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Unstationary Electromechanical Systems with modes, Closely similar to the sliding modes

By

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1. ABSTRACT:-

In the present paper, **characteristics** of transient processes, appearing in an unstationary armature controlled d-c electric drive system, are studied. The study here is based on realizing in the system, a mode closely similar to the sliding modes, in the contour of speed. The derived control algorithm is based on the fact that, there is no complete information about the system state variables.

A mathematical model for a ningth order electro-mechanical system is represented. The designed regulator insures the invariance property of the system against the internal disturbances, resulting from parameters variations.

Computing results, provide a new trend for the possibility of practical utilization of variable structure control methods in electro-mechanical systems.

2. INTRODUCTION:-

Analysis of the operational conditions in electro-mechanical systems, shows that there is a set of reasons, leading to the system parameters variations. Informations about the effect of these reasons, in most cases are undetermined. However, the speach can run about the parameters variation with time.

From another point of view, control methods based on the sliding modes theory(1), require synthesis of control algorithms. These control algorithms make the system invariant with respect to internal and external disturbances, which affect the system characteristics.

All studies in the field of variable structure systems (VSS) try to overcome the effect of these disturbances, on the characteristics of control systems.

3. Mathematical Model:-

Consider the 9 th order electro-mechanical system shown in Fig.(1). The system contains an armature controlled d.c. motor, fed through a thyristor SCR bridge. In deriving the dynamic equation of the thyristor, the two first terms only, in Pad series are taken into account. This means that the thyristor will repr-sent a pure delay nonlinearity. It is assummed also, that the system has no frictional nonlinearities.

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Then the shown electromechanical system could be described by the following set of linear differential equations, having variable co-efficients

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + H(t)F(t) \qquad (1\text{-a})$$
$$y_1(t) = C_1 x(t) \qquad (1\text{-b})$$

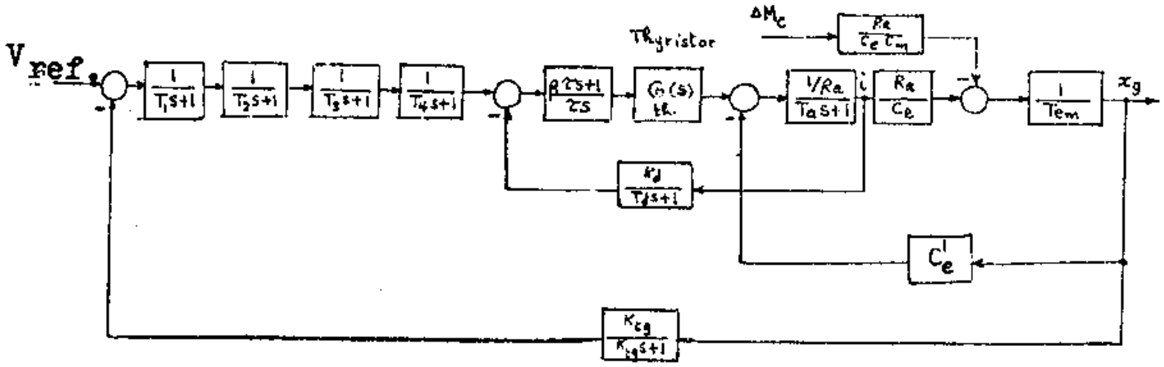

Fig. (1)

where:

$A(t)$ = (9x9) - unstationary matrix of the system;

$x(t) = \left\{x_i\right\}_{i=1}^{9}$ - state vectors of the controlled system;

$B(t)$ = (9x5) - unstationary control matrix;

$U(t) = \left\{U_i\right\}_{i=1}^{5}$ - control vectors;

$U(t)$ = (9x1) - unstationary matrix of the external disturbance;

$F(t)$ = external disturbance on the system;

$Y_1(t)$ = output of the system;

$C_1^1(t)$ = measuring matrix.

4. CONTROL ALGORITHM:

For realizing the sliding control algorithm, the following conditions are necessary:

1- The system must be completely observable. In the given case, this means that all the system states must be measured without increasing the order of the system, i.e. the measuring equation will have the form;-

$$Y(t) = C\,x(t) \qquad (2)$$

Where:

C = (9x9) matrix & $Y(t) = \left\{Y_i(t)\right\}_{i=1}^{9}$ = output vector.

2- All measurements are not exposed to the effect of noise.

3- The assumed dynamic equation could be realized physically, i.e. the motion of the system is assymptotically stable, under sliding mode existence and the variation of control signal, due to the compensator, consisting of a group of a series connected inertial elements with a number of feed-back paths, taken from the co-ordinates $Z_i(t)$ is given by

$$\tilde{B}(t).Z(t) = \left\{\tilde{A}(t)-G\right\}.x(t) \qquad (3)$$

Where:

$\tilde{B}(t) = (1x5)$ - unstationary matrix having elements from $B(t)$;
$\tilde{A}(t) = (1x8)$ - unstationary matrix having elements from $A(t)$;
$G(t) = (1x8)$ - matrix of motion on the surface of discontinuity;

$Z(t) = \left\{Z_i(t)\right\}_{i=1}^{4}$ - correction vector of the controlled plant.

It must be stated that $Z(t)$ & $x(t) \longrightarrow 0$ as $t \longrightarrow \infty$.

The control problem of the forced motion will be formulated for plants with variable parameters, such that, the controller has unstationary parameters.
Realization of the system invariance property requires that the following constraints must be realized:

i - The function related to the class C^5, must contain the derivatives of the control vector.

ii- The equation of motion of the controller, must contain also the derivatives of the control vector.

In general, the control vector at the entrance of summating point, colud be given by the following linear combination, having discontinuous co-efficients(2)

$$U_o(t) = \sum_{i=1}^{8} \alpha_i^x \left|x_i\right| \text{sign}\left\{Gx(t)\right\} + \sum_{i=1}^{4} \alpha_i^z \left|Z_i\right| \text{sign}\left\{Gx(t)\right\} + \sum_{i=1}^{5} \alpha_i^i \left|i\right| \text{sign}\left\{Gx(t)\right\} \qquad (4)$$

Where:

α_i^x, α_i^z & α_i^i are the co-efficients of the state vectors $x(t)$, correction co-ordinates $Z_i(t)$ and the internal co-ordinates $i'(t)$ respectively;

$i = \left\{i_i(t)\right\}_{i=1}^{5}$ - internal co-ordinates of current.

The above control vector, having discontinuous co-efficients, verifies the invariance property of the system against the external disturbances and forces the system - (representative point) - to move on the discontinuity surface.

The controller of the given electro-mechanical system, based on the control vector relation (4), is shown in Fig. (2).

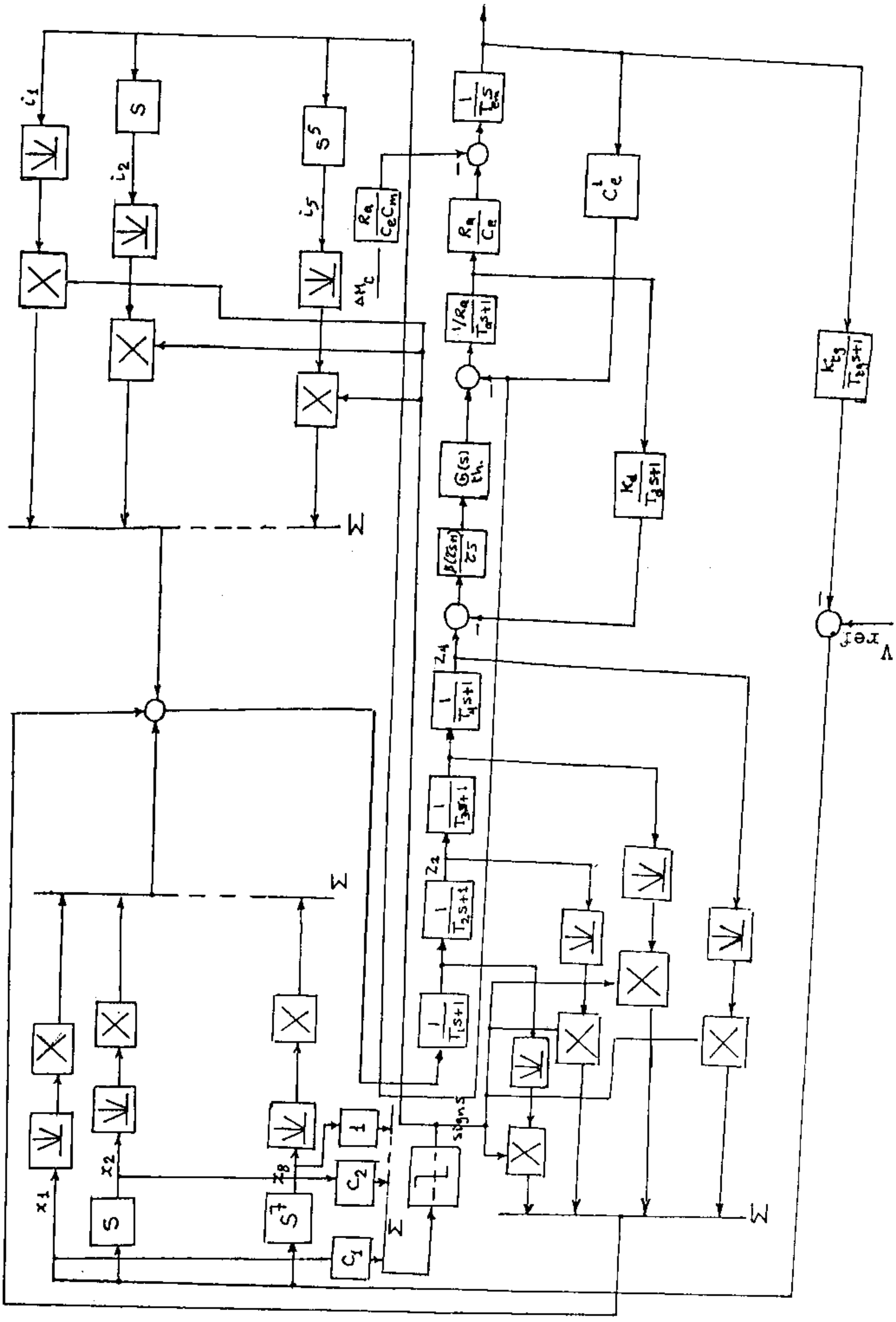

Fig. (2)

Practical realization of that controller , must take into account the derivatives existence problem. This problem leads to the attempt of obtaining the canonical form for the system dynamic equations. The specifications of real differential structures state that, there is no need to add new states without increasing the number of poles of the dynamic system, i.e. in practice, the motion of (VSS) takes place, with incomplete information about the system state.

Representing the given system by an ideal system in addition to a dynamic subsystem, the block diagram of the given system, could be represented in the canonical form as shown in fig.(3). The order of the ideal system is equal to the number of state variables, that could be measured.

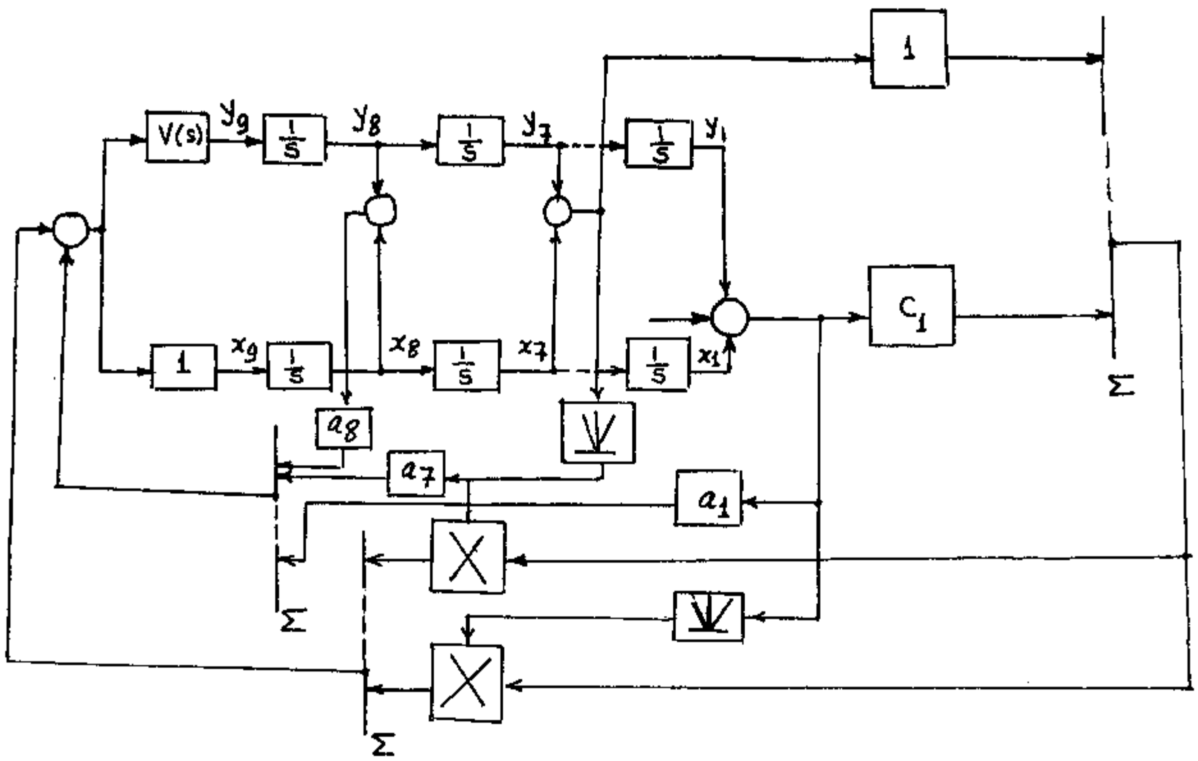

Fig. (3)

In this case, it is clear that the switching moment from one structure to another is given by the relation

$$Gx(t) + GY(t) = 0 \qquad (5)$$

Where:

$Y(t)$ = state vector of the dynamic subsystem at the switching moment.

In different moments of time, the previous relation could be true and hence, the following relations could be obtained at certain moments of time

$$Gx(t_1) = - GY(t_1) < 0 \qquad (6\text{-a})$$

$$Gx(t_1) = - GY(t_1) > 0 \qquad (6\text{-b})$$

If the following constraints, on the motion of the representative point were carried - out

$$G \lim_{\delta_i \to +0} x(t) > 0 \qquad (7\text{-a});$$

When $G\,x(t) > 0$, $\delta_i = t - t_i$

$$G \lim_{\delta_i \to -0} x(t) < 0 \qquad (7\text{-b});$$

$$G \lim_{\delta_i \to +0} x(t) < 0 \qquad (7\text{-c});$$

When $G\,x(t) < 0$, $\delta_i = t - t_i$

$$G \lim_{\delta_i \to -0} x(t) > 0 \qquad (7\text{-d}).$$

then, the motion of the representative point will be periodical, with respect to the switching surface of the ideal system and this motion is deseribed by the equality[1]

$$G\,x(t) = 0 \;,\quad \text{i.e.}\quad i = j + 1 \qquad (8)$$

Thus, if there is incomplete information about the state variables of the control system, a quasi-sliding motion[3] is generated on a discontinuous surface, known as the hyper - switching plane. In this case, the structure of the dynamic subsystem, can be determined separately. The envelope of this quasi-sliding motion determines the switching surface and the transfer of the representative point from state $x(t_i)$ to state $x(t_j)$ occurs at a time , such that

$$\delta = \delta_i + \delta_j$$

In practice, measuring of the output co-ordinates in electric drive systems is subjected to noise effect, only, under the high levels of this noise.
Hence; in real electric drive systems it is sufficient to use only the first derivative. Figure (4) represents the block diegram of the given system according to the previous analysis. The transfer function V(S) of the dynamic subsystem contains the inertial current detector, thyristor and the tachogenerator. The relation between V(S) and the transfer function of the dotted part U(S), is given by

$$V(S) = U(S) - 1 \qquad (9)$$

If the inertia of the current detector, thyristor & tachogenerator was neglected and taking the time constant of the current regulator (τ_c) equal to the armature circuit time constant (T_a), then

$$U(S) = 1 \qquad (10)$$

and the ideal electric drive system, could be represented by a group of blocks connected in series with a time constant (T_u), given by

$$T_u = \frac{R_a \; T_{em} \; T_a}{T_{em} \; K_d \; K_{th}\beta \; + \; \tau_c R_a} \qquad (11)$$

and an integrator having a time constant (T_i), given by

$$T_i = \frac{C_e (K_{th}\beta\,\tau_c T_{em} K_d + \tau_c R_a T_a + \tau_c R_a T_{em})(T_{em} K_d K_{th}\beta + \tau_c R_a)}{K_{th}\beta R_a (\tau_c T_{em} K_d \beta + \tau_c^2 R_a + R_a T_{em} T_a)} \qquad (12)$$

where:

T_{em} = electromechanical time constant;

k_d, k_{th} & β = transfer co-efficients of the current detector, thyristor & the current regulator, respectively;

R_a & T_a = armature circuit resistance, time constant respectively;

C_e' = motor contructional constant

Analysing the forced motion of the system, the transfer function due to an external disturbance, can be written as

$$\phi(S) = \phi_M(S) \frac{R_a T_e S}{C_e' (T_{em}S+1)} \qquad (13)$$

where:

$$\phi_M(S) = \frac{1}{C_M} \left[1 + \frac{K_d C_e' K}{(T_d S + 1)R_a (T_u S + 1) T_e S} \right] \quad ;$$

C_M = motor constructional constant.

Assumming that $\left| T_d(jw) \right| \ll 1$, $\left| T_{em}jw + 1 \right| \gg 1$, then

$$\phi(S) \simeq \frac{1}{C_M} \left[1 + \frac{K_d K}{(T_u S + 1)(T_{em} S + 1)} \right] \qquad (14)$$

The function $f_1(t)$, represents the system response to a step variation in the moment $M_c(t)$. This function is not referred to the class of continuous responses. Introducing an additional control vector in the form

$$U_i = \alpha^i \left| i \right| \text{ sign } (G\, x\, (t)) \qquad (15)$$

will not insure the independance of the system response, of the step variation as an external disturbance[2]. However, this new control vector, improves the invariant properties of the system. This improvement takes place when the compensation of continuous differentiated components of $f_1(t)$ is calculated.

These continuous differentiated components of $f_1(t)$ can be written in the form:

$$f_2(t) = \frac{K_d K}{C_M'} M_c + \Delta M_c \left(1 + \frac{T_{em}\, e^{-t/T_{em}} - T_u\, e^{-t/T_u}}{(T_u - T_{em})}\right) \qquad (16)$$

Thus, it is easy to write

$$\dot{f}_2(t) = \theta(t)\, f_2(t) \tag{17}$$

where

$$\theta(t) = \frac{e^{-t/T_u} - e^{-t/T_{em}}}{(T_u - T_{em})\left(1 + \frac{M_c}{\Delta M_c}\right) + T_{em}\, e^{-t/T_{em}} - T_u e^{-t/T_u}} \tag{18}$$

The function $\theta(t)$ is limited by the value

$$\operatorname{Sup} \theta(t) = \frac{\Delta M_c}{M_c} \cdot \frac{1}{T_u}\, e^{\, T_{em} \frac{\ln T_{em}/T_u}{T_u - T_{em}}}, \quad \Delta M_c \ll M_c \tag{19}$$

Thus, the ideal system, which is invariant with respect to the disturbance producing $f_2(t)$, can be described by the following equations:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \theta \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K}{T_u T_e} \end{bmatrix} U + \begin{bmatrix} \frac{\theta(t)}{T_e} + \frac{1}{T_u T_e} + \frac{K \operatorname{sign}(\delta i)}{T_u T_e} \end{bmatrix} i$$

Using the necessary and sufficient conditions, for realizing the sliding mode(1), the following computing relations for the required controller, can be written as:

$$\alpha^x \geq \operatorname{Sup}_t \left| \left(\frac{C_e\, T_{em} T_a}{K_{th}\,\beta}\right)\left(\frac{\Delta M_c}{M_c} \cdot \frac{T_{em} K_d K_{th} \beta + \tau_c R_a}{R_a\, T_{em}\, T_a} \cdot \exp. \ln \frac{\frac{T_{em} K_d K_{th} \beta + \tau_c R_a}{R_a\, T_a} +}{\frac{R_a\, T_a}{T_{em} K_d K_{th} \beta + \tau_c R_a} - 1}\right. \right|$$

$$\alpha^i \geq \operatorname{Sup}_t \left| \frac{1}{K} \left(\frac{\Delta M_c}{M_c} \exp. \ln \frac{\frac{T_{em} K_d K_{th} \beta + \tau_c R_a}{R_a\, T_a}}{\frac{R_a\, T_a}{T_{em} K_d K_{th} \beta + \tau_c R_a}} + 1 \right) \right|$$

The analysis of the invariant properties of ideal systems against the internal and external disturbance show that(4), the motion of the system is independent of the values of the main parameters. The dynamic subsystem has the following transfer function

$$V(S) = \left\{ \frac{\beta \frac{\tau_c S+1}{\tau_c S} \cdot \frac{K_{th}}{T_{th}S+1} \cdot \frac{(\tau^2 S^2 - 6s\tau + 12)}{(\tau^2 S^2 + 6s\tau + 12)}}{1+ \beta \frac{\tau_c S+1}{\tau_c S} \cdot \frac{K_{th} \cdot (\tau^2 S^2 - 6 s\tau + 12)}{(T_{th}S+1)(\tau^2 S^2 + 6s\tau + 12)}} \right\} \cdot \left\{ \frac{\frac{1}{R_a(T_a S+1)} \cdot \frac{(T_u S+1)\, T_e S}{C_e(T_{em}S+1)\, K} \cdot \frac{K_{tg}}{(T_{tg}S+1)}}{\frac{1}{R_a(T_a S+1)} \cdot \frac{T_{em}S}{(T_{em}S+1)} \cdot \frac{K_d}{(T_d S + 1)}} \right\}$$

5. COMPUTATIONAL RESULTS:

To describe the motion of the above dynamic subsystem, it is necessary to solve(1) two sets of differential equations instantaneously. The difficulties of this analytical solution, lead directly to use the computer. Hence, for a model of the given system, the relation between the parameters of the oscillating system & the factors of invariance under the variation of τ, T_{em}, T_a, K_d, τ_c, T_d & T_{tg} is derived using the computational relations that were previously mentioned.

It has been shown that any internal disturbance, due to variation of one or more of the previously mentioned parameters, has no effect on the character of the system motion. However, the existence of a dynamic subsystem, causes oscillations with a frequency η and amplitude ϵ. The results also show that, the degree of effect on the main system due to the dynamic subsystem, depends mainly on the value of τ, while the effects due to the parameters T_{em}, T_a, K_d & τ_c are relatively small. The time of transient process (t), is not affected by the variation of these parameters. Figure (5) illustrates the relation between t, η & ϵ with τ. Figures(6), (7) & (8) illustrate the shape of transient processes for different values of τ. In these figures, the transients of the output ω, internal co-ordinate i & their representation on the phase plane are illustrated. The forced motion of the system could be explained through Figures (9) & (10), which are taken from the oscilligraph.

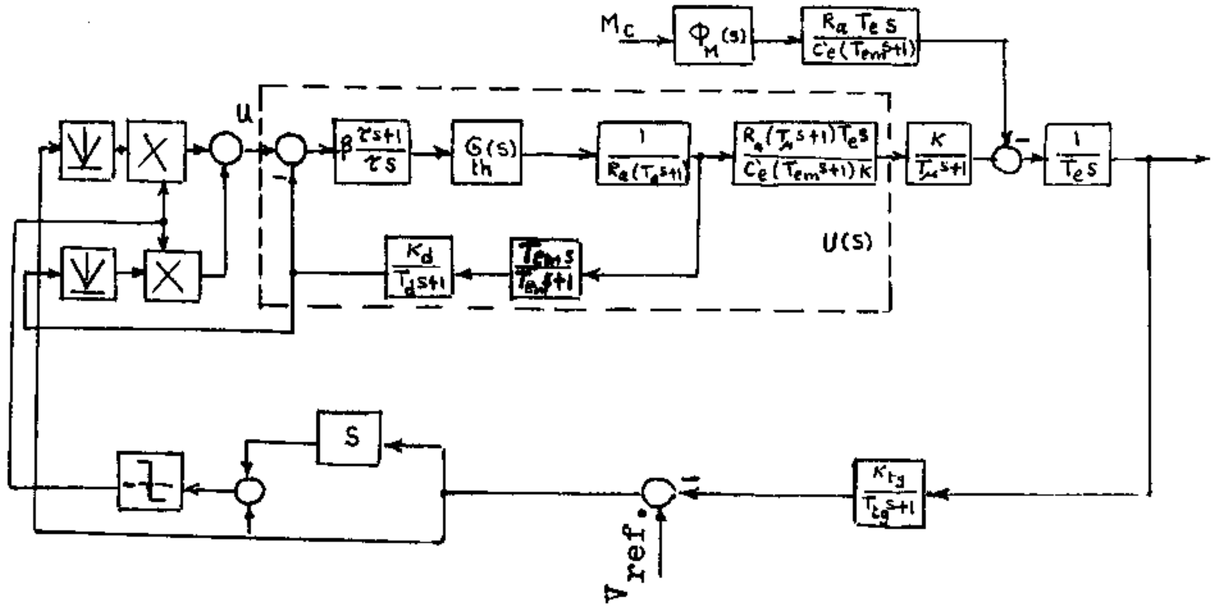

Fig. (4)

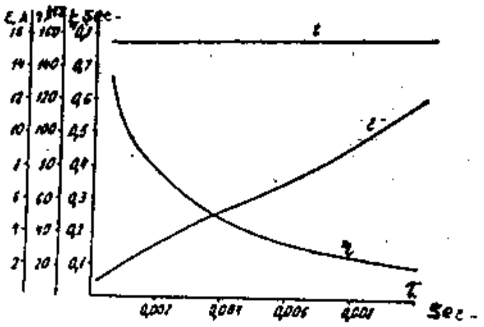

Fig. (5)

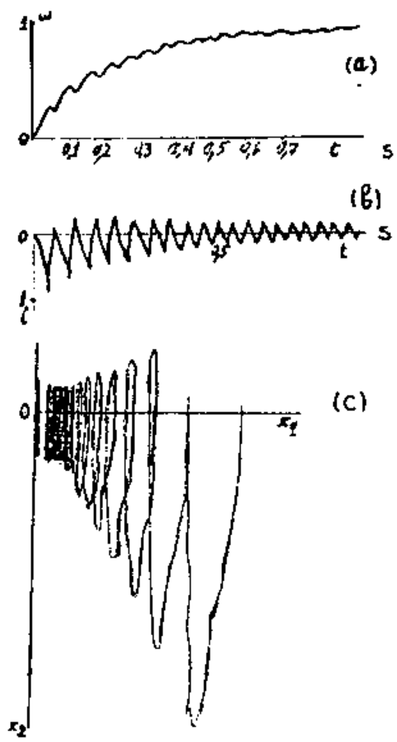

Fig. (8)

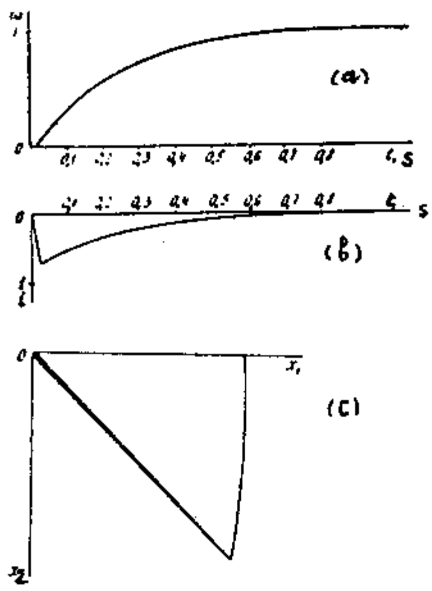

Fig. (6)

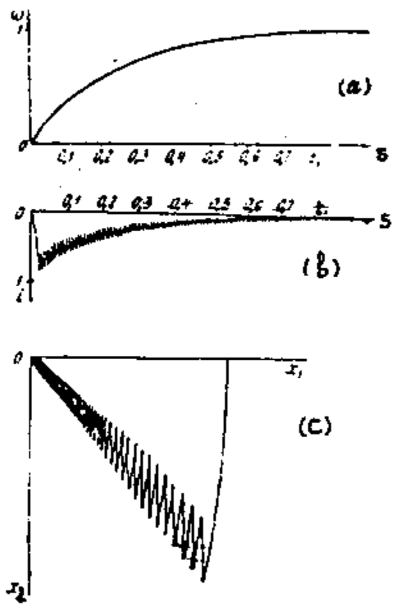

Fig. (7)

Transient Processes: a) Speed b) Current c) Phase plane

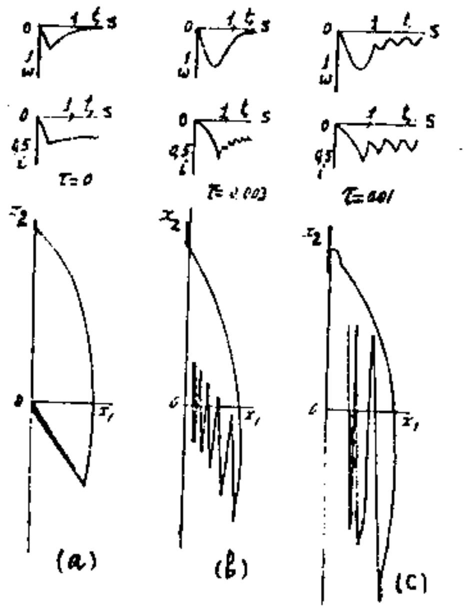

Fig. (9)

Effect of disturbances

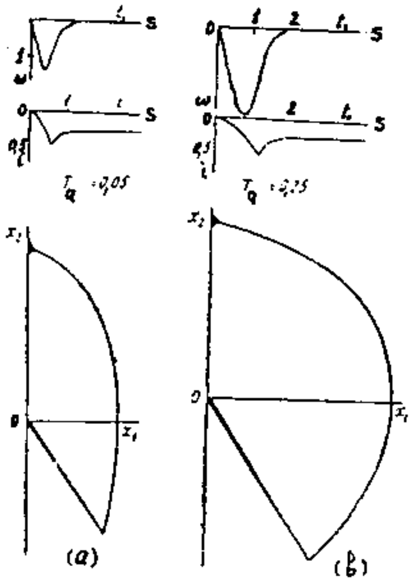

Fig. (10)

6. CONCLUSIONS:-

In the present paper, the following main conclusions are obtained :

1- The small inertia in electric drives has a very small effect on the character of motion of electromechanical systems, if a sliding mode was initiated inside the system. This requires a simplified control algorithm in comparison with those suggested in (1).

2- The suggested control algorithm, makes the system invariant against any internal disturbance.

3- Due to a step external disturbance, forced oscillations take place & this requires a further research.

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