A CLOSED FORM APPROXIMATION OF CARSON'S FORMULAE

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INTRODUCTION:

This paper presents a closed form transmission line impedance formula which is simple and accurate[1]. The closed form can be used with hand hold calculators for simple applications or, because of its analytical form, it can be used for deriving new expressions such as, for instance, for magnetic field intensity in the vicinity of overhead lines.

The availability of the closed form formulae permits to write simple programs, which are also very cost effective, for the solution of power frequency problems and of transients on overhead transmission lines.[3]

THE CLOSED FORM IMPEDANCE FORMULA :

The calculation of line impedances according to Carson[2] is based on equations which contain infinite integrals with complex arguments. For their evaluation Carson has proposed infinite series and also some convenient approximations for low and high frequencies. While these approximations are relatively simple they are valid each for a limited range of frequencies only, and medium frequencies are not covered.

Self and mutual impedances of parallel wires above homogeneous conductive earth can be calculated by using the complex depth

$$p = \frac{1}{\sqrt{j w \mu_o \sigma}} \tag{1}$$

of a fictitious ("complex") mirroring surface; see Figure 1. The image of conductor C is then a "complex image" C^*. The distance D' to a neighbouring conductor C' is real:

$$D' = \sqrt{(h-h')^2 + d^2} \tag{2'}$$

but the distance to the image is complex:

$$D'' = \sqrt{(h+h'+2p)^2 + d^2} \tag{2''}$$

The self and mutual impedances are

$$Z_s = j w \frac{\mu_o}{2\pi} \ln \frac{2(h+p)}{r} Z_c \tag{3'}$$

$$Z_m = j w \frac{\mu_o}{2\pi} \ln \frac{D''}{D'} \tag{3''}$$

where r is the radius and Z_c the impedance of the current carrying conductor. We note that because p and D'' are complex the logarithmic expressions are also complex: thus these terms reflect not only ground return reactances but resistances as well.

It is interesting to note that Z_s and Z_m include both the resistance and the reactance of the current return loop. Formally this results from p being complex and physically it is due to the fact that the flux in the earth is not in phase with the current.

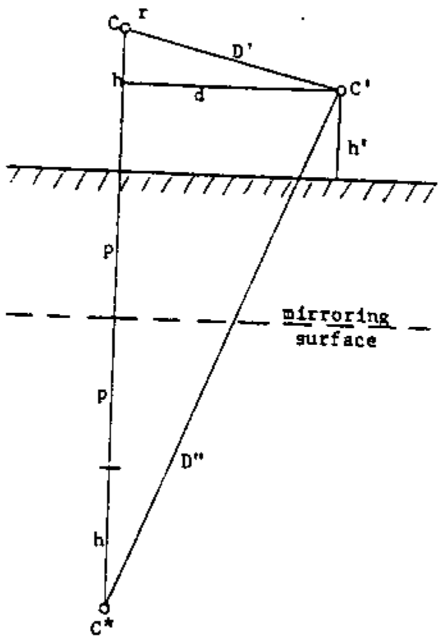

Figure 1: Complex depth and related quantities.

PLAUSIBILITY ARGUMENT FOR THE CLOSED FORM APPROXIMATION:

The field in the earth is not in phase with the current in the wire. Therefore, the magnetic flux will have an appropriate phase shift. If the wire is replaced by a sheet conductor then the geometry becomes plane-paralled and p of (1) appears as the equivalent depth for the return current to produce the same resultant flux. It is assumed, and then proved by numerical evaluation, that a complex return plane at depth p is still valid for the actual linear conductor.

ACCURACY OF CLOSED FORM APPROXIMATION:

Carson's correction P and Q are closely approximated by corrections P' and Q' which are implicit in equations (3). Figure 2 portrays these corrections in terms of Carson's parameter r (for $\theta = 0°$, and $90°$).

CONCLUSIONS:

The availability of simple and accurate analytical expressions in the complex frequency s for transmissions line impedances permits straightforward calculation of line parameters.

REFERENCES:

1 Deri A.; Tevan G.; Semlyen A. and Castanheira A., "The Complex Ground Return Plane, A Simplified Model for Homogeneous and Multi-Layer Earth Return", IEEE Paper No. 81 WM 222-9, presented at the 1981 Winter Power Meeting, Atlanta, Georgia.

2 Carson J.R., "Wave Propagation in Overhead Wires, with Ground Return", Bell Syst, Tech. J., 1926, Vol. 5, pp. 539 - 554.

3 Semlyen, A., and Abdel-Rahman, M.H., "A state variable approach for the calculation of switching transients on a transmission line", IEEE Trans. on Circuits and systems, Vol. CAS-29, 1982, pp. 624-633.

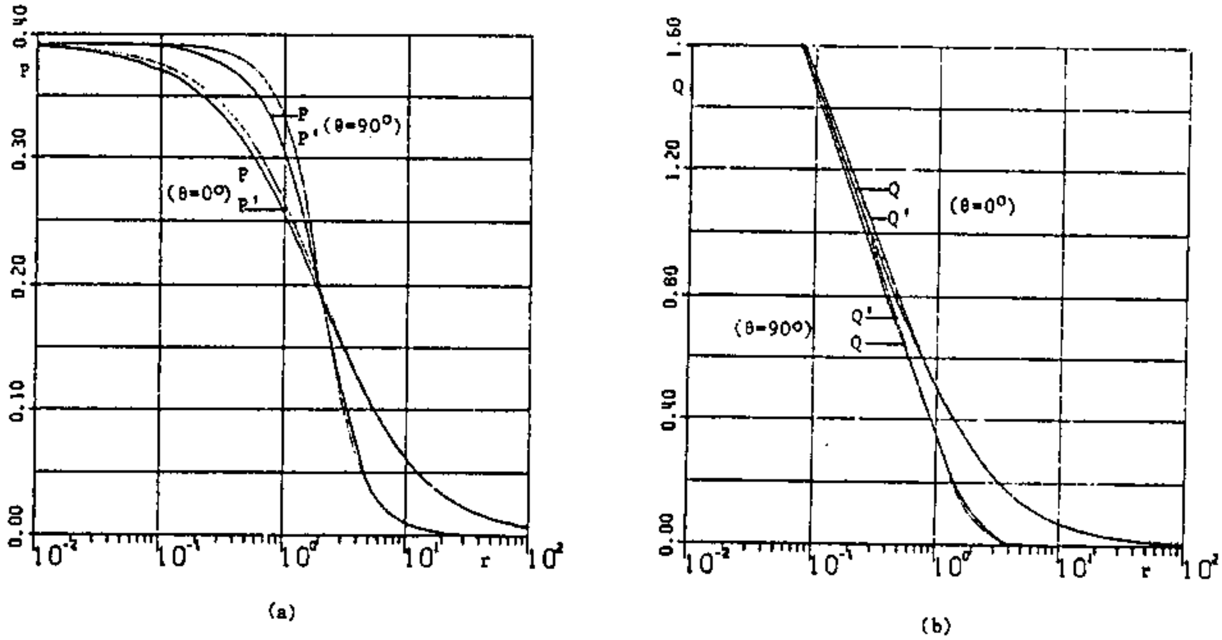

Fig. 2. Impedance correction term.

P,Q - Carson's values

P',Q' - Analytically obtained values