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ROBUST TRACKING CONTROL FOR SLOSH-CONTAINER SYSTEM AGAINST VISCOUS FRICTION UNCERTAINTY

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ABSTRACT
In this paper, control design techniques are proposed for trajectory tracking of slosh-container system. In particular we assume that only partial state measurements are available for feedback and we synthesize observer-based control law such that the controlled output successfully tracks a pre-defined trajectory. Moreover, we take into account the uncertainty issue in the estimate of the viscous friction coefficient and we enhance the controller with a robustness property against such uncertainty. The proposed approach is demonstrated by numerical simulations on the dynamical model of slosh-container system.

KEYWORDS
Slosh-container system, Observer-based control, Friction model.

INTRODUCTION
Nowadays, the slosh-container system become very important and involved in a lot of industrial applications, [1, 2] like steel industries, rocket launches systems, and liquid carriers. This system can be defined as the liquid movement inside a container or tank during the transfer of the container. For example in the steel industry, the transfer of molten steel in molds is considered a typical slosh-container problem that needs to move the container in minimum time on the other hand this causes the molten steel to slosh in the internal container sides. So the control of the slosh-container system is very important to avoid the overflow of molten steel and non-need cooling for the molten steel, which reduces the quality of the product and is dangerous.

The research interest on control of the slosh-container system has been spread out over the last decades all over the world. This is obvious according to the rapid increase in the number of research papers and projects in that field, [3, 4]. In the previous literature review, the first step for the researchers is to develop the dynamic mathematical model of the slosh-container system. This model usually is very complex and nonlinear, as it can be represented as two degrees of the freedom pendulum system, [5, 6]. Then they design their proposed controller techniques. Sandhra et al., [7] designed a Sliding Mode Controller using a non-linear sliding surface to provide a better response compared to a Sliding Mode Controller with a linear sliding surface. Also, Mohammad Abdulrahman, [8] developed an adaptive robust control-based wavelet network to approximate the nonlinearity of the system. Furthermore, Rigatos et al., [9], proposed an H-infinity feedback controller to

solve the problem of the nonlinearity optimal control for the system which gives fast and accurate tracking for all state variables.

The main objective of this work is to synthesize a simple and robust feedback law to achieve successful tracking of a reference tracking with respect to uncertainty in the estimate of the viscous friction coefficient. This issue has not been considered before in the literature, to the best of our knowledge. Considering that only part of the state can be measured, we also design a full order observer, [10] to estimate the full state. Then the key point of the proposed technique is to decouple the tracking error from the estimation error to ensure that the observer performance is not degraded.

This paper is organized as follows; section (1) is an introduction about the slosh-container system and the motivations of the proposed work. The dynamic model of the slosh-container system is presented in Section (2), while section (3) describes the control strategy used to control the slosh-container system. The simulation results are explained in Section (4) Finally, Section (5) contains the summary of the proposed work.

MATHEMATICAL MODEL

In this section, we preset the dynamic model of the slosh-container system. The system consists of a cart attached with a container through a pivot to transfer it from some starting point to an end point. The sloshing of the fluid inside the container leads to a tilting motion of the container. To eliminate such sloshing, the systems has to be of two degrees of freedom represented by the linear motion of the cart and the tilting motion of the container. In other words, two electric motors are employed to control the cart position and the angular displacement of the container in order to prevent sloshing as shown in Fig.1.

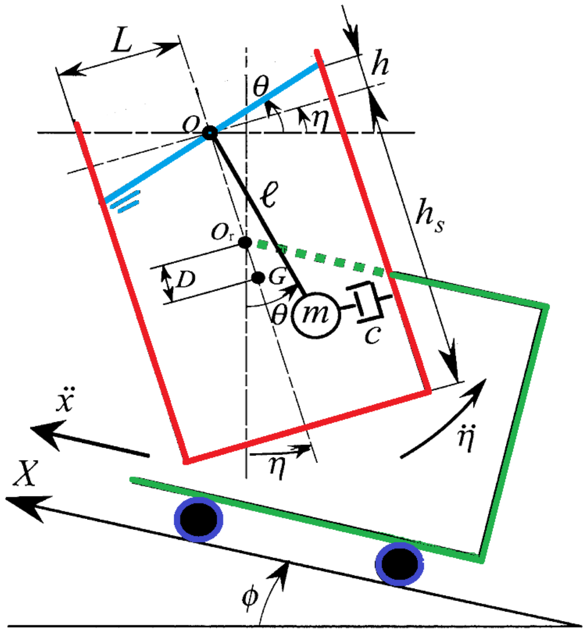

Fig. 1 Slosh-container system moving over inclined plane

The sloshing motion of the fluid can be equivalently modeled as two degrees of freedom pendulum system. In other words, the fluid motion is similar to a pendulum attached to surface and both the pendulum and the surface are allowed to exhibit angular displacements, denoted by θ and η respectively in Fig. 1. It is also considered that the cart

attached to the container has a translational motion x over an inclined path with some angle $\emptyset$. In this setup, we assume that only two variables are available for measurements, which are the container position x and the displacement h of fluid surface with respect to the nominal level h_s Note that from Fig.1 the displacement of the fluid level h is a function of the difference between the angle of rotation of the fluid θ and the angle of rotation of the container η. More precisely, we have

$$h = L \sin(\Theta - \eta) \qquad [1]$$

Hence, in order to suppress sloshing, we need to maintain $h = 0$ while the cart is transferring through a desired trajectory.

SLOSH DYNAMICS

Let O denotes the center of rotation of the pendulum, O_r is the center of rotation of the container, G is the center of gravity and D is the distance between O_r and G. Also, it is assumed that the equivalent pendulum of sloshing motion has length ℓ and lumped mass m Then, by applying Netwons' laws of motion, the dynamic model of the system is given by Yano et al., [11].

$$m\ell^2\ddot{\Theta} = -\varepsilon\ell^2(\dot{\Theta} - \dot{\eta})cos^2\Theta - mg\ell\sin\Theta + m\ell\ddot{\Theta}\cos\phi\cos\Theta - m\ell\ddot{x}\sin\phi\sin\Theta - m\ell D\ddot{\eta}\cos\Theta \qquad [2]$$

where $m\ell^2$ is the moment of inertia, ε denotes the damping effect due to viscosity of the fluid and friction with the container walls, $\ddot{x}$ is the linear acceleration of the cart, $\ddot{\Theta}$ the angular acceleration of the pendulum and $\ddot{\eta}$ is the angular acceleration of the container. Dividing both sides on $m\ell^2$ and linearizing (2) around the equilibrium point $\Theta \simeq 0$, we obtain

$$\ddot{\Theta} = -\frac{\varepsilon}{m}(\dot{\Theta} - \dot{\eta}) - \frac{g}{\ell}\Theta + \frac{1}{\ell}\ddot{x}\,cos\,\phi - \frac{1}{\ell}\ddot{x}\Theta\,sin\,\phi - \frac{D}{\ell}\ddot{\eta} \qquad [3]$$

The terms $\frac{D}{\ell}\ddot{\eta}$ and $\ddot{x}\Theta\,sin\,\phi$ are typically very small and can be neglected. Hence, (3) becomes

$$\ddot{\Theta} = -\frac{\varepsilon}{m}(\dot{\Theta} - \dot{\eta}) - \frac{g}{\ell}\Theta + \frac{1}{\ell}\ddot{x}\,cos\,\phi \qquad [4]$$

Moreover, based on the previous assumptions, equation (1) becomes

$$h = L(\Theta - \eta) \qquad [5]$$

FRICTION UNCERTAINTY

In this section, we give more insight on the estimation of the friction coefficient ε in (4), that encodes fluid viscosity and friction of the fluid with the container walls. This factor has a great importance and need to be carefully estimated. The reason is that since the free surface of the fluid can oscillate, we have to make sure that the equivalent damping coefficient ε is sufficient to prevent sloshing near natural frequencies. Otherwise, resonance may occur and the resulting hydrodynamic forces on the container can be very destructive. In such cases, practical solutions need to be implemented such as introducing baffles or dividing the tank into compartments.

Basically, the damping coefficient ε depends on three main parameters: viscosity of the fluid, liquid level inside the tank and the dimensions of the tank cross-section. Assume that the tank in cylindrical, then according to Abramson, [12], the friction coefficient ε is estimated based on the following empirical formula:

$$\varepsilon = \frac{2.89}{\pi}\sqrt{\frac{\nu}{R^{3/2}g^{1/2}}}\left[1 + \frac{0.318}{sinh(1.84H/R)}\left(\frac{1-(H/R)}{cosh(1.84h/R)} + 1\right)\right] \qquad [6]$$

where ν is the fluid viscosity, g is the gravity acceleration, H is the tank height and *R* is the tank radius. When the tank is deep, i.e., *H/R* > *1*, the previous relation reduces to

$$\varepsilon = \frac{2.89}{\pi}\sqrt{\frac{\nu}{R^{3/2}g^{1/2}}} \qquad [7]$$

It is important to note here that the above formula is only empirical. Hence, the estimated value of the friction coefficient ε suffers from uncertainly. This requires that the designed controller need to be robust against the uncertainty in ε in order to ensure that the desired output successfully tracks the reference trajectory, which is our objective in this study.

ACTUATORS MODEL

As stated before, the translation motion of the cart and the angular motion of the container are driven by electric motors, typically DC servo motors. Hence, we have two control inputs u_1 and u_2, which are the input voltages to derive circuits of the DC motors. We adopt the same models of the DC motors as in Yano et al., [11], where the transfer function relating the input u1 with the cart linear velocity $\dot{x}$ is given by

$$\frac{sX(s)}{U_1(s)} = \frac{k_1}{\mathcal{T}_1 + 1} \qquad [8]$$

and transfer function relating the input u2 with the container angular velocity $\dot{\eta}$ is given by

$$\frac{s\eta(s)}{U_2(s)} = \frac{k_2\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad [9]$$

where k_1, $\mathcal{T}_1$ are the static gain and the time constant of cart's motor, k_2, ζ, ω_n are the static gain, damping ratio and natural frequency of the container's motor.

We need now to derive the state space model for the closed-loop system (4)-(9). We start our derivation from equation (8), which can be written as

$$(s^2 + \frac{1}{\mathcal{T}_1}s)X(s) = \frac{k_1}{\mathcal{T}_1}U_1(s) \qquad [10]$$

Let

$$q_1 = x \Longrightarrow Q_1(s) = X(s)$$

$$q_2 = \dot{x} \Longrightarrow Q_2(s) = sX(s) \qquad [11]$$

Consequently, in view of (10)

$$sQ_1(s) = Q_2(s)$$
$$sQ_2(s) = s^2X(s) \qquad [12]$$
$$= -\frac{1}{\mathcal{T}_1}Q_2(s) + \frac{k_1}{\mathcal{T}_1}U_1(s)$$

Then, we have

$$s\begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{\mathcal{T}_1} \end{bmatrix} \begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_1}{\mathcal{T}_1} \end{bmatrix} U_1(s) \qquad [13]$$

Taking the inverse of Laplace transform, we get

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{\mathcal{T}_1} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_1}{\mathcal{T}_1} \end{bmatrix} u_1 \qquad [14]$$

By following similar steps for equation (9), we obtain

$$\frac{d}{dt}\begin{bmatrix} \eta \\ \dot{\eta} \\ \ddot{\eta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} \eta \\ \dot{\eta} \\ \ddot{\eta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K_2\omega_n^2 \end{bmatrix} u_2 \qquad [15]$$

MODEL OF THE OVERALL SYSTEM

Let $\boldsymbol{x} = [\Theta \quad \dot{\Theta} \quad \boldsymbol{\eta} \quad \dot{\boldsymbol{\eta}} \quad \ddot{\boldsymbol{\eta}} \quad \boldsymbol{x} \quad \dot{\boldsymbol{x}}]^{\boldsymbol{T}}$ be the state vector, $\boldsymbol{u} = [\boldsymbol{u_1} \quad \boldsymbol{u_2}]^{\boldsymbol{T}}$ be the input vector and $\boldsymbol{y} = [\boldsymbol{y_1} \quad \boldsymbol{y_2}]^{\boldsymbol{T}}$ be the output vector with $y_1 = L(\theta - \eta)$ and $y_2 = x$. Then, in view of (4), (14) and (15), we obtain the state space model of the overall system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{g}{\ell} & -\frac{\varepsilon}{\ell} & 0 & \frac{\varepsilon}{m} & 0 & 0 & -\frac{1}{\ell\mathcal{T}_1}\cos\phi \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\omega_n^2 & -2\zeta\omega_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\mathcal{T}_1} \end{bmatrix} \begin{bmatrix} \Theta \\ \dot{\Theta} \\ \eta \\ \dot{\eta} \\ \ddot{\eta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{k_1}{\ell\mathcal{T}_1}\cos\phi & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & k_2\omega_n^2 \\ 0 & 0 \\ \frac{k_1}{\mathcal{T}_1} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$=: Ax + Bu$$

$$y = \begin{bmatrix} L & 0 & -L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Theta \\ \dot{\Theta} \\ \eta \\ \dot{\eta} \\ \ddot{\eta} \\ x \\ \dot{x} \end{bmatrix} =: Cx \qquad [16]$$

ROBUST CONTROL DESIGN

In this section, we present a simple design approach to avoid sloshing while following a specified trajectory. In particular, we need the output $y_1 = L(\theta - \eta)$ to be maintained at 0 to prevent sloshing and the output $y_2 = x$ to move the container from any starting to an end point. For ease of presentation, let us first assume that the full state can be measure. Then, the closed-loop system can be written as

$$\dot{x} = Ax + Bu$$
$$y = -Cx \qquad [17]$$

In the absence of uncertainty in the plant parameters, we can simply design the control law as $u = -Kx + Nr$, where K is the controller gain for stability, N is a feedforward pre-compensation gain to achieve tracking of the reference r. However, as mentioned in Section 2.2, the equivalent damping coefficient ε may subject to uncertainty a robust controller need to be designed.

A simple approach to tackle this problem is to add an integral action to the controller to eliminate steady state error on tracking. To that end, we augment the plant with an auxiliary state that represents the tracking error. Then, a state feedback of the form $u = -Kx$ is designed for the extended state system such that the overall system is stable. This consequently ensures that the tracking error approaches zero. To better clarify this idea, let us define the additional state z with the dynamic

$$\dot{z} = r - y = r - Cx \qquad [18]$$

Hence, z represents the integral of the tracking error. Then, the control law for the augmented plant is modified to

$$u = -\begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} \qquad [19]$$

Where, K_1, K_2 are gain matrices with appropriate dimensions. Substituting of u in (17) yields

$$\dot{x} = Ax - BK_1x - BK_2z$$

$$y = -Cx \qquad [20]$$

In view of (18) and (20), the augmented system becomes

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} - \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$\dot{\bar{x}} =: (\bar{A} - \bar{B}\bar{K})\bar{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \qquad [21]$$

If $\bar{K}$ is appropriately designed to render the closed-loop system $(\bar{A} - \bar{B}\,\bar{K})$ stable, then it holds that

$$\lim_{t\to\infty} \dot{z} = 0 \implies \lim_{t\to\infty} y = r \implies \text{achieving tracking.} \qquad [22]$$

So far we have assumed that the full state measurements are available. Now we take into account that only part of the state are measured, we need to design an observer-based controller to estimate the unmeasured state. Then, the control law becomes $u = -K\hat{x}$, where $\hat{x}$ denotes the estimated state by the observer. Many techniques can be used for the observer design. We synthesize full order Luenberger observer of the form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \quad [23]$$

where L is the observer gain to derive the estimation error to 0.
In order to appropriately design the observer, gain *L*, we need to study the dynamics of the estimation error e := $x - \hat{x}$, which is given by

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} \\ &= [Ax - BK_1\hat{x} - BK_2 z] - [A\hat{x} - BK_1\hat{x} - BK_2 z + L(Cx - C\hat{x})] \\ &= A(x - \hat{x}) - LC(x - \hat{x}) \\ &= (A - LC)e \end{aligned} \quad [24]$$

Hence, assuming that the pair (*A*, *C*) is observable, to ensure that the estimation error eventually tends to 0, we design the observer gain L such that the eigenvalues of λ *(A − LC)* < 0 are strictly negative. Note that the dynamics of the total system [*x z e*] will be

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK_1 & -BK_2 & -BK_1 \\ -C & 0 & 0 \\ 0 & A - LC & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ e \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} r \quad [25]$$

SIMULATION RESULT

We used the same numerical values of the parameters in Yano et al.[12] .Assume that our desired trajectories are to keep slosh oscillations ($\theta - \eta$) at zero, and the container position *x* moves from its initial position $x = x_0$ to a final position *x = 5*. We set the initial condition of the state as x_0 = (10, 20, 30, −10, −20, 10, 40) and we run simulation for 5 seconds. The open-loop response is shown in Figs 2-5, where we note that the fluid exhibits oscillations and the output *y* does not converge to the desired trajectory *r*.

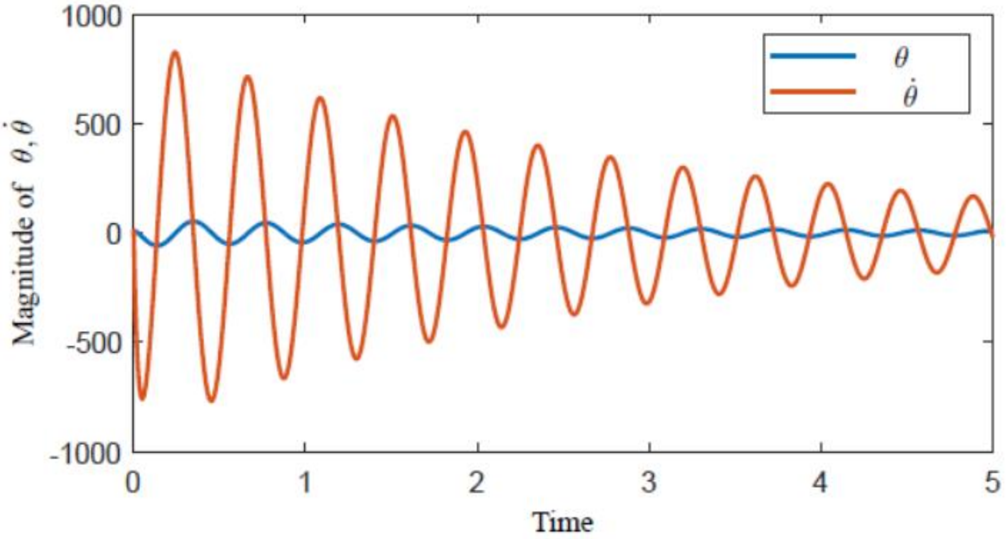

Fig. 2 Open-loop response.

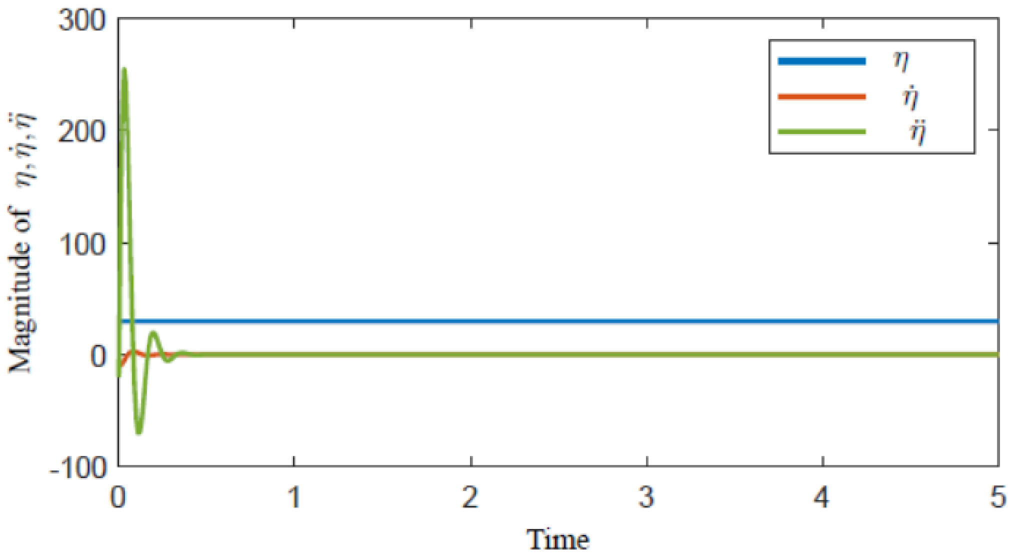

Fig. 3 Open-loop response.

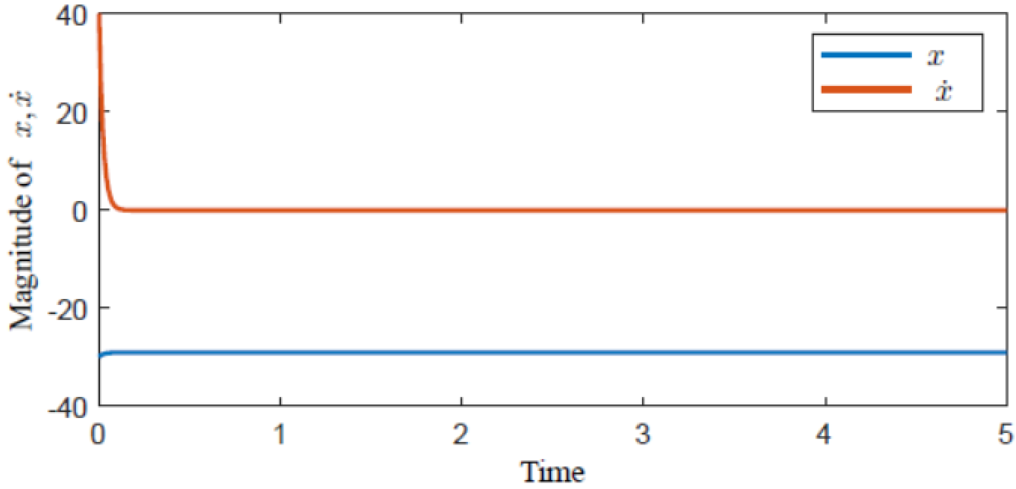

Fig. 4 Open-loop response.

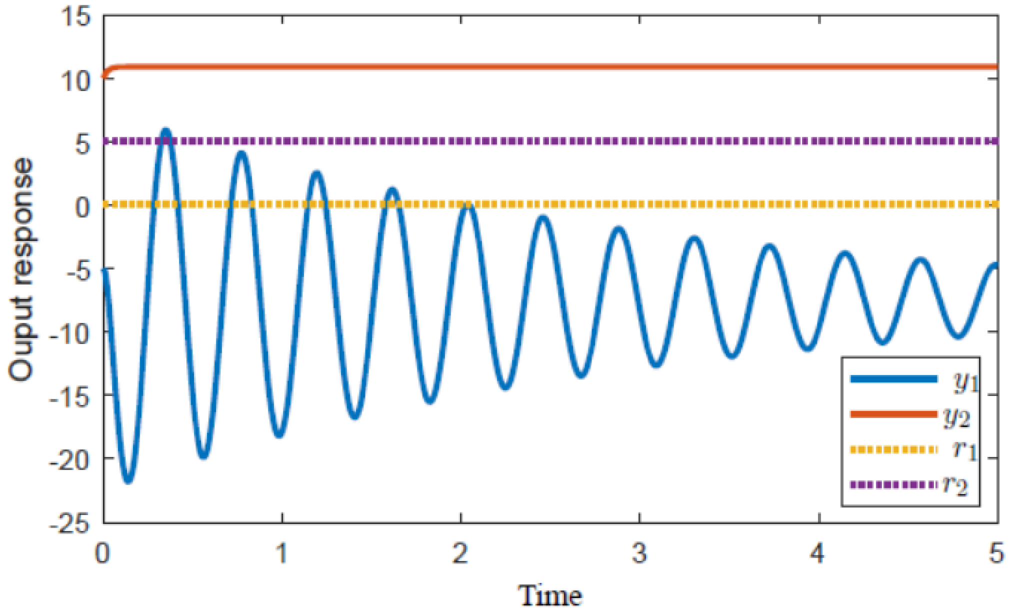

Fig. 5 Open-loop response.

Next we apply the proposed control technique in Section 3. The closed-loop response is shown in Fig. 6, where we note slosh behavior vanishes after short time and the output *y* follow successfully the reference trajectory *r*.

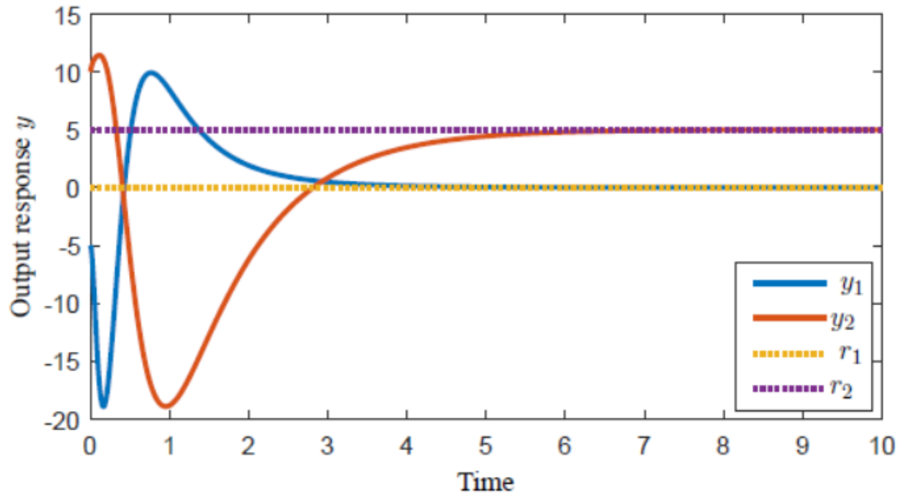

Fig. 6 Closed-loop response.

Finally, we check the robustness of the proposed methodology with respect to uncertainty in the damping coefficient $\varepsilon \pm \Delta$. The obtained results for uncertainty $\Delta = 50\%$ is shown in Fig. 7. It is clear that the designed controller achieves desirable robustness and maintains tracking of the reference trajectory.

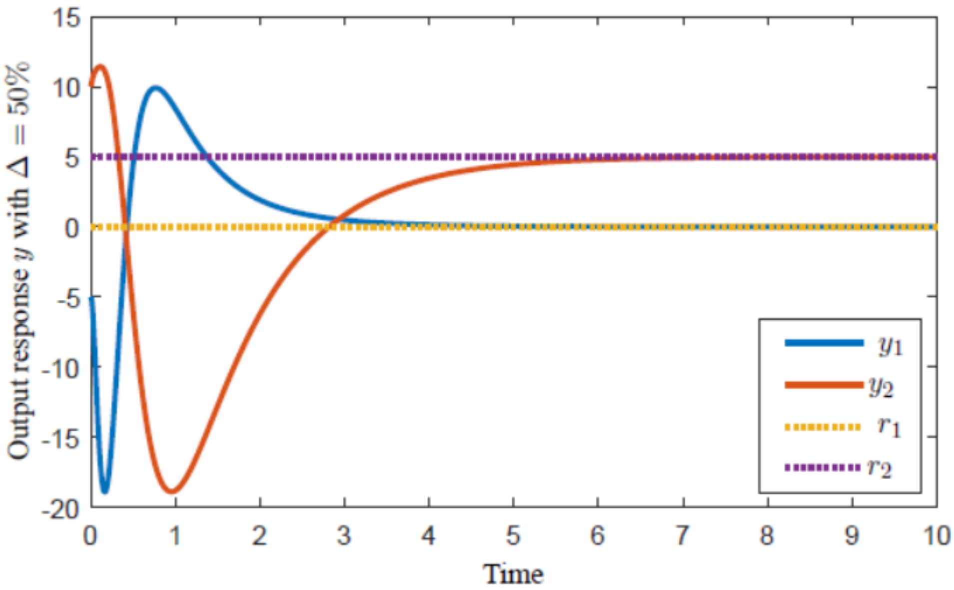

Fig. 7 Closed-loop response.

CONCLUSIONS

In this work, we presented an observer-based control technique. This controller success in tracking a pre-defined trajectory of the slosh-container system. The performance of the controller is tested and validated by a simulation program based on a frictional model of the slosh-container system. The simulation results show a good trajectory tracking of the slosh-container system. Furthermore, it gives robustness performance against the uncertainty of estimating the viscous friction coefficient. Future work includes the extension of the technique to the nonlinear case and considers digital implementation of the control law.

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