THE NODAL LINE FINITE DIFFERENCE METHOD WITH ITERATION PROCEDURE IN THE ANALYSIS OF ELASTIC PLATES IN BENDING

BY

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INTRODUCTION

The rigorous analysis of plate bending problems is limited to relatively simple plate geometry, load and boundary conditions. If these conditions are more complex, the analysis becomes increasingly tedious and even impossible. In such cases numerical methods are the available approaches that can be used. The economy of the solution is an important factor to be consider in selecting the numerical method to be used in the analysis. While the use of finite difference technique is very simple and the method is quite general,it is characterized by slow convergence. Furthermore, extremely fine mesh, and the resulting large number of simultaneous equations may create round-off errors in computer solutions and adversely affect the accuracy and economy of the method. The finite element method as the most powerful and versatile tool of solution in structural analysis, is now well-known and established. This numerical method has the drawback that it involves large number of simultaneous algebric equations which has to be solved.

Therefore for certain class of problems, it is worthwhile to develop simplified semi analytical methods, of which the finite strip is one. The finite strip method was introduced by CHEUNG [1] who used a trigonometric series as a basic function in the analysis of elastic plates with two opposite simply supported ends. These trigonometric series possess the valuable properties of orthogonality that lead to the uncoupling of the static equilibrium equations. Basic functions other than trigonometric series, are used by CHEUNG [2] to analyze plates with two opposite edge conditions other than simply supported. Unfortunatly, with these basic functions the uncoupling property mentioned before cannot occur. The auther [4,5] developed a simplified iteration solution technique, in order to overcome the coupling property which occurs when using basic functions other than trigonometric series.

A newly developed semi analytical method named " The nodal line finite difference method "(N.L.F.D) was introduced by the auther [6] in the analysis of elastic plates with two opposite simply supported ends. The basic functions which fitted the boundary conditions in one direction of the plate are used in this method at a mesh of nodal lines in conjunction with simple finite difference approach in the other direction. This method is similar

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to that of the finite strip method since the same basic functions are used in each of them to express the displacement variation along the nodal lines. These basic functions, which are derived from the solution of beam vibration differential equation, have been worked out explicitly by VLAZOV [3] for the various end conditions.

The object of the present work is to extend the application of the nodal line finite difference method developed by the auther [6] to include end conditions other than simply supported ends. In order to overcome the coupling property of the equilibrium difference equations, a simplified iteration solution technique has been introduced. The basic idea of this iteration solution is similar to that used by the auther [4,5] in the analysis of plates by finite strip method. The proposed iteration technique emphasises the role of the dominant terms of the equilibrium nodal line finite difference equation. Accordingly each term of the basic function can be solved individually as in the case of trigonometric series.

The first iteration solution considers the original load vector and results in good approximate values for the unknown nodal line parameters. These parameters can be utilized with the non dominant terms of the equilibrium nodal line finite difference equation to obtain the modified load vector of the plate. This modified load vector can be used in the second iteration solution to give more improved values for the unknown nodal line parameters. The same procedure is repeated in the subsequent iteration solution until the required accuracy is obtained. The iteration procedure presented herein is applied for the basic function of the case of clamped - clamped edge condition. The results obtained are in very close agreement with those of the same conditions worked out by TIMOSHENKO [7]

METHOD OF ANALYSIS

a - Nodal line finite difference equation

In applying the nodal line finite difference method for the analysis of plate bending problems, we first divide the plate into a mesh of fictitious nodal lines as shown in Fig.1. According to the boundary conditions of the two opposite ends perpendicular to the nodal lines, a basic function expresses the displacement variation along the nodal lines is to be chosen. The displacement function at each nodal line is expressed as a summation of the chosen basic function terms multiplied by a single variable functions as a nodal parameters. Therefore the two dimensional plate bending problems are reduced to one dimensional problems.

The bending problems of constant thickness isotropic plates have been represented by the differential equation derived from the equilibrium condition of internal and external forces. This differential equation has the following form

$$w^{\cdot\cdot\cdot\cdot} + 2\, w^{\cdot\cdot\prime\prime} + w^{\prime\prime\prime\prime} = \frac{q}{B} \qquad (1)$$

where $(\ )^{\cdot} = \frac{\partial}{\partial x}$, $(\ )^{\prime} = \frac{\partial}{\partial y}$ and

$B = \frac{E\ t^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate

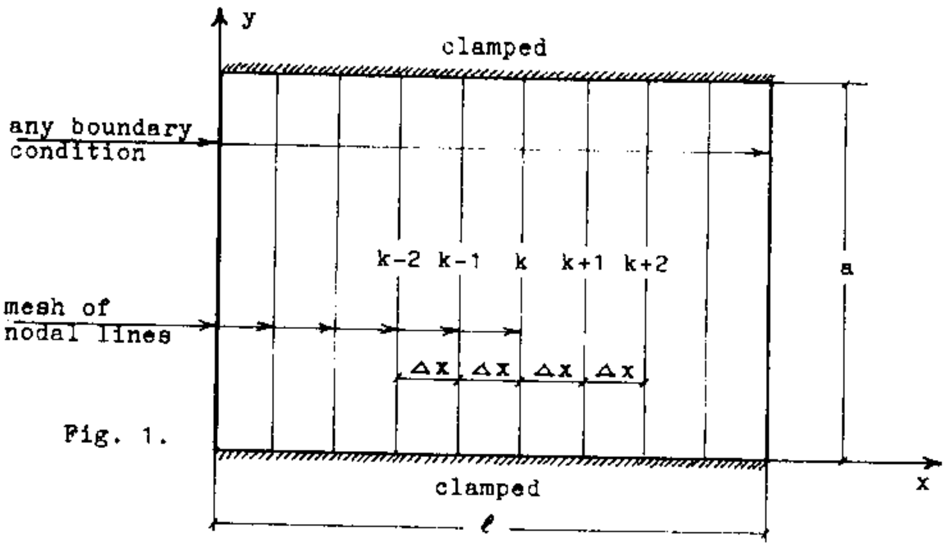


Fig. 1.

The displacement function at any nodal line labelled k (Fig.1) is proposed in a series form as follows

$$w_k = \sum_{m=1}^{r} f_{m,k}(x) .\ Y_m(y) \qquad (2)$$

For the case of plates with two opposite clamped ends, the basic function fitting the boundary conditions is expressed in the form

$$Y_m = \sin k_m y - \sinh k_m y - \alpha_m(\cos k_m y - \cosh k_m y) \qquad (3)$$

where $k_m = \frac{\mu_m}{a}$, $\alpha_m = \frac{\sin\mu_m - \sinh\mu_m}{\cos\mu_m - \cosh\mu_m}$

$$\mu_m = 4.73,\ 7.8532,\ 10.996,\ \ldots \frac{2m+1}{2}\pi$$

The second and fourth derivatives of the basic function in the y direction can be written as

$$\left.\begin{array}{l} Y_m'' = k_m^2\ \overset{*}{Y}_m'' \quad , \quad Y_m'''' = k_m^4\ Y_m \\ \text{where} \\ \overset{*}{Y}_m'' = -\sin k_m y - \sinh k_m y + \alpha_m(\cos k_m y + \cosh k_m y) \end{array}\right\} \qquad (4)$$

For the purpose of iteration procedure presented here, the second derivative function $\overset{*}{Y}''_m$ must be resolved into a series of the basic function Y_m

$$\overset{*}{Y}''_m = \sum_{n=1}^{r} \beta_{mn} Y_n \tag{5}$$

where

$$\left.\begin{aligned} &\text{for } m = n \qquad \beta_{mm} = \left(-\frac{1}{\alpha_m^2} + \frac{2}{\alpha_m \mu_m} \right) \\ &\text{for } m = n \qquad \beta_{mn} = [1+(-1)^{m+n}] \frac{4\mu_n^2}{\alpha_n^2(\mu_m^4-\mu_n^4)} (\mu_m\alpha_n - \mu_n\alpha_m) \end{aligned}\right\} \tag{6}$$

Resolving the load into series similar to the basic function and substituting equations (3) and (4) into equation (1) at any nodal line labelled k leads to this relation

$$\sum_{m=1}^{r} [f''''_{m,k} Y_m + 2 k_m^2 f''_{m,k} \overset{*}{Y}''_m + k_m^4 f_{m,k} Y_m] = \frac{1}{B} \sum_{m=1}^{r} q_{m,k} Y_m \tag{7}$$

Considering equations (5) and (6), equation (7) can be written as

$$\sum_{m=1}^{r} [f''''_{m,k} Y_m + 2 k_m^2 f''_{m,k} (\sum_{n=1}^{r} \beta_{mn} Y_n) + k_m^4 f_{m,k} Y_m] = \frac{1}{B} \sum_{m=1}^{r} q_{m,k} Y_m \tag{8}$$

After rearrangement of the terms, equation (8) takes the form

$$\sum_{m=1}^{r} [f''''_{m,k} + 2(\sum_{n=1}^{r} k_n^2 \beta_{nm} f''_{n,k}) + k_m^4 f_{m,k}] \; Y_m = \frac{1}{B} \sum_{m=1}^{r} q_{m,k} Y_m \tag{9}$$

For each term of the basic function, equation (9) takes the form

$$[f''''_{m,k} + 2(\sum_{n=1}^{r} k_n^2 \beta_{nm} f''_{n,k} + k_m^4 f_{m,k}] = \frac{1}{B} q_{m,k} \tag{10}$$

Equation (10) can be rewritten in the following form

$$[(f''''_{m,k} + 2 k_m^2 \beta_{mm} f''_{m,k} + k_m^4 f_{m,k}) + 2(\sum_{n=1}^{r} k_n^2 \beta_{nm} f''_{n,k} - k_m^2 \beta_{mm} f''_{m,k})] = \frac{1}{B} q_{m,k} \tag{11}$$

By applying the central finite difference technique in the direction perpendicular to the nodal lines, equation (11) can be written as

$$\frac{1}{\Delta x^4} [\{f_{m,k-2} + C_m^1 f_{m,k-1} + C_m^2 f_{m,k} + C_m^1 f_{m,k+1} + f_{m,k+2}\} + 2\{\sum_{n=1}^{r} \psi_n^2 \beta_{nm} (f_{n,k-1} - 2f_{n,k} + f_{n,k+1}) + \psi_m^2 \beta_{mm} (f_{m,k-1} - 2f_{m,k} + f_{m,k+1})\}] = \frac{1}{B} q_{m,k} \tag{12}$$

where $C_m^1 = -(4-2\beta_{mm}\psi_m^2)$

$C_m^2 = (6-4\beta_{mm}\psi_m^2+\psi_m^4)$

$\psi_m = \frac{k_m a}{\lambda}$, $\lambda = \frac{a}{\Delta x}$

$\Delta x = \frac{a}{\lambda}$ is the constant interval between the nodal lines

After separation of the dominant and the non-dominant terms, equation (12) takes the form

$$[f_{m,k-2}+C_m^1 f_{m,k-1}+C_m^2 f_{m,k}+C_m^1 f_{m,k+1}+f_{m,k+2}] = \frac{a^4}{B\lambda^4} q_{m,k} - \bar{q}_{m,k} \quad (13)$$

Where $\bar{q}_{m,k}$ represents the non-dominant terms of the central nodal line finite difference equation

$$\bar{q}_{m,k} = 2[\sum_{n=1}^{r} \psi_n^2 \beta_{nm}(f_{n,k-1}-2f_{n,k}+f_{n,k+1})$$
$$+\psi_m^2 \beta_{mm}(f_{m,k-1}-2f_{m,k}+f_{m,k+1})] \quad (14)$$

The general form of equation (13) can be written as

$$[1 \quad C_m^1 \quad C_m^2 \quad C_m^1 \quad 1]\{f_{m,k-2} \quad f_{m,k-1} \quad f_{m,k} \quad f_{m,k+1} \quad f_{m,k+2}\}$$
$$= \frac{a^4}{B}\left(\frac{1}{\lambda^4} q_{m,k} - \frac{B}{a^4} \bar{q}_{m,k}\right) \quad (15)$$

Equation (15) represents the central nodal line finite difference equation in matrix form

In the first iteration solution, the non-dominant term $\bar{q}_{m,k}$ is neglected. Accordingly, the central nodal line finite difference equation can be written as

$$[1 \quad C_m^1 \quad C_m^2 \quad C_m^1 \quad 1]\{f_{m,k-2} \quad f_{m,k-1} \quad f_{m,k} \quad f_{m,k+1} \quad f_{m,k+2}\}$$
$$= \frac{a^4}{B\lambda^4} q_{m,k} \quad (16)$$

Application of equation (16) at each nodal line of the plate gives uncoupled system of simultaneous algebric equations, which can be written in the matrix form as follows

$$[S]_m \{f\}_m = \{P\}_m \quad (17)$$

where $[S]_m$ is a square band matrix,

$\{f\}_m$ is the vector of the unknown nodal line parameters,

and $\{P\}_m$ is the original load vector

Solution of equation (17) results in good approximate values for the unknown nodal line parameters $\{f\}_m$ of each term of the basic function. These parameters can be utilized to determine the non dominant term $\bar{q}_{m,k}$ at each nodal line and to obtain the modified load vector. The modified load vector can be used in the second iteration solution to give more improved values for the unknown nodal parameters. The same procedure can be repeated in the subsequent iteration solution. The general form of equation (17) takes the form

$$[S]_m \{f\}_m = \{\bar{P}\}_m \tag{18}$$

where $\{\bar{P}\}_m$ is the modified load vector

Due to the uncoupling property of the resulting difference equations, each term of the basic function can be solved individually such as that in the case of trigonometric series. Moreover, the resulting square matrix $[S]_m$ has the nice property of banded matrices with small band width equal to 5. This matrix can be stored in a reduced rectangular matrix of dimension (Mx5). Where M is the number of the fictitious nodal lines. Thus requiring small core storage and short computer time for execution.

b - Internal forces

The internal forces per unit length at any point of an elastic isotropic plate are connected to the displacement through the following relations

$$\left.\begin{aligned} M_x &= -B\,(w^{\cdot\cdot} + \nu w'') \\ M_y &= -B\,(w'' + \nu w^{\cdot\cdot}) \\ M_{xy} &= -M_{yx} = B\,(1-\nu)\,w^{\cdot\prime} \\ Q_x &= -B\,(w^{\cdot\cdot\cdot} + w^{\cdot\prime\prime}) \\ Q_y &= -B\,(w^{\cdot\cdot\prime} + w^{\prime\prime\prime}) \end{aligned}\right\} \tag{19}$$

By applying the central nodal line finite difference technique, the internal forces at any nodal line labelled k can be written as the following

$$\left.\begin{aligned}
M_{x,k} &= -\frac{B\lambda^2}{a^2}\sum_{m=1}^{r}\left[Y_m f_{m,k-1} - (2Y_m - \nu\psi_m^2 \overset{*}{Y}{}_m'') f_{m,k} + Y_m f_{m,k+1}\right] \\
M_{y,k} &= -\frac{B\lambda^2}{a^2}\sum_{m=1}^{r}\left[\nu Y_m f_{m,k-1} - (2\nu Y_m - \psi_m^2 \overset{*}{Y}{}_m'') f_{m,k} + \nu Y_m f_{m,k+1}\right] \\
M_{xy,k} &= -M_{yx,k} = \frac{B\lambda^2}{2a}(1-\nu)\sum_{m=1}^{r}\psi_m \overset{*}{Y}{}_m' \left[-f_{m,k-1} + f_{m,k+1}\right] \\
Q_{x,k} &= -\frac{B\lambda^3}{2a^3}\sum_{m=1}^{r}\left[-Y_m f_{m,k-2} + (2Y_m - \psi_m^2 \overset{*}{Y}{}_m'') f_{m,k-1}\right. \\
&\qquad \left. - (2Y_m - \psi_m^2 \overset{*}{Y}{}_m'') f_{m,k+1} + Y_m f_{m,k+2}\right] \\
Q_{y,k} &= -\frac{B\lambda^3}{a^3}\sum_{m=1}^{r}\psi_m\left[\overset{*}{Y}{}_m' f_{m,k-1} + (\psi_m^2 \overset{*}{Y}{}_m''' - 2\overset{*}{Y}{}_m') f_{m,k} + \overset{*}{Y}{}_m' f_{m,k+1}\right]
\end{aligned}\right\} \quad (20)$$

where $\overset{*}{Y}{}_m' = \frac{1}{k_m} Y_m'$, $\overset{*}{Y}{}_m'' = \frac{1}{k_m^2} Y_m''$, $\overset{*}{Y}{}_m''' = \frac{1}{k_m^3} Y_m'''$

c - Boundary conditions

Solution of the governing differential equation of the plate bending problems by the nodal line finite difference method requires propre finite difference representation of the boundary conditions. Consequently, we replace the derivatives in the expressions of various boundary conditions with the pertinent finite difference expressions. When central finite difference technique is used at the edge nodal line, the introduction of two fictitious nodal lines outside of the plate is required. According to the prescribed boundary conditions at the edge nodal line, the parameters of the exterior nodal lines have to be expressed in terms of parameters of the edge and the two adjacent interior nodal lines. For each term of the basic function, the parameters of the exterior nodal lines are connected to those of the edge and the interior nodal lines through the following relations

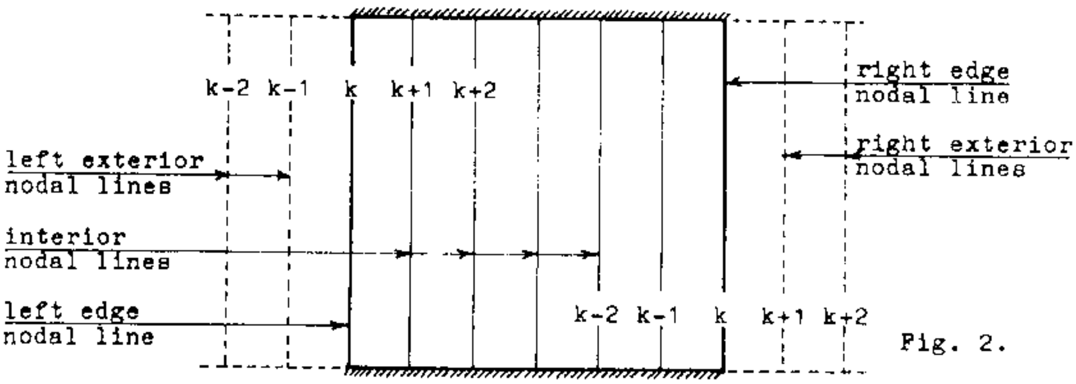
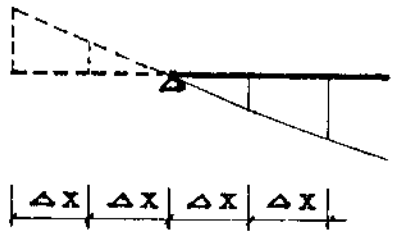


Fig. 2.

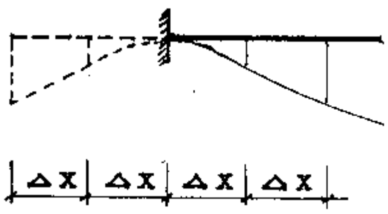
1 - Simply supported [$w_k = 0$, $(w^{\cdot\cdot} + \nu w'')_k = 0$]

$$\left.\begin{aligned} f_{m,k} &= 0 \\ f_{m,k-1} &= - f_{m,k+1} \\ f_{m,k-2} &= - f_{m,k+2} \end{aligned}\right\} \qquad (21)$$



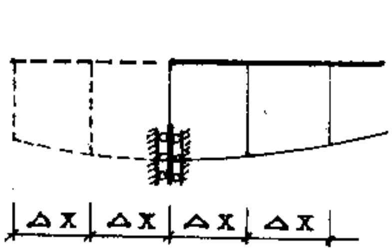
2 - Clamped edge [$w_k = 0$, $w^{\cdot}_k = 0$]

$$\left.\begin{aligned} f_{m,k} &= 0 \\ f_{m,k-1} &= f_{m,k+1} \\ f_{m,k-2} &= f_{m,k+2} \end{aligned}\right\} \qquad (22)$$

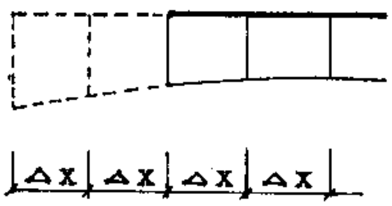


3 - Guided edge [$w^{\cdot}_k = 0$, $(w^{\cdot\cdot\cdot} + w^{\cdot\prime\prime})_k = 0$]

$$\left.\begin{aligned} f_{m,k} &\neq 0 \\ f_{m,k-1} &= f_{m,k+1} \\ f_{m,k-2} &= f_{m,k+2} \end{aligned}\right\} \qquad (23)$$



4 - Free edge [$(w^{\cdot\cdot} + \nu w'')_k = 0$, $\{ w^{\cdot\cdot\cdot} + (2-\nu)\, w^{\cdot\prime\prime} \}_k = 0$]



The direct formulation of a free boundary condition when using a trigonometric series as a basic function has been easly achieved by the author [6]. The treatment of the free boundary condition in the case of basic functions other than trigonometric series requires further investigation not implemented in the present work.

NUMERICAL EXAMPLES

Two main factors have to be considered when dealing with any iterative procedure. The first of which is the convergence criteria, the second is the accuracy. In order to demonstrate the convergence and the accuracy of the proposed technique presented here, some selected plate bending problems have been analized. The study of convergence is carried out for square plates subjected to uniform distributed load and having the boundary conditions illustrated in Figs.3-a and 3-b. The study deals with the effect of both number of terms of the basic function and the mesh interval Δx on the convergence of the proposed method. The results obtained are summarized in tables 1 and 2

To check the accuracy of the proposed method, analysis of rectangular plates with different ratios of rectangularity has been achieved. As far as the loading is concerned, uniform distributed and triangular load are considered. The results obtained from the fourth iteration solution are presented in tables 3, 4, 5, 6 and 7. The analysis is carried out for seven terms of the basic function and with Δx equal to $\ell/40$

Due to symmetry of loading in the nodal lines direction, only odd terms of the basic function contribute to the results of the above mentioned examples

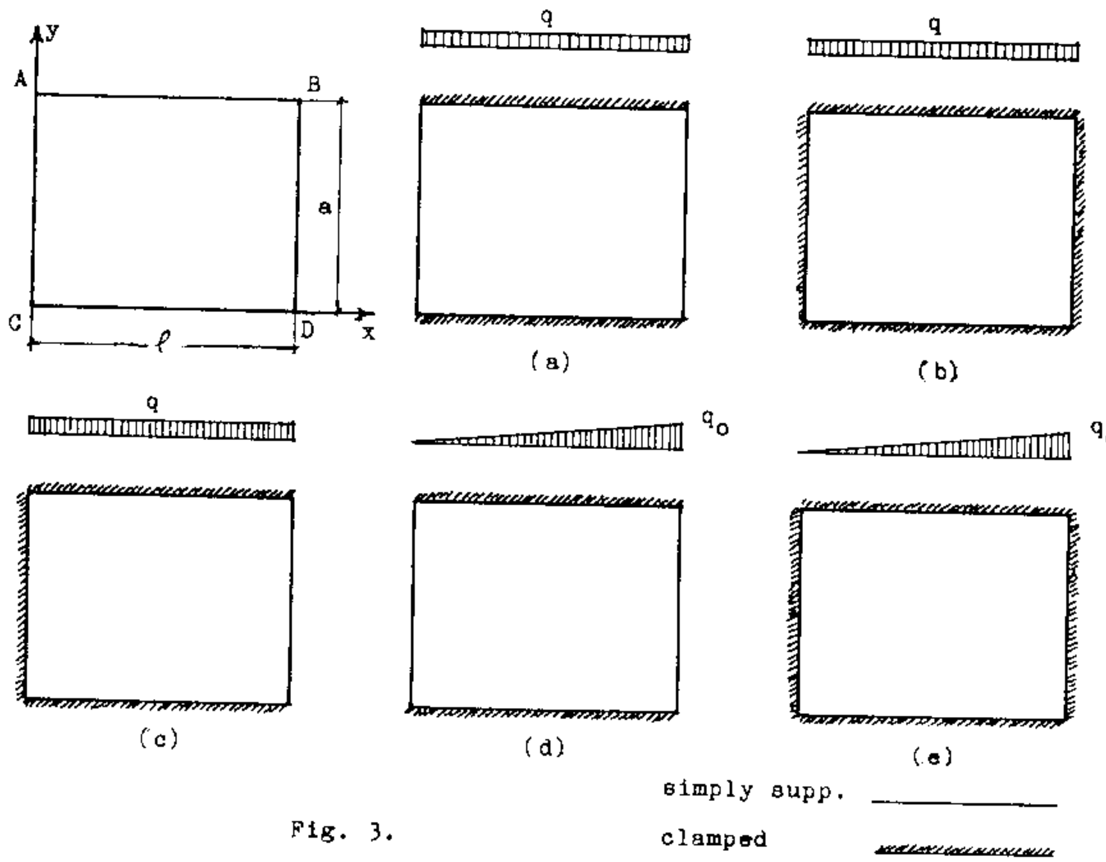


Fig. 3.

Table 1. Study of convergence. Square plate clamped on sides AB and CD. simply supported on the two other sides subjected to uniform distributed load of intensity q. Fig. 3-a $\nu = 0.3$

Δx	No of Terms	Iteration Order	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB
$\ell/20$	4	1st Iter.	19.1544	2.4743	3.4533	-6.2942
		2nd "	19.0887	2.4215	3.2859	-6.7920
		3 d "	19.1432	2.4265	3.2957	-6.8126
		4th "	19.1434	2.4264	3.2954	-6.8134
	5	1st Iter.	19.1568	2.4806	3.4746	-6.3243
		2nd "	19.0933	2.4328	3.3238	-6.8458
		3 d "	19.1481	2.4379	3.3339	-6.8669
		4th "	19.1383	2.4377	3.3336	-6.8676
	6	1st Iter.	19.1558	2.4771	3.4627	-6.3412
		2nd "	19.0918	2.4264	3.3023	-6.8765
		3 d "	19.1466	2.4314	3.3122	-6.8978
		4th "	19.1468	2.4312	3.3119	-6.8984
	7	1st Iter.	19.1563	2.4793	3.4703	-6.3517
		2nd "	19.0926	2.4305	3.3162	-6.8957
		3 d "	19.1475	2.4356	3.3262	-6.9170
		4th "	19.1477	2.4354	3.3259	-6.9176
	8	1st Iter.	19.1563	2.4793	3.4703	-6.3587
		2nd "	19.0926	2.4305	3.3162	-6.9084
		3 d "	19.1475	2.4356	3.3262	-6.9298
		4th "	19.1477	2.4354	3.3259	-6.9303
	9	1st Iter.	19.1563	2.4793	3.4703	-6.3635
		2nd "	19.0926	2.4305	3.3162	-6.9173
		3 d "	19.1475	2.4356	3.3263	-6.9388
		4th "	19.1477	2.4354	3.3259	-6.9392
$\ell/40$	4	1st Iter.	19.1728	2.4782	3.4571	-6.3001
		2nd "	19.1070	2.4253	3.2892	-6.7991
		3 d "	19.1617	2.4304	3.2990	-6.8198
		4th "	19.1619	2.4302	3.2987	-6.8206
	5	1st Iter.	19.1752	2.4846	3.4784	-6.3302
		2nd "	19.1116	2.4367	3.3272	-6.8529
		3 d "	19.1666	2.4418	3.3373	-6.8740
		4th "	19.1668	2.4416	3.3370	-6.8747
	6	1st Iter.	19.1743	2.4810	3.4664	-6.3471
		2nd "	19.1100	2.4302	3.3056	-6.8836
		3 d "	19.1651	2.4353	3.3156	-6.9049
		4th "	19.1653	2.4351	3.3153	-6.9055
	7	1st Iter.	19.1747	2.4833	3.4741	-6.3576
		2nd "	19.1109	2.4344	3.3195	-6.9028
		3 d "	19.1660	2.4395	3.3296	-6.9242
		4th "	19.1662	2.4393	3.3293	-6.9247
	8	1st Iter.	19.1747	2.4833	3.4741	-6.3645
		2nd "	19.1109	2.4344	3.3195	-6.9155
		3 d "	19.1660	2.4395	3.3296	-6.9370
		4th "	19.1662	2.4393	3.3293	-6.9375
	9	1st Iter.	19.1747	2.4833	3.4741	-6.3693
		2nd "	19.1109	2.4344	3.3195	-6.9244
		3 d "	19.1660	2.4395	3.3296	-6.9459
		4th "	19.1662	2.4393	3.3293	-6.9464
Exact [7]			19.20	2.44	3.32	-6.97
Multiplier			$10^{-4}.q\ a^4/B$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$

Table 2. Study of convergence. Square plate clamped on four sides subjected to uniform distributed load of intensity q. Fig. 3-b $\nu = 0.3$

Δx	No of Terms	Iteration Order	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB	M_x at Middle of AC
$\frac{\ell}{20}$	4	1st Iter.	12.8171	2.3278	2.4274	-4.4968	-4.9524
		2nd "	12.7378	2.2611	2.2729	-4.9542	-5.0314
		3rd "	12.7826	2.2730	2.2829	-4.9829	-5.0342
		4th "	12.7831	2.2728	2.2822	-4.9858	-5.0349
	5	1st Iter.	12.8195	2.3342	2.4486	-4.5269	-4.9626
		2nd "	12.7424	2.2732	2.3125	-5.0101	-5.0296
		3rd "	12.7876	2.2857	2.3244	-5.0421	-5.0302
		4th "	12.7883	2.2856	2.3239	-5.0453	-5.0307
	6	1st Iter.	12.8186	2.3306	2.4367	-4.5438	-4.9580
		2nd "	12.7407	2.2663	2.2898	-5.0422	-5.0312
		3rd "	12.7859	2.2785	2.3005	-5.0762	-5.0333
		4th "	12.7866	2.2783	2.2999	-5.0796	-5.0339
	7	1st Iter.	12.8190	2.3329	2.4443	-4.5543	-4.9604
		2nd "	12.7415	2.2707	2.3045	-5.0624	-5.0301
		3rd "	12.7868	2.2832	2.3160	-5.0976	-5.0313
		4th "	12.7875	2.2830	2.3155	-5.1011	-5.0318
	8	1st Iter.	12.8190	2.3329	2.4443	-4.5612	-4.9604
		2nd "	12.7415	2.2707	2.3045	-5.0758	-5.0301
		3rd "	12.7869	2.2832	2.3160	-5.1119	-5.0313
		4th "	12.7876	2.2830	2.3155	-5.1155	-5.0318
	9	1st Iter.	12.8190	2.3329	2.4443	-4.5660	-4.9604
		2nd "	12.7415	2.2707	2.3045	-5.0852	-5.0301
		3rd "	12.7869	2.2832	2.3160	-5.1219	-5.0313
		4th "	12.7876	2.2830	2.3155	-5.1255	-5.0318
$\frac{\ell}{40}$	4	1st Iter.	12.7167	2.3350	2.4140	-4.4684	-5.0131
		2nd "	12.6382	2.2680	2.2590	-4.9286	-5.1109
		3rd "	12.6827	2.2799	2.2690	-4.9576	-5.1140
		4th "	12.6832	2.2798	2.2683	-4.9605	-5.1147
	5	1st Iter.	12.7191	2.3414	2.4352	-4.4985	-5.0298
		2nd "	12.6427	2.2801	2.2988	-4.9848	-5.1018
		3rd "	12.6878	2.2927	2.3107	-5.0170	-5.1024
		4th "	12.6884	2.2926	2.3102	-5.0203	-5.1029
	6	1st Iter.	12.7182	2.3378	2.4233	-4.5154	-5.0213
		2nd "	12.6410	2.2732	2.2760	-5.0170	-5.1089
		3rd "	12.6861	2.2855	2.2867	-5.0513	-5.1113
		4th "	12.6867	2.2853	2.2861	-5.0547	-5.1119
	7	1st Iter.	12.7186	2.3401	2.4309	-4.5259	-5.0262
		2nd "	12.6418	2.2777	2.2907	-5.0372	-5.1037
		3rd "	12.6870	2.2902	2.3023	-5.0728	-5.1049
		4th "	12.6876	2.2901	2.3017	-5.0763	-5.1054
	8	1st Iter.	12.7186	2.3401	2.4309	-4.5329	-5.0362
		2nd "	12.6418	2.2777	2.2907	-5.0507	-5.1037
		3rd "	12.6870	2.2902	2.3023	-5.0871	-5.1049
		4th "	12.6877	2.2901	2.3018	-5.0907	-5.1054
	9	1st Iter.	12.7186	2.3401	2.4309	-4.5377	-5.0262
		2nd "	12.6418	2.2777	2.2907	-5.0601	-5.1037
		3rd "	12.6870	2.2902	2.3023	-5.0972	-5.1048
		4th "	12.6877	2.2901	2.3018	-5.1008	-5.1054
Exact [7]			12.60	2.31	2.31	-5.13	-5.13
Multiplier			$10^{-4}.q\ a^4/B$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$

Table 3. Analysis of rectangular plates clamped on sides AB and CD, simply supported on the two other sides subjected to uniform distributed load of intensity q. Fig. 3-a ($\Delta x=\ell/40$, 7 Terms, 4th Iter.), $\nu = 0.3$

ℓ/a	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB	
1.0	19.1662 19.20	2.4393 2.44	3.3293 3.32	-6.9247 -6.97	N.L.F.D Exact [7]
1.1	20.8769 20.90	2.2990 2.30	3.5652 3.55	-7.3364 -7.39	N.L.F.D Exact [7]
1.2	22.2359 22.30	2.1559 2.15	3.7473 3.75	-7.6490 -7.71	N.L.F.D Exact [7]
1.3	23.2995 23.40	2.0191 2.03	3.8857 3.88	-7.8825 -7.94	N.L.F.D Exact [7]
1.4	24.1210 24.00	1.8934 1.92	3.9892 3.99	-8.0539 -8.10	N.L.F.D Exact [7]
1.5	24.7465 24.70	1.7813 1.79	4.0651 4.06	-8.1772 -8.22	N.L.F.D Exact [7]
2.0	26.0990 26.00	1.4193 1.42	4.2084 4.20	-8.3915 -8.42	N.L.F.D Exact [7]
Multiplier	$10^{-4}.q\ a^4/B$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	

Table 4. Analysis of rectangular plates clamped on four sides subjected to uniform distributed load of intensity q. Fig. 3-b ($\Delta x=\ell/40$, 7 Terms, 4th Iter.), $\nu = 0.3$

ℓ/a	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB	M_x at Middle of AC	
1.0	12.6876 12.60	2.2901 2.31	2.3017 2.31	-5.0763 -5.13	-5.1054 -5.13	N.L.F.D Exact [7]
1.1	15.1151 15.00	2.3132 2.31	2.6795 2.64	-5.7558 -5.81	-5.3486 -5.38	N.L.F.D Exact [7]
1.2	17.2812 17.20	2.2816 2.28	3.0071 2.99	-6.3390 -6.39	-5.4982 -5.54	N.L.F.D Exact [7]
1.3	19.1454 19.10	2.2131 2.22	3.2815 3.27	-6.8228 -6.87	-5.5819 -5.63	N.L.F.D Exact [7]
1.4	20.7063 20.70	2.1231 2.12	3.5054 3.49	-7.2136 -7.26	-5.6223 -5.68	N.L.F.D Exact [7]
1.5	21.9854 22.00	2.0231 2.03	3.6840 3.68	-7.5223 -7.57	-5.6363 -5.70	N.L.F.D Exact [7]
1.6	23.0151 23.00	1.9214 1.93	3.8241 3.81	-7.7615 -7.80	-5.6353 -5.71	N.L.F.D Exact [7]
1.7	23.8311 23.80	1.8235 1.82	3.9320 3.92	-7.9435 -7.99	-5.6263 -5.71	N.L.F.D Exact [7]
1.8	24.4683 24.50	1.7327 1.74	4.0137 4.01	-8.0793 -8.12	-5.6135 -5.71	N.L.F.D Exact [7]
1.9	24.9584 24.90	1.6509 1.65	4.0744 4.07	-8.1786 -8.22	-5.5993 -5.71	N.L.F.D Exact [7]
2.0	25.3295 25.40	1.5787 1.58	4.1185 4.12	-8.2493 -8.29	-5.5848 -5.71	N.L.F.D Exact [7]
Multiplier	$10^{-4}.q\ a^4/B$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	

Table 5. Analysis of rectangular plates simply supported on side BD, clamped on the three other sides subjected to uniform distributed load of intensity q. Fig. 3-c ($\Delta x = \ell/40$, 7 Terms, 4th Iter.) $\nu = 0.3$

ℓ/a	Central Deflection	M_y at Middle of AB	M_x at Middle of AC	
0.5	44.9952 44.90	-7.5604 -7.86	-11.4316 -11.48	N.L.F.D Exact [7]
0.75	28.6859 28.60	-7.1714 -7.30	-8.3358 -8.38	N.L.F.D Exact [7]
1.0	15.7209 15.70	-5.9423 -6.01	-5.4720 -5.51	N.L.F.D Exact [7]
4/3	21.6296 21.50	-7.4497 -7.50	-5.6633 -5.71	N.L.F.D Exact [7]
2.0	25.7145 25.70	-8.3205 -8.37	-5.5809 -5.71	N.L.F.D Exact [7]
Multiplier	$10^{-4}.q\ L^4/B$	$10^{-2}.q\ L^2$	$10^{-2}.q\ L^2$	

L is the smallest value of ℓ and a

Table 6. Analysis of rectangular plates clamped on sides AB and CD, simply supported on the two other sides subjected to triangular load Fig.3-d, ($\Delta x = \ell/40$, 7 Terms, 4th Iter.) $\nu = 0.3$

a/ℓ	$x=\frac{\ell}{2}$, $y=\frac{a}{2}$		$x=\frac{3\ell}{4}$, $y=\frac{a}{2}$		$x=\frac{\ell}{2}$, $y=0$	$x=\frac{3\ell}{4}$, $y=0$	
	M_x	M_y	M_x	M_y	M_y	M_y	
0.5	0.7096 0.70	2.1041 2.10	1.5306 1.80	2.7135 2.90	-4.1958 -4.20	-5.5494 -6.20	N.L.F.D Exact [7]
0.75	0.9879 1.10	1.9617 2.00	1.6970 1.80	2.0820 2.10	-3.9729 -4.00	-4.4045 -4.50	N.L.F.D Exact [7]
1.0	1.2196 1.30	1.6644 1.70	1.6459 1.70	1.5683 1.50	-3.4624 -3.50	-3.4263 -3.50	N.L.F.D Exact [7]
1.25	2.0600 2.10	2.0753 2.10	2.3687 2.40	1.8380 1.90	-4.4664 -4.50	-4.1461 -4.30	N.L.F.D Exact [7]
1.5	2.9250 3.00	2.3049 2.30	3.0490 3.10	1.9820 2.00	-5.1568 -5.10	-4.6202 -4.80	N.L.F.D Exact [7]
2.0	4.3464 4.30	2.3828 2.40	4.1084 4.20	2.0196 2.00	-5.7884 -6.00	-5.0188 -5.30	N.L.F.D Exact [7]
Multiplier	$10^{-2}.q_o\ L^2$						

L is the smallest value of ℓ and a

Table 7. Analysis of rectangular plates clamped on four sides subjected to triangular load Fig.3-e , ($\Delta x = \ell/40$, 7 Terms, 4th Iter.) $\nu = 0.3$

a/ℓ	$x=\frac{\ell}{2}$, $y=\frac{a}{2}$			$x=\ell$, $y=\frac{a}{2}$	$x=0$, $y=\frac{a}{2}$	$x=\frac{\ell}{2}$, $y=0$	
	w	M_x	M_y	M_x	M_x	M_y	
0.5	0.7915 0.80	0.1973 0.198	0.5148 0.515	-1.1223 -1.15	-0.2739 -0.28	-1.0312 -1.04	N.L.F.D Exact [7]
2/3	2.1714 2.17	0.4496 0.451	0.8186 0.817	-1.8487 -1.87	-0.6564 -0.66	-1.6716 -1.68	N.L.F.D Exact [7]
1.0	6.3438 6.30	1.1450 1.15	1.1506 1.15	-3.3251 -3.34	-1.7805 -1.79	-2.5382 -2.57	N.L.F.D Exact [7]
1.5	11.0292 11.00	1.8416 1.84	1.0235 1.02	-4.6015 -4.62	-2.9459 -2.95	-2.7776 -2.85	N.L.F.D Exact [7]
Multiplier	$10^{-4} q_o L^4/B$	$10^{-2} q_o L^2$					

L is the smallest value of ℓ and a

CONCLUSION

A new development of the nodal line finite difference method is introduced for the analysis of plates with two opposite ends other than simply supported. In the present work, plates with two opposite clamped ends have been analyzed. In order to overcome the coupling property of the equilibrium difference equations in the case of basic functions other than trigonometric series, a simplified iteration procedure has been developed. This procedure emphasises the role of the dominant terms of the central nodal line finite difference equation. Accordingly,each term of the basic function can be solved individually such as that in the case of trigonometric series. The results obtained demonstrate the rapid convergence and the high accuracy of the method. This method can be extended to include other end conditions and other material properties.

NOTATION

w	= transverse deflection.
a	= length of the nodal lines.
ℓ	= length or width of the plate.
Δx	= distance between the nodal lines.
E	= modulus of elasticity.
t	= thickness of the plate.
ν	= poisson's ratio.
B	= flexural rigidity.
$f_{m,k}$	= nodal line parameters.
Y_m	= basic function.
q	= load intensity.
$[S]_m$	= square band matrix.
$\{f\}_m$	= nodal line parameters vector.
$\{P\}_m$	= original load vector.
$\{\bar{P}\}_m$	= modified load vector.

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