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ON SOFT PRE IDEAL OPEN SETS IN SOFT IDEAL TOPOLOGICAL SPACES

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Abstract. The aim of this paper is to study some concepts in soft ideal topological spaces related to soft pre ideal open sets. Also, the concepts of a soft pre ideal neighborhood, a soft pre ideal limit point, a soft pre ideal frontier, a soft pre ideal exterior and a soft pre ideal border of a soft set are investigated**.**

Keywords. Soft sets; soft ideal topological spaces; soft pre - $\tilde{I}$- neighborhoods; soft pre- $\tilde{I}$- limit points; soft pre - $\tilde{I}$- frontiers; soft pre - $\tilde{I}$- exteriors; soft pre - $\tilde{I}$- borders.

INTRODUCTION

In 1999, Molodtsov [9] introduced the concept of a soft set and started to develop basic of the theory as a new approach for modeling uncertainties. In [12], Shabir and Naz introduced soft topological spaces. Consequently, the notions of soft open sets, soft closed sets, soft closure, soft nbd of a point, soft separation axioms and soft subspace were introduced and several of their properties were investigated. Hussain and Ahmed [4] discussed some properties of soft interior, soft exterior and soft boundary of soft set. Cagman et.al [2] introduced soft limit points. Nazmul et al [10] studied properties of soft neighborhood systems. EL-Sheikh [3] introduced the concept of soft pre - $\tilde{I}$- open set using the notion of γ- operation. Kandil et.al [6] introduced the concept of soft ideal and soft local function. They generate a soft topological space finer than the given soft topological space $(X, \tilde{\tau}, E)$ on the same set X with fixed set of parameters E by using the soft ideal notion.

This paper is organized as follows: In section 2, some preliminary results are given. In section 3, some more results on soft pre - $\tilde{I}$- open sets are introduced. Also, the notions of soft pre -$\tilde{I}$- interior and soft pre - $\tilde{I}$-neighborhood of a soft point are investigated. In section 4, the soft pre- $\tilde{I}$-limit point of a soft subset in a soft ideal space are studied. In section 5, the soft pre - $\tilde{I}$- border of a soft set is given. In section 6, the soft pre - $\tilde{I}$-frontier of a soft set is introduced. In section 7, the soft pre - $\tilde{I}$- exterior of a soft set is studied. Finally, a conclusion is given.

2. Preliminaries

Now we recall some definitions and results which are useful in the sequel.

Definition 2.1 ([9]). Let X be an initial universe, E be a set of parameters, $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, E) is called a soft set over $\tilde{X}$, where F is a mapping given by $F: A \to P(X)$. In other words, a soft set over $\tilde{X}$ is a parameterized family of subsets of the universe X. For $e \in E$, $F(e)$ may be considered as the set of e – approximate elements of the soft set (F, E).

Definition 2.2 ([8]).

I. Let $(F, A), (G, B) \in SS(X)_E$.Then (F, A) is a soft subset of (G, B), denoted by $(F, A) \tilde{\subseteq} (G, B)$, if

(i) A ⊆ B, and

(ii) $F(e) \subseteq G(e)$, for every $e \in A$.

II. Two soft subsets (F, A) and (G, B) over a common universe set X are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

III. A soft set (F, A) over $\tilde{X}$ is said to be a null soft set denoted by $\tilde{\emptyset}$ or $\emptyset_A$ if for all $e \in A$, $F(e) = \emptyset$ (null set).

IV. A soft set (F, A) over $\tilde{X}$ is said to be an absolute soft set denoted by $\tilde{X}$ or X_A if for all $e \in A$, $F(e) = X$. Clearly we have $X_A^c = \emptyset_A$ and $\emptyset_A^c = X_A$.

V. The soft union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$$

VII. The soft intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C) where $C = A \cap B$ and for all $e \in C, H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F, E) over a universe X with the same set of parameter E. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.3 ([1]). The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined by $(F, E)^c = (F^c, E)$, $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$, for every $e \in E$ and F^c is called the soft complement function of F.

Definition 2.4 ([12]).

I. The difference of two soft sets (F, E) and (G, E) over the common universe, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E, H(e) = F(e) - G(e)$.

II. Let (F, E) be a soft set over X *and* $x \in X$. We say that $x \in (F, E)$ read as x belong to the soft set (F, E), if $x \in F(e)$ for all e $\in E$.

III. Let $\tilde{\tau}$ be a collection of soft sets over a universe X with a fixed set of parameters, then $\tilde{\tau}$ is called a soft topology on X if,

(i) $\tilde{X}$, $\tilde{\emptyset} \tilde{\in} \tilde{\tau}$, where $\tilde{\emptyset}(e) = \emptyset$ and $\tilde{X}$ (e) = X for every $e \in E$.

(ii) If (F, E) , $(G, E) \tilde{\in} \tilde{\tau}$, then $(F, E) \tilde{\cap} (G, E) \tilde{\in} \tilde{\tau}$.

(iii) If $\{(F_i, E)\}_{i\in I} \tilde{\in} \tilde{\tau}$, then $\tilde{\bigcup}_{i\in I} (F_i, E) \tilde{\in} \tilde{\tau}$.

The triple $(X, \tilde{\tau}, E)$ is called a soft topological space over $\tilde{X}$.

IV. Let $(X, \tilde{\tau}, E)$ be a soft topological space over$\tilde{X}$. The members of $\tilde{\tau}$ are called soft open sets in $\tilde{X}$ and their complements are called soft closed sets in $\tilde{X}$. We denote the set of all soft open (resp. soft closed) sets by $\tilde{S}O(\tilde{X})$(resp. $\tilde{S}C(\tilde{X})$).

V. Let $(X, \tilde{\tau}, E)$ be a soft topological space and $(F, E) \tilde{\in} SS(X)_E$. The soft closure of (F, E), denoted by $\tilde{s}cl(F, E)$ is the soft intersection of all soft closed supersets of (F, E) i.e

$\tilde{s}cl(F, E) = \tilde{\bigcap} \{ (H, E) : (H, E)$ is soft closed set and $(F, E) \tilde{\subseteq} (H, E)\}$.

Clearly $\tilde{s}cl(F, E)$ is the smallest soft closed set which contains(F, E).

Definition 2.5 ([13]**).**

I. Let J be an arbitrary indexed set and $L = \{(F_i, E),\ i \in J\}$ be a subfamily of $SS(X)_E$.

(1) The soft union of L is the soft set(H, E), where $H(e) = \tilde{\bigcup}_{i\in I} F_i(e)$for each$e \in E$. We write $\tilde{\bigcup}_{i\in I} (F_i, E) = (H, E)$.

(2) The soft intersection of L is the soft set (M,E), where $M(e) = \widetilde{\bigcap}_{i\in I} F_i(e)$ for each $e \in E$. We write $\widetilde{\bigcap}_{i\in I}(F_i,E) = (M,E)$.

II. Let $(X,\tilde{\tau},E)$ be a soft topological space and $(G,E) \tilde{\in} SS(X)_E$. The soft interior of (G,E), denoted by $\tilde{s}int(G,E)$ is the soft union of all soft open subset of (G,E) i.e

$\tilde{s}int(G,E) = \widetilde{\bigcup}\{(H,E) : (H,E)$ is soft open set and $(H,E) \tilde{\subseteq} (G,E)\}$

Clearly $\tilde{s}int(G,E)$ is the largest soft open set which contained in (G,E).

III. The soft set $(F,E) \tilde{\in} SS(X)_E$ is called a soft point in $\tilde{X}$ if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \emptyset$ for each $e' \in E - \{e\}$, and the soft point (F,E) is denoted by x_e.

IV. The soft point x_e is said to be soft belongs to the soft set (G,E), denoted by $x_e \tilde{\in} (G,E)$, if for the element $e \in E$, $F(e) \subseteq G(e)$.

V. A soft set (G,E) in a soft topological space $(X,\tilde{\tau},E)$ is called a soft neighborhood of the soft point $x_e \tilde{\in} \tilde{X}$ if there exists a soft open set (H,E) such that $x_e \tilde{\in} (H,E) \tilde{\subseteq} (G,E)$. The neighborhood system of a soft point x_e denoted by $N_\tau(x_e)$ is the family of all it's neighborhood.

A soft set (G,E) in a soft topological space $(X,\tilde{\tau},E)$ is called a soft neighborhood of the soft set (F,E) if there exists a soft open set (H,E) such that $(F,E) \tilde{\subseteq} (H,E) \tilde{\subseteq} (G,E)$.

Theorem 2.1 ([13]**).** Let $(X,\tilde{\tau},E)$ be a soft topological space and $(F,E),(G,E) \tilde{\in} SS(X)_E$. Then

(i) $\tilde{s}int(\tilde{\emptyset}) = \tilde{\emptyset}$ and $\tilde{s}int(\tilde{X}) = \tilde{X}$.

(ii) If $(F,E) \tilde{\subseteq} (G,E)$, then $\tilde{s}int(F,E) \tilde{\subseteq} \tilde{s}int(G,E)$.

(iii) A soft set (F,E) is soft open set if and only if $(F,E) = \tilde{s}int(F,E)$.

(iv**)** $\tilde{s}cl(\tilde{\emptyset}) = \tilde{\emptyset}$ and $\tilde{s}cl(\tilde{X}) = \tilde{X}$.

(v) If $(F,E) \tilde{\subseteq} (G,E)$, then $\tilde{s}cl(F,E) \tilde{\subseteq} \tilde{s}cl(G,E)$.

(vi) A soft set (F,E) is soft closed set if and only if $(F,E) = \tilde{s}cl(F,E)$.

Definition 2.6 ([4]). Let $(X,\tilde{\tau},E)$ be a soft topological space over $\tilde{X}$ then the soft frontier of a soft set (F,E) is $\tilde{s}F_r(F,E) = \tilde{s}cl(F,E) \tilde{\cap} \tilde{s}cl(F,E)^c$**.**

Definition 2.7 ([2]). Let $(X,\tilde{\tau},E)$ be a soft topological space. A soft point $x_e \tilde{\in} \tilde{X}$ is said to be soft limit point of a soft set (F,E) if for each $(U,E) \tilde{\in} \tilde{S}O(\tilde{X})$, $(U,E) \tilde{\cap} ((F,E) - x_e) \neq \tilde{\emptyset}$. The set of all soft limit points of (F,E) is called the soft derived set of (F,E) and is denoted by $\tilde{s}d\,(F,E)$.

Definition 2.8 ([4]). Let $(X, \tilde{\tau}, E)$ be a soft topological space and $(F, E) \tilde{\in} SS(X)_E$**.** A soft point $x_e \tilde{\in} \tilde{X}$ is said to be a soft exterior point of (F, E) if x_e is a soft interior point of $(F, E)^c$. The soft exterior of (F, E) is denoted by $\tilde{s}Ext(F, E)$. Thus $\tilde{s}Ext(F, E) = \tilde{s}int(F, E)^c$.

Theorem 2.2 ([10]). Let $(X, \tilde{\tau}, E)$ be a soft topological space and $(F, E) \tilde{\in}$ SS$(X)_E$. A soft point $x_e \tilde{\in} \tilde{s}cl(F, E)$ if and only if each soft neighborhood of x_e intersects (F, E).

Definition 2.9 ([6]).
I. Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with the same set of parameters E. Then $\tilde{I} \tilde{\subseteq} SS(X)_E$ is called a soft ideal on X with the same set E if
(1) $(F, E) \tilde{\in} \tilde{I}$ and $(G, E) \tilde{\in} \tilde{I}$, $then$ $(F, E) \tilde{\cup} (G, E) \tilde{\in} \tilde{I}$,
(2) $(F, E) \tilde{\in} \tilde{I}$ and $(G, E) \tilde{\subseteq} (F, E)$, $then$ $(G, E) \tilde{\in} \tilde{I}$,
i.e. $\tilde{I}$ is closed under finite soft unions and soft subsets.
II. Let $(X, \tilde{\tau}, E)$ be a soft topological space and $\tilde{I}$ be a soft ideal over X. Then $(X, \tilde{\tau}, E, \tilde{I})$ is called a soft ideal topological space. Let (F, E) $\tilde{\in}$ SS$(X)_E$, The soft operator $* : SS(X)_E \rightarrow SS(X)_E$, defined by
$(F, E)^*$ (Ĩ, $\tilde{\tau}$) or $(F, E)^* = \tilde{\cup} \{x_e \tilde{\in} \tilde{X} : O_{x_e} \tilde{\cap} (F, E) \tilde{\notin} \tilde{I} \; \forall \; O_{x_e} \tilde{\in} \tilde{\tau} \}$,
is called the soft local function of (F, E) with respect to $\tilde{I}$ and $\tilde{\tau}$, where O_{x_e} is a $\tilde{\tau}$ – soft open set containing x_e.

Theorem 2.3 ([6]). Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space. Then the soft closure operator $\tilde{s}cl^* : SS(X)_E \rightarrow SS(X)_E$, defined by $\tilde{s}cl^*(F, E) = (F, E) \tilde{\cup} (F, E)^*$, satisfy Kuratwski's axioms.

Definition 2.10 ([6]). In a soft topological space$(X, \tilde{\tau}, E)$, a soft set (F, E) is said to be soft preopen set if $(F, E) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(F, E))$**.**

Definition 2.11 ([3]).
I. Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \tilde{\in} SS(X)_E$, Then (F, E) is said to be a soft -$\tilde{I}$-open set if $(F, E) \tilde{\subseteq} \tilde{s}int(F, E)^*$ and the complement of soft -$\tilde{I}$-open set is called soft -$\tilde{I}$- closed. We denote the set of all soft -$\tilde{I}$-open sets by $\tilde{S}\tilde{I}O(\tilde{X})$ and the set of all soft -$\tilde{I}$-closed sets by $\tilde{S}\tilde{I}C(\tilde{X})$.
II. Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \tilde{\in} SS(X)_E$, Then (F, E) is said to be a soft pre-$\tilde{I}$-open set if $(F, E) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl^*(F, E))$. The family of all soft pre-$\tilde{I}$- open sets in $(X, \tilde{\tau}, E, \tilde{I})$ is

denoted by $\tilde{S}P\tilde{I}O(\tilde{X})$. The complement of a soft pre-$\tilde{I}$- open set is called soft pre-$\tilde{I}$-closed and the family of all soft pre-$\tilde{I}$- closed sets in $(X, \tilde{\tau}, E, \tilde{I})$ is denoted by $\tilde{S}P\tilde{I}C(\tilde{X})$.

III. Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \tilde{\in} SS(X)_E$. Then,

(i) x_e is called a soft pre -$\tilde{I}$- interior point of (F, E) if there exists $(G, E) \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X})$ such that $x_e \tilde{\in} (G, E) \tilde{\subseteq} (F, E)$, the set of all soft pre -$\tilde{I}$- interior points of (F, E) is called the soft pre -$\tilde{I}$- interior of (F, E) and is denoted by $\tilde{s}p\tilde{I}int(F, E)$. Consequently,

$\tilde{s}p\tilde{I}int(F, E) = \tilde{\bigcup} \{ (G, E) : (G, E) \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X}), (G, E) \tilde{\subseteq} (F, E)\}$

(ii) x_e is called a soft pre -$\tilde{I}$- closure point of (F, E) if $(F, E) \tilde{\cap} (H, E) \neq \tilde{\emptyset}$ for every $(H, E) \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X})$ and $x_e \tilde{\in} (H, E)$. The set of all soft pre -$\tilde{I}$- closure points of (F, E) is called the soft pre -$\tilde{I}$- closure of (F, E) and is denoted by $\tilde{s}p\tilde{I}cl(F, E)$ consequently,

$\tilde{s}p\tilde{I}cl(F, E) = \tilde{\bigcap} \{ (H, E) : (H, E) \tilde{\in} \tilde{s}p\tilde{I}C(\tilde{X}), (F, E) \tilde{\subseteq} (H, E)\}$.

Definition 2.12 ([7]). A soft subset (F, E) of a soft ideal topological space $(X, \tilde{\tau}, E, \tilde{I})$ is said to be $*$ - soft dense if $\tilde{s}cl^*(F, E) = \tilde{X}$.

Remark 2.1.

(i) Every soft open set is a soft preopen set [5].

(ii) Every soft -$\tilde{I}$-open set is a soft pre-$\tilde{I}$-open set [3].

(iii) Every soft pre-$\tilde{I}$-open set is a soft preopen set [3].

Theorem 2.4 ([3]). Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space, and $(F, E), (G, E) \in SS(X)_E$. Then the following hold:

(i) $\tilde{s}p\tilde{I}cl\ (\tilde{X} - (F, E)) = \tilde{X} - \tilde{s}p\tilde{I}int\ (F, E)$.

(ii) $\tilde{s}p\tilde{I}int\ (\tilde{X} - (F, E)) = \tilde{X} - \tilde{s}p\tilde{I}cl\ (F, E)$.

(iii) $\tilde{s}p\tilde{I}int\ [(F, E) \tilde{\cap} (G, E)] \tilde{\subseteq} \tilde{s}p\tilde{I}int\ (F, E) \tilde{\cap} \tilde{s}p\tilde{I}int\ (G, E)$.

(iv) $(F, E) \tilde{\in} \tilde{S}P\tilde{I}C(\tilde{X}) \Leftrightarrow (F, E) = \tilde{s}p\tilde{I}cl\ (F, E)$.

(v) $\tilde{s}p\tilde{I}cl\ (F, E)$ is the smallest soft pre - $\tilde{I}$- closed set in $\tilde{X}$ containing (F, E).

(vi) $(F, E) \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X}) \Leftrightarrow (F, E) = \tilde{s}p\tilde{I}int\ (F, E)$.

(vii) $\tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}int\ (F, E)) = \tilde{s}p\tilde{I}int\ (F, E)$.

(viii) Arbitrary soft union of soft pre - $\tilde{I}$- open sets is soft pre - $\tilde{I}$- open set.

(ix) If $(F, E) \tilde{\subset} (G, E)$, then $\tilde{s}p\tilde{I}int\ (F, E) \tilde{\subset} \tilde{s}p\tilde{I}int\ (G, E)$.

(x) $\tilde{s}p\tilde{I}int\ [(F, E) \tilde{\cup} (G, E)] \tilde{\supseteq} \tilde{s}p\tilde{I}int\ (F, E) \tilde{\cup} \tilde{s}p\tilde{I}int\ (G, E)$.

(xi) $\tilde{s}p\tilde{I}cl[(F, E) \tilde{\cap} (G, E)] \tilde{\subseteq} \tilde{s}p\tilde{I}cl\ (F, E) \tilde{\cap} \tilde{s}p\tilde{I}cl\ (G, E)$.

3. Some results on soft pre - $\tilde{I}$ – Open sets

Now we study some results related to soft pre - $\tilde{I}$- open sets and introduce the concept of soft pre - $\tilde{I}$- neighborhood of a soft point.

Theorem 3.1. Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \tilde{\in} SS(X)_E$. If (F, E) is a $*$ - soft dense set in $(X, \tilde{\tau}, E, \tilde{I})$ then (F, E) is soft pre - $\tilde{I}$- open set.

Proof.
Suppose (F, E) is $*$ - soft dense set in$(X, \tilde{\tau}, E, \tilde{I})$. Then $\tilde{s}cl^*(F, E) = \tilde{X}$ which implies that $\tilde{s}int\,(\tilde{s}cl^*(F, E)) = \tilde{s}int\ \tilde{X} = \tilde{X}$. Thus $(F, E) \tilde{\subseteq} \tilde{s}int\,(\tilde{s}cl^*(F, E))$, therefore (F, E) is soft pre-$\tilde{I}$- open.

Remark 3.1
The converse of Theorem 3.1 need not be true as shown by the following counter example.

Example 3.1
Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space, where $X = \{h_1, h_2, h_3\}$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\emptyset}, \{(e, \{h_1\})\}, \{(e, \{h_2, h_3\})\}\}$ and $\tilde{I} = \{\tilde{\emptyset}, \{(e, \{h_1\})\}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_3\})\}\}$. Then one can deduce that $\tilde{S}P\tilde{I}O(\tilde{X}) = \{\tilde{X}, \tilde{\emptyset}, \{(e, \{h_1\})\}, \{(e, \{h_2, h_3\})\}, \{(e, \{h_2\})\}, \{(e, \{h_1, h_2\})\}\}$. Let $(G, E) = \{(e, \{h_2\})\}$. Then $(G, E)^* = \{(e, \{h_2, h_3\})\}$. Thus $\tilde{s}cl^*(G, E) = \{(e, \{h_2, h_3\})\} \neq \tilde{X}$. Therefore (G, E) is not $*$ - soft dense set.

Theorem 3.2. Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space, and $(F, E), (G, E) \tilde{\in} SS(X)_E$ and (G, E) is a soft open set such that $(F, E) \tilde{\subseteq} (G, E) \tilde{\subseteq} \tilde{s}cl^*(F, E)$. Then (F, E) is soft pre - $\tilde{I}$- open.

Proof.
Since$(F, E) \tilde{\subseteq} (G, E) \tilde{\subseteq} \tilde{s}cl^*(F, E)$, then $\tilde{s}int(F, E) \tilde{\subseteq} \tilde{s}int\,(G, E) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl^*(F, E)$. Thus, $\tilde{s}int(F, E) \tilde{\subseteq} (G, E) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl^*(F, E)$. But, since$(G, E) \tilde{\in} \tilde{\tau}$, we have $(G, E) = \tilde{s}int(G, E)$. Hence, $(F, E) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl^*(F, E)$. Therefore (F, E) is soft pre - $\tilde{I}$- open in $\tilde{X}$.

Definition 3.1. A soft set (G,E) in a soft ideal topological space $(X,\tilde{\tau},E,\tilde{I})$ is called a soft pre - $\tilde{I}$- neighborhood of the soft point $x_e \tilde{\in} \tilde{X}$ if there exists soft pre - $\tilde{I}$- open set $(U,E) \tilde{\in} \widetilde{SPIO}(\tilde{X})$ such that $x_e \tilde{\in} (U,E) \tilde{\subseteq} (G,E)$. Note that (G,E) is a soft pre - $\tilde{I}$- neighborhood of the soft point x_e if and only if x_e is a soft pre -$\tilde{I}$- interior point of (G,E).

Example 3.2
Let $(X,\tilde{\tau},E,\tilde{I})$ as in Example 3.1, then $\{(e,\{h_2,h_3\})\}$ is a soft pre - $\tilde{I}$- neighborhood of the soft point $x_e = \{(e,\{h_2\})\}$. Indeed, $x_e \tilde{\in} \{(e,\{h_2\})\} \tilde{\subset} \{(e,\{h_2,h_3\})\}$ and $\{(e,\{h_2\})\} \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X})$.

Theorem 3.3. Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E) \tilde{\in} SS(X)_E$. Then (F,E) is a soft pre - $\tilde{I}$- open set if and only if it is a soft pre - $\tilde{I}$- neighborhood of each of it's soft points.

Proof.
Let $(G,E) \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X})$. Then, by definition, (G,E) is a soft pre - Ĩ- neighborhood of each of its soft points. Conversely, suppose (G,E) is a soft pre - $\tilde{I}$- neighborhood of each of its soft points. Then for each $x_e \tilde{\in} (G,E)$, there exists $(U,E) \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X})$ such that $x_e \tilde{\in} (U,E) \tilde{\subset} (G,E)$. Clearly $(G,E) = \tilde{\cup}\{(U,E): x_e \tilde{\in} (U,E) \tilde{\subset} (G,E)\}$. It follows that $(G,E) \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X})$.

Remark 3.2
Soft - $\tilde{I}$- openness and soft openness are independent of each other as shown by the following two examples.

Example 3.3
Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space, where $X = \{h_1,h_2,h_3,h_4\}$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\emptyset}, \{(e,\{h_3\})\}, \{(e,\{h_1,h_2\})\}, \{(e,\{h_1,h_2,h_3\})\}\}$ And $\tilde{I} = \{\tilde{\emptyset}, \{(e,\{h_1\})\}\}$. Then $\{(e,\{h_2,h_3,h_4\})\} \tilde{\in} \tilde{S}\tilde{I}O(\tilde{X})$, but $\{(e,\{h_2,h_3,h_4\})\} \tilde{\notin} \tilde{\tau}$.

Example 3.4
Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space, where $X = \{h_1,h_2,h_3,h_4\}$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\emptyset}, \{(e,\{h_4\})\}, \{(e,\{h_1,h_3\})\}, \{(e,\{h_1,h_3,h_4\})\}\}$ and $\tilde{I} = \{\tilde{\emptyset}, \{(e,\{h_3\})\}, \{(e,\{h_4\})\}, \{(e,\{h_3,h_4\})\}\}$. It is clear that $\{(e,\{h_1,h_3,h_4\})\} \tilde{\in} \tilde{\tau}$ but $\{(e,\{h_1,h_3,h_4\})\} \tilde{\notin} \tilde{S}\tilde{I}O(\tilde{X})$.

Remark 3.3
Since every soft open set is a soft pre - $\tilde{I}$- open set, then every soft interior point of (F,E) is a soft pre-$\tilde{I}$-interior point of (F,E) and $\tilde{s}int(F,E) \tilde{\subseteq} \tilde{s}p\tilde{I}int(F,E)$. In general, $\tilde{s}int(F,E) \neq \tilde{s}p\tilde{I}int(F,E)$ as shown by the following example.

Example 3.5:
Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space, where $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$, $\tilde{\tau}$ ={ $\tilde{X}$, $\tilde{\emptyset}$, $\{(e_1,\{h_2\})\}$} and $\tilde{I}$ = { $\tilde{\emptyset}$, $\{(e_1,\{h_1\})\}\}$. One can deduce that, $\tilde{S}P\tilde{I}O(\tilde{X})$ = {$\tilde{X}$, $\tilde{\emptyset}$, $\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$, $\{(e_1,\tilde{X}),(e_2,\{h_1\})\}$, $\{(e_1,\{h_2\}),(e_2,\{h_1\})\}$, $\{(e_1,\tilde{X}),(e_2,\{h_2\})\}$, $\{(e_1,\{h_2\}),(e_2,\tilde{X})\}$, $\{(e_1,\tilde{X})\}$, $\{(e_1,\{h_2\})\}\}$. We have $\tilde{s}int\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$ = $\{(e_1,\{h_2\})\}\}$ and $\tilde{S}P\tilde{I}int\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$ = $\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$. Therefore, $\tilde{s}int\{(e_1,\{h_2\}),(e_2,\{h_2\})\} \neq \tilde{S}P\tilde{I}int\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$.

Remark 3.4
$\tilde{s}p\tilde{I}int(F,E)$= $\tilde{s}p\tilde{I}int(G,E)$ doesn't imply that $(F,E) = (G,E)$ as can be shown by the following example.

Example 3.6
Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space, where$X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$, $\tilde{\tau}$ ={ $\tilde{X}$, $\tilde{\emptyset}$, $\{(e_1,\{h_1\})\}$, $\{(e_2,\{h_2\})\}$, $\{(e_1,\{h_1\}), (e_2,\{h_2\})\}\}$ and $\tilde{I}$ = { $\tilde{\emptyset}$, $\{(e_1,\{h_1\})\}$, $\{(e_1,\{h_1\}), (e_2,\{h_2\})\}\}$. Then one can deduce that $\tilde{S}P\tilde{I}O(\tilde{X})$ = { $\tilde{X}$, $\tilde{\emptyset}$, $\{(e_1,\{h_1\})\}$, $\{(e_2,\{h_2\})\}$, $\{(e_1,\{h_1\}), (e_2,\{h_2\})\}$, $\{(e_1,\{h_1\}), (e_2,\tilde{X})\}$, $\{(e_1,\{h_2\}), (e_2,\tilde{X})\}$, $\{(e_1,\tilde{X}), (e_2,\{h_2\})\}\}$. If we take (F,E) = $\{(e_1,\{h_1\})\}$ and (G,E) = $\{(e_1,\tilde{X})\}$, then we have $\tilde{S}P\tilde{I}int(F,E)$ = $\{(e_1,\{h_1\})\}$ and $\tilde{s}p\tilde{I}int(G,E)$ = $\{(e_1,\{h_1\})\}$. Therefore, $\tilde{s}p\tilde{I}int(F,E)$= $\tilde{s}p\tilde{I}int(G,E)$ but$(F,E) \neq (G,E)$.

Remark 3.5
In general, $\tilde{s}p\tilde{I}int[(F,E) \tilde{\cap} (G,E)] \neq \tilde{s}p\tilde{I}int(F,E) \tilde{\cap} \tilde{s}p\tilde{I}int(G,E)$, as shown by the following example.

Example 3.7

Let $(X,\tilde{\tau},E,\tilde{I})$ be the soft ideal topological space as in Example 3.6. If we take (F,E) = $\{(e_1,\{h_2\}), (e_2,\tilde{X})\}$ and (G,E)= $\{(e_1,\tilde{X}), (e_2,\{h_2\})\}$ so that $(F,E) \tilde{\cap} (G,E) = \{(e_1,\{h_2\})\}$, $\tilde{s}p\tilde{I}int[(F,E) \tilde{\cap} (G,E)] = \tilde{\emptyset}$, also we have $\tilde{s}p\tilde{I}int(F,E) = \{(e_1,\{h_2\}), (e_2,\tilde{X})\}$ and $\tilde{s}p\tilde{I}int(G,E) = \{(e_1,\tilde{X}), (e_2,\{h_2\})\}$, then we have $\tilde{s}p\tilde{I}int(F,E) \tilde{\cap} \tilde{s}p\tilde{I}int(G,E) = \{(e_1,\{h_2\})\}$, thus $\tilde{S}P\tilde{I}int[(F,E) \tilde{\cap} (G,E)] \neq \tilde{s}p\tilde{I}int(F,E) \tilde{\cap} \tilde{s}p\tilde{I}int(G,E)$.

Theorem 3.4. Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E),(H,E) \tilde{\in} SS(X)_E$. Then,
$\tilde{s}p\tilde{I}int[(F,E)-(H,E)] \tilde{\subseteq} \tilde{s}p\tilde{I}int(F,E) - \tilde{s}p\tilde{I}int(H,E)$.

Proof.
Let $x_e \tilde{\in} \tilde{s}p\tilde{I}int[(F,E)-(H,E)]$. Then there exists a soft pre - Ĩ- open set (G,E) containing x_e such that
$x_e \tilde{\in} (G,E) \tilde{\subseteq} (F,E)-(H,E) \tilde{\subseteq} (F,E)$. This shows that $(G,E) \tilde{\cap} (H,E) = \tilde{\emptyset}$. Hence $x_e \tilde{\notin} \tilde{s}p\tilde{I}int(H,E)$ and $x_e \tilde{\in} \tilde{s}p\tilde{I}int(F,E)$. Therefore, $\tilde{s}p\tilde{I}int[(F,E)-(H,E)] \tilde{\subseteq} \tilde{s}p\tilde{I}int(F,E) - \tilde{s}p\tilde{I}int(H,E)$. The equality in the above theorem doesn't hold in general, as illustrated by the following example.

Example 3.8
Let$(X,\tilde{\tau},E,\tilde{I})$ be the soft ideal topological space as in Example 3.5. Take $(F,E) = \{(e_1,\{h_2\}),(e_2,\{h_2\})\}$ and (G,E)=$\{(e_1,\tilde{X}),(e_2,\{h_1\})\}$, then $\tilde{s}p\tilde{I}int(F,E) = \{(e_1,\{h_2\}),(e_2,\{h_2\})\}$ and $\tilde{s}p\tilde{I}int(G,E) =$
$\{(e_1,\tilde{X}),(e_2,\{h_1\})\}$. Therefore $\tilde{s}p\tilde{I}int(F,E) - \tilde{s}p\tilde{I}int(G,E) =$
$\{(e_2,\{h_2\})\}$ and $\tilde{s}p\tilde{I}int[(F,E)-(G,E)] = \tilde{s}p\tilde{I}int\{(e_2,\{h_2\})\} = \tilde{\emptyset}$.
Therefore, $\tilde{s}p\tilde{I}int[(F,E)-(G,E)] \neq \tilde{s}p\tilde{I}int(F,E) - \tilde{s}p\tilde{I}int(G,E)$.

Theorem 3.5 Let$(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and$(F,E),(G,E) \tilde{\in} SS(X)_E, x_e \tilde{\in} \tilde{X}$. Then $x_e \tilde{\in} \tilde{s}p\tilde{I}cl\,(F,E)$ if and only if $(G,E) \tilde{\cap} (F,E) \neq \tilde{\emptyset}$ for every a soft pre- $\tilde{I}$- open set (G,E) containingx_e.

Proof.
Suppose $x_e \tilde{\notin} \tilde{s}p\tilde{I}cl\,(F,E)$, then there exists a soft pre - $\tilde{I}$- closed set (H,E) such that $(F,E) \tilde{\subseteq} (H,E)$ and $x_e \tilde{\notin} (H,E)$. Hence $(H,E)^c$ is a soft pre- $\tilde{I}$- open set containing x_e and $(F,E) \tilde{\cap} (H,E)^c \tilde{\subseteq} (F,E) \tilde{\cap} (F,E)^c = \tilde{\emptyset}$. This is a contradiction, and hence $x_e \tilde{\in} \tilde{s}p\tilde{I}cl\,(F,E)$. Conversely, if

there exists a soft pre- $\tilde{I}$- open set (G,E) containing x_e which doesn't intersect (F,E) implies that $(G,E)\,\tilde{\cap}\,(F,E) = \tilde{\emptyset}$. By definition we get $x_e \tilde{\notin}\ \tilde{s}p\tilde{I}cl\,(F,E)$, a contradiction, so $(G,E)\,\tilde{\cap}\,(F,E) \neq \tilde{\emptyset}$ for every a soft pre- $\tilde{I}$- open set (G,E) containing x_e.

Remark 3.6
It obvious that $\tilde{s}p\tilde{I}cl\,(F,E) \tilde{\subseteq}\ \tilde{s}cl\,(F,E)$. The converse is false as shown by the following example

Example 3.9
Let$(X,\tilde{\tau},E,\tilde{I})$ be the soft ideal topological space as in Example 3.5, if we take$(F,E) = \{(e_1,\{h_1\})\}$, then $\tilde{s}cl\,(F,E)$= $\{(e_1,\{h_1\}),(e_2,\tilde{X})\}$ and $\tilde{s}p\tilde{I}cl\,(F,E)$ = $\{(e_1,\{h_1\})\}$. Hence $\tilde{s}cl\,(F,E) \tilde{\not\subseteq}\ \tilde{s}p\tilde{I}cl(F,E)$.

4. Soft pre - $\tilde{I}$- limit point

In this section we introduce the concept of soft pre - $\tilde{I}$- limit point of a soft set and study some of its properties.

Definition 4.1 Let $(X,\tilde{\tau},E,\tilde{I}\ )$ be a soft ideal topological space. A soft point $x_e \tilde{\in} \tilde{X}$ is said to be soft pre- $\tilde{I}$- limit point of a soft set (F,E) if for each $(U,E)\ \tilde{\in}\ \tilde{S}P\tilde{I}O(\tilde{X})$, $(U,E)\ \tilde{\cap}\,((F,E)$- $x_e) \neq \tilde{\emptyset}$. The set of all soft pre - $\tilde{I}$- limit points of (F,E) is called the soft pre - $\tilde{I}$- derived set of (F,E) and is denoted by$\tilde{s}p\tilde{I}d\,(F,E)$.

Example 4.1
Let$(X,\tilde{\tau},E,\tilde{I})$ be the soft ideal topological space as in Example 3.7, if we take $(F,E) = \{(e_1,\{h_2\}),\ (e_2,\tilde{X})\}$, then $\tilde{s}p\tilde{I}d\,(F,E)$ =$\{\{(e_1,\{h_2\})\},\{(e_2,\{h_1\})\}\}$.

Remark 4.1 Since every soft open set is soft pre - $\tilde{I}$- open set, it follows that every soft pre - $\tilde{I}$- limit point of (F,E) is a soft limit point of (F,E).

Theorem 4.1 Let $(X,\tilde{\tau},E,\tilde{I}\ )$ be a soft ideal topological space and $(F,E)\ \tilde{\in}\ SS(X)_E$. Then $(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d(F,E)$ is a soft pre - $\tilde{I}$-closed set.

Proof.
Let $x_e\ \tilde{\in}\ (\tilde{X} - [(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d(F,E)])$. Since $x_e \tilde{\notin}\ \tilde{s}p\tilde{I}d\,(F,E)$, there exists a soft pre- $\tilde{I}$- open set (G,E) such that $x_e \tilde{\in} (G,E)$ and

$(G,E)\ \tilde{\cap}\ (F,E) = \tilde{\emptyset}$ or $(G,E)\ \tilde{\cap}\ (F,E) = x_e$. However, $x_e \tilde{\notin} (F,E)$; hence, in particular, $(G,E)\ \tilde{\cap}\ (F,E) = \tilde{\emptyset}$. We also claim that $(G,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}d\ (F,E) = \tilde{\emptyset}$. For if $y_e \tilde{\in} (G,E)$, then $y_e \tilde{\in} (G,E)$ and $G,E)\ \tilde{\cap}\ (F,E) = \tilde{\emptyset}$. So $y_e \tilde{\notin} \tilde{s}p\tilde{I}d\ (F,E)$ and thus $(G,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}d\ (F,E) = \tilde{\emptyset}$. Accordingly, $(G,E)\ \tilde{\cap}\ ((F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d(F,E))=$ $\left((G,E)\ \tilde{\cap}\ (F,E)\right) \tilde{\cup} \left((G,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}d\ (F,E)\right) = \tilde{\emptyset}$ and so $(G,E) \tilde{\subseteq} \tilde{X} - [(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d(F,E)]$. Thus x_e is soft pre - $\tilde{I}$- interior point of $\tilde{X} - [(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d(F,E)]$ which is therefore a soft pre- $\tilde{I}$- open set. Hence $(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d(F,E)$ is a soft pre - $\tilde{I}$-closed set.

Theorem 4.2 Let $(X,\tilde{\tau},E,\tilde{I}\ )$ be a soft ideal topological space and $(F,E) \tilde{\in} SS(X)_E$. Then $\tilde{s}p\tilde{I}cl\ (F,E)$= $(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d\ (F,E)$.

Proof.
From Theorem 4.1, $(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d(F,E)$ is a soft pre - Ĩ-closed set containing (F,E)**.** Therefore $\tilde{s}p\tilde{I}cl\ (F,E) \tilde{\subseteq} ((F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d\ (F,E))$. Conversely, let $x_e \tilde{\notin} \tilde{s}p\tilde{I}cl\ (F,E)$, then there exists a soft pre- $\tilde{I}$- open set (G,E) such that $x_e \tilde{\in} (G,E)$ and $(G,E)\ \tilde{\cap}\ (F,E) = \tilde{\emptyset}$. Then $x_e \tilde{\notin} (F,E)$ and $x_e \tilde{\notin} \tilde{s}p\tilde{I}d\ (F,E)$. Therefore $x_e \tilde{\notin} ((F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d\ (F,E))$ which shows that $(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d\ (F,E) \tilde{\subseteq} \tilde{s}p\tilde{I}cl\ (F,E)$. This completes the proof.

Corollary 4.1 For any soft set $(F,E) \tilde{\in} SS(X)_E$, we have $\tilde{s}p\tilde{I}d\ (F,E) \tilde{\subseteq} \tilde{s}p\tilde{I}cl\ (F,E)$.

Theorem 4.3 Let $(X,\tilde{\tau},E,\tilde{I}\ )$ be a soft ideal topological space and $(F,E) \tilde{\in} SS(X)_E$. Then (F,E) is a soft pre - $\tilde{I}$-closed set if and only if it contains all of it's soft pre - $\tilde{I}$- limit points.

Proof.
Follows directly from theorem 4.3.

Theorem 4.4 Let $(X,\tilde{\tau},E,\tilde{I}\ )$ be a soft ideal topological space and $\{(F_i,E) : i \in I\}$ be any family of soft subsets of $(X,\tilde{\tau},E,\tilde{I}\ )$. If $\tilde{\bigcup}_{i\in I}\tilde{s}p\tilde{I}cl(F_i,E)$ is soft pre - $\tilde{I}$- closed, then $\tilde{\bigcup}_{i\in I}\tilde{s}p\tilde{I}cl(F_i,E) = \tilde{s}p\tilde{I}cl(\tilde{\bigcup}_{i\in I}(F_i,E))$.

Proof.

Since $(F_i, E) \tilde{\subseteq} \tilde{\bigcup}_{i\in I}(F_i, E)$, $\tilde{s}p\tilde{I}cl(\tilde{\bigcup}_{i\in I}(F_i, E)) \tilde{\subseteq} \tilde{s}p\tilde{I}cl(\tilde{\bigcup}_{i\in I}(F_i, E))$ and hence $\tilde{\bigcup}_{i\in I}\tilde{s}p\tilde{I}cl(F_i, E) \tilde{\subseteq} \tilde{s}p\tilde{I}cl(\tilde{\bigcup}_{i\in I}(F_i, E))$. We will show that $\tilde{s}p\tilde{I}cl(\tilde{\bigcup}_{i\in I}(F_i, E)) \tilde{\subseteq} \tilde{\bigcup}_{i\in I}\tilde{s}p\tilde{I}cl(F_i, E)$. Let $x_e \tilde{\notin} \tilde{\bigcup}_{i\in I}\tilde{s}p\tilde{I}cl(F_i, E)$. Since $\tilde{\bigcup}_{i\in I}\tilde{s}p\tilde{I}cl(F_i, E)$ is soft pre - $\tilde{I}$- closed it contains all it's pre - $\tilde{I}$-limit points and so, there exists a soft pre-$\tilde{I}$-neighborhood (U, E) of x_e such that $(U, E) \tilde{\cap}(\tilde{\bigcup}_{i\in I}\tilde{s}p\tilde{I}cl(F_i, E)) = \tilde{\emptyset}$ this implies that $(U, E) \tilde{\cap} \tilde{s}p\tilde{I}cl(F_i, E) = \tilde{\emptyset}$ for every $i \in I$ and hence $(U, E) \tilde{\cap} (F_i, E) = \tilde{\emptyset}$ then we have $x_e \tilde{\notin} \tilde{s}p\tilde{I}cl(\tilde{\bigcup}_{i\in I}(F_i, E))$. Therefore,
$\tilde{s}p\tilde{I}cl(\tilde{\bigcup}_{i\in I}(F_i, E)) \tilde{\subseteq} \tilde{\bigcup}_{i\in I}\tilde{s}p\tilde{I}cl(F_i, E)$. Which completes the proof.

Theorem 4.5 Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E), (G, E) \tilde{\in} SS(X)_E$, then the following hold.
(i) $\tilde{s}p\tilde{I}d(F, E) \tilde{\subseteq} \tilde{s}d(F, E)$, where $\tilde{s}d(F, E)$ is the soft derived set of (F, E).
(ii) If $(F, E) \tilde{\subseteq} (G, E)$, then $\tilde{s}p\tilde{I}d(F, E) \tilde{\subseteq} \tilde{s}p\tilde{I}d(G, E)$.
(iii) $\tilde{s}p\tilde{I}d((F, E) \tilde{\cup} \tilde{s}p\tilde{I}d(F, E)) \tilde{\subseteq} (F, E) \tilde{\cup} \tilde{s}p\tilde{I}d(F, E)$.
(iv) $\tilde{s}p\tilde{I}d(\tilde{\emptyset}) = \tilde{\emptyset}$.
(v) If $x_e \tilde{\in} \tilde{s}p\tilde{I}d(F, E)$, then $x_e \tilde{\in} \tilde{s}p\tilde{I}d((F, E) - x_e)$.
(vi) $\tilde{s}p\tilde{I}d(F, E) \tilde{\cup} \tilde{s}p\tilde{I}d(G, E) \tilde{\subseteq} \tilde{s}p\tilde{I}d[(F, E) \tilde{\cup} (G, E)]$.
(vii) $\tilde{s}p\tilde{I}d[(F, E) \tilde{\cap} (G, E)] \tilde{\subseteq} \tilde{s}p\tilde{I}d(F, E) \tilde{\cap} \tilde{s}p\tilde{I}d(G, E)$.
(viii) $\tilde{s}p\tilde{I}int(F, E) = (F, E) - \tilde{s}p\tilde{I}d(\tilde{X} - (F, E))$.

Proof.
(i) Follows from the fact that every soft open set is soft pre- $\tilde{I}$-open set.
(ii) Let $x_e \tilde{\in} \tilde{X}$ be a soft pre - $\tilde{I}$- limit point of (F, E). Then for each $(U, E) \tilde{\in} \tilde{S}P\tilde{I}O(\tilde{X})$, we have $(U, E) \tilde{\cap} [(F, E) - x_e] \neq \tilde{\emptyset}$ and hence it follows that $(U, E) \tilde{\cap} [(G, E) - x_e] \neq \tilde{\emptyset}$, thus $x_e \tilde{\in} \tilde{S}P\tilde{I}d(G, E)$. Thus $\tilde{s}p\tilde{I}d(F, E) \tilde{\subseteq} \tilde{s}p\tilde{I}d(G, E)$.
(iii) By Theorem 4.1, $(F, E) \tilde{\cup} \tilde{s}p\tilde{I}d(F, E)$ is a soft pre - Ĩ-closed set, and by Theorem 4.3, we get $(F, E) \tilde{\cup} \tilde{s}p\tilde{I}d(F, E)$ contains all its soft pre - $\tilde{I}$-limit points, i.e. $\tilde{s}p\tilde{I}d((F, E) \tilde{\cup} \tilde{s}p\tilde{I}d(F, E)) \tilde{\subseteq} (F, E) \tilde{\cup} \tilde{s}p\tilde{I}d(F, E)$.
(iv) Obvious.
(v) If $x_e \tilde{\in} \tilde{s}p\tilde{I}d(F, E)$, then x_e is soft pre - $\tilde{I}$- limit point of (F, E). So, every soft pre- $\tilde{I}$- neighborhood of x_e contains at least one soft point of (F, E) other than x_e. Consequently, x_e is soft pre - $\tilde{I}$- limit point of $((F, E) - x_e)$. Thus $x_e \tilde{\in} \tilde{s}p\tilde{I}d((F, E) - x_e)$.
(vi) and (vii) Follow directly by (ii)

(viii) If $x_e \tilde{\in} (F,E) - \tilde{s}p\tilde{I}d(\tilde{X} - (F,E))$, then $x_e \tilde{\notin} \tilde{s}p\tilde{I}d(\tilde{X} - (F,E))$ and so there exists a soft pre- $\tilde{I}$- open set (G,E) containing x_e such that (G,E) $\tilde{\cap}$ $(\tilde{X} - (F,E)) = \tilde{\emptyset}$. Thus $x_e \tilde{\in} (G,E) \tilde{\subseteq} (F,E)$ and hence $x_e \tilde{\in} \tilde{s}p\tilde{I}int(F,E)$. This shows that $(F,E) - \tilde{s}p\tilde{I}d(\tilde{X} - (F,E)) \tilde{\subseteq} \tilde{s}p\tilde{I}int(F,E)$. Now let $x_e \tilde{\in} \tilde{s}p\tilde{I}int(F,E)$. Since $\tilde{s}p\tilde{I}int(F,E)$ is soft pre- $\tilde{I}$- open, then $\tilde{s}p\tilde{I}int(F,E) \tilde{\cap} (\tilde{X} - (F,E)) = \tilde{\emptyset}$. So we have $x_e \tilde{\notin} \tilde{s}p\tilde{I}d(\tilde{X} - (F,E))$. Therefore $\tilde{s}p\tilde{I}int(F,E) = (F,E) - \tilde{s}p\tilde{I}d(\tilde{X} - (F,E))$.

5. Soft pre - $\tilde{I}$- border of a soft set

In this section we introduce the concept of soft pre - $\tilde{I}$- border of a soft set and study some of its properties.

Definition 5.1 Let $(X,\tilde{\tau},E,\tilde{I})$be a soft ideal topological space and $(F,E) \tilde{\in} SS(X)_E$. The soft Pre - $\tilde{I}$- border of (F,E), denoted by $\tilde{s}p\tilde{I}b(F,E)$, is defined by $\tilde{s}p\tilde{I}b(F,E) = (F,E) - \tilde{s}p\tilde{I}int(F,E)$.

Example 5.1

Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space, where $X = \{h_1,h_2,h_3,h_4\}$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\emptyset}, \{(e,\{h_1,h_2\})\}, \{(e,\{h_1,h_2,h_3\})\}\}$ and $\tilde{I} = \{\tilde{\emptyset}, \{(e,\{h_4\})\}\}$. Then one can deduce that
$\tilde{S}P\tilde{I}O(\tilde{X})=\{\tilde{X}, \tilde{\emptyset}, \{(e,\{h_1\})\}, \{(e,\{h_1,h_2\})\}, \{(e,\{h_1,h_3\})\},$
$\{(e,\{h_1,h_2,h_3\}\}, \{(e,\{h_1,h_3,h_4\})\}, \{(e,\{h_1,h_4\})\},$
$\{(e,\{h_1,h_2,h_4\})\}\}$. Let $(F,E) = \{(e,\{h_2,h_3,h_4\})\}$. Then $\tilde{s}p\tilde{I}int(F,E) = \tilde{\emptyset}$. Then $\tilde{s}p\tilde{I}b(F,E) = \{(e,\{h_2,h_3,h_4\})\}$.

Theorem 5.1 Let $(X,\tilde{\tau},E,\tilde{I})$be a soft ideal topological space and $(F,E) \tilde{\in} SS(X)_E$. Then the following hold.
(i) $\tilde{s}p\tilde{I}b(\tilde{\emptyset}) = \tilde{s}p\tilde{I}b(\tilde{X}) = \tilde{\emptyset}$.
(ii) $\tilde{s}p\tilde{I}b(F,E) \tilde{\subseteq} (F,E)$.
(iii) $(F,E) = \tilde{s}p\tilde{I}int(F,E) \tilde{\cup} \tilde{s}p\tilde{I}b(F,E)$.
(iv) $\tilde{s}p\tilde{I}int(F,E) \tilde{\cap} \tilde{s}p\tilde{I}b(F,E) = \tilde{\emptyset}$.
(v) $\tilde{s}p\tilde{I}int(F,E) = (F,E) - \tilde{s}p\tilde{I}b(F,E)$.
(vi) $\tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}b(F,E)) = \tilde{\emptyset}$.
(vii) (F,E) is soft pre- $\tilde{I}$- open if and only if $\tilde{s}p\tilde{I}b(F,E) = \tilde{\emptyset}$.
(viii) $\tilde{s}p\tilde{I}b(\tilde{s}p\tilde{I}int(F,E)) = \tilde{\emptyset}$.
(ix) $\tilde{s}p\tilde{I}b(\tilde{s}p\tilde{I}b(F,E)) = \tilde{s}p\tilde{I}b(F,E)$.

(x) $\tilde{s}p\tilde{I}b(F,E) = (F,E) \tilde{\cap} \tilde{s}p\tilde{I}cl(\tilde{X} - (F,E))$.
(xi) $\tilde{s}p\tilde{I}b(F,E) = (F,E) \tilde{\cap} \tilde{s}p\tilde{I}d(\tilde{X} - (F,E))$.

Proof.
(i), (ii), (iii), (iv) and (v) follow from the definition.
(vi) If possible let $x_e \tilde{\in} \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}b(F,E))$. Then $x_e \tilde{\in} \tilde{s}p\tilde{I}b(F,E)$, since $\tilde{s}p\tilde{I}b(F,E) \tilde{\subset} (F,E)$, then $x_e \tilde{\in} \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}b(F,E)) \tilde{\subset} \tilde{s}p\tilde{I}int(F,E)$. Therefore, $x_e \tilde{\in} \tilde{s}p\tilde{I}int(F,E) \tilde{\cap} \tilde{s}p\tilde{I}b(F,E)$ which is a contradiction to (iv). Thus $\tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}b(F,E)) = \tilde{\emptyset}$.
(vii) (F,E) is soft pre- $\tilde{I}$- open if and only if $(F,E) = \tilde{s}p\tilde{I}int\,(F,E)$(Theorem 2.4(vi)). But $\tilde{s}p\tilde{I}b(F,E) = (F,E) - \tilde{s}p\tilde{I}int(F,E)$ implies $\tilde{s}p\tilde{I}b(F,E) = \tilde{\emptyset}$.
(viii) Since $\tilde{s}p\tilde{I}int(F,E)$ is soft pre - $\tilde{I}$- open, it follows from (vii) that $\tilde{s}p\tilde{I}b(\tilde{s}p\tilde{I}int(F,E)) = \tilde{\emptyset}$.
(ix) Since $\tilde{s}p\tilde{I}b(F,E) = (F,E) - \tilde{s}p\tilde{I}int(F,E)$, then $\tilde{s}p\tilde{I}b(\tilde{s}p\tilde{I}b(F,E)) = \tilde{s}b\tilde{I}d(F,E) - \tilde{s}p\tilde{I}int(\tilde{s}b\tilde{I}d(F,E))$ using (vi), we get $\tilde{s}p\tilde{I}b(\tilde{s}p\tilde{I}b(F,E)) = \tilde{s}p\tilde{I}b(F,E)$.
(x) Using Theorem 2.4(i), we have $\tilde{s}p\tilde{I}b(F,E) = (F,E) - \tilde{s}p\tilde{I}int(F,E)$
$= (F,E) \tilde{\cap} (\tilde{X} - \tilde{s}p\tilde{I}int(F,E)) = (F,E) \tilde{\cap} \tilde{s}p\tilde{I}cl(\tilde{X} - (F,E))$**.** Hence $\tilde{s}p\tilde{I}b(F,E) = (F,E) \tilde{\cap} \tilde{s}p\tilde{I}cl(\tilde{X} - (F,E))$.
(xi) Applying (x) and Theorem 4.2, we have $\tilde{s}p\tilde{I}b(F,E) = (F,E) \tilde{\cap} \tilde{s}p\tilde{I}cl(\tilde{X} - (F,E)) = (F,E) \tilde{\cap}$ **[**$(\tilde{X} - (F,E)$**)** $\tilde{\cup}\ \tilde{s}p\tilde{I}d(\tilde{X} - (F,E))] = (F,E) \tilde{\cap} \tilde{s}p\tilde{I}d(\tilde{X} - (F,E)$.

6. Soft pre - $\tilde{I}$- Frontier of a soft set

Now we introduce the concept of soft pre-$\tilde{I}$- frontier of a soft set and study some of it's properties.

Definition 6.1 Let $(X,\tilde{\tau},E,\tilde{I}\,)$be a soft ideal topological space and $(F,E) \tilde{\in} SS(X)_E$. The soft pre-$\tilde{I}$- frontier of (F,E), denoted by $\tilde{s}p\tilde{I}F_r(F,E)$, is defined as $\tilde{s}p\tilde{I}F_r(F,E) = \tilde{s}p\tilde{I}cl(\,F,E)$ - $\tilde{s}p\tilde{I}int(F,E)$.

Remark 6.1
It is obvious that $\tilde{s}p\tilde{I}F_r(F,E) \tilde{\subset} \tilde{s}F_r(F,E)$, the soft frontier of (F,E). But the converse need not be true as can be shown by the following example.

Example 6.1

Let $(X, \tilde{\tau}, E, \tilde{I})$ as in Example 5.1. Take $(F, E) = \{(e, \{h_1, h_2, h_4\})\}$, $\tilde{s}F_r(F, E) = \{(e, \{h_3, h_4\})\}$, $\tilde{s}p\tilde{I}F_r(F, E) = \{(e, \{h_3\})\}$, this shows that $\tilde{s}F_r(F, E) \tilde{\not\subset} \tilde{s}p\tilde{I}F_r(F, E)$.

Theorem 6.1 Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \tilde{\in} SS(X)_E$. (F, E) is soft Pre - $\tilde{I}$- closed set if and only if $\tilde{s}p\tilde{I}F_r(F, E) \tilde{\subset} (F, E)$.

Proof.
Assume that (F, E) is soft pre - $\tilde{I}$- closed. Then, $\tilde{s}p\tilde{I}F_r(F, E) = \tilde{s}p\tilde{I}cl(F, E) - \tilde{s}p\tilde{I}int(F, E) = (F, E) - \tilde{s}p\tilde{I}int(F, E) \tilde{\subseteq} (F, E)$. Conversely suppose that $\tilde{s}p\tilde{I}F_r(F, E) \tilde{\subseteq} (F, E)$. Then $\tilde{s}p\tilde{I}cl(F, E)$ - $\tilde{s}p\tilde{I}int(F, E) \tilde{\subseteq} (F, E)$, and so $\tilde{s}p\tilde{I}cl(F, E) \tilde{\subseteq} (F, E)$. Noticing that $(F, E) \tilde{\subseteq} \tilde{s}p\tilde{I}cl(F, E)$, so we have $(F, E) = \tilde{s}p\tilde{I}cl(F, E)$. Therefore (F, E) is soft pre - $\tilde{I}$- closed.

Theorem 6.2 Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \tilde{\in} SS(X)_E$. Then the following hold.

(i) $\tilde{s}p\tilde{I}cl(F, E) = \tilde{s}p\tilde{I}int(F, E) \tilde{\cup} \tilde{s}p\tilde{I}F_r(F, E)$.

(ii) $\tilde{s}p\tilde{I}int(F, E) \tilde{\cap} \tilde{s}p\tilde{I}F_r(F, E) = \tilde{\emptyset}$.

(iii) $\tilde{s}p\tilde{I}F_r(F, E) = \tilde{s}p\tilde{I}cl(F, E) \tilde{\cap} \tilde{s}p\tilde{I}cl(\tilde{X} - (F, E))$.

(iv) $\tilde{s}p\tilde{I}F_r(F, E) \tilde{\subset} \tilde{s}F_r(F, E)$, where $\tilde{s}F_r(F, E)$ is the soft frontier of (F, E).

(v) $\tilde{s}p\tilde{I}F_r(F, E)$ is soft pre - $\tilde{I}$- closed set.

(vi) $\tilde{s}p\tilde{I}int(F, E) = (F, E) - \tilde{s}p\tilde{I}F_r(F, E)$.

(vii) $\tilde{s}p\tilde{I}b(F, E) \tilde{\subset} \tilde{s}p\tilde{I}F_r(F, E)$.

(viii) $\tilde{s}p\tilde{I}F_r(F, E) = \tilde{s}p\tilde{I}b(F, E) \tilde{\cup} (\tilde{s}p\tilde{I}d(F, E) - \tilde{s}p\tilde{I}int(F, E))$.

Proof.
(i) $\tilde{s}p\tilde{I}int(F, E) \tilde{\cup} \tilde{s}p\tilde{I}F_r(F, E) = \tilde{s}p\tilde{I}int(F, E) \tilde{\cup} (\tilde{s}p\tilde{I}cl(F, E) - \tilde{s}p\tilde{I}int(F, E)) = \tilde{s}p\tilde{I}cl(F, E)$.

(ii) $\tilde{s}p\tilde{I}int(F, E) \tilde{\cap} \tilde{s}p\tilde{I}F_r(F, E) = \tilde{s}p\tilde{I}int(F, E) \tilde{\cap} (\tilde{s}p\tilde{I}cl(F, E) - \tilde{s}p\tilde{I}int(F, E)) = (\tilde{s}p\tilde{I}int(F, E) \tilde{\cap} \tilde{s}p\tilde{I}cl(F, E)) - (\tilde{s}p\tilde{I}int(F, E) \tilde{\cap} \tilde{s}p\tilde{I}int(F, E)) = \tilde{s}p\tilde{I}int(F, E) - \tilde{s}p\tilde{I}int(F, E) = \tilde{\emptyset}$

(iii) $\tilde{s}p\tilde{I}F_r(F, E) = \tilde{s}p\tilde{I}cl(F, E) - \tilde{s}p\tilde{I}int(F, E) = \tilde{s}p\tilde{I}cl(F, E) \tilde{\cap} (\tilde{X} - \tilde{s}p\tilde{I}int(F, E)) = \tilde{s}p\tilde{I}cl(F, E) \tilde{\cap} \tilde{s}p\tilde{I}cl(\tilde{X} - (F, E))$ (Theorem 2.4(i)).

(iv) Obvious

(v) $\tilde{s}p\tilde{I}cl(\tilde{s}p\tilde{I}F_r(F,E))$ = $\tilde{s}p\tilde{I}cl[\tilde{s}p\tilde{I}cl(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl(\tilde{X}-(F,E))]$ (by using (iii)) $\tilde{\subseteq}\ \tilde{s}p\tilde{I}cl(\tilde{s}p\tilde{I}cl(F,E))\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl(\tilde{s}p\tilde{I}cl(\tilde{X}-(F,E)))$ (Theorem 2.4, (xi)) = $\tilde{s}p\tilde{I}cl(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl(\tilde{X}-(F,E))$ = $\tilde{s}p\tilde{I}F_r(F,E)$. Therefore $\tilde{s}p\tilde{I}F_r(F,E)$ is soft pre - $\tilde{I}$- closed set (Theorem 2.4(iv)).

(vi) (F,E) - $\tilde{s}p\tilde{I}F_r(F,E)$ = $(F,E)-(\tilde{s}p\tilde{I}cl(F,E)-\tilde{s}p\tilde{I}int(F,E))$
= $(F,E)-[\tilde{s}p\tilde{I}cl(F,E)\ \tilde{\cap}(\tilde{X}$- $\tilde{s}p\tilde{I}int(F,E))]$
= $(F,E)-[\tilde{s}p\tilde{I}cl(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl(\tilde{X}-(F,E))]$
= $(F,E)\ \tilde{\cap}\ [\tilde{s}p\tilde{I}int(\tilde{X}-(F,E))\ \tilde{\cup}\ \tilde{s}p\tilde{I}int(F,E)]$
= $[(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}int(\tilde{X}-(F,E))]\ \tilde{\cup}\ [(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}int(F,E)]$ = $\tilde{s}p\tilde{I}int(F,E)$.

(vii) Since $(F,E)\ \tilde{\subseteq}\ \tilde{s}p\tilde{I}cl(F,E)$, we have $\tilde{s}p\tilde{I}b(F,E)$ = $(F,E)-\tilde{s}p\tilde{I}int(F,E)\ \tilde{\subseteq}\ \tilde{s}p\tilde{I}cl(F,E)-\tilde{s}p\tilde{I}int(F,E)$ = $\tilde{s}p\tilde{I}F_r(F,E)$.
(viii) Using Theorem 4.2, we obtain
$\tilde{s}p\tilde{I}F_r(F,E)$ = $\tilde{s}p\tilde{I}cl(F,E)-\tilde{s}p\tilde{I}int(F,E)$=**(**$(F,E)\ \tilde{\cup}\ \tilde{s}p\tilde{I}d\,(F,E)$**)** $\tilde{\cap}$ $(\tilde{X}-\tilde{s}p\tilde{I}int\ (F,E))$ = **[**$(F,E)\ \tilde{\cap}(\tilde{X}-\tilde{s}p\tilde{I}int\ (F,E)$**]** $\tilde{\cup}$**[**$\tilde{s}p\tilde{I}d\,(F,E)\ \tilde{\cap}\ (\tilde{X}-\tilde{s}p\tilde{I}int\ (F,E)$**]** = **[**$(F,E)-\tilde{s}p\tilde{I}int(F,E)$**]** $\tilde{\cup}$ **[**$\tilde{s}p\tilde{I}d\,(F,E)-\tilde{s}p\tilde{I}int\,(F,E)$**]** = $\tilde{s}p\tilde{I}b(F,E)\ \tilde{\cup}\ (\tilde{s}p\tilde{I}d\,(F,E)-\tilde{s}p\tilde{I}int(F,E))$.

Theorem 6.3 Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E)\ \tilde{\in}\ SS(X)_E$**.** Then the following hold.
(i) $\tilde{s}p\tilde{I}F_r(F,E)=\tilde{s}p\tilde{I}F_r(\tilde{X}-(F,E))$.
(ii) $(F,E)\ \tilde{\in}\ \tilde{S}P\tilde{I}O(\tilde{X})$ if and only if $\tilde{s}p\tilde{I}F_r(F,E)\ \tilde{\subset}\ (\tilde{X}-(F,E))$.
(iii) $(F,E)\ \tilde{\in}\ \tilde{S}P\tilde{I}C(\tilde{X})$ if and only if $\tilde{s}p\tilde{I}F_r(F,E)\ \tilde{\subset}\ (F,E)$.

Proof.
(i) $\tilde{s}p\tilde{I}F_r(\tilde{X}-(F,E))=\tilde{s}p\tilde{I}cl(\tilde{X}-(F,E))-\tilde{s}p\tilde{I}int(\tilde{X}-(F,E))$ = $(\tilde{X}-\tilde{s}p\tilde{I}int(F,E))-(\tilde{X}-\tilde{s}p\tilde{I}cl(F,E))=\tilde{s}p\tilde{I}cl(F,E)-\tilde{s}p\tilde{I}int(F,E)$ $=\tilde{s}p\tilde{I}F_r(F,E)$.
(ii) Suppose $(F,E)\ \tilde{\in}\ \tilde{S}P\tilde{I}O(\tilde{X})$. Then by definition, $\tilde{s}p\tilde{I}F_r(F,E)$= $sp\tilde{I}cl(F,E)-\tilde{s}p\tilde{I}int(F,E)$= $\tilde{s}p\tilde{I}cl(F,E)-(F,E)$ (Theorem 2.4(vi)), Therefore $(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}F_r(F,E)$ =$(F,E)\ \tilde{\cap}\ (\tilde{s}p\tilde{I}cl(F,E)-(F,E))$ = $(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl(F,E)\ \tilde{\cap}(\tilde{X}-(F,E))$= $\tilde{\emptyset}$.Conversely, suppose $(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}F_r(F,E)$ = $\tilde{\emptyset}$. Then $(F,E)\ \tilde{\cap}\ (\tilde{s}p\tilde{I}cl(F,E)-\tilde{s}p\tilde{I}int(F,E)$ = $\tilde{\emptyset}$. So, $(F,E)\ \tilde{\cap}\ [\tilde{s}p\tilde{I}cl(F,E)\ \tilde{\cap}\ (\tilde{X}-\tilde{s}p\tilde{I}int(F,E))]$ = $[(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl(F,E)]\ \tilde{\cap}\ [\tilde{X}-\tilde{s}p\tilde{I}int(F,E)]$ = $(F,E)\ \tilde{\cap}\ [\tilde{X}-\tilde{s}p\tilde{I}int(F,E)]$ = $\tilde{\emptyset}$. Thus

$(F,E)\ \tilde{\subseteq}\ \tilde{s}p\tilde{I}int(F,E)$. But $\tilde{s}p\tilde{I}int(F,E)\ \tilde{\subseteq}\ (F,E)$. Therefore, $(F,E)\ \tilde{\in}\ \tilde{S}P\tilde{I}O(\tilde{X})$.

(iii) Suppose $(F,E)\ \tilde{\in}\ \tilde{S}P\tilde{I}C(\tilde{X})$. Then, $\tilde{s}p\tilde{I}F_r(F,E) = \tilde{s}p\tilde{I}cl(F,E) - \tilde{s}p\tilde{I}int(F,E) = (F,E) - \tilde{s}p\tilde{I}int(F,E)$(Theorem 2.4(iv)), and $(\tilde{X} - (F,E))\ \tilde{\cap}\ \tilde{s}p\tilde{I}F_r(F,E) = (\tilde{X} - (F,E))\ \tilde{\cap}\ [[(F,E) - \tilde{s}p\tilde{I}int(F,E)] = (\tilde{X} - (F,E))\ \tilde{\cap}\ (F,E)\ \tilde{\cap}\ [\tilde{X} - \tilde{s}p\tilde{I}int(F,E)] = \tilde{\emptyset}$, so $\tilde{s}p\tilde{I}F_r(F,E)\ \tilde{\subseteq}\ (F,E)$. Conversely, if $(\tilde{X} - (F,E))\ \tilde{\cap}\ \tilde{s}p\tilde{I}F_r(F,E) = \tilde{\emptyset}$, then from (i) we have $(\tilde{X} - (F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}F_r(\tilde{X} - (F,E)) = \tilde{\emptyset}$. Hence, $(\tilde{X} - (F,E))\ \tilde{\cap}\ [\tilde{s}p\tilde{I}cl(\tilde{X} - (F,E)) - \tilde{s}p\tilde{I}int(\tilde{X} - (F,E))] = \tilde{\emptyset}$, or $\left[\left(\tilde{X} - (F,E)\right)\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl\left(\tilde{X} - (F,E)\right)\right]\ \tilde{\cap}\ [\tilde{X} - \tilde{s}p\tilde{I}int(\tilde{X} - (F,E))] = \tilde{\emptyset}$. Thus, $(\tilde{X} - (F,E))\ \tilde{\cap}\ [\tilde{X} - \tilde{s}p\tilde{I}int(\tilde{X} - (F,E))] = \tilde{\emptyset}$. Therefore, $(\tilde{X} - (F,E))\ \tilde{\subseteq}\ \tilde{s}p\tilde{I}int(\tilde{X} - (F,E)$. It follows that $(\tilde{X} - (F,E))\ \tilde{\in}\ \tilde{S}P\tilde{I}O(\tilde{X})$. So $(F,E)\ \tilde{\in}\ \tilde{S}P\tilde{I}C(\tilde{X})$.

Remark 6.2 Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E),(G,E)\ \tilde{\in}\ SS(X)_E$. Then $(F,E)\ \tilde{\subseteq}\ (G,E)$ doesn't imply either $\tilde{s}p\tilde{I}F_r(F,E)\ \tilde{\subseteq}\ \tilde{s}p\tilde{I}F_r(G,E)$ or $\tilde{s}p\tilde{I}F_r(G,E)\ \tilde{\subseteq}\ \tilde{s}p\tilde{I}F_r(F,E)$. This can be verified by the following example.

Example 6.2

Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space, where $X = \{h_1,h_2,h_3\}$, $E = \{e\}$, $\tilde{\tau} = \{\tilde{X}, \tilde{\emptyset}, \{(e,\{h_2\})\}, \{(e,\{h_1,h_2\})\}, \{(e,\{h_2,h_3\})\}$ and $\tilde{I} = \{\tilde{\emptyset}, \{(e,\{h_3\})\}, \{(e,\{h_2,h_3\})\}\}$. One can deduce, $\tilde{S}P\tilde{I}O(\tilde{X}) = \{\tilde{X}, \tilde{\emptyset}, \{(e,\{h_2\})\}, \{(e,\{h_1,h_2\})\}, \{(e,\{h_2,h_3\})\}$. Let $(F,E) = \{(e,\{h_3\})\}$, $(G,E) = \{(e,\{h_2,h_3\})\}$, then $(F,E)\ \tilde{\subseteq}\ (G,E)$. Now, $\tilde{s}p\tilde{I}int(F,E) = \tilde{\emptyset}$, therefore $\tilde{s}p\tilde{I}int(G,E) = \{(e,\{h_2,h_3\})\}$, $\tilde{s}p\tilde{I}cl(F,E) = \{(e,\{h_3\})\}$, $\tilde{s}p\tilde{I}cl(G,E) = \tilde{X}$, $\tilde{s}p\tilde{I}F_r(F,E) = \{(e,\{h_3\})\}$, $\tilde{s}p\tilde{I}F_r(G,E) = \{(e,\{h_1\})\}$, so $\tilde{s}p\tilde{I}F_r(F,E)\ \tilde{\nsubseteq}\ \tilde{s}p\tilde{I}F_r(G,E)$ and $\tilde{s}p\tilde{I}F_r(G,E)\ \tilde{\nsubseteq}\ \tilde{s}p\tilde{I}F_r(F,E)$.

Theorem 6.4 Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space, we have $\tilde{s}p\tilde{I}F_r(\tilde{s}p\tilde{I}F_r(F,E))\ \tilde{\subseteq}\ \tilde{s}p\tilde{I}F_r(F,E)$.

Proof.

By Theorem 6.2 (iii), $\tilde{s}p\tilde{I}F_r(F,E) = \tilde{s}p\tilde{I}cl(F,E)\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl(\tilde{X} - (F,E))$. Therefore, $\tilde{s}p\tilde{I}F_r(\tilde{s}p\tilde{I}F_r(F,E)) = \tilde{s}p\tilde{I}cl(\tilde{s}p\tilde{I}F_r(F,E))\ \tilde{\cap}\ \tilde{s}p\tilde{I}cl(\tilde{X} - \tilde{s}p\tilde{I}F_r(F,E))\ \tilde{\subseteq}\ \tilde{s}p\tilde{I}cl(\tilde{s}p\tilde{I}F_r(F,E) = \tilde{s}p\tilde{I}F_r(F,E)$, since $\tilde{s}p\tilde{I}F_r(F,E)$ is soft pre - $\tilde{I}$- closed set.

Theorem 6.5 Let $(X, \tilde{\tau}, E, \tilde{I})$be a soft ideal topological space and (G, E) $\tilde{\in}$ $SS(X)_E$. Then the following hold.
(i) $\tilde{s}p\tilde{I}F_r(\tilde{s}p\tilde{I}int(F, E)) \tilde{\subseteq} \tilde{s}p\tilde{I}F_r(F, E)$.
(ii) $\tilde{s}p\tilde{I}F_r(\tilde{s}p\tilde{I}cl(F, E)) \tilde{\subseteq} \tilde{s}p\tilde{I}F_r(F, E)$.

Proof.
(i) $\tilde{s}p\tilde{I}F_r(\tilde{s}p\tilde{I}int(F, E)) = \tilde{s}p\tilde{I}cl(\tilde{s}p\tilde{I}int(F, E)) - \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}int(F, E)) = \tilde{s}p\tilde{I}cl(\tilde{s}p\tilde{I}int(F, E)) - \tilde{s}p\tilde{I}int(F, E) \tilde{\subseteq} \tilde{s}p\tilde{I}cl(F, E) - \tilde{s}p\tilde{I}int(F, E) = \tilde{s}p\tilde{I}F_r(F, E)$.
(ii) $\tilde{s}p\tilde{I}F_r(\tilde{s}p\tilde{I}cl(F, E)) = \tilde{s}p\tilde{I}cl(\tilde{s}p\tilde{I}cl(F, E)) - \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}cl(F, E)) = \tilde{s}p\tilde{I}cl(F, E) - \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}cl(F, E)) \tilde{\subseteq} \tilde{s}p\tilde{I}cl(F, E) - \tilde{s}p\tilde{I}int(F, E)$ $=\tilde{s}p\tilde{I}F_r(F, E)$.

Theorem 6.6 Let $(X, \tilde{\tau}, E, \tilde{I})$be a soft ideal topological space and $(G, E) \tilde{\in} SS(X)_E$. (F, E) is soft pre - $\tilde{I}$- open if and only if $\tilde{s}p\tilde{I}F_r(F, E)$ $= \tilde{s}p\tilde{I}b(\tilde{X} - (F, E))$.

Proof.
Assume that (F, E) is soft pre - $\tilde{I}$- open. Then $\tilde{s}p\tilde{I}F_r(F, E) = \tilde{s}p\tilde{I}b(F, E) \tilde{\cup} (\tilde{s}p\tilde{I}d\ (F, E)$- $\tilde{s}p\tilde{I}int(F, E)) = \tilde{\emptyset} \tilde{\cup} (\tilde{s}p\tilde{I}d\ (F, E) - (F, E)) = \tilde{s}p\tilde{I}d\ (F, E) - (F, E) = \tilde{s}p\tilde{I}b(\tilde{X} - (F, E))$, by using Theorem 6.2(viii), Theorem 5.1(vii),(xi). Conversely, suppose that $\tilde{s}p\tilde{I}F_r(F, E) = \tilde{s}p\tilde{I}b(\tilde{X} - (F, E))$. Then $\tilde{\emptyset} = \tilde{s}p\tilde{I}F_r(F, E) - \tilde{s}p\tilde{I}b(\tilde{X} - (F, E)) = [\tilde{s}p\tilde{I}cl(F, E) - \tilde{s}p\tilde{I}int(F, E)] - [\left(\tilde{X} - (F, E)\right) - \tilde{s}p\tilde{I}int(\tilde{X} - (F, E))] = (F, E) - \tilde{s}p\tilde{I}int(F, E)$. (by Theorem 4.3(viii) and Theorem 2.4(i)), and so (F, E) $\tilde{\subseteq}$ $\tilde{s}p\tilde{I}int(F, E)$. Since $\tilde{s}p\tilde{I}int(F, E) \tilde{\subseteq} (F, E)$ in general, it follows that $\tilde{s}p\tilde{I}int(F, E) = (F, E)$ so (F, E) is soft pre - $\tilde{I}$- open.

7. Soft pre - $\tilde{I}$- Exterior of a soft set

In this section we define the soft pre -$\tilde{I}$- exterior of a soft set and study some of it's properties.

Definition 7.1 Let $(X, \tilde{\tau}, E, \tilde{I})$ be a soft ideal topological space and $(F, E) \tilde{\in} SS(X)_E$**.** A soft point $x_e \tilde{\in} \tilde{X}$ is said to be a soft pre -$\tilde{I}$- exterior point of (F,E) if x_e is a soft pre -$\tilde{I}$- interior point of $(F,E)^c$. The soft pre -$\tilde{I}$- exterior of (F,E) is denoted by $\tilde{s}p\tilde{I}Ext(F,E)$. Thus $\tilde{s}p\tilde{I}Ext(F,E) = \tilde{s}p\tilde{I}int(F,E)^c$.

Example 7.1
Let $(X,\tilde{\tau},E,\tilde{I})$ as in Example 5.1. Let $(F,E) = \{(e,\{h_2,h_3\})\}$, $(F,E)^c = \{(e,\{h_1,h_4\})\}$. Then $\tilde{s}p\tilde{I}int(F,E)^c = \{(e,\{h_1,h_4\})\}$. Hence $\tilde{s}p\tilde{I}Ext(F,E) = \{(e,\{h_1,h_4\})\}$.

Theorem 7.1 Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E),(G,E) \tilde{\in} SS(X)_E$, then the following hold.
(i) If $(F,E) \tilde{\subseteq} (G,E)$, then $\tilde{s}p\tilde{I}Ext(G,E) \tilde{\subseteq} \tilde{s}p\tilde{I}Ext(F,E)$.
(ii) $\tilde{s}p\tilde{I}Ext((F,E) \tilde{\cup} (G,E)) \tilde{\subseteq} \tilde{s}p\tilde{I}Ext(F,E) \tilde{\cap} \tilde{s}p\tilde{I}Ext(G,E)$.
(iii) $\tilde{s}p\tilde{I}Ext((F,E) \tilde{\cap} (G,E)) \tilde{\supseteq} \tilde{s}p\tilde{I}Ext(F,E) \tilde{\cup} \tilde{s}p\tilde{I}Ext(G,E)$.

Proof.
(i) If $(F,E) \tilde{\subseteq} (G,E)$, then $(G,E)^c \tilde{\subseteq} (F,E)^c$, and hence $\tilde{s}p\tilde{I}int(G,E)^c \tilde{\subseteq} \tilde{s}p\tilde{I}int(F,E)^c$ (Theorem 2.4(ix)). This implies that $\tilde{s}p\tilde{I}Ext(G,E) \tilde{\subseteq} \tilde{s}p\tilde{I}Ext(F,E)$.
(ii) $\tilde{s}p\tilde{I}Ext((F,E) \tilde{\cup} (G,E)) = \tilde{s}p\tilde{I}int[\tilde{X} - ((F,E) \tilde{\cup} (G,E))]$
$= \tilde{s}p\tilde{I}int[(\tilde{X} - (F,E)) \tilde{\cap} (\tilde{X} - (G,E))]$
$\tilde{\subseteq} \tilde{s}p\tilde{I}int(\tilde{X} - (F,E)) \tilde{\cap} \tilde{s}p\tilde{I}int(\tilde{X} - (G,E))$ (Theorem 2.4(iii))
$= \tilde{s}p\tilde{I}Ext(F,E) \tilde{\cap} \tilde{s}p\tilde{I}Ext(G,E)$.
(iii) $\tilde{s}p\tilde{I}Ext((F,E) \tilde{\cap} (G,E)) = \tilde{s}p\tilde{I}int[\tilde{X} - ((F,E) \tilde{\cap} (G,E))]$
$= \tilde{s}p\tilde{I}int((\tilde{X} - (F,E)) \tilde{\cup} (\tilde{X} - (G,E)))$
$\tilde{\supseteq} \tilde{s}p\tilde{I}int(\tilde{X} - (F,E)) \tilde{\cup} \tilde{s}p\tilde{I}int(\tilde{X} - (G,E))$ (Theorem 2.4(x))
$= \tilde{s}p\tilde{I}Ext(F,E) \tilde{\cup} \tilde{s}p\tilde{I}Ext(G,E)$

Theorem 7.2 Let $(X,\tilde{\tau},E,\tilde{I})$ be a soft ideal topological space and $(F,E),(G,E) \tilde{\in} SS(X)_E$. Then the following hold.
(i) $\tilde{s}p\tilde{I}Ext(F,E)$ is soft pre - $\tilde{I}$- open.
(ii) $\tilde{s}Ext(F,E) \tilde{\subset} \tilde{s}p\tilde{I}Ext(F,E)$, where $\tilde{s}Ext(F,E)$ is the soft exterior of (F,E).
(iii) $\tilde{s}p\tilde{I}Ext(F,E) = \tilde{X} - \tilde{s}p\tilde{I}cl(F,E)$.
(iv) $\tilde{s}p\tilde{I}Ext(\tilde{s}p\tilde{I}Ext(F,E)) = \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}cl(F,E))$.
(v) $\tilde{s}p\tilde{I}Ext(\tilde{X}) = \tilde{\emptyset}$ and $\tilde{s}p\tilde{I}Ext(\tilde{\emptyset}) = \tilde{X}$.
(vi) $\tilde{s}p\tilde{I}Ext(F,E) = \tilde{s}p\tilde{I}Ext(\tilde{X} - \tilde{s}p\tilde{I}Ext(F,E))$.

(vii) $\tilde{s}p\tilde{I}int \ \tilde{\sqsubseteq} \ \tilde{s}p\tilde{I}Ext\ (\tilde{s}p\tilde{I}Ext(F,E))$.
(viii) $\tilde{X} = \tilde{s}p\tilde{I}int \ \tilde{\cup} \ \tilde{s}p\tilde{I}Ext(F,E) \ \tilde{\cup} \ \tilde{s}p\tilde{I}F_r(F,E)$.
(ix) $(F,E) \ \tilde{\cap} \ \tilde{s}p\tilde{I}Ext(F,E) = \ \tilde{\emptyset}$.

Proof.
(i) Follows form the definition.
(ii) Let $x_e \ \tilde{\in} \ \tilde{s}Ext(F,E)$, then $x_e \ \tilde{\in} \ \tilde{s}int(F,E)^c$ since every soft open is soft pre - $\tilde{I}$- open. Then we have $x_e \ \tilde{\in} \ \tilde{s}p\tilde{I}int(F,E)^c$ and then $x_e \ \tilde{\in} \ \tilde{s}p\tilde{I}Ext(F,E)$. Hence $\tilde{s}Ext(F,E) \ \tilde{\sqsubseteq} \ \tilde{s}p\tilde{I}Ext(F,E)$.
(iii) Since $\tilde{s}p\tilde{I}Ext(F,E) = \tilde{s}p\tilde{I}int(F,E)^c$, by Theorem 2.4, we have $\tilde{s}p\tilde{I}int(F,E)^c = \tilde{X} - \tilde{s}p\tilde{I}cl(\ F,E)$. Hence $\tilde{s}p\tilde{I}Ext(F,E) = \tilde{X} - \tilde{s}p\tilde{I}cl(\ F,E)$.
(iv) $\tilde{s}p\tilde{I}Ext\ (\tilde{s}p\tilde{I}Ext(F,E)) = \tilde{s}p\tilde{I}int(\tilde{X} - \ \tilde{s}p\tilde{I}Ext(F,E))$, by (iii) we have $\tilde{s}p\tilde{I}int(\tilde{X} - \ \tilde{s}p\tilde{I}Ext(F,E)) = \tilde{s}p\tilde{I}int(\tilde{X} - (\tilde{X} - \ \tilde{s}p\tilde{I}cl(\ F,E))) = \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}cl(\ F,E))$.
(v) Follows form the definition.
(vi) $\tilde{s}p\tilde{I}Ext(\tilde{X} - \tilde{s}p\tilde{I}Ext(F,E)) = \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}Ext(F,E)) = \tilde{s}p\tilde{I}int(\tilde{s}p\tilde{I}int(F,E)^c) = \tilde{s}p\tilde{I}int(F,E)^c$(Theorem 2.4(vii)) $= \tilde{s}p\tilde{I}Ext(F,E)$.
(vii) $\tilde{s}p\tilde{I}Ext(F,E) = \tilde{s}p\tilde{I}int(F,E)^c \ \tilde{\sqsubseteq} \ (F,E)^c$**,** so from Theorem 7.1, (i) we get $\tilde{s}p\tilde{I}Ext(F,E)^c \ \tilde{\sqsubseteq} \ \tilde{s}p\tilde{I}Ext(\tilde{s}p\tilde{I}Ext(F,E))$. Hence $\tilde{s}p\tilde{I}int(F,E) \ \tilde{\sqsubseteq} \ \tilde{s}p\tilde{I}Ext\ (\tilde{s}p\tilde{I}Ext(F,E))$.
(viii) By Theorem 6.2(i), we get $\tilde{s}p\tilde{I}int \ \tilde{\cup} \ \tilde{s}p\tilde{I}Ext(F,E) \ \tilde{\cup} \ \tilde{s}p\tilde{I}F_r(F,E) = \tilde{s}p\tilde{I}Ext(F,E) \ \tilde{\cup} \ \tilde{s}p\tilde{I}cl(\ F,E) = (\ \tilde{X} - \ \tilde{s}p\tilde{I}cl(\ F,E)) \ \tilde{\cup} \ \tilde{s}p\tilde{I}cl(\ F,E) = \tilde{X}$.
(ix) $\tilde{s}p\tilde{I}Ext(F,E) = \tilde{s}p\tilde{I}int(F,E)^c \ \tilde{\sqsubseteq} \ (F,E)^c$, then $(F,E) \ \tilde{\cap} \ \tilde{s}p\tilde{I}Ext(F,E) = (F,E) \ \tilde{\cap} \ (F,E)^c = \tilde{\emptyset}$

8. Conclusion

This work goes on to study some concepts in soft ideal topological spaces related to soft pre ideal open sets. We introduced the notion of soft pre - $\tilde{I}$- neighborhood, soft pre- $\tilde{I}$- limit point , soft pre- $\tilde{I}$- border , soft pre- $\tilde{I}$- frontier and soft pre- $\tilde{I}$- exterior.

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