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Parametric Study of the Lateral-Torsional Buckling Behaviour for Steel Girders with Corrugated Webs

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ABSTRACT

The steel plate girder with the corrugated web is widely used in many applications because it has many advantages. It has a high resistance against lateral-torsional buckling compared with another flat web girder with the same steel weight. The design method for calculating the lateral-torsional buckling strength of corrugated web girders is not provided in the current standards and specifications. This paper focuses on the strength of corrugated web girders against lateral-torsional buckling and presents the theoretical and finite element analysis to study the behavior of this type of girders. The Nonlinear finite element analysis is carried out using ANSYS. The influence of various parameters (web thickness, web height, corrugation angle, corrugation width, and corrugation shape) is considered in this study. Where there are a few approaches such as *Linder*, *Moon*, and *Zhang* that explain how to determine the warping constants of corrugated web beams therefore, the applicability of these approaches is evaluated in this study. It is found that the formulas of *Moon* and *Zhang* are more accurate than the *Linder* formula except in the case of beams with a ratio between web height and flange width equal or less than 2.0. The applicability of using the *Euro-code* equations of the lateral-torsional buckling moment for plate girders with flat webs in cases of girders with corrugated webs is studied. It is concluded that the *Euro-code* equations can be used for corrugated web girders with a good agreement after using *Moon's* or *Zhang's* formula to calculate the warping constant.

1. Introduction

The steel plate girder with the corrugated web has been used on a large scale since the automation welding technologies are available in the late 1980s/early 1990s. It has many advantages such as a high ability to carry loads compared to the weight of the steel used. The previous researches state that the critical buckling moment is strongly influenced by the torsion and warping constants which are higher in the case of corrugated web beams than that for flat web beams. Therefore, a rather recent method to increase the resistance of girders against lateral torsion buckling is to have a corrugated shape of the web instead of a flat web.

The current standards and specifications do not provide the design method to calculate the strength of girders with corrugated webs against lateral-torsional buckling. It didn't give any information about the accuracy of using the given equations for steel beams with flat webs (FW) for beams with corrugated webs (CW). These equations depend on the value of the warping section constant of steel beams. There isn't

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found a recommended method or equation in different codes that can be used to determine the warping constant for (CW) steel beams.

Lindner [1] took precedence in the investigation of the lateral-torsional buckling strength of (CW) steel beams. It is assumed that the shear and the moment were carried only by the web and the flanges respectively. It is also concluded that the torsional section constant (J) for (CW) steel beams doesn't differ from those for (FW) steel beams, but the warping section constant (C_w) is different. *Lindner* proposed a formula to determine the warping constant ($C_{w,Linder}$) of steel beams based on the numerical derivations verified according to experimental results. *Sayed-Ahmed* [2, 3] carried out a series of finite element analyses to study the (CW) steel beams. It is concluded that the lateral-torsional buckling resistance of the (CW) steel beams is higher by about (15-37%) than those of (FW) beams, and the equations used to determine the strength of (FW) beams against critical lateral-torsional buckling be less accurate when used in case of the (CW) steel beams. However, the behavior of lateral-torsional buckling of the (CW) steel beams wasn't explained and investigated clearly, but *J. Moon et. al.* [4] carried out finite element analysis models to investigate it. They used a numerical method to suggest an approximate method for determining the location of the shear center and calculating the warping constant $(C_{w,Moon})$of this type of steel beam. *H.R. Kazemi et.al [5]* performed finite element analysis models to study the lateral-torsional buckling moment for simply supported (CW) steel beams under a uniform moment. They found that the lateral-torsional buckling moment of (CW) steel beams increased than that for (FW) steel beams up to 40%. *Fatimah De'nan et. al.* [6] performed numerically finite element analyses to investigate the behavior in the minor and the major axes of the triangular (CW) due to pure bending and the effect of the variation in the corrugation angle for the triangular web. They compared between the results of triangular (CW) and the equivalent (FW) steel beams and concluded that the values of moments of inertia about minor and major axes for the (CW) steel beams are in ranges from (1.523 to 1.686) and (0.754 to 0.818) times of that for the same beams with (FW) respectively. *Elgaaly et.al. [7]* performed analytical and experimental studies to investigate the behavior of (CW) steel beams due to bending. They concluded that the flexural strength of (CW) steel beams is only contributed from its flanges and the effect of the web is negligible due to the accordion action. *Y.A Khalid et. al. [8]* carried out experimental and analytical studies to investigate the influence of the ratio between the thicknesses of web and flange, the panel aspect ratio, and the corrugation configuration in the bending strength of the (CW) steel beams. *Limaye A. A. et. al. [9]* performed a finite element analysis of a plate girder using ANSYS to compare the buckling strength – weight ratio for (CW) steel beams with rectangular corrugated web plate and (FW) steel beams. They concluded that the (CW) steel beams have high buckling strength and sufficient reduction in weight with light gauge elements than that for (FW) steel beams. Z. *Zhang et. al. [10]* performed analytical models, as *J. Moon [4]*, to investigate the strength of (CW) steel beams under uniform bending against lateral-torsional buckling and carried out a Zhang's formula to determine the warping constant of (CW) steel beams based on the concept of a prismatic girder with an eccentric flat web. *Alia M. Abd Elaziz [11]* carried out a finite element numerical analysis to study the strength of (CW) steel beams with different corrugation shapes against lateral-torsional buckling and shear buckling. *Elamary A. S. et. al. [12]* carried out analytical and experimental models to study the flexural behavior of (CW) steel and composite beams. They compared the values of nominal moment capacity for the tested beams with that obtained by using a limit state design process taking into consideration the effect of (CW) in flexural behavior. They found that the comparison between the designed values of bending moment agreed to an acceptable degree of accuracy with the values obtained experimentally. *Basiński1 W [13]* concerned experimental investigations and finite element numerical analysis to study the shear buckling of (CW) steel beams with a sine wave without and with semi-rigid and rigid end stiffeners. He estimated a new method to determine the shear buckling resistance for this type of girders. *Lopes et. al. [14]* carried out analytical finite element models to investigate the behavior of a wide range of sinusoidal (CW) steel beams with different corrugation profiles and lengths. *Chenpu Guo et. al. [15]* studied the lateral-torsional behavior of trapezoidal (CW) steel beams subjected to uniform and non-uniform torsion by performing finite element models. They concluded that the torsional constant of corrugated web girders is significantly greater than those of flat web due to the difference between the warping constants. *SAVE E. et. al. [16]* have investigated the fatigue resistance of (CW) steel beams and they concluded that the fatigue strength of (CW) steel beams is slightly better than what is currently suggested in the design standard EN 1993-1-9. *B. Jáger and L. Dunai [17]* performed advanced nonlinear finite element numerical models to study the lateral-torsional buckling behavior of steel trapezoidal (CW) girders. They based design methods for this type of beams including initial geometric imperfections and residual stresses.

The *Euro-code (EC3) [18]* gave an equation to calculate the elastic lateral-torsional buckling moment for steel girders with flat webs, it is shown in Equ. (1).

$$M_{cr,EC3} = C_1 \frac{\pi^2 E I_z}{L_{cr}^2} \sqrt{\frac{C_w}{I_z} + \frac{L_{cr}^2 G I_T}{\pi^2 E I_z}} \qquad \text{......... (1)}$$

Where, C_w is the warping constant for steel girder, G is the shear modulus, I_T is the torsion constant, I_z is the second moment of area about the minor axis, E is the modulus of elasticity, C_1 is the equivalent moment factor, and L_{cr} is the unsupported length of compression flange.

The European code, like the rest of the international codes, did not provide any information about the warping constant C_w in the case of steel girders with corrugated webs, while many previous kinds of research provide different methods for calculating this constant for this type of girders such as *Linder [1], Moon et al. [4], and Zhang et al. [10]*.

Therefore, the aims of this paper are:

1- Verifying the accuracy of warping constant formulas for (CW) steel beams which were proposed from previous researches.
2- Check the accuracy of using the given equation of elastic lateral-torsional buckling moment in the design procedure of *Euro-code [18]* in the case of (CW) steel beams.
3- Studying the effect of the variations in the dimensions and corrugation shape of the (CW) steel beams on the lateral-torsional buckling behavior.

2. Modeling of nonlinear Finite elements

The finite element commercial package ANSYS [19] was used to perform advanced nonlinear computations. It has a good efficiency in representing the geometric and material nonlinearities as it is considered in all analyzed girders studied in this research. The finite nonlinear Shell element 281 has eight nodes and each node has six degrees of freedom. So, this element is used to model the flanges and webs of all analyzed girders.

2.1 Properties of material

The properties of steel material considered in all analyzed girders are following the data of the bilinear stress-strain curve, as shown in Fig. 1. The initial elastic modulus of elasticity is *210 GPa*. After that, it assumed that the material followed the linear hardening with a reduced hardening modulus E_r equal to *0.01E*. The used steel with specified yield stress f_y and ultimate strength f_u of *240 MPa* and *370 MPa*, respectively. Poisson's ratio is taken equal to 0.3.

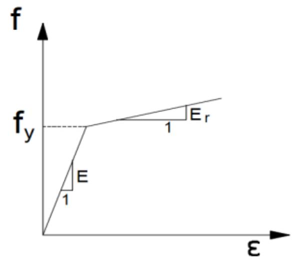


Figure (1) Bilinear stress-strain curve for steel

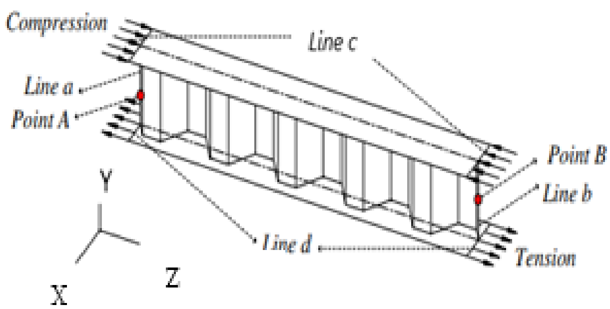


Figure (2) Boundary conditions of the analyzed girder

2.2 Supports and Loads

The considered girders in this study are simply supported in torsion and flexure. The vertical displacement (U_y) in direction Y at lines (c, d) and the out translation (U_x) in direction X at lines (a, b) are restrained. Point (A) and point (B) are considered as hinge and roller supports, respectively. Where for the two points the rotation about the Z-axis and the translations in Y, and X directions are restrained. But the translation in Z is restrained only for point (A). All boundary conditions for analyzed girders are shown in Figure (2).

Eigenvalue buckling analysis was performed on all analyzed girders to evaluate the theoretical buckling loads where girders become unstable. All analyzed girders were subjected to equal two end moments to study the lateral-torsional buckling due to pure moment without any shear force acting on girders. This case of loading is represented as compression and tension forces at the upper and lower flanges at the ends of the studied girder, as shown in Figure (2).

2.3 Geometry of the analyzed girders

The analyzed studied girders are assumed simply supported at both ends with span (L=15600mm), consisting of two flange plates with width (b_f) and thickness (t_f) equal 500 mm and 40mm, respectively. The web is considered a corrugated web with height (h_w) and thickness (t_w) equal to 2000 mm and 12 mm, respectively. The corrugation angle (Θ) is assumed to equal 45º. The width (b) and depth (d) of corrugation are taken equal 330mm and 270mm, respectively, as shown in Figure (3).

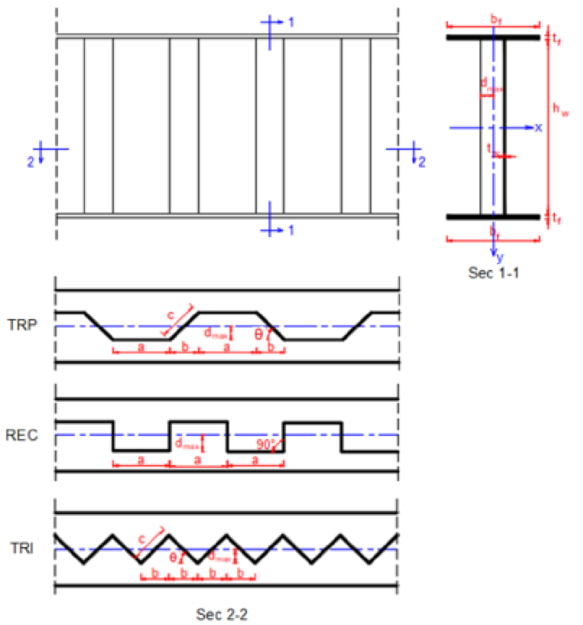


Figure (3) The studied web corrugation profiles

3. Verification of the finite element model

The results of the proposed F.E models are verified with the results of previous experimental and analytical studies as follows:

3.1 Verification with experimental study

The experimental model was performed by *Nikolaus L. et. al. [20]* to investigate the maximum patch load of a (CW) girder at different locations. The girder consists of two equal spans (3.0m) with hinge support at point (a) and roller supports at points (b) and (c). Vertical stiffeners were arranged at each support. The web thickness and height of the analyzed girder are 3mm and 578mm respectively. Both flanges are made from 160mm wide and 12mm thick steel plates. The corrugation angle equals 45º and the width (b) and the depth (h) are equal to 140mm and 50mm, respectively, as shown in Figures (4) and (5).

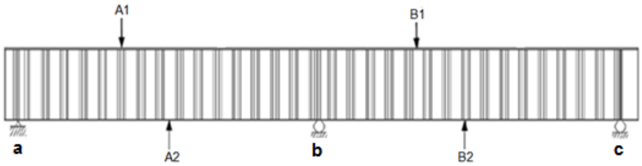


Figure (4) An illustration showing the tested girder and the load cases by *Nikolaus L. et. al. [20]*

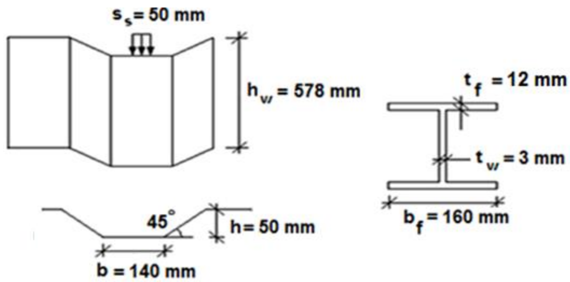


Figure (5) Dimensions and geometry of the tested girder by *Nikolaus L. et. al. [20]*.

A finite element model is performed by the authors using element 281 in *ANSYS* with the same dimensions; corrugation profile of the web, material properties, and boundary conditions as that considered in the experimental model by *Nikolaus L. et. al. [20]* as shown in Figure (6).

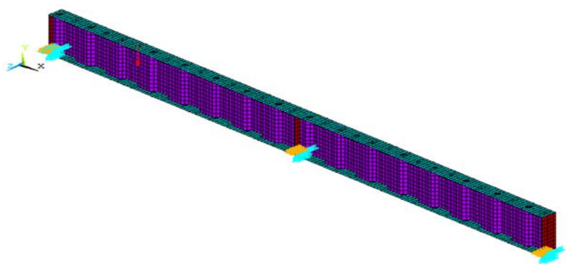


Figure (6) F.E.M by authors for verification with the experimental model by *Nikolaus L. et. al. [20]*

Table (1) shows the comparison between the results of the F.E model and the results of experimental tests by *Nikolaus L. et. al. [20]*. It is shown that the values of the patch loading resistances for the analyzed girder which are obtained from the FEM are in good agreement with that obtained experimentally by *Nikolaus L. et. al. [20]*, where the differences ranged from 3 to 12.7%.

Table (1) Comparison between results of patch loading resistance from the experimental model by *Nikolaus L. et. al. [20]* and FEM by Authors

Position of applied load	Results of the experimental tests by Nikolaus L. et. al. [20]	Results of FEM by Authors	% Difference
A1	219 KN	226 KN	3 %
A2	181 KN	204 KN	12.7 %
B1	188 KN	204 KN	8.5 %
B2	218 KN	234 KN	7.3 %

3.2 Verification with the analytical study

H.R. Kazemi et. al. [5] used finite element software *ANSYS* to investigate the strength of (CW) steel beams against lateral-torsional buckling and the effect of web corrugation profiles. The Four side shell elements SHELL 43 were used to model the web, top, and bottom flanges. The analyzed girder was simply a supported beam with a span of 15000mm, length of a horizontal panel of 100mm. The inclined panel has a projected length equal to 150mm and the corrugation depth is 250mm. while the width and thickness of both flanges of the analyzed girder were 500mm and 40mm, respectively. The web was made from 2000mm height and 12mm thickness. The girder was subjected to two end moments, as shown in Figure (7).

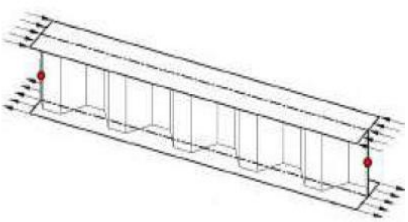


Figure (7) Shows the boundary conditions and case of loading which considered by *H.R. Kazemi et. al.* [5]

The author's finite element model was carried out by using a nonlinear shell element 281 in *ANSYS* with the same dimensions; corrugation profile of the web, material properties, and boundary conditions as that considered in the analytical model by *H.R. Kazemi et. al. [5]*, as shown in Figure (8). The evaluation of the critical moment (M_{cr}) which initiated the lateral instability was performed by using eigenvalue buckling analysis.

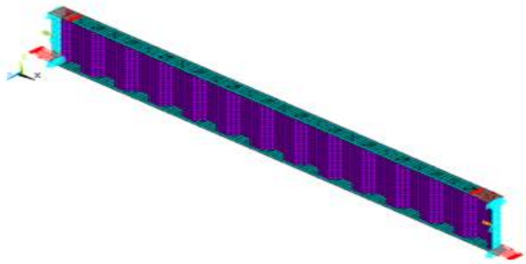


Figure (8) Shows the F.E.M by authors for verification with an analytical model by *H.R. Kazemi et. al.* [5].

Table (2) explain the comparison between the results of the critical moment from the analytical models of *H.R. Kazemi et. al. [5]* and Authors

Buckling mode	Results of the analytical model by H.R. Kazemi et. al. *[5]*	Results of FEA by Authors	% Difference
Mode (1)	9867 KN.m	9494 KN.m	3.9 %
Mode (2)	32977 KN.m	31637 KN.m	4.2 %
Mode (3)	70145 KN.m	67870 KN.m	3.3 %

It is detected from Table (2) that the F.E. model's results of critical moment initiating lateral-torsional buckling are very close to that obtained by *Kazemi et. al.*, where the difference ranged from 3.3 to 4.2 %.

4. PARAMETRIC STUDY

An extensive parametric study is conducted using the finite element model which was described earlier to study the warping constant for girders with corrugated webs and their lateral-torsional buckling behavior as follows:

4.1 Warping constant for girders with corrugated web

There is not found any formula in any international steel code such as the Euro-code (EC3) can be used to calculate the warping constant of (CW) steel girders while, it was investigated in previous researches such as *Linder [1]*, *J. Moon et al. [4] and Zhang et al. [10]*. Therefore, one of the main objects of this search is a determination of the more accurate technique from these methods. So, the values of the warping constant calculated according to these methods are compared with the results of F.E models as follows:

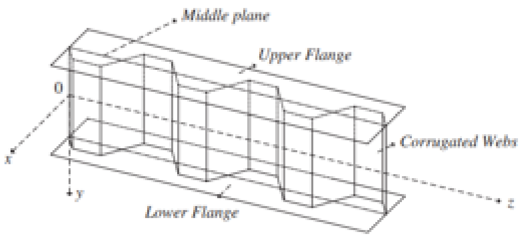
Firstly, *Linder [1]* presented an approach to determine the critical moment of (CW) girders by using analytical analysis, verified by experimental testing. He assumed that the torsion constant [I_t] and also the moment of inertia about the weak axis [I_y] are the same as for (FW) girders. He derived an empirical formula to find the warping constant for (CW) girders $C_{w,Linder}$ which was defined as follows:

$$C_{w,Linder} = C_{w,f} + \frac{c_w L^2}{E\pi^2} \qquad \text{.................. (2)}$$

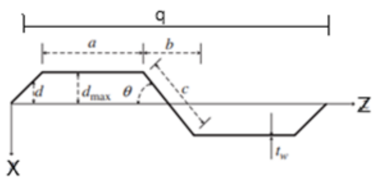
Where $$c_w = \frac{(2d_{max})^2 h_w^{\ 2}}{8u_x(a+b)} \qquad \text{........... (3-a)}$$

$$u_x = \frac{h_w}{2Gat_w} + \frac{h_w^{\ 2}(a+b)^3(I_{x,co}+I_{y,co})}{600\,a^2 E I_{x,co} I_{y,co}} \qquad \text{...... (3-b)}$$

Where $\boldsymbol{C_{w,f}}$ is the warping constant of (FW) girder, $\boldsymbol{L}$ is the girder span, $\boldsymbol{E}$ is the modulus of elasticity, $\boldsymbol{h_w}$ is the web height, $\boldsymbol{t_f}$ is the flange thickness and $\boldsymbol{G}$ is the shear modulus, $\boldsymbol{I_{x,co}}$, $\boldsymbol{I_{y,co}}$ are the moment of inertia about strong and weak axes for (CW) girder, respectively. Figure (9) shows different dimensions of the corrugated web which were used in *Linder's* formula.



(9-a) Isometric for *Linder [1]* corrugated web beam



(9-b) The definition of different symbols

Figure (9) shows the considered corrugation profile by *Linder [1]*.

After that, *J. Moon et al. [4]* created an approach to determine the critical lateral torsion buckling moment $M_{cr,Moon}$ of (CW) girders. They assumed, as Lindner, that all sectional properties of (CW) girders are equal to those of (FW) girders, except the warping constant.

So, they derived quite complex analytical expressions to calculate it. They considered that the section of (CW) girders was composed of a series of interconnected plate elements. Then the constant of warping $C_{w,Moon}$ of (CW) girders can be determined according to Moon's method as detailed in Equ. (4):

$$C_{w,Moon} = \frac{1}{3}\Sigma(W_{ni}^{\ 2} + W_{nj}W_{ni} + W_{nj}^{\ 2})t_{ij}L_{ij} \qquad \text{....(4)}$$

Where t_{ij} and L_{ij} are the thickness and length of an element between node i and node j, respectively. The normalized unit warping W_{ni} and W_{nj} at the ends of each element (i-j) is defined according to Equ. (5).

$$W_{ni} = \frac{1}{2A}\Sigma_0^n(W_{0i} + W_{0j})t_{ij}L_{ij} - W_{0i} \qquad \text{....(5-a)}$$

$$W_{nj} = \frac{1}{2A}\Sigma_0^n(W_{0i} + W_{0j})t_{ij}L_{ij} - W_{0j} \qquad \text{(5-b)}$$

Where: A is the cross-section area$=\Sigma_0^n t_{ij}L_{ij}$

W_{0i} and W_{0j} is the unit warping with respect to the centroid at point i and j respectively

$$W_{0i} = \rho_{ij}L_{ij} \qquad \text{.................. (5-c)}$$

$$W_{0j} = W_{0i} + \rho_{ij}L_{ij} \qquad \text{.................. (5-d)}$$

J. Moon et al. [4] gave an example, as shown in Fig. (10), to simplify the use of their approach in the calculation of the constant of warping for an eccentric web girder. Equ. (6) shows in detail how the normalized unit warping for each node that was mentioned in Equ. (5) can be calculated.

$$W_{n1} = \frac{2b_f^{\ 2}h_w t_f + b_f h_w^{\ 2} t_w}{8b_f t_f + 4h_w t_w} = W_{n6} \qquad \text{..... (6-a)}$$

$$W_{n2} = \frac{2b_f^{\ 2}h_w t_f + b_f h_w^{\ 2} t_w}{8b_f t_f + 4h_w t_w} - \left(\frac{b_f}{4} - \frac{d_{avg}}{2}\right)h_w \qquad \text{..... (6-b)}$$

$$W_{n3} = \frac{2b_f^{\ 2}h_w t_f + b_f h_w^{\ 2} t_w}{8b_f t_f + 4h_w t_w} - \left(\frac{b_f}{4} + \frac{d_{avg}}{2}\right)h_w \qquad \text{..... (6-c)}$$

$$W_{n4} = \frac{2b_f^{\ 2}h_w t_f + b_f h_w^{\ 2} t_w}{8b_f t_f + 4h_w t_w} - \frac{1}{2}b_f h_w = W_{n5} \qquad \text{..... (6-d)}$$

Where d_{avg} is an average eccentricity $= \frac{(2a+b)d_{max}}{2(a+b)}$

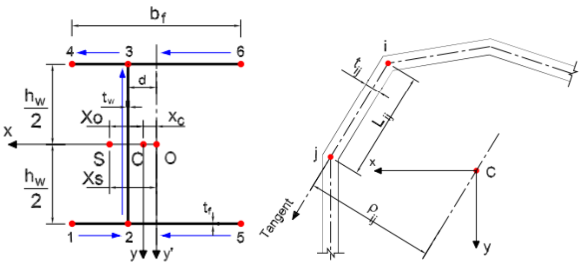


Figure (10) The path directions for calculating the warping constant of (CW) girders according to the *Moon* approach, *J. Moon et al. [4]*

Z. Zhang et al. [10] performed another approach to calculate the critical lateral-torsional buckling moment. They assumed also the same assumptions that assumed by *Linder and Moon* as mentioned before. They mentioned that the increase in warping constant $C_{w,ecc}$ for (CW) girder is the only cause for the increase in the value of its critical buckling moment. Zhang's method is dependent on the concept of a prismatic (FW)girder with an eccentric web in the calculation of a (CW) girder as defined in equation (7).

$$C_{w,zhang} = \frac{t_f b_f^{\,3} h_w^{\,2}}{24} + \frac{t_w h_w^{\,3} d^2}{12} \qquad \text{........ (7)}$$

Equ. (7) consists of two terms, the first term represents the warping constant of asymmetric (FW) girder, $C_{w,f}$, while the eccentricity of the (CW) web is represented by the second term. *Z. Zhang et. al.* considered one corrugation wavelength, [$q=2a+2b$] as shown in Fig. (9), in the integration of equation (7). So, they found that to account for the varying in the eccentricity of the corrugated web, the second term must be divided by (q) as shown in equation (8).

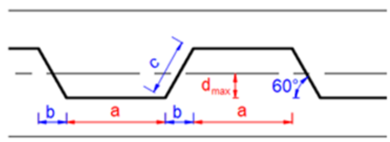
$$C_{w,zhang} = \frac{1}{q}\int_0^q \left[\frac{t_f b_f^{\,3} h_w^{\,2}}{24} + \frac{t_w h_w^{\,3} d^2}{12}\right] dx$$

$$= C_{w,f} + \frac{t_w h_w^{\,3} d_{max}^{\,2}}{12} \frac{(a+\frac{b}{3})}{2q} \qquad \text{........ (8)}$$

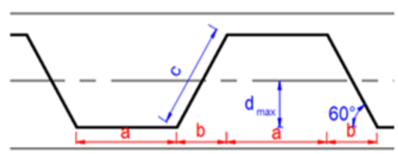
In this paper, the accuracy of *Linder*, *Moon*, and *Zhang* formulas to calculate the warping constant of corrugated web girders was compared using numerical models based on the finite element technique. Where the warping constant value does not appear in the output results of F.E. models so, it is calculated by substituting the value of elastic lateral-torsional buckling strength which is resulted from the finite element analysis, $M_{cr,\,FEM}$, in the Euro code equation, Equ. (1). So, the value of warping constant for (CW) girder can be calculated from the finite element analysis $C_{w,\,FEM}$ as detailed in Equ. (9).

$$C_{w,F.E.M} = \left(\frac{M_{cr,F.E.M}^{\;2} L_{cr}^{\;2}}{C_1^{\,2} E^2 \pi^2 I_z} - \frac{GJ}{E}\right)\left(\frac{L_{cr}}{\pi}\right)^2 \qquad \text{............ (9)}$$

There are many parameters considered in the comparison between the different formulas, such as web thickness; t_w, web height; h_w, corrugation angle; ϴ, corrugation width; b and the ratio between the applied two end moments. The thickness of the web is considered from 8 mm to 18 mm, while its height is ranged from 500 mm to 3000 mm. Additionally, the influence of corrugation angle; ϴ is studied by considering two cases for the variation in the corrugation angle. In case {1}, the corrugation depth (d) is assumed to have a constant value, 270 mm, and the increase in the corrugation angle is a result of the decrease of the inclined panel length (c) which is led to increasing the number of corrugation waves along beam span in this case as shown in Fig. (11) and table (2). While, in case {2}, the length of the horizontal projection of the inclined panel (b) is assumed to have a constant value, 270 mm, and the increase in the corrugation angle is a result of increasing the inclined panel length (c) as shown in figure (12) and table (2).



Case 1: Web with trapezoidal corrugation; CW1-60



Case 2: Web with trapezoidal corrugation; CW2-60

Figure (11) The difference between the two cases 1 and 2 for the variations in corrugation angle; ϴ,

The corrugation width is considered to increase from 130mm to 530mm. Finally, the influence of the ratio between the two applied end moments (M_1/M_2)

is studied to range from -1.0 to +1.0 with increment 0.5, where M_1, M_2 are the smallest and the largest end moment, respectively.

Figure (12) illustrates the influence of each parameter on the ratio between each warping constant resulted according to the different studied formals ($C_{w,Linder}$, $C_{w,Moon}$, $C_{w,Zhang}$) compared with another value which was resulted from finite element models, $C_{w,F.E.M}$.

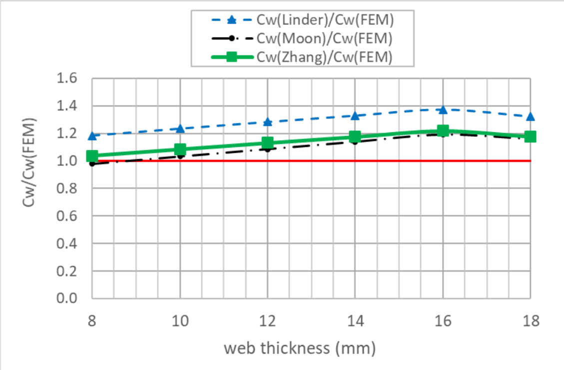


Figure (12-a) Comparison of the warping constant for different web thickness

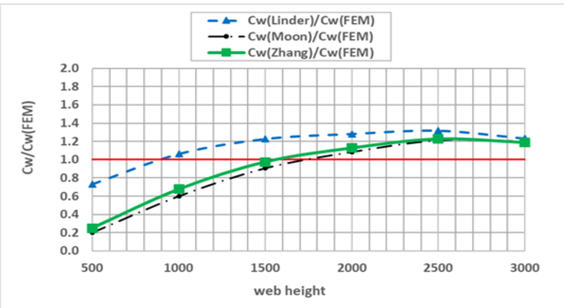


Figure (12-b) Comparison of the warping constant for different web height

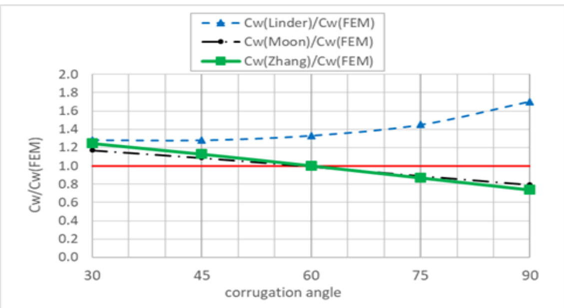


Figure (12-c) Comparison of the warping constant for different corrugation angles (case 1)

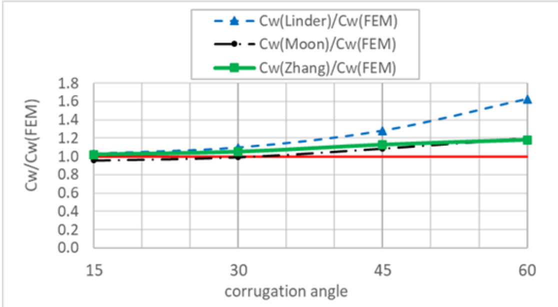


Figure (12-d) Comparison of the warping constant for different corrugation angles (case 2)

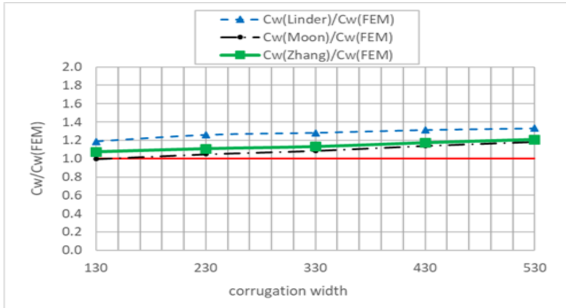


Figure (12-e) Comparison of the warping constant for different corrugation width

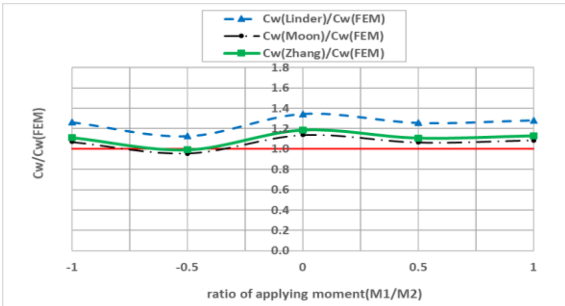


Figure (12-f) Comparison of the warping constant for different applied moment ratios

It can be noted from Figure (12) that the results of the *Linder* formula are slightly far when compared with the F.E.M. results for different parameters. Because Linder suggested his formula by using a series of experimental results under transverse loading, so the out-of-plane displacement along the compression flange length occurred. Therefore, *Linder's* formula for warping constant $C_{w,Linder}$ includes the effect of the lateral displacement component added to the lateral-torsional buckling. In addition, the accuracy of using *Linder's* formula for triangular corrugation web, because in this case the corrugation width; a, equals zero and then *Linder's* formula gives $u_x = \infty$, $C_w = 0$ and $C_{w,Linder} = C_{w,f}$ i.e according to *Linder's* formula: the warping constant of the beam with triangular corrugated web equals to that for flat web beam, this is illogical.

On the other hand, Figure (12) shows that the results of *Moon* and *Zhang* formulas are closed to the results of F.E. models for almost all parameters except the models which have web height ($h_w \leq 1000mm$) with flange width ($b_f = 500mm$) as shown in Fig.(12-b). Therefore, it is preferable not to use the *Moon* and *Zhang* formulas only in the case of the beam with a ratio between web height (h_w) and flange width (b_f) equal to or less than 2.0.

In general, it is noticeable that the accuracy of *Zhang's and Moon's* formulas is much better than *Linder's* formula, especially when the corrugation angle is greater than 30°. So, the warping constant formula, proposed by *Zhang* or *Moon*, can be used to determine the elastic lateral-torsional buckling moment according to the Euro-code equation.

4.2 Lateral torsional buckling behavior

Little previous research was conducted on the lateral-torsional buckling of (CW) girders still does not cover much of this behavior. So, one of the main aims of this paper is the check the applicability of using the *Euro-code* equation which was used to determine the elastic lateral-torsional buckling moment for (FW) girders; Equ. (1); in the case of (CW) girders after using *Zhang's* or *Moon's* formulas in the determination of its warping constant. The accuracy of using the Euro-code equation was examined by comparing the resulting values of the critical lateral-torsional buckling moment with the results of the F.E. models.

In this part, the same parameters which were considered in the previous part of this paper are considered added to the corrugation shape. The corrugation shape is presented in three types trapezoidal, triangular, and rectangular corrugated web. Figure (13) illustrates the influence of each parameter on the lateral-torsional buckling moment.

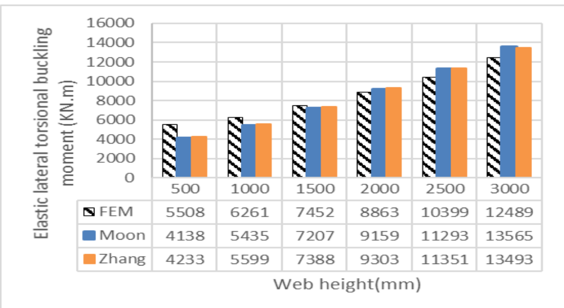


Figure (13-a) Influence of web height on the lateral-torsional buckling moment

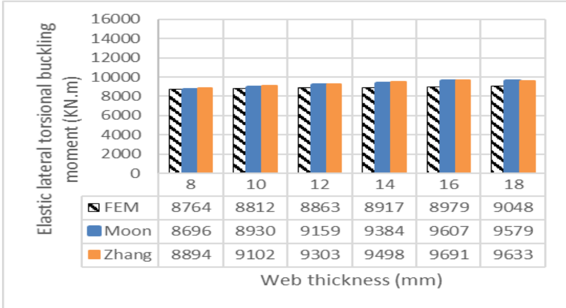


Figure (13-b) Influence of web thickness on the lateral-torsional buckling moment

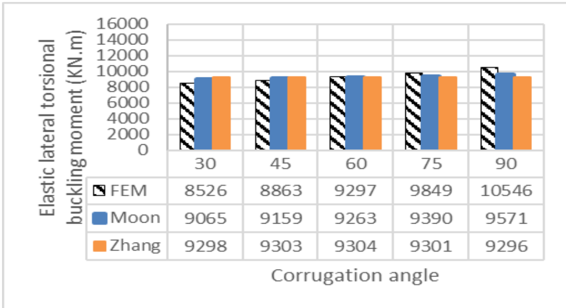


Figure (13-c) Influence of corrugation angle on the lateral-torsional buckling moment (Case 1)

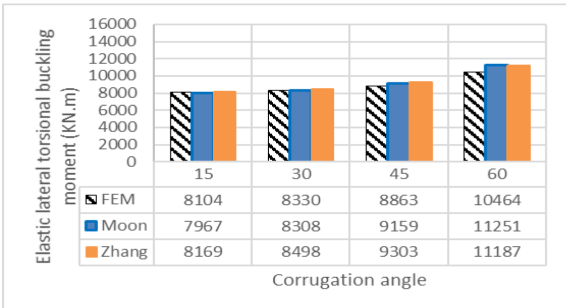


Figure (13-d) Influence of corrugation angle on the lateral-torsional buckling moment (Case 2)

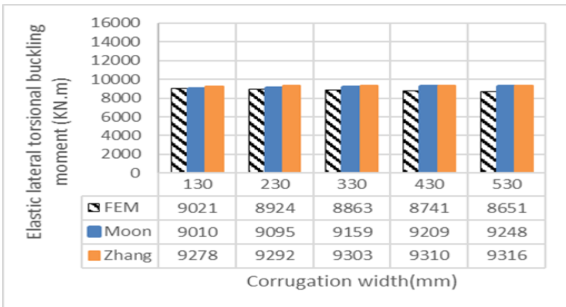
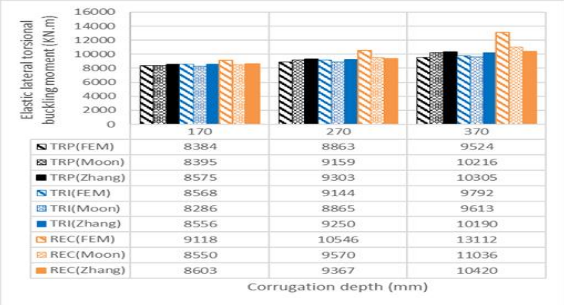


Figure (13-e) Influence of corrugation width on the lateral-torsional buckling moment



	170	270	370
TRP(FEM)	8384	8863	9524
TRP(Moon)	8395	9159	10216
TRP(Zhang)	8575	9303	10305
TRI(FEM)	8568	9144	9792
TRI(Moon)	8286	8865	9613
TRI(Zhang)	8556	9250	10190
REC(FEM)	9118	10546	13112
REC(Moon)	8550	9570	11036
REC(Zhang)	8603	9367	10420

Figure (13-f) Influence of corrugation shape on the lateral-torsional buckling moment

It is obvious from Figure (13-a) that the increase of web height increases the values of the elastic lateral-torsional buckling moment, Mcr; For (CW) beams because the increase in web height leads to an increase in the moment of inertia about the weak axis (I_y). This note is confirmed from Figures (14-a), (14-b), and (14-c) due to the eigenvalues for the out-of-plane displacement (Uz) for girders CW1500, CW2000, and CW2500, respectively. It is shown that the eigenvalues for girders with web height 1500, 2000, and 2500mm are equal to 483891, 434437, and 409390 respectively.

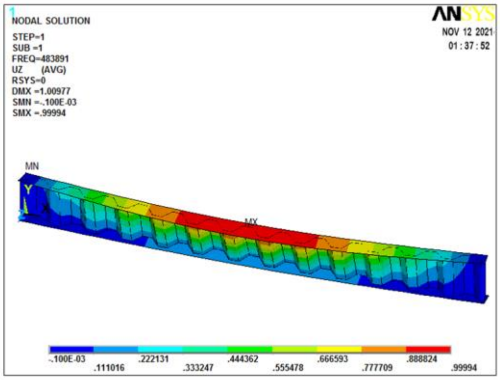


Figure (14-a) The lateral-torsional buckling mode for CW-1500

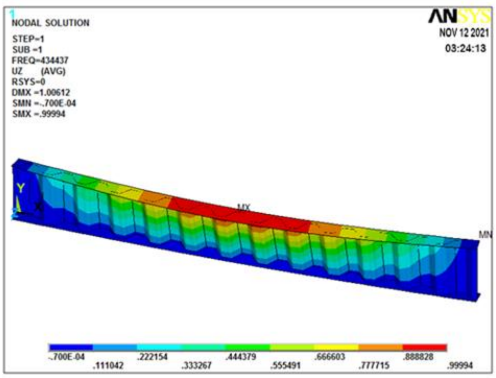


Figure (14-b) The lateral-torsional buckling mode for CW-2000

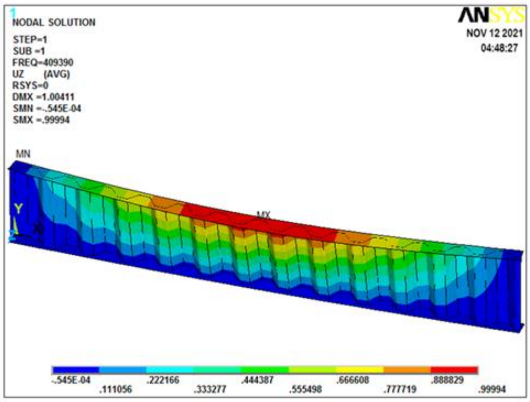


Figure (14-c) The lateral-torsional buckling mode for CW-2500

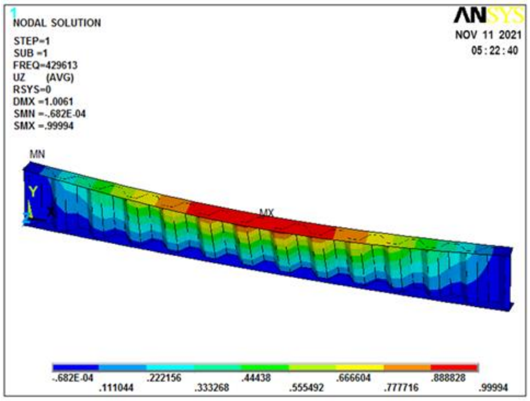


Figure (14-d) The lateral-torsional buckling mode for CW-8

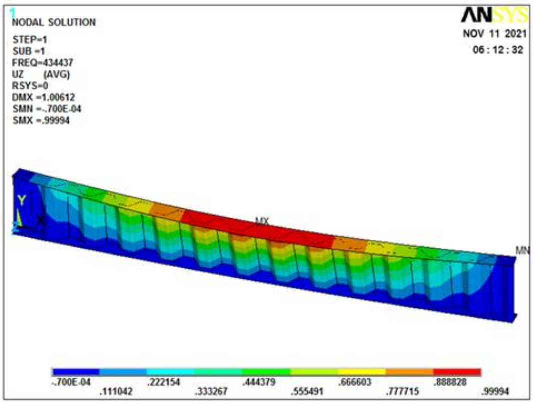


Figure (14-e) The lateral-torsional buckling mode for CW-12

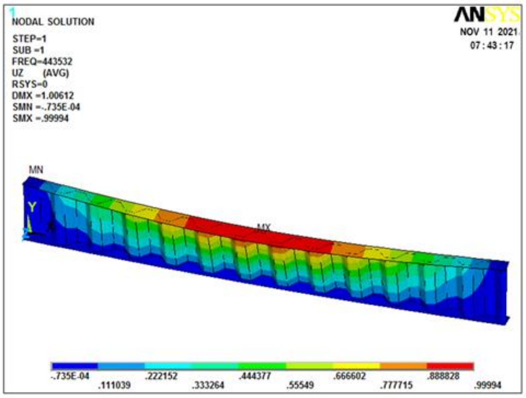


Figure (14-f) The lateral-torsional buckling mode for CW-18

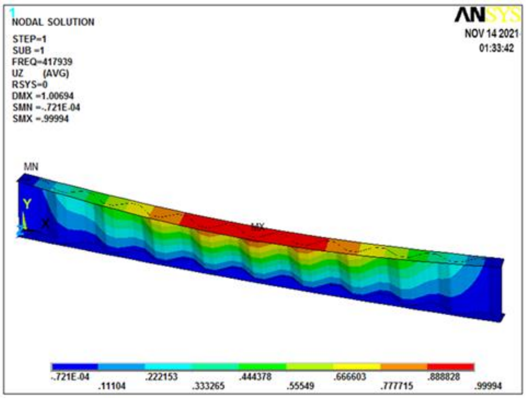


Figure (14-g) The lateral-torsional buckling mode for CW1-30

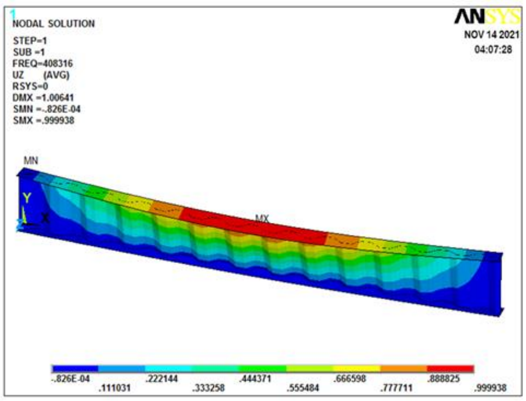


Figure (14-h) The lateral-torsional buckling mode for CW2-30

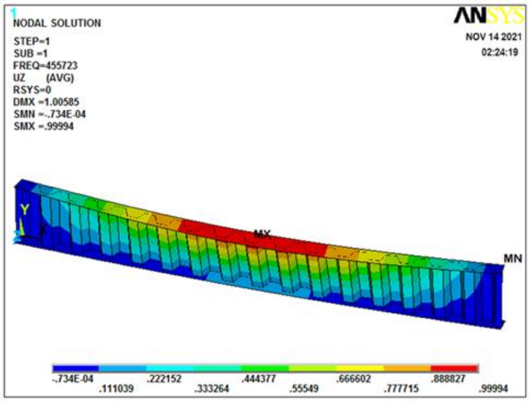


Figure (14-i) The lateral-torsional buckling mode for CW1-60

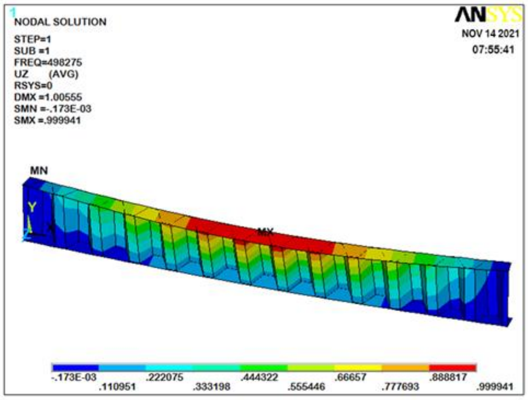


Figure (14-j) The lateral-torsional buckling mode for CW2-60

Additionally, Figure (13-a) shows that the F.E.M results are in good agreement with the Euro-code, except the cases of models with web height equal to 500 mm and 1000 mm because the accuracy of *Moon's* or *Zhang's* formulas in the calculation of warping constant is decreased when plate girders with the corrugated web have ratios between web height and flange width equal or less than 2.0.

Figure (13-b) shows that the values of elastic lateral-torsional buckling moment obtained from F.E. models are very close to those obtained according to the Euro-code equation with using *Moon's* or *Zhang's* formulas in the calculation of warping constant. It is obvious from Figure (13-b) that the influence of web thickness in the elastic lateral-torsional buckling moment M_{cr} is very small; as shown in Figures (14-d), (14-e), and (14-f) where the eigenvalues for the out of plane displacement (U_z) for girders CW8, CW12, and CW18 with web thickness 8, 12, and 18mm are equal to 429613, 434437, and 443532 respectively.

It can be observed from Figures (13-c), (13-d) that the value of M_{cr} in Case {1} is higher than that in Case {2} for corrugation angle 30°. Contrarily, at angle 60° the values of M_{cr} in Case {1}is lower than that in Case {2}. The reason for this is that the corrugation depth for girders CW1-30 and CW2-30 is equal to 270mm and 155mm, respectively, while the corrugation depth for girders CW1-60 and CW2-60 equal 270mm and 450mm, respectively. As shown in Figures (14-g), (14-h), (14-i), and (14-j) the eigenvalues for the out of plane displacement (U_z) for girders CW1-30 and CW2-30 equal 417939 and 408316, respectively. While it equals 455723 and 498276, respectively for girders CW1-60 and CW2-60. This means that the girders with large corrugation depth have a moment of inertia about the minor axis more than that for the girders with small corrugation depth.

It can be noticed from Figure (13-e) that the variation in the values of the lateral-torsional buckling moment of (CW) due to the increase in the corrugation width from 130mm to 530mm is very small. This note is observed from the results of F.E. analysis and also the Euro-code equation. According to the logical concept, the increase in the corrugation width leads to a decrease in the number of corrugations along the girder span, and then the constraint of the compression against lateral buckling is decreased as it has already happened diminutively in the results of F.E. analysis.

From Figure (13-f), it can be noticed that the lateral-torsional buckling resistance of girders with rectangular (CW) is higher than that of girders with the triangular or trapezoidal (CW) with average ratios of 18% and 21%, respectively. In the case of rectangular corrugation, the restraint of the compression flange against lateral deformations is higher than that for the two other cases due to the effect of the existence of vertical panels. From the comparison between the results of F.E. analysis and Euro-code, it is found that the results of lateral-torsional buckling according to Euro-code equations are in good agreement with the results from the finite model analysis for triangular and trapezoidal webs.

On the other hand, for rectangular (CW) beam, the results of the finite element model are higher than that found according to Euro-code because the moment of inertia about the weak axis for the rectangular corrugated web as shown in Figure (15) where at section (A-A) it depends on web thickness while in section (B-B) depends on the corrugation depth. This is not taken into account in the calculation of the moment of inertia about the weak axis in the case of Euro-code equations. Therefore, the difference between the values of the lateral-torsional buckling moment for the rectangular corrugated web which is obtained from F.E. analysis and Euro-code is increased due to the increase in corrugation depth.

Additionally, it is found that the values of the lateral-torsional buckling moment, M_{cr}, calculated according to Euro-code for the triangular (CW) beam is lower than that for the beam with trapezoidal corrugation with the same corrugation depth because the warping constant; which is calculated according to either *Moon's* or *Zhang's* formula; is decreased due to the decrease in the corrugation width (***b***). While, the values of the lateral-torsional buckling moment, M_{cr}, which are obtained from finite element analysis; for the triangular (CW) beam are higher than that of the trapezoidal (CW) beam because of the restriction of the compression flange against lateral deformation in the case of the triangular corrugated web is higher.

After careful inspection of Figures (13) and (14), it can be observed that the results of finite element analysis and Euro-code equations, using the warping constant of Moon's or Zhang's formulas, are in good agreement.

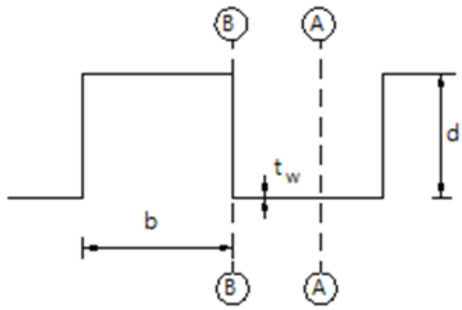


Figure (15) Corrugation profile for rectangular corrugated web girder

CONCLUSIONS

From an analytical study on the lateral-torsional buckling behavior of the steel girders with corrugated webs, it can be concluded the following:

1- The accuracy of *Zhang's and Moon's* formulas in the calculation of warping constant for corrugated web beams is much better than *Linder's* formula, especially when the corrugation angle is greater than 45°.

2- It is preferable not to use *Zhang's and Moon's* formulas only in case of the beam with the ratio between web height and flange width equal or less than 2.0 and then

Linder's formula is more accurate in the calculation of warping constant for corrugated web beams.

3- The accuracy of using Linder's formula for triangular corrugation web, because according to Linder's formula: the warping constant of the beam with triangular corrugated web equals to that for flat web beam, this is illogical.

4- *Linder's* formula cannot be used in the calculation of the warping constant for triangular corrugation web beams, because in this case, the corrugation width equals zero and then *Linder's* formula gives undefined values.

5- The equations of the lateral-torsional buckling moment for plate girders with flat webs in the *Euro-code* can be used for the corrugated web girders with a good agreement after using either *Moon's* or *Zhang's* formula in the calculation of warping constant.

6- The lateral-torsional buckling resistance for the corrugated web girders is increased according to the increase in web height and corrugation angle with noticeable rates while the increase is very small due to the increase in web thickness and corrugation width.

7- The strength of corrugated web plate girders against lateral torsional buckling increased due to the increase in corrugation depth.

8- Girders with rectangular webs have a lateral-torsional buckling resistance more than that of girders with triangular or trapezoidal webs with average ratios of 18% and 21%, respectively.

9- Euro-code equations of lateral-torsional buckling resistance give convergent results to that of the finite element models for girders with the triangular or trapezoidal web. But for girders with rectangular web, the results of the Euro-code are relatively far away than that for the finite element model.

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