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Particular Investigation of Energy Backbending Phenomena for Even-Even Nuclei ^{102}Mo and ^{102}Pd Modeling Approach

Samir ELkhamisy[1], Diaa Atta[2*]

[1] *Department of Physics, Faculty of Science, Ain Shams University, Cairo, Egypt*

[2]*Spectroscopy department, Physics Research Institute, National Research Centre, 33 El Behooth St., Dokki, Giza, Egypt, Affiliation ID: 60014618.*

Nuclear energy level has been calculated using several models depending on the least square fitting and the experimental data. The energy level distribution have been also calculated. In addition the Yrast line has been plotted. Palladium and Molybdenum isotopes ^{102}Mo and ^{102}Pd energy level distribution have been calculated by the nuclear softness model, variable moment of inertia model, Anharmonic vibrator- model (AVM1 and AVM2), exponential model (exp- model) and interaction boson model (IBM) in O6. Experimental results have been used to calculate the unknown factors of the used equations. The backbending of the energy against the momentum have been also calculated and discussed.

Keywords: nuclear modeling- Pd and Mo isotopes- system crossing- backbending

Introduction

One from the most important issues in nuclear physics is how the protons and neutrons are arranged inside the nucleus. In this regime the nucleus geometry is one from the important parameters. The classical idea is the spherical shape. By increasing the population of the protons and neutrons such deformation takes place to the geometrical shape of the spherical shape.

The nuclei deformation comes from the deformation in the nuclear density. It is found that this deformation take place in the region of atomic mass from 150 upto 180 also that nuclii with atomic mass greater than 224. But not the large masses only that it is noticed also in the region around 24.

According to the deformation from the ideal spherical shape, such rotational and vibrational breathing modes take place producing nuclear spectra. Rotational spectrums of any deformed nuclei consist of different rotational bands, ground state band and excited as super band.

Rotational modes could be regarded as as it were nuclei with non-spherical equilibrium shapes (which is the case of vibrational modes).

By plotting the relation the excitation energy with the angular momentum of the ground state, what so called a YARST line is produced. It is noticed that Yarst lines get crossed in some cases and always with deformed nuclei. This crossing at a specific spin quantum number have been introduced by Johnson et. al. as a back bending phenomenon.

Many models have been developed to calculate and predict the distribution of the nuclear energy levels. By the mid of the last century J. Rainwater [2] suggests that the single particle deforms the whole nucleus, and the observed quadrupole results from the collective deformation of many orbits. Hence it could be considered that nuclear quadrupole moment gives the opportunity to calculate the deviation of nuclear shape from a sphere. The mathematical development of the modeling ideas [4-7] is the basis for the collective model which has been particularly successful for A=24, 150<A<190 and A>230.

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The collective model could describe both the rotational states and the vibrational states, in molecular spectra, [8-10] one could easily see the evidences about three types of electronic excitation [11-13] through the electronic transitions, vibrational levels transitions and/or rotational states. On the other hand, inside the nuclei, one could be easily notice that the nucleons suffer the same transitions.

Flexing the surface of such nucleus causes vibrations affecting the nuclear levels and forming vibrational states which have more complicated nature than that outer molecular vibrational states. The rotational motion is more complicated than the rotation of rigid body. It could be regarded as such rotation of deformed body surface enclosing free particles.

Both nuclear vibrational motion and rotational motion involve orderly displacements of many nucleons, and both types are therefore classified as nuclear collective motion.

The rotational energy level distribution could be interpreted by the following equation,

$$\boldsymbol{E = \frac{\hbar}{2\Phi}\left(J(J+1) - K^2\right)}$$

Where ϕ is the ground state moment of inertia for the nucleus, K is the band head angular momentum, and J is the total angular momentum.

While the nucleus is not a pure sphere such correction factor (B) should be included in the previous equation, hence the new form of the rotational energy levels will be

$$\boldsymbol{E = \frac{\hbar}{2\Phi}\left(J(J+1) + B(J^2(J+1)^2\right)}$$

The collective model is not the only model but there are other different models like nuclear softness and variable moment of inertia also the anharmonic vibrator model. The challenge in computing the energy level like any modeling work is always the selection of the appropriate model.

By regarding the nuclei [14-20]populated to higher states by heavy ion bombardment it is noticed that the angular frequency (ω) of the rotational bands increasing with the increase of moment of inertia (Φ) of the level till a point the proportionality get reversed and (ω) get decreases with (Φ) increasing for some states then the regular case returned. This phenomenon called the Backbending; also we notice that by J increasing we have another backbends called the second, third, etc… Backbending, by more studying this phenomenon we can explain it as it happen when the nucleus bands get crosses for two bands or more. We can formulate the frequency and inertia in units of ħ as following

$$(\hbar\omega)^2 = [(J^2\text{-}J+1)/\ (2J\text{-}1)2]\ [EJ\text{-}EJ\text{-}2]^2 \quad (1)$$

$$(2\Phi/\hbar) = (4J\text{-}2)\ /\ (EJ\text{-}EJ\text{-}2) \quad (2)$$

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In the present work an attempt is made to study the behavior of the back-bending phenomena in the ground state bands of ^{102}Mo and ^{102}Pd making use of the VMI-model,NS3-model and exponential model compared with the experimental data..

Methods:

In this work the variable moment of inertia model (VMI1, VMI2), nuclear softness (NS3), exponential model (exp- model) and interaction boson model (IBM) in O6 were utilized to calculate the energy levels. A computer subroutine was developed to calculate the unknown parameters and computing the back pending data.

Results and discussion:

Using the least square fitting and numerical methods the experimental data has been utilized to calculate the Parameters of both models VMI and NS3 as it is clear in table 1, also the parameters of AVM1 and AVM2 models have been calculated and presented in table 2.

Table (1) The parameters of the VMI-model and NS3-model as calculated by least squares fitting using a computer program and the known experimental data

	VMI-model		NS3-model		
Isotope	C (Kev^2)	φ_o (Kev^{-1}) $\times 10^{-3}$	A (Kev) $\times 10^{-3}$	σ_1	$\sigma_{2\times 10^{-3}}$
^{102}Mo	3261417	6.81	6.6	0.2979	-10.67
^{102}Pd	6992718	1.002234	.85	0.4683	-8.764

Table (2) The parameters of the AVM1 and ANM2-model as calculated by least squares fitting using a computer program and the known experimental data

	AVM 1&2-model		
isotope	φ_o (Kev^{-1})	X	Y $\times 10^{-3}$
^{102}Mo	148.3	0.12	-1.898
^{102}Pd	278.215	0.0732	-3.52

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As models NS3, AVM and VMI both exponential model and IBM model parameters also calculated and tabulated in table 3.

Table (3) The parameters of the IBM (O6) and exp.-model as calculated by least squares fitting using a computer program and the known experimental data

	IBM (O6)		Exp.-model		
isotope	B	C	φ_o (Kev^{-1})	Δ_o	J_c
^{102}Mo	73.475	0.4495	3.916	2.95365	12
^{102}Pd	173.669	-23.04	0.074	7.646466	22

After determining the model's parameters the energy levels of ^{102}Mo and ^{102}Pd have been calculated and tabulated in tables 4 and 5 respectively.

Table (4) Experimental and calculated energy levels of ^{102}Mo by different models, the energy is given in Kev.

^{102}Mo	**[21]**	**[22]**	**Present work**					
Spin(J)	**Exp**	**VMI**	**VMI**	**NS3**	**AVM1**	**AVM2**	**exp**	**O6**
2	**296.597**	**296.6**	**292.75**	**292.6**	**296.59**	**296.59**	**348.35**	**296.59**
4	**743.74**	**743.7**	**757.4**	**749.7**	**743.7**	**743.7**	**873.5**	**743.7**
6	**1327.92**	**1284**	**1328.2**	**1323.8**	**1341.4**	**1327.9**	**1327.9**	**1341.4**
8	**2018.82**	**1898**	**1979.3**	**2019.8**	**2089.6**	**2035.6**	**1551.7**	**2089.6**
10	**2481.12**	**2571**	**2696.4**	**2861.4**	**2988.5**	**2853.4**	**1438.6**	**2988.5**
12	**3632.2**	**3294**	**3470**	**3889.1**	**4037.7**	**3767.6**		**4037.7**

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14		**4093**	**4293.5**	**5165.48**	**5237.6**	**4764.8**		**5237.6**
16		**4872**	**5161.8**	**6787.9**	**6588**	**5831.6**		**6588**
18		**5719**	**6071.2**	**8915.4**	**8089**	**6954.3**		**8089**
20			**7018**	**11823**	**9740.5**	**8119.5**		**9740**
22			**8000.5**	**16032.5**	**11542**	**9313.6**		**11542**
24			**9015.7**	**22666**	**13495**	**10523**		**13495**
26			**10061.8**	**34657.5**	**15598**	**11734.8**		**15598**
28			**11137**	**62882.7**	**17852**	**12934.8**		**17852**
30			**12240**	**209269.6**	**20256**	**14109.7**		**20256**

Table (5) Experimental and calculated energy levels of ^{102}Pd by different models, the energy is given in Kev.

^{102}Pd	**[23]**	**[20]**	**[22]**	**Present work**					
Spin(J)	**Exp**	**VMI**	**AVM2**	**VMI**	**NS3**	**AVM1**	**AVM2**	**exp**	**O6**
2	**556.43**	**556.43**	**556.43**	**546.1**	**552.9**	**556.43**	**556.43**	**651.24**	**556.42**
4	**1275.87**	**1275.87**	**1275.87**	**1254.75**	**1282**	**1275.87**	**1275.87**	**1493.2**	**1275.87**
6	**2111.35**	**2117**	**2111.35**	**2084.5**	**2106**	**2158**	**2111.35**	**2111.4**	**2158.3**
8	**3013.06**	**3054**	**3015.8**	**3008.6**	**3014.5**	**3203.8**	**3015.9**	**2375.4**	**3203.8**

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10	**3992.71**	**4071**	**3942.5**	**4011.4**	**4010.3**	**4412**	**3942.5**	**2308.3**	**4412**
12	**5055.1**	**5156**	**4844.5**	**5082**	**5102.5**	**5783.7**	**4844.3**	**2001.4**	**5783.7**
14	**6138.63**	**6302**	**5674.9**	**6213.5**	**6303**	**7318**	**5674.3**	**1563.4**	**7318**
16	**7428.8**	**7504**		**7399.6**	**7627**	**9015.7**	**6385.4**	**1091.7**	**9015.7**
18	**8707**	**8757**		**8635.9**	**9094**	**10876**	**6930.8**	**659.7**	**10876**
20	**9892.7**			**9918.6**	**10728**	**12899.7**	**7263**	**311.8**	**12899.7**
22	**11227.5**			**11244.7**	**12558**	**15086**	**7336**		**15086**
24	**13592.9**			**12611**	**14621**	**17435.8**	**7102.4**		**17435.8**
26	**15414.3**			**14017**	**16964.7**	**19948**	**6514.9**		**19948**
28				**15458**	**19649.8**	**22623.9**	**5526.8**		**22623.9**
30				**16934.8**	**22756.8**	**25462.5**	**4091**		**25462.5**

As it's clear from the presented data that comparing with the other work our calculations were more converging to the experimental data, also the best model among the studied models were the Variable moment of inertia model.

The Swedish-Copenhagen approximation [24] of VMI model could be formulated as

$$(\hbar\omega)^2 = 4(J_i^2 - J_i + 1)\left[\frac{E(J_i) - E(J_i - 2)}{4J_i - 2}\right]^2$$

The moment of inertia anomalies are derived in this manner and plotted in figure (1) for the ^{102}Mo and figure (2) for ^{102}Pd as a function of the square of the rotational frequency.

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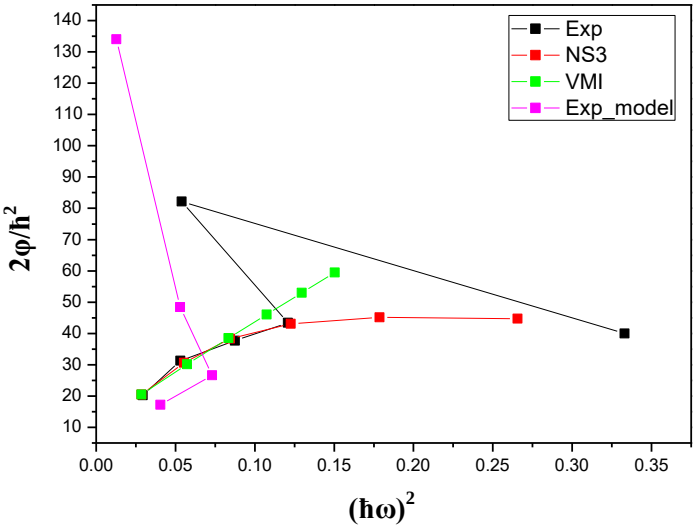

Figure 1 The backbending of ^{102}Mo

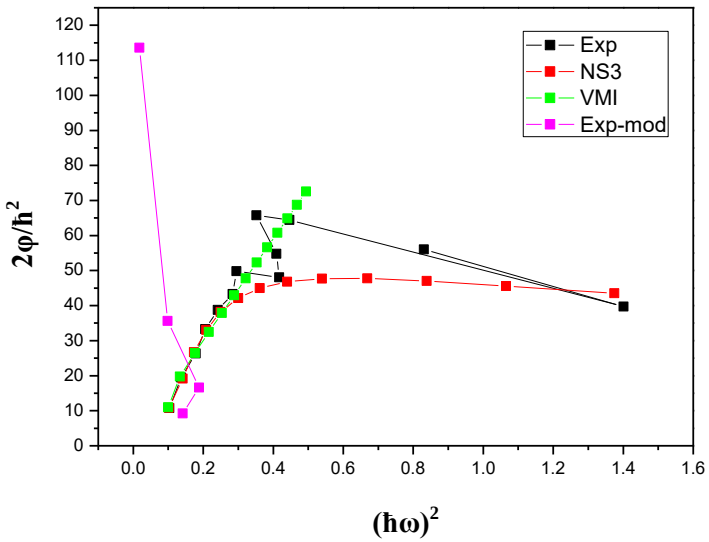

Figure 2 The backbending of ^{102}Pd

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It is clear from figures 1 and 2 that the backbending phenomena illustrated from the calculated energy levels and rotational frequencies take place only in the exponential model with vast divergence from the experimental data. This means that while there are convergence in the energy levels but still some modifications and correlation parameters should be done on the models to coincide with the experimental data.

Conclusion

The nuclear models of even-even nuclei have been utilized to calculate the energy levels of the Molybdenum 102 and palladium 102 isotopes nuclei. It is clear that the VMI model is the most converging calculated data among the studied models. The Backbending phenomena have been studied also. It is clear that there are divergence between the calculated and experimental rotational frequencies. Understanding the energy level distribution leads to understanding how the radiation take place and how to control the radiation and the nuclear reactions. The results shows that more efforts needs to exert on the models to produce more convergence with the experimental data and some correlation parameters were in need to do that.

References

1. J. F. Sharpey-Schafer, Failures of Nuclear Models of Deformed Nuclei, 50th International Winter Meeting on Nuclear Physics, Bormio, Italy 23-27 January 2012.
2. J. Rainwater, Nuclear Energy Level Argument for a Spheroidal Nuclear Model, Phys. Rev. 79(1950)432.
3. A. Gargano, L. Coraggio, A. Covello, N. Itaco, Realistic shell-model calculations and exotic nuclei, Journal of Physics: Conference Series, 527 (2014) 012004.
4. A. Boher, B. Motelson, R.; Nuclear Structure, New York, Amsterdam, (Benjamin W. A. Inc.), 1975.
5. J. Cseh, On the relation of the shell, collective and cluster models, Journal of Physics: Conference Series 580 (2015) 012046.
6. N. A. Son, L. V. Huy, P. N. Son, Application of the collective model to determine some rotational bands of ^{239}U nucleus, Science and Technology Development Journal - Natural Sciences, 2(2019)65-70.
7. V.E. Mitroshin, Dynamical collective model: even-even nuclei, Phys. Atom. Nuclei, 68 (2005) 1314–1351.
8. Farrage, N.M., Oraby, A.H., Abdelrazek, E.M.M., Atta, D.,Synthesis, characterization of Ag@PANI core-shell nanostructures using solid state polymerization method, Biointerface Research in Applied Chemistry, 2019, 9(3), pp. 3934–3941.
9. Atta, D., Gomaa, F., Elhaes, H., Ibrahim, M.,Effect of hydrated dioxin on the physical and geometrical parameters of some amino acids, Journal of Computational and Theoretical Nanoscience, 2017, 14(5), pp. 2405–2408.
10. D Atta, A Fakhry, M Ibrahim, Chitosan membrane as an oil carrier: Spectroscopic and modeling analyses, Der Pharma Chemica 7 (10), 357-361
11. D Atta, A Okasha, M Ibrahim, Setting up and calibration of simultaneous dual color wide field microscope for single molecule imaging, Der pharma chemica 8 (19), 76-82
12. Atta, D., Okasha, A., Single molecule laser spectroscopy, Spectrochimica Acta - Part A: Molecular and Biomolecular Spectroscopy, 2015, 135, pp. 1173–1179
13. Okasha, A., Atta, D., Badawy, W.M., ...Elhaes, H., Ibrahim, M., Modeling the coordination between Na, Mg, Ca, Fe, Ni, and Zn with organic acids, Journal of Computational and Theoretical Nanoscience, 2017, 14(3), pp. 1357–1361

ISSN: 2785-9541 VOLUME 3, ISSUE 2, 2020, 1 – 9. www.egyptfuture.org/ojs/

14. K. A. Gado (2020) Macroscopic investigation of rotations for some deformed even–even nuclei, Journal of Radiation Research and Applied Sciences, 13:1, 37-40, P. C. Sood and A. K. Jain, phys. Rev., 18C, 1906(1978).
15. R. F. casten, P. Von Brentano, K. Heyde, P. Van Isacker and J. Jolie, Nucl. Phys., A439, 289 (1985).
16. S. ELkhamisy, D. Atta, I. J. A. S. R. I.,2,1-7 (2021)
17. Abraham Klein, Nucl. Phys., A347, 3 (1980).
18. Dennis Bonatsos and A. Klein, Atomic Data and Data Tables 30, 27 (1984).
19. J. Timar, J. Gizon, D. Sohler, B. M. Nyako, L. Zolnai, A. J. Boston,D. T. Joss, E. S. Paul, A. T. Semple, C. M. Parry and I. Ragnarsson, Pyhs. Rev., 62C, 044317 (2000).
20. Rechard B. Firston, Table of Isotopes 8^{th} edition, Wiley-Interscince (1998).
21. Abraham Klein, Nucl. Phys., A347, 3 (1980).
22. Rechard B. Firston, Table of Isotopes 8th edition, Wiley-Interscince (1998).
23. Dennis Bonatsos and A. Klein, Atomic Data and Data Tables 30, 27 (1984).
24. Nuclear Data Center, http://bnlnd2.bnl.gov/htbin/nudat.cgi (2002
25. A. Johnson, H.Ryde and J. Sztarkier, Phys. Lett. 34 B, 605, (1971).