Assiut Univ. J. of Mathematics and Computer Science *Printed ISSN 1687-4897*
47(2), pp74- 86 (2018)

MOTION OF CURVES USING QUASI FRAME

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Received: 3/9/2018 **Accepted:** 3/10/2018 **Available Online:** 20/12/2018

In this paper, evolution of space curves using quasi frame are considered. We derive a pair of coupled partialdifferential equations that governing the time evolution for the quasi curvatures of the evolving curve. Exact solutions of these equations are obtained. Also we have reconstructed the evolving curve from their quasi curvatures via the numerical integration of quasi Serret-Frenet equations.

Keywords: *Quasi frame, Quasi curvature, Frenet frame, Time evolution equation, Vector field.*

1. INTRODUCTION

Da Rios [1] in 1906 established the geometric link between a completely integrable equation and a moving curve in an analysis by on the spatial evolution of an isolated vortex filament in an unbounded invoiced liquid. Da Rios invoked what is now known as the localized induction approximation to derive a pair of coupled nonlinear equations governing the time evolution of the curvature and torsion of the vortex filament. Kambe and Takao [2], showed that the Da Rios equations can be converted to producethe celebrated nonlinear Schrodinger equation. Lamb [3], later in 1977, linked the motion of curves with, the sine-Gordon, modified Korteweg-de Vries and nonlinear Schrodinger equations.

Lakshmanan et al. [4], derived the Heisenberg spin chain equation via the spatial motion of a curve. Recently, Nakayama, et al. [5] obtained the sine-Gordon equation by considering a nonlocal motion. AlsoNakayama and Wadati [6] presented a general formulation of evolving curves in two dimensions and its connection to mKdV hierarchy. R. Mukherjee and R. Balakrishnan [7] applied their method to the sine-Gordon equation and obtained links to five new classes of space curves, in addition to the two which were found by Lamb [3]. For each class, they displayed the rich variety of moving curves associated with the one-soliton, the breather, the two-soliton and the soliton-antisoliton solutions.

All above authors are used Frenet frame that associated to a space curve to mobilize the curve in three dimensional space while T. Korpinar and E. Turhan [8] used type-2 Bishop Frame in E^3 to study inextensible flows of a space curve according to type-2 Bishop Frame. They obtained time evolution equation for type-2 Bishop Frame and type-2 Bishop curvatures which governing the evolution of the curve.

The article is organized as follows. In section (2) we introduce differential geometry of curves focusing on Serret-Frenet frame and quasi frame along a space curve. In section (3), the evolution of curves is represented by two sets of quasi frame. By applying compatibility condition on these vectors, partial differential equations for the curvatures κ_1, κ_2 are derived. Exact solutions for these equations have been obtained. Also we have reconstructed the evolving curve from its quasi curvatures via the numerical integration of quasi Serret-Frenet equations.

2. Quasi frame along a space curve

Let $\vec{\mathbf{r}} = \vec{\mathbf{r}}(s)$ be a natural representation of a regular curve. The moving Frenet frame is defined as follows,

$$\vec{\mathbf{t}} = \frac{\vec{\mathbf{r}}'(s)}{\|\vec{\mathbf{r}}'(s)\|},$$

$$\vec{\mathbf{n}} = \frac{\vec{\mathbf{t}}'(s)}{\|\vec{\mathbf{t}}'(s)\|}, \tag{1}$$

$$\vec{\mathbf{b}} = \vec{\mathbf{t}} \times \vec{\mathbf{n}},$$

The frame evolves along the curve according to the well-known Frenet formulas [16]

$$\frac{d}{ds}\begin{pmatrix} \vec{\mathbf{t}} \\ \vec{\mathbf{n}} \\ \vec{\mathbf{b}} \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}\begin{pmatrix} \vec{\mathbf{t}} \\ \vec{\mathbf{n}} \\ \vec{\mathbf{b}} \end{pmatrix}. \tag{2}$$

The curvature κ and the torsion τ are given by

$$\kappa = \vec{\mathbf{n}}.\frac{d\vec{\mathbf{t}}}{ds},$$
$$\tau = -\vec{\mathbf{n}}.\frac{d\vec{\mathbf{b}}}{ds} \tag{3}$$

There are many frames attached to the curve in the space such as Frenet frame [16], Bishop frame [18–20], Kepler frame [21] . In this section we, introduce the deferential geometry of quasi frame [22] as an alternant to Frenet frame. The quasi frame of a regular space curve $\vec{r} = \vec{r}(s)$ is given by

$$\vec{\mathbf{t}} = \frac{\vec{\mathbf{r}}'(s)}{\|\vec{\mathbf{r}}'(s)\|},$$
$$\overrightarrow{\mathbf{n_q}} = \frac{\vec{\mathbf{t}}\times\vec{\mathbf{k}}}{\|\vec{\mathbf{t}}\times\vec{\mathbf{k}}\|}, \tag{4}$$
$$\overrightarrow{\mathbf{b_q}} = \vec{\mathbf{t}}\times\overrightarrow{\mathbf{n_q}},$$

where $\vec{\mathbf{k}}$ is the projection vector. In this paper, we have chosen the projection vector $\vec{\mathbf{k}}$ = (0, 0, 1).

Let ϕ is the angel between the normal $\vec{\mathbf{n}}$ and quasi normal $\overrightarrow{\mathbf{n_q}}$. Then, the relation between two frames is given by

$$\begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi(s) & \sin\phi(s) \\ 0 & -\sin\phi(s) & \cos\phi(s) \end{pmatrix} \begin{pmatrix} \vec{\mathbf{t}} \\ \vec{\mathbf{n}} \\ \vec{\mathbf{b}} \end{pmatrix}. \tag{5}$$

Thus,

$$\begin{pmatrix} \vec{\mathbf{t}} \\ \vec{\mathbf{n}} \\ \vec{\mathbf{b}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi(s) & -\sin\phi(s) \\ 0 & \sin\phi(s) & \cos\phi(s) \end{pmatrix} \begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix}. \tag{6}$$

Denoting by

$$F = \begin{pmatrix} \vec{\mathbf{t}} \\ \vec{\mathbf{n}} \\ \vec{\mathbf{b}} \end{pmatrix}, A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi(s) & -\sin\phi(s) \\ 0 & \sin\phi(s) & \cos\phi(s) \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi(s) & \sin\phi(s) \\ 0 & -\sin\phi(s) & \cos\phi(s) \end{pmatrix},$$
$$Q = \begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix}, K = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}. \tag{7}$$

Equations (2), (6) and (5) can be written as

$$\frac{\partial F}{\partial s} = KF,$$
$$Q = AF, \tag{8}$$
$$F = BQ.$$

By using the above equation the rate of change of quasi frame is given by

$$\frac{dQ}{ds} = (\frac{dA}{ds}.B + A.K.B)Q \tag{9}$$

by using equation (7) we have

$$\frac{d}{ds}\begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix} = \begin{pmatrix} 0 & \kappa\cos\phi(s) & \kappa\sin\phi(s) \\ -\kappa\cos\phi(s) & 0 & (\tau - \frac{d\phi}{ds}) \\ -\kappa\sin\phi(s) & -(\tau - \frac{d\phi}{ds}) & 0 \end{pmatrix} \begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix} \tag{10}$$

Thus equation (10) can be written as,

$$\frac{\mathbf{d}}{\mathbf{d}s}\begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix} = \begin{pmatrix} 0 & \kappa_1 & \kappa_2 \\ -\kappa_1 & 0 & 0 \\ -\kappa_2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix}$$

(11)

where quasi curvatures are

$$\kappa_1 = \kappa\cos\phi(s),$$
$$\kappa_2 = \kappa\sin\phi(s) \tag{12}$$

, where

$$\phi(s) = \int_0^s \tau(t)dt. \tag{13}$$

The frame $(\vec{\mathbf{t}}, \overrightarrow{\mathbf{n_q}}, \overrightarrow{\mathbf{b_q}})$ is properly oriented, We shall call the set $(\vec{\mathbf{t}}, \overrightarrow{\mathbf{n_q}}, \overrightarrow{\mathbf{b_q}}, \kappa_1, \kappa_2)$ as quasi invariants of the curve $\vec{\mathbf{r}} = \vec{\mathbf{r}}(s)$.

The variation of an orthonormal quasi frame $(\vec{\mathbf{t}}, \overrightarrow{\mathbf{n_q}}, \overrightarrow{\mathbf{b_q}})$ defined along the curve may be specified by its angular velocity $\vec{\boldsymbol{\omega}}$ through the relations

$$\frac{d\vec{\mathbf{t}}}{ds} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{t}},$$
$$\frac{d\overrightarrow{\mathbf{n_q}}}{ds} = \vec{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{n_q}}, \qquad (14)$$
$$\frac{d\overrightarrow{\mathbf{b_q}}}{ds} = \vec{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{b_q}}.$$

Since $(\vec{\mathbf{t}}, \overrightarrow{\mathbf{n_q}}, \overrightarrow{\mathbf{b_q}})$ comprise a basis for R^3 we can write

$$\vec{\boldsymbol{\omega}} = \omega_1 \vec{\mathbf{t}} + \omega_2 \overrightarrow{\mathbf{n_q}} + \omega_3 \overrightarrow{\mathbf{b_q}}, \qquad (15)$$

thus equations (14) becomes

$$\frac{d\vec{\mathbf{t}}}{ds} = -\omega_2 \overrightarrow{\mathbf{b_q}} + \omega_3 \overrightarrow{\mathbf{n_q}},$$
$$\frac{d\overrightarrow{\mathbf{n_q}}}{ds} = \omega_1 \overrightarrow{\mathbf{b_q}} - \omega_3 \vec{\mathbf{t}}, \qquad (16)$$
$$\frac{d\overrightarrow{\mathbf{b_q}}}{ds} = -\omega_1 \overrightarrow{\mathbf{n_q}} + \omega_2 \vec{\mathbf{t}}.$$

Comparing equations (16) with equations (11) we have

$$\omega_1 = 0,$$
$$\omega_2 = \kappa_2, \qquad (17)$$
$$\omega_3 = \kappa_1,$$

, thus the Darbuox vector for quasi frame which represent total rotation of quasi frame is given by

$$\vec{\boldsymbol{\omega}} = \kappa_2 \overrightarrow{\mathbf{n_q}} + \kappa_1 \overrightarrow{\mathbf{b_q}}. \qquad (18)$$

In the next example we introduce geometric visualization for Frenet frame and quasi frame for
a space curve.

1.1 Example 1

Let, the space curve is given by,

$$\vec{\mathbf{r}} = (s\cos(s), s\sin(s), s). \qquad (19)$$

The Frenet frame and curvatures are calculated and given by

$$\begin{cases}\vec{\mathbf{t}}=(\frac{\cos(s)-s\sin(s)}{\sqrt{s^2+2}},\frac{\cos(s)+s\sin(s)}{\sqrt{s^2+2}},\frac{1}{\sqrt{s^2+2}})\\ \vec{\mathbf{n}}=(\frac{-(s^2+4)\sin(s)-(s^3+3s)\cos(s)}{\sqrt{(s^2+2)(s^4+5s^2+8)}},\frac{(s^2+4)\sin(s)-(s^3+3s)\cos(s)}{\sqrt{(s^2+2)(s^4+5s^2+8)}},\frac{-s}{\sqrt{(s^2+2)(s^4+5s^2+8)}})\\ \vec{\mathbf{b}}=(\frac{s\sin(s)-2s\cos(s)}{(s^4+5s^2+8)},\frac{-2\sin(s)+2s\cos(s)}{(s^4+5s^2+8)},\frac{s^2+2}{(s^4+5s^2+8)})\\ \kappa=\sqrt{\frac{8+5s^2+s^4}{(2+s^2)}}\\ \tau=\frac{(6+s^2)^{3/2}}{8+5s^2+s^4}\end{cases} \tag{20}$$

For the projection vector $\vec{\mathbf{k}}=(0,0,1)$ the quasi Frenet and quasi curvatures are calculated and given by

$$\begin{cases}\vec{\mathbf{t}}=(\frac{\cos(s)-s\sin(s)}{\sqrt{s^2+2}},\frac{\cos(s)+s\sin(s)}{\sqrt{s^2+2}},\frac{1}{\sqrt{s^2+2}})\\ \overrightarrow{\mathbf{n_q}}=(\frac{\cos(s)+s\sin(s)}{\sqrt{s^2+2}},\frac{-\cos(s)+s\sin(s)}{\sqrt{s^2+2}},0)\\ \overrightarrow{\mathbf{b_q}}=(\frac{\cos(s)-s\sin(s)}{\sqrt{s^2+2}},\frac{\cos(s)+s\sin(s)}{\sqrt{s^2+2}},-\sqrt{\frac{s^2+1}{s^2+2}})\\ \kappa_1=1/\sqrt{\frac{s^2+1}{2+s^2}}\\ \kappa_2=-\frac{s}{\sqrt{1+s^2}\,(2+s^2)}\\ \kappa_3=\frac{\sqrt{2+s^2}}{1+s^2}\end{cases} \tag{21}$$

The geometric visualization for both Frenet frame and quasi frame are displayed in figure (1).

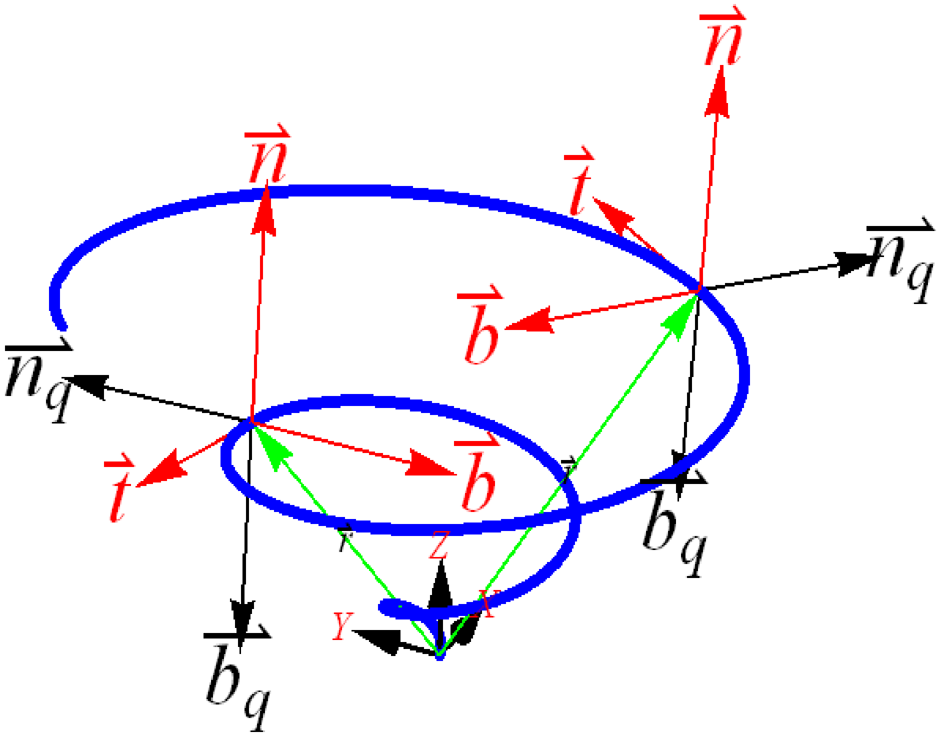

Fig.1: Frenet frame and quasi frame along the curve $\vec{\mathbf{r}} = (s\cos(s), s\sin(s), s)$

3. Evolution of a space curve with time by quasi frame

Serret-Frenet equations of a curve are basic in the development of the theory of curves. However, if the curve has in inflection points or straight segments or if it fails to be at least three times continuously differentiable, the Frenet frame becomes either discontinuous or may not even exist. In such cases there can be great motivations to think about the choice to present an elective framing, for example, Bishop frame [18-20], Kepler frame [21] and quasi frame [22]. In this section we study the evolution of aregular space curve using quasi frame. We derive time evolution equation for quasi frame and quasi curvatures.

Let, the curve evolving in the space according to

$$\frac{\partial \vec{\mathbf{r}}}{\partial t} = \lambda_1 \vec{\mathbf{t}} + \lambda_2 \overrightarrow{\mathbf{n_q}} + \lambda_3 \overrightarrow{\mathbf{b_q}}. \tag{22}$$

The evolution of quasi Seret-Frenet with respect to *s* and *t* are similar to [23, 24]

$$\frac{\partial}{\partial s}\begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix} = \begin{pmatrix} 0 & \kappa_1 & \kappa_2 \\ -\kappa_1 & 0 & 0 \\ -\kappa_2 & 0 & 0 \end{pmatrix}\begin{pmatrix} \overrightarrow{\mathbf{t_q}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix}, \tag{23}$$

$$\frac{\partial}{\partial t}\begin{pmatrix} \overrightarrow{\mathbf{t_q}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix} = \begin{pmatrix} 0 & \beta_1 & \beta_2 \\ -\beta_1 & 0 & 0 \\ -\beta_2 & 0 & 0 \end{pmatrix}\begin{pmatrix} \overrightarrow{\mathbf{t_q}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix}, \tag{24}$$

where, the parameters $\beta_1, \beta_2, \beta_3$ are function of s, t. Applying the compatibility condition,

$$\frac{\partial}{\partial s}\frac{\partial}{\partial t}\begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix} = \frac{\partial}{\partial t}\frac{\partial}{\partial s}\begin{pmatrix} \vec{\mathbf{t}} \\ \overrightarrow{\mathbf{n_q}} \\ \overrightarrow{\mathbf{b_q}} \end{pmatrix}, \tag{25}$$

a short calculation using Eqs. (23), (24) and (25) leads to,

$$\begin{aligned} \frac{\partial \kappa_1}{\partial t} &= \frac{\partial \beta_1}{\partial s}, \\ \frac{\partial \kappa_2}{\partial t} &= \frac{\partial \beta_2}{\partial s}, \\ 0 &= \beta_2\kappa_1 - \beta_1\kappa_2. \end{aligned} \tag{26}$$

Applying the compatibility condition

$$\frac{\partial}{\partial s}\frac{\partial}{\partial t}\vec{\mathbf{r}} = \frac{\partial}{\partial t}\frac{\partial}{\partial s}\vec{\mathbf{r}}, \tag{27}$$

yields,

$$\frac{\partial \lambda_1}{\partial s} = \lambda_2\kappa_1 + \lambda_3\kappa_2,$$
$$\frac{\partial \lambda_2}{\partial s} = \beta_1 - \lambda_1\kappa_1, \tag{28}$$
$$\frac{\partial \lambda_3}{\partial s} = \beta_2 - \kappa_2\lambda_1.$$

For a given velocity vector $(\lambda_1, \lambda_2, \lambda_3)$, equations (26) and (28) form a set of 6 nonlinear first
order partial differential equations which governing the evolution of the quasi curvatures of the evolving curve in the space.Equation (26) together gives

$$\frac{\partial \kappa_1}{\partial t} = \frac{\partial \beta_1}{\partial s},$$
$$\frac{\partial \kappa_2}{\partial t} = \frac{\kappa_2}{\kappa_1}\frac{\partial \beta_1}{\partial s} + \beta_1\frac{\partial}{\partial s}(\frac{\kappa_2}{\kappa_1}) \tag{29}$$

From equation (28) β_1 is given by

$$\beta_1 = \frac{\partial \lambda_2}{\partial s} + \lambda_1\kappa_1. \tag{30}$$

The time evolution of the curvatures κ_1 and κ_2 of the curve may now be expressed in terms of the components of velocity $(\lambda_1, \lambda_2, \lambda_3)$ by substitution (30) into (29) to obtain

$$\frac{\partial \kappa_1}{\partial t} = \frac{\partial}{\partial s}(\frac{\partial \lambda_2}{\partial s} + \lambda_1\kappa_1),$$
$$\frac{\partial \kappa_2}{\partial t} = \frac{\kappa_2}{\kappa_1}\frac{\partial}{\partial s}(\frac{\partial \lambda_2}{\partial s} + \lambda_1\kappa_1) + (\frac{\partial \lambda_2}{\partial s} + \lambda_1\kappa_1)\frac{\partial}{\partial s}(\frac{\kappa_2}{\kappa_1}), \tag{31}$$

While, the time evolution of the quasi moving frame is given by,

$$\frac{\partial \vec{\mathbf{t}}}{\partial t} = (\frac{\partial \lambda_2}{\partial s} + \lambda_1\kappa_1)\overrightarrow{\mathbf{b_q}} + \frac{\kappa_2}{\kappa_1}(\frac{\partial \lambda_2}{\partial s} + \lambda_1\kappa_1)\overrightarrow{\mathbf{n_q}},$$
$$\frac{\partial \overrightarrow{\mathbf{n_q}}}{\partial t} = -(\frac{\partial \lambda_2}{\partial s} + \lambda_1\kappa_1)\vec{\mathbf{t}}, \tag{32}$$
$$\frac{\partial \overrightarrow{\mathbf{b_q}}}{\partial t} = -\frac{\kappa_2}{\kappa_1}(\frac{\partial \lambda_2}{\partial s} + \lambda_1\kappa_1)\vec{\mathbf{t}}.$$

3.1 model 1

If, the velocity vector $\vec{\boldsymbol{\lambda}} = (\lambda_1, \lambda_2, \lambda_3)$ is given by

$$\lambda_1 = 0,$$
$$\lambda_3 = \kappa_1, \qquad (33)$$
$$\lambda_2 = \kappa_2,$$

, then the quasi curvatures evolves according to

$$\frac{\partial \kappa_1}{\partial t} = \frac{\partial}{\partial s}\left(\frac{\partial \kappa_2}{\partial s}\right),$$
$$\frac{\partial \kappa_2}{\partial t} = \frac{1}{\kappa_1^2}\left(\kappa_1 \frac{\partial \kappa_2}{\partial s} - \kappa_2 \frac{\partial \kappa_1}{\partial s}\right)\left(\frac{\partial \kappa_2}{\partial s}\right) + \frac{\kappa_2}{\kappa_1}\frac{\partial}{\partial s}\left(\frac{\partial \kappa_2}{\partial s}\right). \qquad (34)$$

On solution of the above system is given by

$$\kappa_1(s,t) = c_4(1 + \tanh(c_1 s + c_2 t + c_3)),$$
$$\kappa_2(s,t) = c_4(-1 + \tanh(c_1 s + c_2 t + c_3)). \qquad (35)$$

The figure (2 represent curve evolving in the space determined by $\kappa_1 = 1 + \tanh(s+t)$ and $\kappa_2 = -1 + \tanh(s+t)$ obtained by solving the quasi SerretFrenet equations (23) for a specified κ_1 and κ_2 using Mathematica [15]. At every fixed t, we clearly have a representation of the corresponding (static) space curve at that instant. The program [15] generates static space curves corresponding to Serret-Frenet equations. It was extended slightly to generate space curves corresponding to quasi SerretFrenet equations.

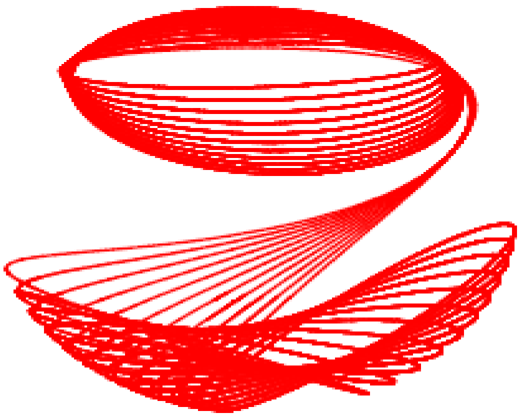

Fig.2: Curve evolving in the space with $\kappa_1 = 1 + \tanh(s+t)$ and $\kappa_2 = -1 + \tanh(s+t)$.

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في هذا البحث قمنا بدراسة حركة المنحنيات باستخدام اطار مماثل لاطار سيرية فرينية وتم اشتقاق زوج من المعادلات التفاضلية الجزئية التي تحكم معدل التغير لدوال الإنحناء وتم حل هذة المعادلات تحت شروط خاصة وحصلنا علي دوال الانحناء وبعد ذلك قمنا بإعادة تكوين المنحني في الفراغ بإستخدام دوال الإنحناء، وذلك عن طريق تكامل معادلات مماثلة لمعادلات سرية فرينية