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CLASSICAL DISTRIBUTION FUNCTIONS FOR RELATIVISTIC PURE ELECTRON PLASMA AND DUSTY PLASMA IN CURVILINEAR COORDINATES

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The aim of this study is to find the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy equations in Curvilinear Coordinates. These equations were used to calculate plasma distribution functions at very high temperatures. The analytical form of the classical two and three particles distribution functions were obtained for two models; relativistic pure electron plasma and dusty plasma. Our results were compared with others.

Keywords: *Dusty plasma; (BBGKY) hierarchy, Curvilinear Coordinates.*

1. INTRODUCTION

Calculating the distribution functions of plasma is of paramount importance in the field of statistical mechanics. As the identification of the final form for distribution functions, will assist in calculating the thermodynamic functions of plasma, for example, pressure, energy, and many other quantities. In statistical mechanics, the BBGKY hierarchy is a number of equations interpreting the dynamics of a system of many interacting particles [1]. Many researchers have been interested in identifying the optimal form of classical distribution functions. Hansen [2] calculated the distribution functions for one component plasma in the classical form and quantum corrections.

Illner and Pulvirenti [3] derived the BBGKY-hierarchy for hard sphere particle systems. Also, Polyakov [4] studied the BBGKY hierarchy in classical relativistic electrodynamics. Bose in 2016 used BBGKY hierarchy for deriving an equation of pair correlation function [5]; he solved the first two equations of BBGKY hierarchy in the absence of velocity-space correlation. Vereshchagin and Aksenov in 2017 [6] produced a very well-written and interesting book on relativistic kinetic theory, starting from first principles of statistical mechanics. Also, they obtained the BBGKY hierarchy for relativistic plasma and they derived fundamental equations such as the Vlasov-Maxwell system, and Landau equations. The BBGKY hierarchy equations of large classical systems were studied by Zessin [7]. Moreover, Prasad and O'Neil [8] obtained the classical distribution of pure electron plasma.

Relativistic plasma is objects encountered in many astrophysical situations. For instance, they occur in the magnetosphere of pulsars where they are strongly magnetized, or in the quasar jets [9]. Such plasma can be created by heating a gas to very high temperatures. In the relativistic plasma, the relativistic corrections to a particle's mass and velocity are important. Such corrections typically become important when a significant number of electrons reach speeds greater than 0.86c where c is the speed of light.

Curvilinear coordinates play an important role in many types of knowledge such as fluid mechanics, quantum mechanics, and statistical mechanics. The formulation of physical equations in a generalized form in curvilinear coordinates facilitates the processing of many complex physical problems. We try in our paper to find distribution functions generally in curvilinear coordinates which can be later used in special situations to solve specific problems for systems with an appropriate type of spatial symmetry. The main idea is that a curvilinear coordinate frame can innately capture more information about the true motion of an orbit than a Cartesian coordinate frame. Classical mechanics, Quantum mechanics and statistical mechanics can be written in any coordinate system, and the usual Hamiltonian methods apply. Stern [10] formulated the Elliptical Cylindrical Coordinate System and showed that the curvilinear system has definite advantages for studying motion along a known, fixed elliptical trajectory. Jones [11] continued Stern's work by developing a state transition matrix for the cylindrical coordinate system derived by Stern. Berreen and Crisp [12] formulated special relative motion equations in curvilinear coordinates for a probe ejecting from a space station and found that by employing curvilinear coordinates, their approximate solution could be used over larger relative displacements than earlier solutions in a rectangular coordinate frame [13]. In Ref. [14], the nonlinear equations of relative orbital motion in a cylindrical frame are presented.

Curvilinear coordinates would be able to handle fixed discontinuity, moving discontinuity in more natural way. Namely, a curvilinear coordinates could generate fine mesh in the neighborhood of fixed and moving discontinuities and increase the accuracy of the solution. Furthermore, if we introduce curvilinear coordinates, we can transform a non-square region into square one and can introduce a regular mesh in the transformed coordinates. Kjaergaard and Mortensen in 1990 [15] made a simple derivation of the quantum mechanical Hamiltonian in curvilinear coordinates. In 2012 a generalized, curvilinear-coordinate formulation of Poisson's equations to solve the electrostatic fields in plasma was given by Fichtl et al. [16]. Calculating the distribution functions in curvilinear coordinates of the dusty plasma model facilitates the selection of the appropriate coordinates of the model studied. Błaszak and Domanski [17] give a definition of a quantum canonical coordinate system for classical mechanics in curvilinear coordinates. We will pay attention to the formulation of BBGKY-hierarchy in curvilinear coordinates and try to use it to calculate the distribution functions of plasma at very high temperature.

Dusty Plasma plays an important role in experimental physics and in experimental physics and in many astrophysical situations [18]. In dusty plasma, a dust particle can be charged to about 10^4electrons due to greater electron mobility than ion mobility. It differs from ordinary plasma in the presence of dust particles along with an even number of positive and negative charges moving at a high speed if compared to the speed of dust [19]. The previous research studies have shown that the distance between every two particles is a straight line. But what can we get if we imagine the distances between the particles are curved lines and get its distribution function in curvilinear coordinates. We used the results to calculate the distribution functions for relativistic electron plasma and dusty relativistic plasma.

2 Curvilinear formulation of BBGKY hierarchy

First we introduce apoint transformation to curvilinear formulation system

$$T(\xi^1, \xi^2, \xi^3, P_{\xi^1}, P_{\xi^2}, P_{\xi^3}) = (x^1, x^2, x^3, P_{x^1}, P_{x^2}, P_{x^3}) \tag{1}$$

between the physical space X with coordinates $X = (\ x^1, x^2, x^3)$ to the logical space Z with coordinates $\xi = (\xi^1, \xi^2, \xi^3)$. By define th Jacobi matrix as :

$$j_{\alpha\beta} = \frac{\partial x^\alpha}{\partial \xi^\beta} \qquad , \alpha, \beta = 1, 2, 3. \tag{2}$$

While its inverse is

$$K^{\alpha\beta} = \frac{\partial \xi^\alpha}{\partial x^\beta} \qquad , \alpha, \beta = 1, 2, 3. \tag{3}$$

The Jacobian of the transformation is the determinant of Jacobi matrix

$$J(\xi) = \det[j_{\alpha\beta}] \tag{4}$$

Now suppose we have a problem of N particles moving in the Hamiltion (switching back to variables (X, ξ)) :

$$H = \sum_{i=1}^{N} \frac{P_{\xi_i}^2}{2m} + \sum_{i=1}^{N} U_i(\xi_i) + \sum_{i<j}^{N} v_{ij} \tag{5}$$

where $U_i(\xi_i)$ is the potential from every particle in curvilinear coordinates and $v_{ij} = v_{ji} = v(|\xi^i - \xi^j|)$ is the potential between two particles in curvilinear coordinates.

Then Liouville's equation is given by:

$$\frac{\partial g}{\partial t} = \sum_{i=1}^{N} (-\overrightarrow{F_i} - \sum_{i=1, i\neq j}^{N} \overrightarrow{K_{ij}}).\nabla_{P_{\xi^i}} g - \frac{\overrightarrow{p_i}}{m}.\nabla_{\xi^i} g \tag{6}$$

where $F_i = -\nabla U_i(\xi_i)$, $K_{ij} = \nabla_{\xi^i} v_{ij}$ and $g(\xi_i, P_{\xi_i})$ is refer to the distribution function for N particle phase space.

Liouville's theorem can be restated as

$$[\frac{\partial}{\partial t} + h_N(\xi^1, \xi^2, \xi^3, P_{\xi^1}, P_{\xi^2}, P_{\xi^3})]g = 0. \tag{7}$$

where the differential operator h_N is defind as

$$\begin{aligned} h_N(\xi^1, P_{\xi^1}, \xi^2, P_{\xi^2}, ..., \xi^N, P_{\xi^N}) &= \sum_{i=1}^{N}[\frac{\overrightarrow{P_{\xi^i}}}{m}.\nabla_{\xi^i} + \overrightarrow{F_i}.\nabla_{p_{\xi^i}}] \\ &+\frac{1}{2}\sum_{i,j=1}^{N}\overrightarrow{K_{ij}}\,.\,(\nabla_{p_{\xi^i}} - \nabla_{p_{\xi^j}}). \end{aligned} \quad (8)$$

Now define the single particle distribution function in curvilinear coordinates with $Z = (\xi, P_\xi)$

$$\begin{aligned} f_1(\xi, P_\xi, t) &= \left\langle \sum_{i=1}^{N}\delta(\xi - \xi_i)\delta(P_\xi - P_{\xi_i}) \right\rangle \\ &= N\int g(Z_1, Z_2, ..., Z_N)J(\xi)\ \ dZ_2...dZ_N, \end{aligned} \quad (9)$$

where $J(\xi)$ is the determinant of Jacobi matrix between coordinates.
To obtain an equation for evolution of f_i, it is useful to write

$$h_N = h_n + h_{N-n} + \sum_{i=1}^{n}\sum_{j=n+1}^{N}\overrightarrow{K_{ij}}\,.\,(\nabla_{p_{\xi^i}} - \nabla_{p_{\xi^j}}), \quad (10)$$

where we have suppressed the coordinate dependence. And finally we can get

$$\begin{aligned} &\int(\frac{\partial}{\partial t} + h_n(z_1, z_2, ..., z_n))f_n(z_1, z_2, ..., z_n)dz_n \\ &= -\int\sum_{i=1}^{N}\overrightarrow{K_{i,n+1}}.\nabla_{p_i}f_{n+1}(z_1, z_2, ..., z_n)J(\xi)dz_{n+1}, \end{aligned} \quad (11)$$

and this equation is known as BBGKY hierarchy [11]. Here, for each value of n we have n equations which can be solved to get the distribution functions. For example for n=2 we have two equations; one of them gives the relation between f_1 and f_2 and the other gives the relation between binary distribution function f_2 and triplet distribution function f_3.

For equilibrium plasma in homogeneous case the one-particle distribution function f_1 is the relativistic Maxwellian distribution in curvilinear coordinates [17] as

$$f_1(\xi^i, p_{\xi^i}) = \frac{\mu_i}{4\pi(m_i c^3)K_2(\mu_i)} \exp(\frac{-\mu_i m_i c}{\sqrt{(m_i c)^2 - p_{\xi^i}^2}} - q_i \phi(\xi^i)). \qquad (12)$$

Where $K_2(\mu_i)$ denotes the modified Bessel function and $\mu_i = \frac{mc^2}{KT}$. Also, $\phi(\xi^i)$ is the electrostatic potential see [18]. This approximation is only suitable for the relativistic diluted plasma if there are no external fields. With taking into account that the potential between each particle depended on the curved distance between them in curvilinear coordinate.

3 Classical distribution functions in curvilinear coordinates

For n=1 the first equation of BBGKY is

$$\begin{aligned}
&(\frac{\partial}{\partial t} + \frac{\overrightarrow{P_{\xi^1}}}{m}.\nabla_{\xi^1} + \overrightarrow{F_1}.\nabla_{p^1}) f_1(\xi^1, P_{\xi^1}, t) \\
= \; & -\int \overrightarrow{K_{12}}.\nabla_{P_{\xi^1}} f_2(\xi^1, P_{\xi^1}, \xi^2, P_{\xi^2}, t) J(\xi) dz_2. \qquad (13)
\end{aligned}$$

The second equation for n=2 has a different structure

$$\begin{aligned}
&(\frac{\partial}{\partial t} + \frac{\overrightarrow{P_{\xi^1}}}{m}.\nabla_{\xi^1} + \overrightarrow{F_1}.\nabla_{P_{\xi^1}} + \frac{\overrightarrow{P_{\xi^2}}}{m}.\nabla_{\xi^2} + \overrightarrow{F_2}.\nabla_{P_{\xi^2}} \\
&+\frac{1}{2}\overrightarrow{K_{12}}.(\nabla_{P_{\xi^1}} - \nabla_{P_{\xi^2}})) f_2(\xi^1, P_{\xi^1}, \xi^2, P_{\xi^2}, t) \\
= \; & -\int (\overrightarrow{K_{13}}.\nabla_{P_{\xi^1}} + \overrightarrow{K_{23}}.\nabla_{P_{\xi^2}}) f_3(\xi^1, P_{\xi^1}, \xi^2, P_{\xi^2}, \xi^3, P_{\xi^3}, t) J(\xi) dz_3. (14)
\end{aligned}$$

Let us now study the model of dusty three component plasma ($3^{th}CP$) which is consists of positive and negative charges such as electrons, positirons and dust particles like ions. For the numerical calculation we use $q_e = -q_p = -e$. Dust particles are heavier and slower in their velocities than electrons and positrons.

For three component plasma, we can use the relativistic binary correlation function $G(1,2)$ which is given by

$$G(1,2) = g_1(\xi_{12})[1 + \frac{\overrightarrow{p_{\xi^1}}.\overrightarrow{p_{\xi^2}}}{(mc)^2} + \frac{(\overrightarrow{\xi_{12}}.\overrightarrow{p_{\xi^1}})(\overrightarrow{\xi_{12}}.\overrightarrow{p_{\xi^2}})}{2(mc)^2\xi_{12}^2}] \tag{15}$$

where

$$g_1(\xi_{12}) = -\frac{e_1 e_2}{KT\xi_{12}} e^{-\kappa\xi_{12}}, \tag{16}$$

$g_1(r_{12})$ is the Debye-Hückel solution [19] and the relativistic triplet correlation function $G(1,2,3)$ which is given by

$$\begin{aligned} G(1,2,3) = {} & g_1(\xi_{12})g_1(\xi_{13})g_1(\xi_{23})[1 + \frac{\overrightarrow{p_{\xi^1}}.\overrightarrow{p_{\xi^2}}}{(mc)^2} + \frac{\overrightarrow{p_{\xi^1}}.\overrightarrow{p_{\xi^3}}}{(mc)^2} \\ & + \frac{\overrightarrow{p_{\xi^2}}.\overrightarrow{p_{\xi^3}}}{(mc)^2} + \sum_{i,j=1, i\neq j}^{3} \frac{(\overrightarrow{\xi_{ij}}.\overrightarrow{p_{\xi^i}})(\overrightarrow{\xi_{ij}}.\overrightarrow{p_{\xi^j}})}{2(mc)^2\xi_{ij}^2} \\ & + \frac{(\overrightarrow{p_{\xi^1}}.\overrightarrow{p_{\xi^2}})(\overrightarrow{p_{\xi^1}}.\overrightarrow{p_{\xi^3}})}{(mc)^4} + \frac{(\overrightarrow{p_{\xi^1}}.\overrightarrow{p_{\xi^3}})(\overrightarrow{p_{\xi^2}}.\overrightarrow{p_{\xi^3}})}{(mc)^4} \\ & + \frac{(\overrightarrow{p_{\xi^1}}.\overrightarrow{p_{\xi^2}})(\overrightarrow{p_{\xi^2}}.\overrightarrow{p_{\xi^3}})}{(mc)^4} + \frac{(\overrightarrow{p_{\xi^1}}.\overrightarrow{p_{\xi^3}})(\overrightarrow{\xi_{12}}.\overrightarrow{p_{\xi^1}})(\overrightarrow{\xi_{12}}.\overrightarrow{p_{\xi^2}})}{2(mc)^4\xi_{12}^2} + \ldots]. \end{aligned} \tag{17}$$

Whatever particles 1, 2, 3 are, we can write

$$F^{(2)}(1,2) = F^{(1)}(1)F^{(1)}(2)[1 + G(1,2) + \frac{1}{2}G^2(1,2) + \frac{1}{3!}G^3(1,2)] \tag{18}$$

$$\begin{aligned} F^{(3)}(1,2,3) = {} & F^{(1)}(1)F^{(1)}(2)F^{(1)}(3)[1 + G(1,2) + G(1,3) + G(2,3) \\ & + \frac{1}{2}(G(1,2) + G(1,3) + G(2,3))^2 + \frac{1}{3!}(G(1,2) \\ & + G(1,3) + G(2,3))^3 + G(1,2,3)]. \end{aligned} \tag{19}$$

where the binary correlation function $G(i,j) \ll 1$.

When equation (15) is substituted into equation (18), the two particles distribution function appears in the following form:

$$
\begin{aligned}
F^{(2)}(1,2) \;=\; & F^{(1)}(1)F^{(1)}(2)\{1-\sum_{i,j=1,i\neq j}^{2}(\frac{e_i e_j}{m_i \xi_{ij}}[e^{-\kappa\xi_{ij}}+\frac{\overrightarrow{p_{\xi^i}}\cdot\overrightarrow{p_{\xi^j}}}{2KT} \\
&+\frac{(\overrightarrow{\xi_{ij}}\cdot\overrightarrow{p_{\xi^i}})(\overrightarrow{\xi_{ij}}\cdot\overrightarrow{p_{\xi^j}})}{2KT\xi_{ij}^2}]+\frac{e_i^2 e_j^2}{2m_i^2\xi_{ij}^2}+[e^{-\kappa m_i\xi_{ij}}+\frac{\overrightarrow{p_{\xi^i}}\cdot\overrightarrow{p_{\xi^j}}}{2c^2} \\
&+\frac{(\overrightarrow{\xi_{ij}}\cdot\overrightarrow{p_{\xi^i}})(\overrightarrow{\xi_{ij}}\cdot\overrightarrow{p_{\xi^j}})}{2m_i^2c^2\xi_{ij}^2}]^2\mu_i^2-\frac{e_i^3 e_j^3}{6m_i^3\xi_{ij}^3}[e^{-\kappa m_i\xi_{ij}}+\frac{\overrightarrow{p_{\xi^i}}\cdot\overrightarrow{p_{\xi^j}}}{2c^2} \\
&+\frac{(\overrightarrow{\xi_{ij}}\cdot\overrightarrow{p_{\xi^i}})(\overrightarrow{\xi_{ij}}\cdot\overrightarrow{p_{\xi^j}})}{2m_i^2c^2\xi_{ij}^2}]^3\mu_i^3)+\ldots\}. \qquad (20)
\end{aligned}
$$

where $F^{(1)}$ is given by equation (12).

Substituting equations (19),(17) and (12) into (14) for $n = 1,2,3$ we obtain the triplet distribution function $F^{(3)}(1,2,3)$ as

$$
\begin{aligned}
F^{(3)}(1,2,3) \;=\; & F^{(1)}(1)F^{(1)}(2)F^{(1)}(3)\{1-\sum_{i,j=1,i\neq j}^{3}(\frac{e_i e_j}{m_i\xi_{ij}}e^{-\kappa\xi_{ij}}\mu_i \\
&+\frac{e_i e_j}{4m_j\pi^2}\psi_{ij}\ \mu_j+\frac{e_i^2 e_j^2}{2m_i^2\xi_{ij}}e^{-2\kappa\xi_{ij}}\mu_i^2+\frac{e_i^2 e_j^2\mu_j\mu_i}{2m_i m_j\pi^2\xi_{ij}}e^{-\kappa\xi_{ij}}\psi_{ij} \\
&-\frac{3e_i^3 e_j^3}{8m_i^2 m_j\pi^2\xi_{ij}}e^{-2\kappa\xi_{ij}}\mu_i^2\mu_j\psi_{ij}\ -\frac{3e_i^3 e_j^3}{16m_i m_j^2\pi^4\xi_{ij}}e^{-\kappa\xi_{ij}}\mu_i\mu_j^2\psi_{ij} \\
&+\frac{e_i^2 e_j^2}{16m_j^2\pi^4}\psi_{ij}\mu_j^2-\frac{e_i^3 e_j^3}{6m_i^3\xi_{ij}}e^{-3\kappa\xi_{ij}}\mu_i^3-\frac{e_i^3 e_j^3}{64m_j^3\pi^6}\psi_{ij}\mu_j^3) \\
&+\ldots\}, \qquad (21)
\end{aligned}
$$

where

$$
\psi_{ij} \;=\; \frac{\overrightarrow{p_{\xi^i}}\cdot\overrightarrow{p_{\xi^j}}}{2KT}+\frac{(\overrightarrow{\xi_{ij}}\cdot\overrightarrow{p_{\xi^i}})(\overrightarrow{\xi_{ij}}\cdot\overrightarrow{p_{\xi^j}})}{2KT\xi_{ij}^2}. \qquad (22)
$$

The triplet and binary functions can be assimilated for a more accurate and full analysis of macroscopic equilibrium properties.We also can used the Kirkwood superposition approximation (KSA) [20]; which is comprised of the assumption that the potential in a set of three particles is the sum of the three pair potentials, this is equivalent to assume that the three particles distribution function is the product of the three radial distribution functions

$$F^{(3)}(1,2,3) = F^{(2)}(1,2)F^{(2)}(2,3)F^{(2)}(1,3). \tag{23}$$

Substituting equation (20) into (23)

$$\begin{aligned} F^{(3)}(1,2,3) &= (F^{(1)}(1)F^{(1)}(2)F^{(1)}(3))^2\{1 - \sum_{i,j=1,i\neq j}^{2} (\frac{e_i e_j}{m_i \xi_{ij}}[e^{-\kappa\xi_{ij}} + \frac{\overrightarrow{p_{\xi^i}}.\overrightarrow{p_{\xi^j}}}{2KT} \\ &\quad + \frac{(\overrightarrow{\xi_{ij}}.\overrightarrow{p_{\xi^i}})(\overrightarrow{\xi_{ij}}.\overrightarrow{p_{\xi^j}})}{2KT\xi_{ij}^2}] + \frac{e_i^2 e_j^2}{2m_i^2\xi_{ij}^2} + [e^{-\kappa m_i \xi_{ij}} + \frac{\overrightarrow{p_{\xi^i}}.\overrightarrow{p_{\xi^j}}}{2c^2} \\ &\quad + \frac{(\overrightarrow{\xi_{ij}}.\overrightarrow{p_{\xi^i}})(\overrightarrow{\xi_{ij}}.\overrightarrow{p_{\xi^j}})}{2m_i^2 c^2 \xi_{ij}^2}]^2 \mu_i^2 + ...\}. \end{aligned} \tag{24}$$

4 Discussion

In the present study, we consided the classical relativistic plasma at very high temperature. An analytical method to solve Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy in curvilinear coordinates is presented. We calculated the two and three particles distribution functions from the BBGKY hierarchy for relativistic and weakly relativistic plasma. We ploted the two and three distribution functions for weakly relativistic and relativistic plasma in different temperatures. Our results were compared with the results of Prasad and O'Neil [10] who obtained the classical distribution of a pure electron plasma in spherical coordinates.

The values of the distribution functions were increased by increasing the temperature, because as the temperature increases, the speed and dispersion of the particles increases and hence increases the probability of their presence in the phase space. Figure (1) shows the one particle relativistic distribution function for relativistic electron plasma in curvilinear coordinates. Also, we found many variations and deviations in the values of the one particle distribution for weakly relativistic electron plasma, these variation increase with increasing temperature (figure (2)).

In figures (3), (4) and (5) we found the values of both two and three particles relativistic distribution function increase in high temperature more than that of the one particle relativistic distribution function for relativistic and weakly electron plasma. These results are physically acceptable because from the definition of the Phase-space distribution function which is the number of particles per unit volume of space per unit volume of velocity space; at high temperature the velocity of the particles increases and as a result the number of particles increases per unit volume see figures (6) and (8). The pair correlation functions for different values of screening length are presented in figure (7). The Pair correlation functions become more acceptable when the number of particles increases (figure 9).

Figures (10), (11) show the three particle distribution function for relativistic and weakly relativistic dusty plasma in curvilinear coordinates. It seems to us that in high-temperature the distribution function for relativistic dusty plasma is more accurate and clear than that the lower temperature (weakly relativistic dusty plasma).

We used Graphics 3D with color points for three particle distribution function in figures (12) and (13); the warm colors indicate the high values of the distribution function while the cold colors have lower values. As temperatures increase, the velocity increases and the momentum increases accordingly and this affects the shape of the distribution functions. Relativistic plasma represents a high proportion of the star's components in space and it seems to us that spherical shape is what most stars take. Thus, the spherical perception of the distribution functions in three dimensions is physically acceptable. This is what we observed from the figures (12) and (13) for relativistic and weakly relativistic dusty plasma, respectively.

Figures (14) and (15) show comparison between our binary and triplet distribution function from our results, Bose [5] and from Prasad and O'Neil [10] for weakly relativistic electron plasma.We observed the applicability of the results at high temperatures.

The Kirkwood superposition approximation (KSA) [20]; which is consisting of the assumption that the potential in a set of three particles is the sum of the three pair potentials, this is equivalent to assuming that the triplet distribution function is the product of the three radial distribution functions. The comparison between triplet distribution function from our result and from (KSA) equation (23) for weakly relativistic and relativistic electron plasma was given in figures (16) and (17).

We note from all forms of distribution that with the increase in momentum values for all distribution functions are approaching zero. The reason for this is that by increasing the momentum, the speed increases accordingly, and at very high velocities near the speed of light there is no value of the distribution function see [17]. The effect of increasing temperatures is more pronounced in the three particle distribution functions than the binary distribution functions.

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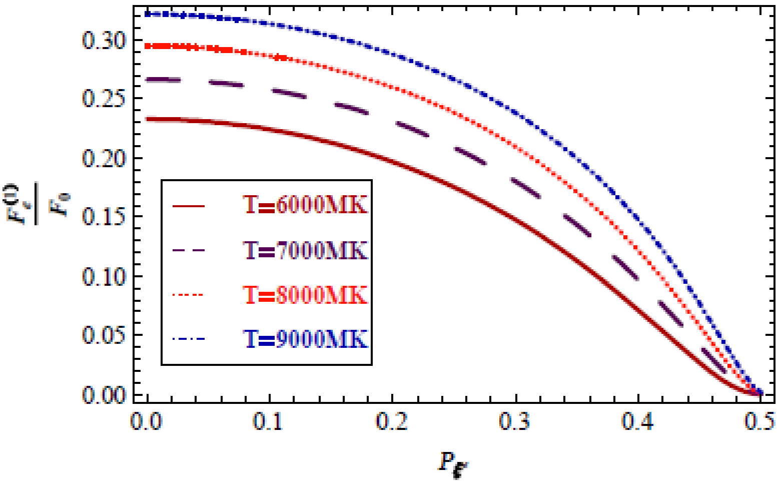

Figure 1: The one particle distribution function for relativistic electron plasma in curvilinear coordinates in different temperatures.

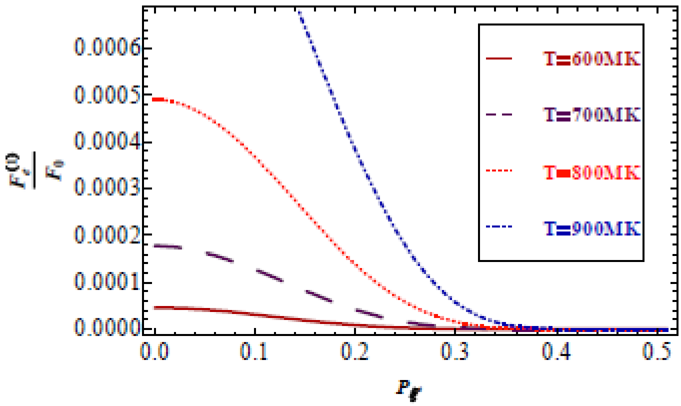

Figure 2: The one particle distribution function for weakly relativistic electron plasma in curvilinear coordinates in different temperatures.

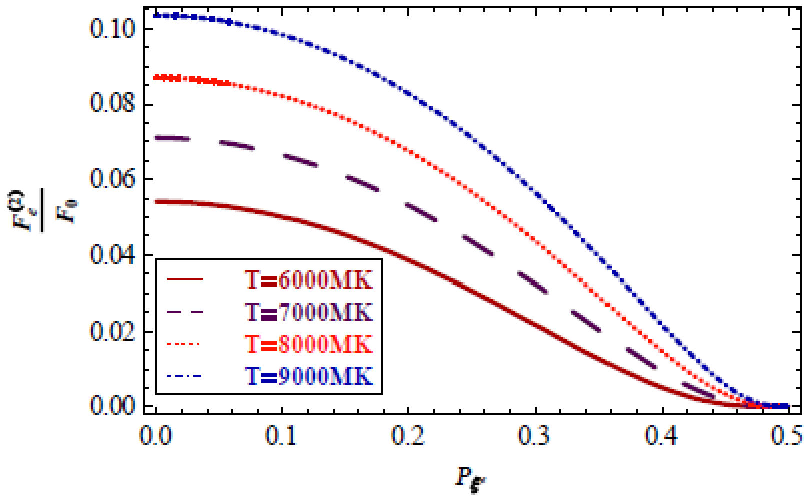

Figure 3: The two particle distribution function for relativistic electron plasma in curvilinear coordinates in different temperatures.

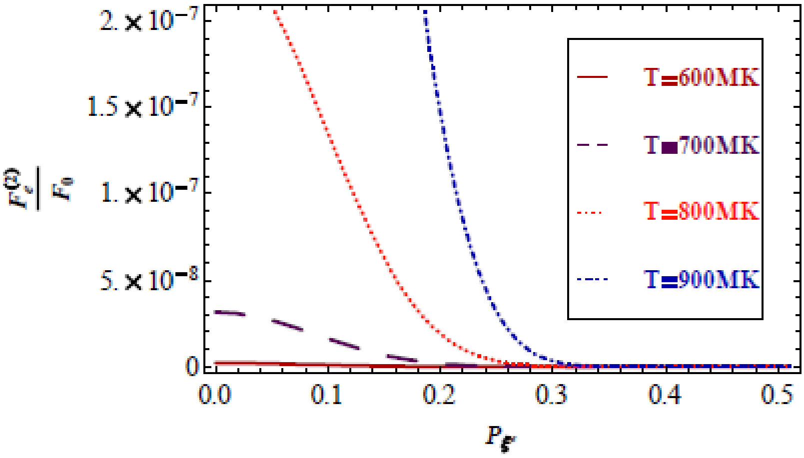

Figure 4: The two particle distribution function for weakly relativistic electron plasma in curvilinear coordinates in different temperatures.

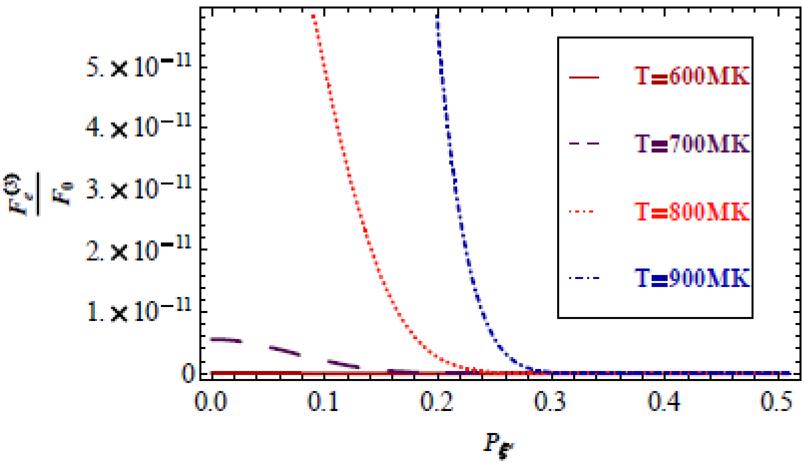

Figure 5: The three particle distribution function for weakly relativistic electron plasma in curvilinear coordinates in different temperatures.

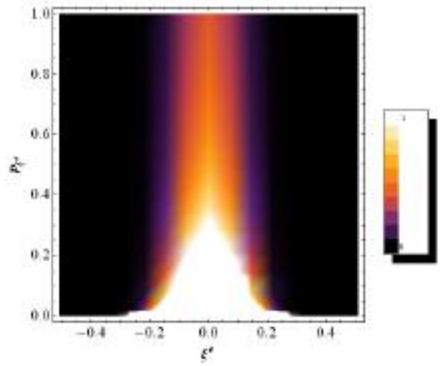

Figure 6: The phase space of weakly relativistic electron plasma.

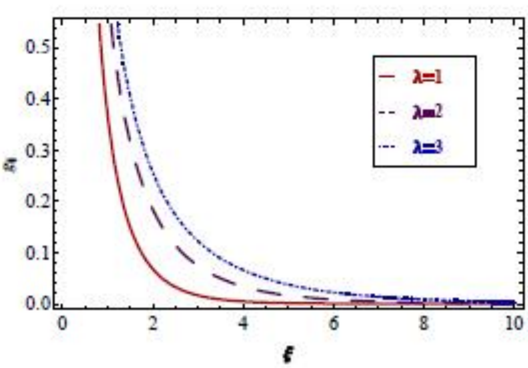

Figure 7: The Pair correlation functions for different values of screening length.

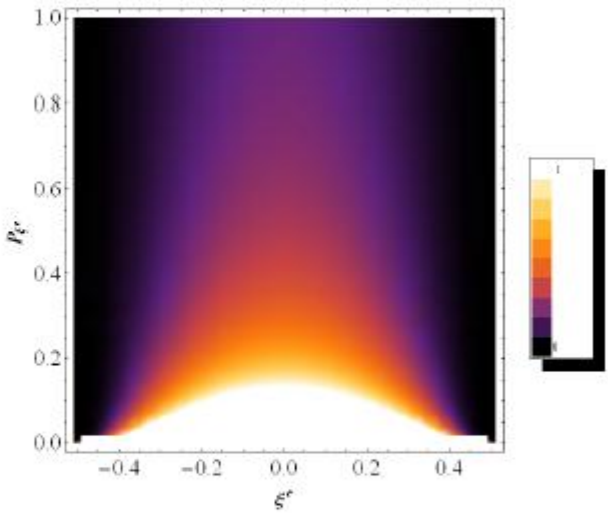

Figure 8: The phase space of relativistic electron plasma.

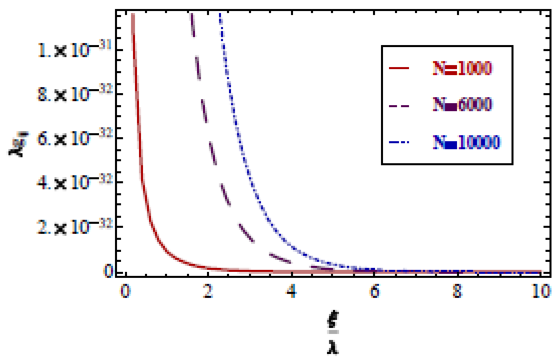

Figure 9: The Pair correlation functions for different values of number of particles

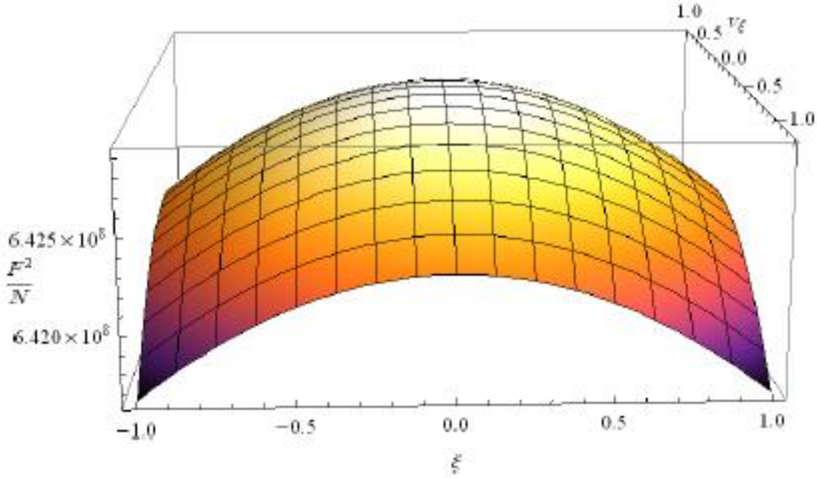

Figure 10: The two particle distribution function for relativistic dusty plasma in curvilinear coordinates.

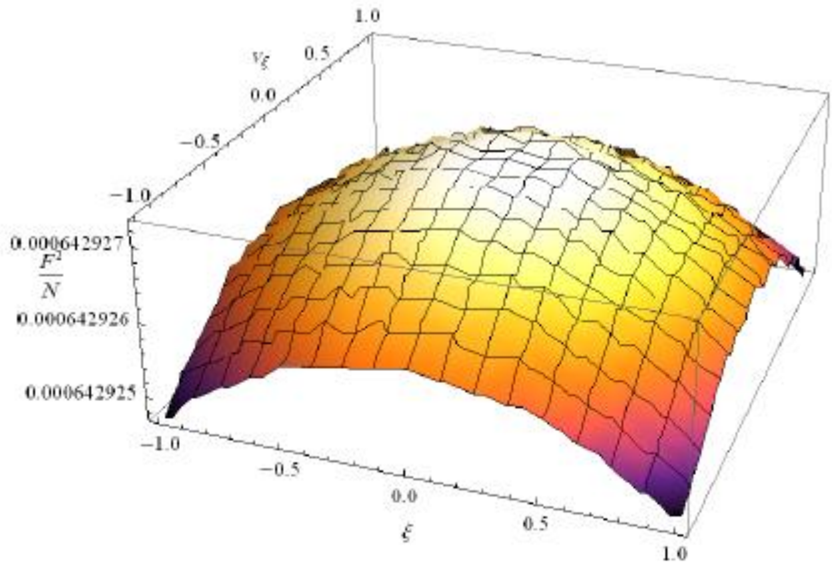

Figure 11: The two particle distribution function for weakly relativistic dusty plasma in curvilinear coordinates.

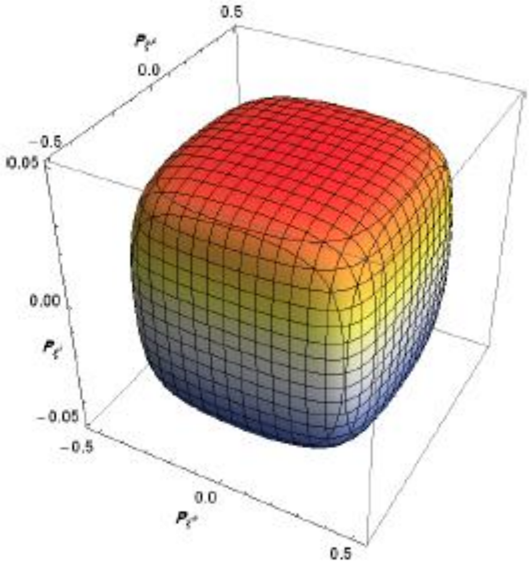

Figure 12: The three particle distribution function for relativistic dusty plasma in curvilinear coordinates.

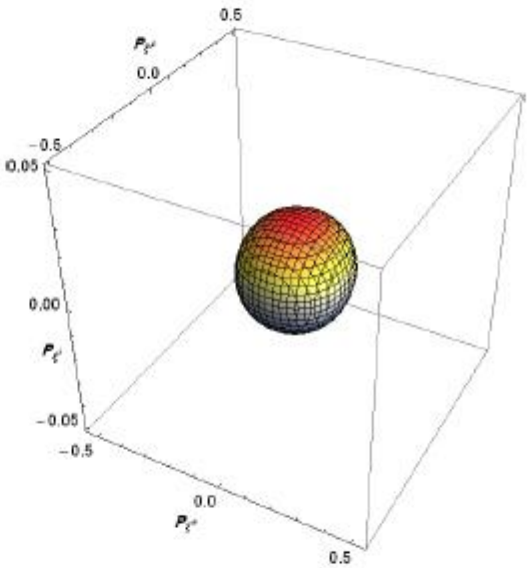

Figure 13: The three particle distribution function for weakly relativistic dusty plasma.

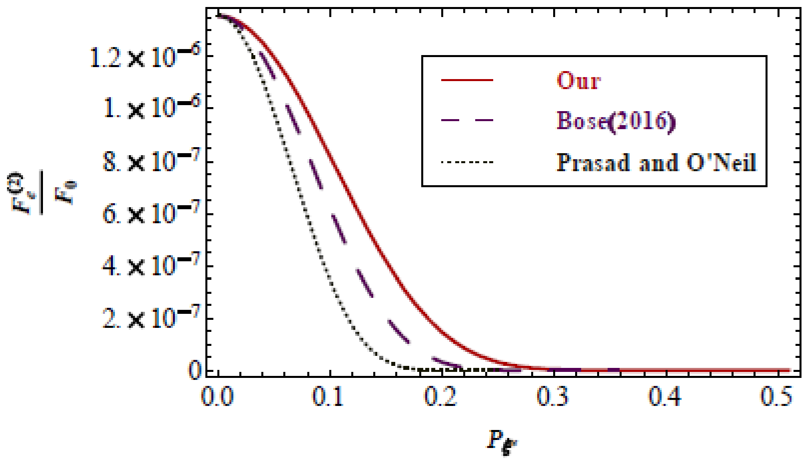

Figure 14: The comparison between binary distribution function from our result , Bose [5] and from Prasad and O'Neil [10] for weakly relativistic electron plasma.

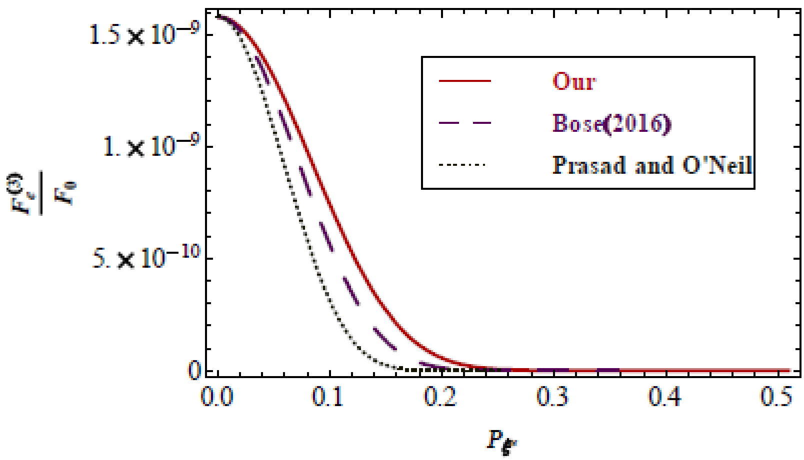

Figure 15: The comparison between triplet distribution function from our result , Bose [5] and from Prasad and O'Neil [10] for weakly relativistic electron plasma.

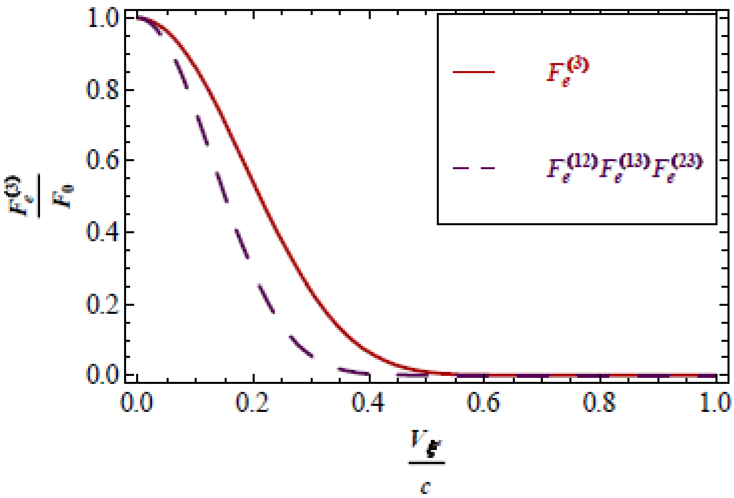

Figure 16: The comparison between triplet distribution function from our result and from (KSA) equation (23) for weakly relativistic electron plasma.

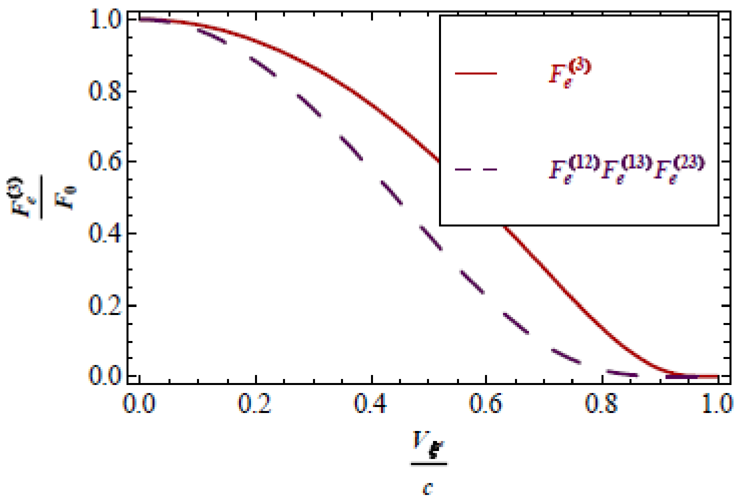

Figure 17: The comparison between triplet distribution function from our result and from (KSA) for relativistic electron plasma.

دوال التوزيع الكلاسيكية لبلازما الإلكترونات النسبية النقيه والبلازما المغبره فى الإحداثيات المنحنيه

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الملخص العربى

تهدف هذه الدراسه لإيجاد معادله التسلسل الهرمي BBGKY hierarchy في الإحداثيات المنحنية. وتم بإستخدام هذه المعادله حساب دوال التوزيع للبلازما في درجات الحرارة العاليه بدلالة معامل التأثير الحراري للبلازما μ. كما تم استنتاج الشكل التحليلي لدوال التوزيع الكلاسيكية الثنائية والثلاثيه لنموذجين هما بلازما الإلكترونات النسبية النقية والبلازما المغبرة. وقد قمنا بمقارنة نتائجنا مع الآخرين.