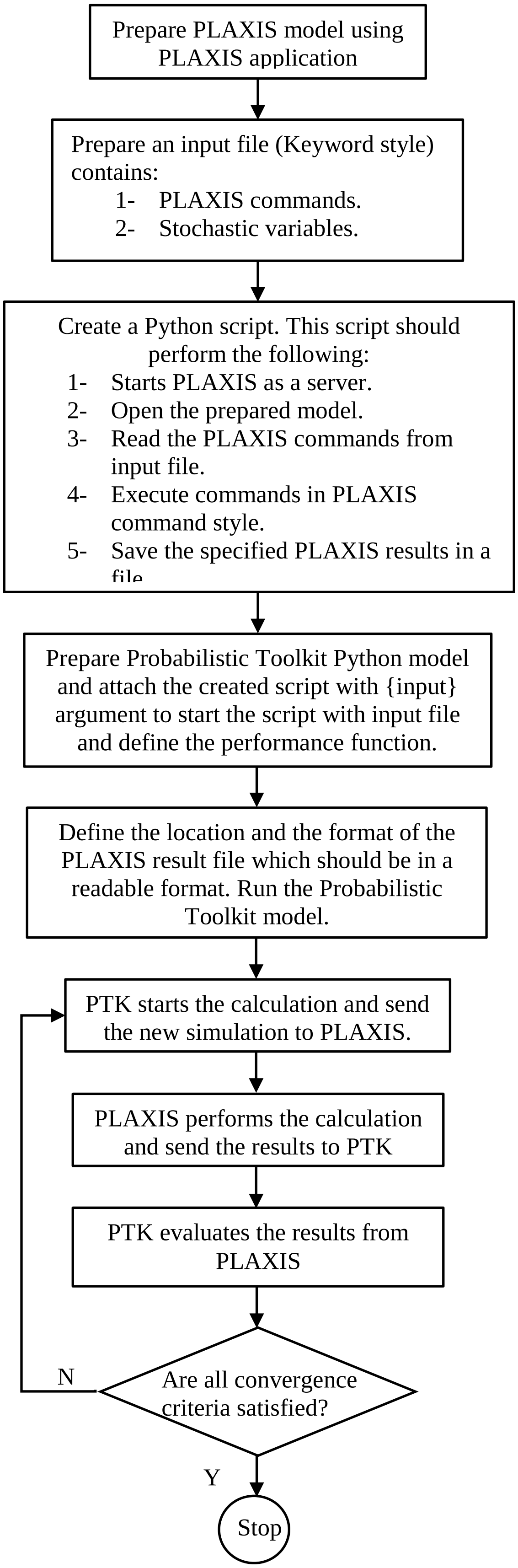

Figure 6: Flow chart of the coupling algorithm.

4. VERIFICATION

This section will check the accuracy of the coupling technique and test the developed python scripts, which link between the program which evaluates the limit state function (PLAXIS) and the reliability library (PTK). This will be done by conducting a reliability analysis for problems used in previous studies and comparing the new technique results with other reliability programs such as (PROBANA from PLAXIS) & (PROB2B from TNO), which were used in previous studies.

4.1 Vertical Cut Stability.

The first problem to be considered is the stability of the vertical cut. The unit weight of soil is 18.00 kN/m3 and, the vertical cut depth is 12.00 m. The problem geometry is shown in Figure 7, and the soil parameters are given in Table 1, according to Manoj 2017 [13].

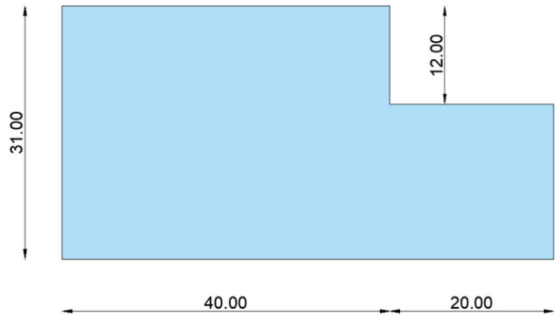

Figure 7: The geometry of the vertical cut problem.

Table 1: Soil Parameters of the vertical cut.

Parameter	Name	Value	Unit
Material Model	Model	Mohr Coulomb	--
Drainage type	Type	Undrained (B)	--
Young's Modulus	E	5000	[kN/m^2]
Poisson's Ratio	ν	0.3	--
Unit weight	γ_{unsat} $/\gamma_{sat}$	18/24	[kN/m^3]
Undrained shear strength	$S_{u,ref}$	133.34	[kN/m^2]
Friction angle	Φ	0	°
Dilatancy angle	ψ	0	°
Tensile strength	σt	10.00 E^6	[kN/m^2]

A reliability analysis is conducted by coupling the deterministic PLAXIS model with PTK. The undrained shear strength parameter, Su is regarded as the stochastic input. The statistical properties of Su are (mean = 133.34 kN/m2 and COV = 0.2) represented by Manoj 2017 [13]. The probability of failure of the slope is computed by using PTK. This requires defining a limit state function in which the safety analysis is considered for this case. Equation 15 defines the threshold of the reliability analysis.

$$Z = MSF - 1 \qquad (15)$$

Where MSF is the mean factor of safety and 1 is the threshold value since F < 1 which means a failure of slope. The reliability analysis and the coupling process gives a probability of failure for the vertical cut equals 0.00168.

Due to the problem linearity, the probability of failure can be calculated manually as:

$$F = \frac{N_O\, Su}{\gamma\, H} \qquad (16)$$

Where No is the stability number which depends on the angle of the slope which could range between 3.83 and 4 for vertical slopes according to the Swedish slip circle method and plane slip surface. The deterministic analysis of PLAXIS gives a factor of safety 2.39. From the obtained factor of safety, the stability number No can be calculated. Hence equation 16 could be written as:

$$F = \frac{3.88\, C}{\gamma\, H} \qquad (17)$$

the statistical properties for the factor of safety can be calculated as shown in Figure 8.

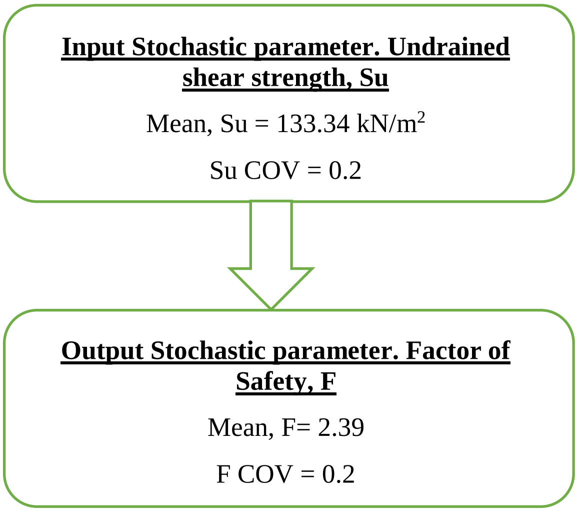

Figure 8: input and output stochastic parameters.

The coefficient of variations for both Su and factor of safety is the same since both of them have a linear relationship. To obtain the probability of failure, the corresponding area under the normal distribution should be calculated. The area below the threshold value 1.0 which corresponds to (μ_f – 2.9σ) which gives a value 0.00186 for the probability of failure.

The resulted probability of failure using manual solution, developed technique and the previous study presented by Manoj 2017 [13] are shown in Table 2.

Table 2: Probability Validation.

Method	Pf
Manual solution	0.00186
Developed technique (PTK)	0.00168 (error = 9%)
Previous study (PROBANA)	0.0017 (error = 8.6%)

Table 2 shows that the coupling technique and the python scripts have an acceptable result concerning the manual solution and the previous study using PROBANA reliability library.

4.2 Foundation Bearing Capacity.

This problem involves the classic Brinch-Hansen bearing capacity example. The main reliability results will first be calculated using the developed technique then the results will be compared with the reliability results obtained from the reliability library PROB2B model found in Schweckendiek 2006 [9]. The problem geometry is shown in Figure 9, and the soil parameters are given in Table 3.

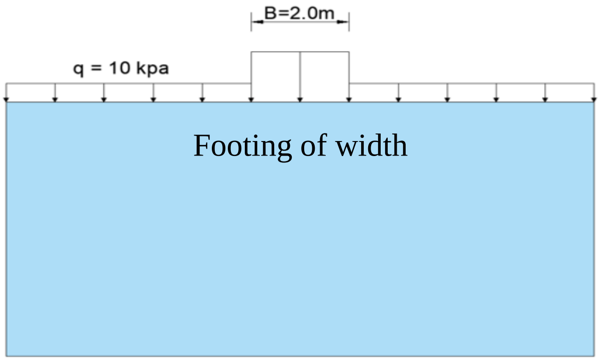

Figure 9: The geometry of the foundation bearing capacity problem.

Table 3: Soil Parameters of the foundation bearing capacity problem.

Parameter	Value	Distribution type	Standard deviation
Unit weight (γ)	15	Normal	1.5
Cohesion (C)	5	Log-Normal	2
Friction angle ($\emptyset$)	25	Normal	2.5

The Brinch-Hansen formula is given by:

$$p = c\,Nc + q\,Nq + 0.5\,\gamma\,B\,N\gamma \qquad (18)$$

where: (C) is the cohesion of soil, (γ) is the volumetric weight of soil, ($\emptyset$) is the friction angle, (q) is the uniform surcharge load, (B) is the width of foundation and (Nc, Nq, $N\gamma$) are a dimensionless coefficient.

PLAXIS will be used to evaluate the limit state, and will be coupled with PTK to perform the reliability analysis through the developed python script. Figure 10 shows the finite element mesh which will be used in the analysis.

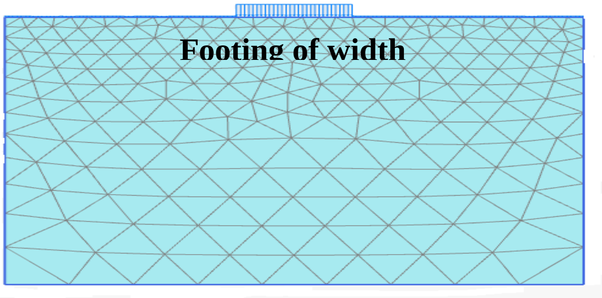

Figure 10: finite element mesh used in the foundation bearing capacity problem.

The PTK analysis results will be compared with PROB2B results. The limit state which will be used is given by equation (19).

$$Z = MSF - 1 \qquad (19)$$

Table 4 shows a comparison between the reliability analysis main results for current study values and the classical values.

Table 4: Reliability analysis results Comparison

	Current study results	Schweckendiek (2006) [9]
$\boldsymbol{\beta}$	3.65	3.75
Pf	8.2×10^{-5}	8.6×10^{-5}
$\alpha_{\emptyset}$	0.82	0.8
α_c	0.57	0.59
α_{γ}	0.06	0.11
LSF evaluation	32	24

The previous table shows that the coupling technique and the python scripts have an acceptable result compared with the previous study using PROB2B reliability library.

4.3 Sheet Pile Wall Without Support.

This problem involves an example of deep excavation in sand using sheet pile without support. The main reliability results will be calculated using PTK and PLAXIS coupling technique then the results will be compared with the PROB2B results found in Schweckendiek 2006 [9]. The problem geometry, soil properties and structural properties are shown in Figure 11.

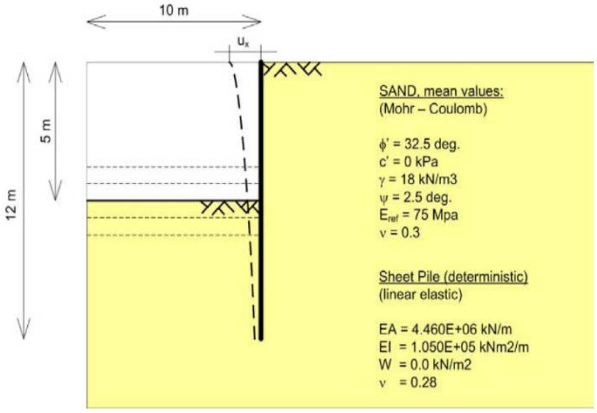

Figure 11: sheet pile wall without support example.

Table 5 illustrates the statistical properties for the soil parameters. Coefficients of variation and distributions types for soil characteristics were chosen according to the values given from the following researches (Schweckendiek 2006 [9], Wolters 2012 [10], Teixeira, et al., 2015 [17]).

Table 5: Soil Parameters of the foundation bearing capacity problem.

Parameter	**Mean Value**	**Distribution type**	**COV**
Eref [kpa]	75000	Normal	25%
Rint [--]	0.5	Normal	20%
∅ [deg.]	32.5	Log-Normal	10%
$\gamma unsat$ [kN/m^3]	18	Normal	5%
ν [--]	0.3	Normal	10%
ψ [deg.]	3.25	Normal	5%

Neither groundwater nor external loads are presented in the verification problem. The structural properties for the sheet pile are treated deterministically. The construction stages are processed as follows:

- Gravity loading of soil
- Excavation until final level (-5.00m)

In this problem, the unacceptable states of the structural system will be considered as failure. The excessive deformation and the exceedance of yield stress for sheet pile will be used as a limit states for this problem.

4.3.1 Top displacement of sheet pile.

The maximum horizontal displacement of the sheet pile (Ux) wall will occur at the top level of the sheet pile. The allowable deformation (Uall) for this type of structures is about 1% of retaining wall height or 10cm. the limit state in this case can be given by the following equation.

$$Z = Uall - Ux \qquad (20)$$

FORM is the reliability method that will employed in this analysis. The reliability main results for the developed coupling technique and the results found in Schweckendiek 2006 [9] are listed in Table 6.

Table 6: Reliability analysis results Comparison

	Current study results	***Schweckendiek (2006)* [9]**
β	2.25	2.33
Pf	1 x 10^{-2}	1.002 x 10^{-2}
LSF evaluation	82	71

4.3.2 Yield strength exceedance in the sheet pile wall.

The bending moment is generated due to the horizontal load on the sheet pile. For the non-anchored structure, the normal force can be neglected. The stresses in the steel should not exceed the yield strength in order to remain safe. Figure 12 shows the expected bending moment for sheet pile without support.

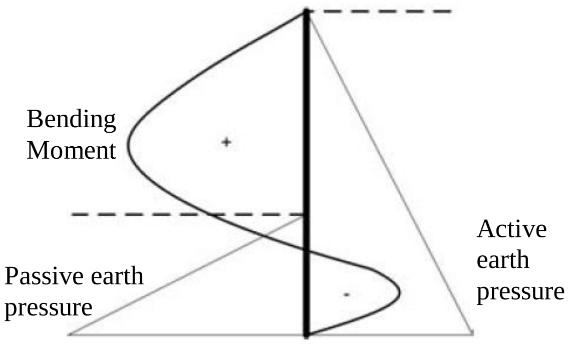

Figure 12: Expected bending moment for sheet pile without support.

The maximum induced value of bending moment (Mmax) will be obtained from FEM. For limit state formulation the maximum allowable moment (Mall) can be given by the following equation:

$$Mall = \sigma_y \, . \, Z \qquad (21)$$

Where: (σ_y) is the yield stress for the sheet pile material and (Z) is the section modulus of the sheet pile wall. The limit state function can be formulated as follows.

$$Z = Mall - Mmax \qquad (22)$$

FORM is the reliability method that will employed in this analysis. The reliability main results for the developed coupling technique and the results found in Schweckendiek 2006 [9] are listed in Table 7.

Table 7: Reliability analysis results Comparison

	Current study results	***Schweckendiek (2006)* [9]**
β	3.81	3.85
Pf	5.84×10^{-5}	5.87×10^{-5}
LSF evaluation	112	106

Table 7 shows that the coupling technique and the python scripts have an acceptable result compared with the previous study result.

5. CONCLUSIONS

The present work demonstrates a developed technique for coupling the newest uncertainty source, PTK, and the finite element program PLAXIS 2D V20 via a Python code interface to perform reliability analysis. The developed technique was checked throughout verification examples found in literature. The results show that the coupling technique and the python scripts have an acceptable result compared with the previous studies.

The developed coupling technique has the following benefits:

- The technique uses FEM advantages through PLAXIS and combines more than one failure mechanism in a single model.
- The technique can access the full probabilistic library of PTK, which contains almost all the probability distribution functions and the most common reliability methods.
- The technique is straightforward and "easy to use" for the researchers interested in the reliability new field.
- The importance factors obtained through the reliability analysis are quite beneficial since they provide more insight into the problem and differentiate between important and less important or even negligible variables.
- The coupling technique could be used for the calibration process.

The developed coupling technique may be considered a research technique that can be used in research projects or advanced assessment of existing structures.

Credit Authorship Contribution Statement:

Mohamed Hamed: Generating the idea, collecting data, Methodology & Original draft preparation, Mohamed El-Gendy: Reviewing & Supervision, Ehab Tolba and Elsayed Galal: Validation, Editing, Reviewing & Supervision

Declaration of competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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LIST OF SYMBOLS:

R :	Resistance component.
S :	Load component.
P_f :	Probability of failure.
ui :	Normalized variable.
μ_{xi} :	Normalized mean.
σ_{xi} :	Normalized standard deviation.
β :	Reliability index.
λ :	Scale factor.
Φ :	Cumulative density function in the standard normal space.
i :	Iteration index number.
N_o :	Stability number.
Su :	Undrained shear strength.
C	Cohesion of soil.
γ :	Volumetric weight of soil.
$\emptyset$:	Friction angle.
q :	Uniform surcharge load.
B :	Width of foundation.
$Nc, Nq, N\gamma$:	Dimensionless coefficient.
$Uall$:	Allowable deformation.
$Mall$:	Allowable moment.
σ_y :	Yield stress.
Z :	Section modulus.