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APPLICATION OF MATHEMATICS IN PERFORMATIVE ARCHITECTURE

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ABSTRACT

Integrating mathematics in architectural design and building construction helped in identifying new methodologies or tools that can generate unpredicted novel forms. Thus, architectural performance had developed with the involving of advanced mathematics fields and computational software. The approach of this research paper is to come up with findings on importance of mathematics in the process of form finding in art and architecture. Consequently, this study will investigate to what extent mathematics forms in architecture had actually a great effect on the architectural performance of building. A discussion of performance from the structural, environmental and aesthetical point of view will show. This will be done through describing how mathematics had been shifted from being an organizing tool into a creative tool then a generative medium.

الملخص العربى :

يتناول البحث مجالات وطرق التصميم التوليدى التى حولت العملية التصميمية من تصميم المجسم الى تصميم المنطق مع اعتبار منهج التحسين الأمثل التصميم البارمترى كأداة فى اكتشاف تصميات متعددة ومختلفة وبالتالى الوصول الى التكوين المعمارى المناسب. وقد ساعد دمج الرياضيات فى التصميم المعمارى وعملية البناء فى توليد حلول التصميم المختلفة، وبناء عليه قد تطور الأداء الانشائى والأداء البيئى والأداء الجمالى مع استخدام مجالات الرياضيات المتقدمة وبرامج الحاسب المتطوره. ويقوم البحث باستكشاف أهمية الرياضيات وتأثيرها فى العمارة. أكد البحث أن أشكال الرياضيات المختلفة فى العمارة كان لها تأثير كبير على الأداء الانشائى والأداء البيئى والأداء الجمالى للمبنى.

1. INTRODUCTION

In usual traditional design, the role of the designer is to explore a solution space. The key relationship between designer and product is a direct one “Figure 1” (even if mediated via a third-party or medium). There is a direct relationship between the designer’s intentions and that of the designed product [9].

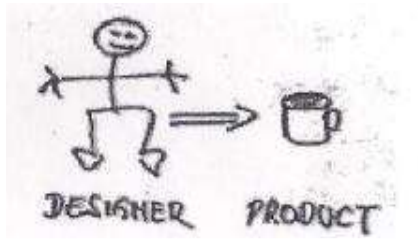


Figure 1.Traditional design approach [9].

In contrast, design using generative methods involves the creation and modification of rules or systems that interact to generate the finished design autonomously “Figure 2” **[18]**.

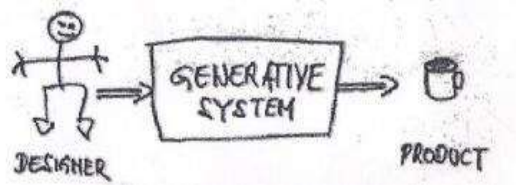


Figure 2. Form generation operation [9].

"Generative design is not about designing a building, it’s about designing the system that designs a building." Thus, Generative systems made a major shift in the concept of design modeling from the modelling of a pre-designed “object” to the modelling of the design “logic” [15].
Generally, the generative systems’ process is based on four main components: the input parameters, the controlled rule, the output generations and the selection of the optimal solution. The design artifact could only be obtained by the fourth phase.

2. GENERATIVE PERFORMANCE DESIGN

Performative design is an approach derived from performance models. It amalgamates form generation and performance, considering both through optimization algorithm and simulation technique. No longer are simulation tools utilized for analysis only, but they are used for both performing analysis and synthesis simultaneously; form is driven by generative processes guided by analytical simulation techniques that automatically modify the model “Figure 3”.

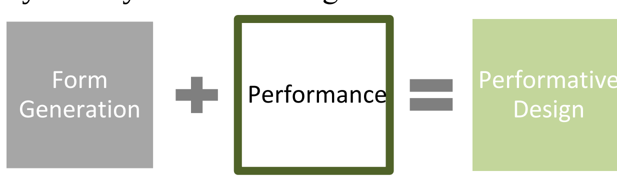


Figure ٣. performative design
Source; by the researcher

Considering architectural field, performance simulation tools have been widely used to establish early design processes, which are effective as alternative solutions to isolate design problems. However, current simulation tools failed to address novel shading systems and façade designs due to its limitation in modeling and exploring wide range of solutions. Thus, integrating new digital generative systems such as parametric design systems as well as Genetic Algorithms with these simulation tools contributes in raising building performance through offering unlimited alternatives in specific ways that ensuring the simulation process is evaluating the resulted in a comprehensive cycle till reaching the optimal solutions. This paper will focus on the latter case.

3. MATHEMATICS IN ARCHITECTURE

For thousands of years, civilized people have used mathematics to investigate sizes, shapes, and the relationships among physical objects. Geometry was carefully organized in about 300 B.C., when Euclid gathered what known at the time, added original work of his own, and arranged 465 propositions into thirteen books, entitled The Elements. Mathematics also speaks through numbers; drawing is the essential language of architecture. Thus, a close relationship has developed between geometry and architecture [22]. As the matrix of ideas contained the creation of shapes, symmetry, patterns and proportions. More novel is the encouragement of creativity through appreciation of mathematics with computer-aided design software, topological and fractal architecture, which provide a variety of tools for the efficient design, analysis, and imagines of complex shapes [4,12].

Form in architecture certainly cannot be separated from the process of form generation and conception. Form generation in architecture is conditioned as an interdependent relationship between two parameters; the aesthetical and technological ones, so it can be defined as a process that balances the intangible feature with the materializing aspect “Figure 4”.

These factors are dependent on the development of the digital technology and naturally on the evolution of mathematical concepts.

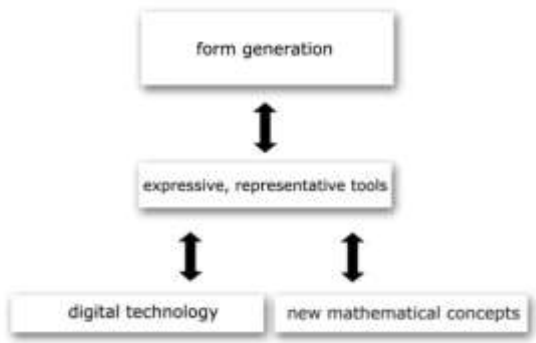
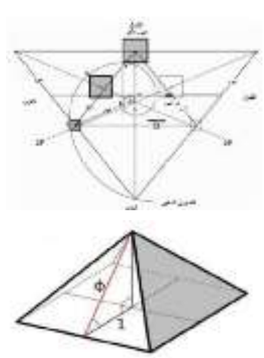
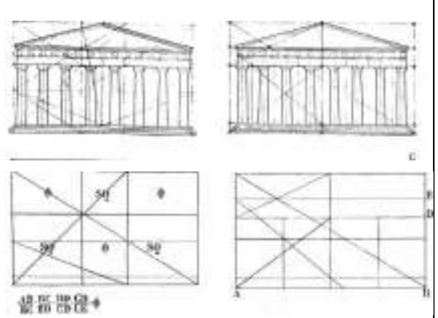
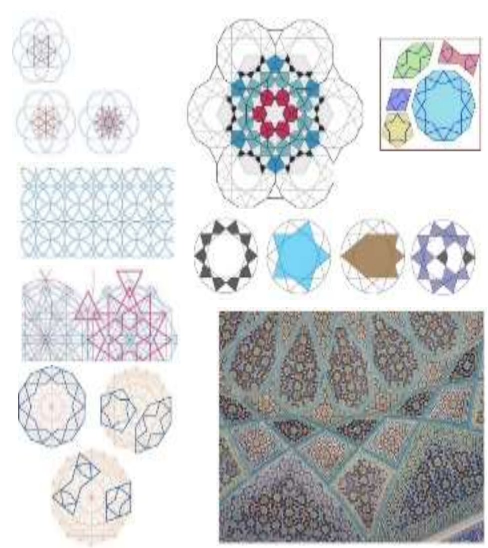


Figure 4. Relationship between form generation, representative methods, and influencing parameters [20].

		
The Great Pyramids of Giza: The Golden Ratio was incorporated into the design of the sides and right triangles were used during the construction process [13].	Two graphic analyses illustrate the use of Golden section in the proportioning of the Parthenon façade (Athens, 447-432 B.C).	Basic of Geometry- Applications of pattern based on the golden mean proportion [4].

4. Historical Review

Historically, the link between math and architecture goes back to Ancient Egyptians times, when the two disciplines were virtually indistinguishable. In the ancient world, the architects who constructed the pyramids, ziggurats and temples were originally mathematicians. In Classical Greece and ancient Rome, architects were required to be mathematicians as well. The Greeks recognized the dominating role the golden section played in the proportioning of the human body. Believing that both man and his temple should belong to higher universal order, these same proportions were reflected in their temple structures. Mathematical systems of proportions originate from the Pythagorean concept of "all is number" and the idea that certain numerical relationships manifest the harmonic structure of the universe.
It is interesting to note that while both analyses begin fitting the façade into a Golden Rectangle, each analysis then varies from the other in its approach to proving the existence of the Golden section and its effect on the facades' dimensions and distribution of elements.
When the Byzantine emperor Justinian decided to construct Hagia Sophia, the building that surpassed everything ever built, he turned to professors of mathematics to do this job. This tradition continued in the Islamic civilization, which created an enormous number of tiling patterns centuries before Western mathematicians devised a complete classification [12].

5. MATHEMATICAL TYPES

The mathematics' importance in architecture was studied via many aspects that helped in designing freeform architecture. On the other hand, some concerned with studying the harmonic relation between geometry and architectural design to keep the background of culture and understand the actual combination between geometrical thinking and architectural designing. In consequence, from many previous studies in various applications, it could be classified into; Geometry, Parametric, Algorithm, and Topology.

5.1 GEOMETRY

The word geometry is derived from the Greek words for earth and measure. Geometric figures first appeared over 15,000 years ago. These geometric figures used as shapes of buildings and decorations on pottery. Thales developed the first general theorems for geometry. Pythagoras tried to explain all aspects of the universe in terms of counting numbers. These counting numbers of Pythagoras were represented by sets of objects arranged in geometric shapes. Plato was another philosopher who was emphasized geometry in his academy. He used the five regular polyhedrons to explain the scientific phenomena of the universe. As a pupil of Plato, Aristotle developed the laws of logical reasoning. Euclid was structured the mathematics into a logical system [10].

TYPES OF GEOMETRY USED IN ARCHITECTURAL DESIGN The most known geometries in mathematic are Euclidean geometry, Non-Euclidean geometry, fractal geometry.

5.1.1 Euclidean geometry

Euclidean geometry is the study of points, lines, planes, and other geometric figures, using a modified version of the assumptions of Euclid. It was of great practical value to the ancient Greeks as they used it to design buildings and survey land. Even we still have used it today [25].

Figure 5. The Malatestian Temple [2].

5.1.2 Non-Euclidean geometry

Non-Euclidean geometry is obtained by replacing Euclid's parallel postulate by one of its contradictory forms. The use of non-Euclidean geometry in architecture is currently an important route to developing the optimum structural forms and in the search for effective engineering solutions [11]. Now Euclidean geometry is well known as the study of flat space based on point, line and plane [10].

Figure 6. *St. Maria Cathedral* in San Francisco, California, design by P.L.Nervi and P. Belluschi [11].

5.1.3 Fractal Geometry

Fractal Geometry is a modern mathematical theory. It is the formal study of mathematical shapes that display a progression of never-ending, self-similar, meandering detail from large to small scales. This means that when such objects are magnified, their parts are seen to bear an exact resemblance to the whole. In other words, the likeness continues with the parts of the parts and so on to infinity. Fractal geometry deals with shapes found in nature that have non-integer dimensions line-like rivers with a fractal dimension of about 1.2 and conelike mountains with a fractal dimension between two and three. Natural shapes and rhythms, such as leaves, tree, mountain ridges, flood levels of a river, wave patterns, and nerve impulses, display this cascading behavior [3].

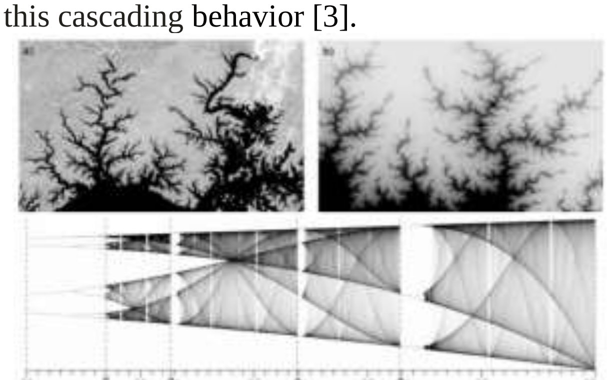


Figure 7. Fractal structures; a) L-System Fractal Plant, b) iterative development of the 'plant-fractal' using the L-system [11].

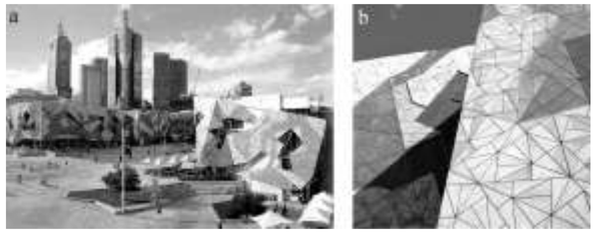


Figure 8. ***Federation Square*, Melbourne, design by Lab Architecture Studio (Peter Davidson, Don Bates); a) birds eye view, b) detail of the Penrose tiling façade [11]**

5.2 PARAMETRIC

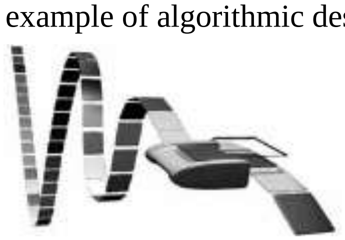
Parametric design is a process in which numerous design alternatives of building models can be generated through the identification of a set of relationships between the geometric entities. Those entities are represented through variables and function(s) that relate them together. Thus, it offers the designer with the ability to change the variables values for creating a number of alternatives, avoiding the exhaustive traditional process.

Parametric Models pose a challenge to expand the design process beyond current limitations of traditional Systems and can manipulate geometry and generate variations by changing its parameters [22].

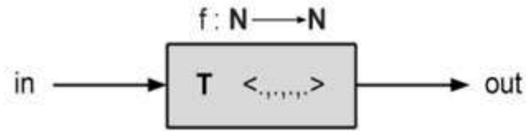
Figure 9. Melbourne stadium concept. Melbourne stadium structure optimization [5].

5.3 ALGORITHM

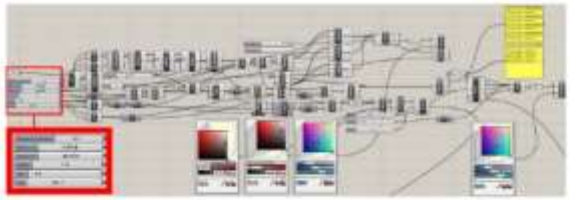
The algorithm is generally a definite set of instructions follow specific procedures to achieve certain targeted criteria. Any Algorithmic process, similarly, to generative systems, starts with a set of input values or parameters which are then subjected to specific mathematical operations in order to get a set of outputs. Terzidis discussed that the algorithms' inductive methodology could be regarded as another human mind not only its digital mirror [24]. Besides, algorithm helps designers to programmed or script the software input to customize the outputs [16]. There is a confusion that algorithmic design is synonymous with parametric design, but an algorithmic design does not have to be parametric at all while a parametric design is always an example of algorithmic design [8].



Visualization of Turing machine.
Source; Kotnik. T, "*Algorithmic Extension of Architecture*", P (19).



Algorithm process description as the Turing machine.
Source; Kotnik. T, "*Algorithmic Extension of Architecture*",P (21).



Illustrates this scenario by showing the application of Wolfram's rule 135 .

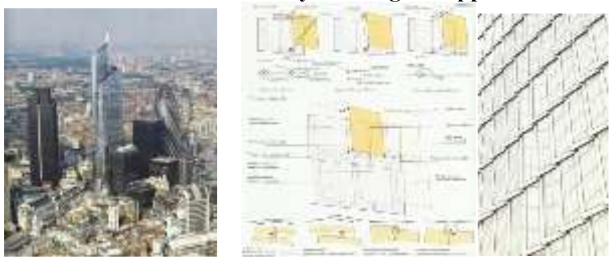


Figure 10. The Pinnacle, London, UK. Transformation of window panels [5].

5.4 TOPOLOGY

It explored the form as a process and not a finite condition, via adjustable mathematical functions or homeomorphism that proposed an interesting metaphoric formal concept "the fluidity of the form" [23].
Figure.11 shows the branches that have been used in architecture in details with clarifying the main differences between them and its application in architecture.
It means the transformations of one object to another, without cutting or splicing, maintains its topological identity [26].

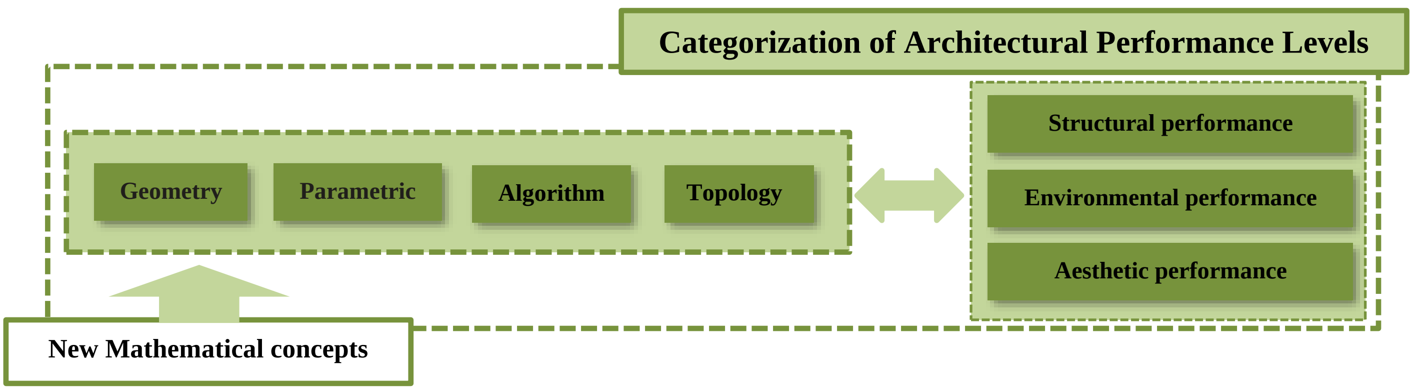
Figure 11. "Island City" is a large, roughly 400 ha artificial island on the eastern side of Hakata Bay [27].

6. BUILDING PERFORMANCE EVALUATION LEVELS

In Merriam-Webster's Collegiate Dictionary [28], the word performance has multiple meanings. It can be explained as the execution of an action, something accomplished, or the manner in which a mechanism performs. It can also be simply understood as the ability to perform. In this sense, performance is the synonym of efficiency. In architectural design, performance is often used as a generic term to describe many design considerations of a building. Almost any term can be put in front of performance and form a phrase that makes sense to architects, e.g., thermal performance, structural performance, fire-resistant performance, etc. A more effective way to understand what performance really means to architectural design is to study what performance issues we need to consider in designing a building.

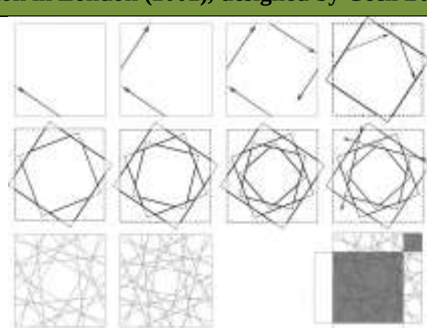
In architectural design, the performance concerns with three topics: Structural performance; is directly linked to the safety of occupants and properties under the shelter of the building. Environmental performance; this category of performance includes solar, thermal, moisture, acoustics, lighting, wind and air, energy, and many others that have an impact on the quality of built environment, both indoor and outdoor. Aesthetic performance, that affected by form, space's organization, material selection, color, and shape, that played a role in forming the aesthetic and cultural performances of a building In consequence, it could be classified into; Formal Aesthetic Principles And Formal Aesthetic Elements.

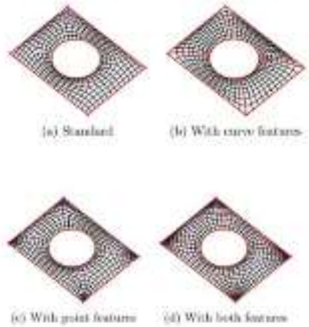
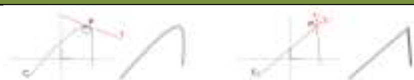
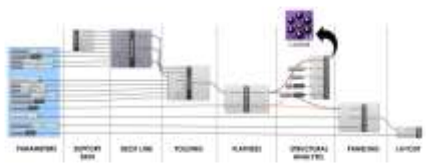
A close connection between mathematics and architecture are limitle



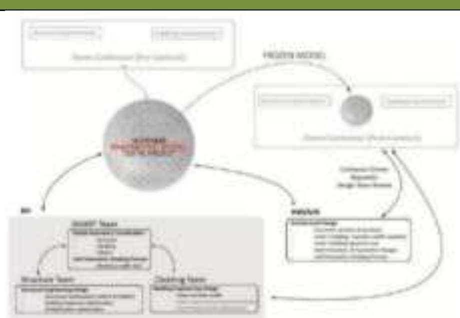
The proposed methodology for applying Mathematical concepts in architecture performance levels
Source; by the researcher

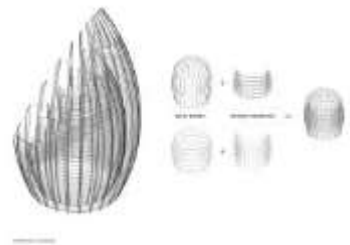
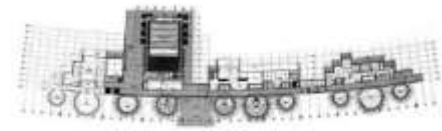
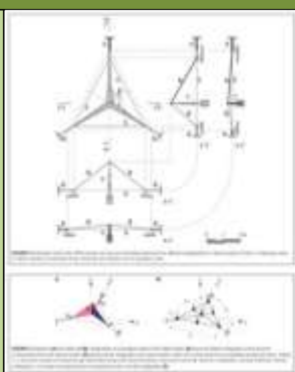
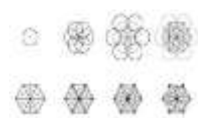
6.1 STRUCTURAL PERFORMANCE

Case Study/ Building	Mathematical Concepts "Logic"	Architecture Style Developing through Application of Mathematics
The Serpentine Pavilion in London (2002), designed by Cecil Balmond and Toyo Ito [14].		
Structural Performance . **Formation 1**	 Serpentine Pavilion: The irregular-looking pattern is based on incremental scaling and rotation of a series of inscribed squares.	 A particularly potent example of a complex-looking design based on simple rules is the design.
	The apparently random patterning that wraps the entire pavilion is produced by incremental scaling and rotation to create a beautiful, seemingly irregular-looking pattern of alternating voids and solids.	
The British Museum Great Court Roof by Foster + Partners, Buro Happold and Waagner-Biro [6].		

Structural Performance . **Formation 2**	 Four patterns for the British Museum roof with different topologies, to assess the relevance of heuristic point and curve features, shown in red.	
	The structural performance is assessed as the ratio of the structural mass over the projected area of the shell. As expected, the buckling requirement is secondary, and deflection is the decisive requirement.	
Interlocking Folded Plate Structure - The Temporary Pavilion [17].		
Structural Performance . **Formation 3**	 Continuous tangent (minimum radius of curvature) - Discontinuous tangent (assembly) -Model of fold  Digital device of design	
	The folding technique is one of the strongest and most common techniques to transform a 2D flat surface into a 3D element to provide geometric structural stability	

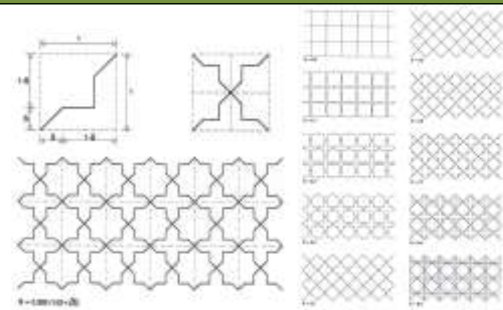
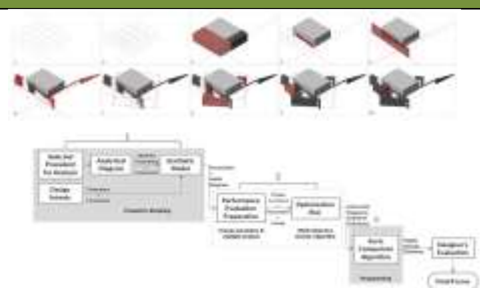
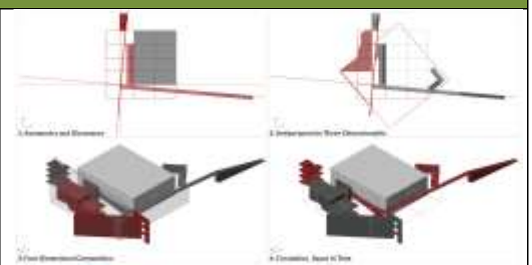
6.2 ENVIRONMENTAL PERFORMANCE

Case Study/ Building	Mathematical Concepts "Logic"	Architecture Style Developing through Application of Mathematics
The Louvre, Ateliers Jean Nouvel, Abu Dhabi, 2017 [25].		
Environmental Performance . **Formation 1**	 Diagram of the parametric workflow, detailing how the various constraints of the project were folded into the central parametric model of the dome.	
	Massive dome displays a complex and seemingly chaotic pattern which creates dynamic light effects inside the museum.	

Jean marie tjibaou cultural center [19].		
Environmental Performance . **Formation 2**	 The Centre followed two main guidelines – Kanak vernacular knowledge in construction competencies, on the other hand making use of modern materials, such as glass, aluminum, steel and advanced lightweight technologies.	 
	These buildings strongly express the harmonious relationship with the environment	
Al Bahar towers in Abu Dhab [1].		
Environmental Performance . **Formation 3**	  	 Detail of a module of the ABS (picture taken during the mounting of the screens).
	The triangular units act as individual shading devices that unfold to various angles in response to the sun's movement in order to obstruct the direct solar radiation.	

6.3 AESTHETICAL PERFORMANCE

Case Study/ Building	Mathematical Concepts "Logic"	Architecture Style Developing through Application of Mathematics
Story Hall, Royal Melbourne Institute of Technology, Australia [5].		
Aesthetical Performance . **Formation 1**	 Story Hall form generation concept.	 Story Hall, Royal Melbourne Institute of Technology, Australia
	The new façade of the hall was emphasized from blurring the historic façade. The used tilling method	

	presents a new decorative language on the exterior textured verdigris.	
Spain Pavilion in Expo 2010 [21].		
Aesthetical Performance . **Formation 2**	 Basic pattern defined parametrically	
	This pattern defined by a basic parameter, can be changed to produce different patterns. The selection of parameters can be implemented in different ways and even more than one parameter can be defined to create various patterns.	
A new architectural design optimization framework [26].		
Aesthetical Performance . **Formation 3**	 Research workflow illustrating phases and processes of prototype.	 .
	This study presents a shift from optimization design methods that deal with technical performance design objectives into optimization that incorporates objectives in aesthetics.	

7. CONCLUSION

Application of mathematics in architecture from ancient to modern architecture age find new possibilities for contemporary architecture design pattern, form and aesthetics. Mathematics and architecture have always enjoyed a close association with each other, not only in the sense that the latter is informed by the former, but also in that both share the search for order and beauty. It is also employed as visual ordering element or as a means to achieve harmony with the universe. Here geometry becomes the guiding principle. Today, using mathematics as a tool of investigation in both the natural and architectural forms gives us an advantage of exploring multiple forms easily and allows us to implement new parameters into the mathematical framework. Architecture, which exists in a dramatically different environment has other parameters to be integrated during the architectural design process concerning its form. These parameters are designed to accommodate the practical requirements of architectural forms.

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