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Delta and Theta Generalized Closed Sets in Bitopological Spaces

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In this paper we introduce new two types of generalized closed sets in bitopological spaces and study some of it's properties and it's relations with other kinds of generalized closed sets. Using these sets we obtain a characterization of some separation axioms in bitopological spaces.

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1- Introduction and preliminaries

Throughout this paper, (X,τ_1,τ_2) (or simply X) always mean a bitopological space (or simply a space) on which no separation axioms are assumed unless explicitly stated. Also, $i,j=1,2$ and $i\neq j$. Let A be a subset of a space (X,τ_1,τ_2). The closure of A and the interior of A in the topological space (X,τ_i) are denoted by $i-Cl(A)$ and $i-Int(A)$, respectively. We write $i-$ open (resp. $i-$closed) set to mean that the set is open (resp. closed) in the topological space (X,τ_i).

Let (X,τ_1,τ_2) be a bitopological space. A point $x\in X$ is said to be in the $ij-\delta-$closure [1] (resp. $ij-\theta-$closure [1]) of a subset A of X if $i-Int(j-Cl(U))\bigcap A\neq\phi$ (resp. $j-Cl(U)\bigcap A\neq\phi$) for every $i-$open set U containing x. The $ij-\delta-$closure (resp. $ij-\theta-$closure) of a subset A is denoted by $Cl^{\delta}_{ij}(A)$ (resp. $Cl^{\theta}_{ij}(A)$). A subset $A\subset X$ is called $ij-\delta-$closed (resp. $ij-\theta-$closed) if $A=Cl^{\delta}_{ij}(A)$ (resp. $A=Cl^{\theta}_{ij}(A)$). The complement of an $ij-\delta-$closed (resp. $ij-\theta-$closed) set is called $ij-\delta-$open (resp. $ij-\theta-$open). The set of all $ij-\delta-$open (resp. $ij-\theta-$open) sets form a topology on X will be denote by τ_i^{δ} [7] (resp. τ_i^{θ} [7]). From the definition it follows that $\tau_i^{\theta} \subset \tau_i^{\delta} \subset \tau_i$ [7]. The space $(X, \tau_1^{\delta}, \tau_2^{\delta})$ is called the pairwise semi-regularization of (X, τ_1, τ_2) [12]. A space (X, τ_1, τ_2) is pairwise semi-regular if $\tau_i = \tau_i^{\delta}$ [12]. A bitopological space (X, τ_1, τ_2) is called pairwise regular [4] if for each $x \in X$ and each $i-$closed set F not containing x, there exist an $i-$open set U and a $j-$open sets V such that $x \in U$, $F \subset V$ and $U \bigcap V = \phi$, equivalently, if for each $i-$open set U and $x \in U$, there exists an $i-$open V such that $x \in V \subset j-Cl(V) \subset U$. A space (X, τ_1, τ_2) is pairwise regular if and only if $\tau_i = \tau_i^{\theta}$ [16]. A subset A of a bitopological space (X, τ_1, τ_2) is called $ij-$regular open [17] (resp. $ij-$regular closed [17]) if $A = i-Int(j-Cl(A))$ (resp. $A = i-Cl(j-Int(A))$). A is called $ij-$nowhere dense if $i-Int(j-Cl(A)) = \phi$. The family of all $ij-$regular open subsets of a bitopological space (X, τ_1, τ_2) form a base for a topology τ_i^* on X. It is shown in [12] that $\tau_i^* = \tau_i^{\delta}$.

Remark 1.1. $Cl_{ij}^{\delta}(A)$ is the $i-$closure of A with respect to $(X, \tau_1^{\delta}, \tau_2^{\delta})$. In general, $Cl_{ij}^{\theta}(A)$ will not be the $i-$closure of A with respect to $(X, \tau_1^{\theta}, \tau_2^{\theta})$.

Remark 1.2.[7] $A \subset i-Cl(A) \subset Cl_{ij}^{\delta}(A) \subset Cl_{ij}^{\theta}(A)$.

Definition 1.3. A subset A of a bitopological space (X, τ_1, τ_2) is called:

(1) $ij-\alpha-$closed [5] if $i-Cl(j-Int(i-Cl(A))) \subset A$.

(2) $ij-\alpha-$open [5] if $X \setminus A$ is $ij-\alpha-$closed or equivalently if $A \subset i-Int(j-Cl(i-Int(A)))$.

(3) $ij-$semi closed [2] if $j-Int(i-Cl(A)) \subset A$.

(4) $ij-$semi open [2] if $X \setminus A$ is $ij-$semi closed or equivalently if $A \subset j-Cl(i-Int(A))$.

(5) $ij-$preclosed [5] if $i-Cl(j-Int(A)) \subset A$.

(6) $ij-$preopen [5] if $X \setminus A$ is $ij-$preclosed or equivalently if $A \subset i-Int(j-Cl(A))$.

(7) $ij-\beta-$closed [5] if $j-In(i-Cl(j-Int(A))) \subset A$.

(8) $ij-\beta-$open [5] if $X\setminus A$ is $ij-\beta-$closed or equivalently if $A\subset j-Cl(i-Int(j-Cl(A)))$.

(9) $ij-g-$closed [3] if $j-Cl(A)\subset U$ whenever $A\subset U$ and U is $i-$open.

(10) $ij-\theta g-$closed [9] if $Cl_{ji}^{\theta}(A)\subset U$, whenever $A\subset U$ and U is $i-$open.

For a subset A of a space (X,τ_1,τ_2), the $ij-\alpha-$closure (resp. $ij-$semiclosure, $ij-$preclosure, $ij-\beta-$closure) of a set $A\subset X$ is the smallest $ij-\alpha-$closed (resp. $ij-$semiclosed, $ij-$preclosed, $ij-\beta-$closed) set containing A. These closures are denoted by $ij-\alpha Cl(A)$ (resp. $ij-sCl(A)$, $ij-pCl(A)$, $ij-\beta Cl(A)$).

Lemma 1.4. [18] Let (X,τ_1,τ_2) be a bitopological space and $A\subset X$, then

(i) $ij-\alpha Cl(A)=A\bigcup i-Cl(j-Int(i-Cl(A)))$.

(ii) $ij-pCl(A)\supset A\bigcup i-Cl(j-Int(A))$.

(iii) $ij-sCl(A)=A\bigcup j-Int(i-Cl(A))$.

(iv) $ij-\beta Cl(A)\supset A\bigcup j-Int(i-Cl(j-Int(A)))$.

2- $ij-\delta\theta-$closed and $ij-\theta\delta-$closed sets

Let $P=\{\tau,\alpha,s,p,\beta,\delta,\theta\}$, where τ denote the set of all $i-$open sets, α denote the set of all $ij-\alpha-$open sets, s denote the set of all $ij-$semi open sets, p the set of all $ij-$preopen sets, β the set of all $ij-\beta-$open sets, δ the set of all $ij-\delta-$open sets and θ the set of all $ij-\theta-$open sets.

Definition 2.1. A subset $A\subset X$ is called $ij-qr-$closed if $ji-qCl(A)\subset U$ whenever $A\subset U$ and U is $ij-r-$open, where $r,q\in P$.

Remark 2.2. If $r,q\in P$, then every $ij-qr-$closed subset of (X,τ_1,τ_2) is $ij-q-$closed if and only if each singleton of X is either $ij-q-$open or $ij-r-$closed.

Definition 2.3. A subset A of a bitopological space (X,τ_1,τ_2) is called:

(1) $ij-\delta g-$closed if $Cl_{ji}^{\delta}(A)\subset U$, whenever $A\subset U$ and U is $i-$open.

(2) $ij-g\delta-$closed if $j-Cl(A)\subset U$, whenever $A\subset U$ and U is $ij-\delta-$open.

(3) $ij-\delta g^{*}-$closed if $Cl_{ji}^{\delta}(A)\subset U$, whenever $A\subset U$ and U is $ij-\delta-$ open.
(4) $ij-g\theta-$closed if $j-Cl(A)\subset U$, whenever $A\subset U$ and U is $ij-\theta-$ open.
(5) $ij-\theta g^{*}-$closed if $Cl_{ji}^{\theta}(A)\subset U$, whenever $A\subset U$ and U is $ij-\theta-$ open.

In the notation of Definition 2.1, we have:
(1) $ij-\delta g-$closed is equivalent to $ij-\delta\tau-$closed.
(2) $ij-g\delta-$closed is equivalent to $ij-\tau\delta-$closed.
(3) $ij-\delta g^{*}-$closed is equivalent to $ij-\delta\delta-$closed.
(4) $ij-\theta g-$closed is equivalent to $ij-\theta\tau-$closed.
(5) $ij-g\theta-$closed is equivalent to $ij-\tau\theta-$closed.
(6) $ij-\theta g^{*}-$closed is equivalent to $ij-\theta\theta-$closed.
(7) $ij-g-$closed is equivalent to $ij-\tau\tau-$closed.

Definition 2.4. A subset A of a bitopological space (X,τ_1,τ_2) is called:
(1) $ij-\delta\theta-$closed, if $ji-\delta-Cl(A)\subset U$ whenever $A\subset U$ and U is $ij-\theta-$ open.
(2) $ij-\theta\delta-$closed, if $ji-\theta-Cl(A)\subset U$ whenever $A\subset U$ and U is $ij-\delta-$open.

Remark 2.5. Obviously every $ji-\delta-$closed set is $ij-\delta\theta-$closed and every $ji-\theta-$closed set is $ij-\theta\delta-$closed. Since $\tau_i^{\theta}\subset\tau_i^{\delta}\subset\tau_i$, every $ij-\theta\delta-$closed set is $ij-\delta\theta-$closed. If $x\in U$ and U is $ij-\theta-$open, then there exists an $i-$open set V such that $x\in V$ and $j-Cl(V)\subset U$. Since $j-C(V)$ is $ji-\delta-$closed, we have $ji-\delta-Cl(\{x\})\subset U$, that is, every singleton in any space is always $ij-\delta\theta-$closed. Let X be an infinite set, $p\in X$ be a particular point and $\tau_1=\tau_2=$ the topology on X consisting of X and all subsets of X not containing p. If $x\neq p$, then $\{x\}$ is not $ij-\delta-$ open and $j-Cl(\{x\})=\{x,p\}\subset ji-\theta-Cl(\{x\})$. Thus $\{x\}$ is $ij-\delta\theta-$ closed but fails to be $ij-\theta\delta-$closed.

Definition 2.6. A bitopological space (X,τ_1,τ_2) is said to be satisfies property

(1) A , if every $ij-\delta\theta-$closed set is $ji-\delta-$closed, i.e., each singleton is either $ji-\delta-$open or $ij-\theta-$ closed.
(2) B , if every $ij-\theta\delta-$closed set is $ji-\theta-$ closed, i.e., each singleton is either $ji-\theta-$open or $ij-\delta-$ closed.

Recall that (X,τ_1,τ_2) is pairwise $T_{\frac{1}{2}}$ [3], if each singleton is either $j-$ open or $i-$closed. (X,τ_1,τ_2) is called pairwise weakly Hausdorff (resp. pairwise almost weakly Hausdorff) if $(X,\tau_1^{\delta},\tau_2^{\delta})$ is pairwise T_1 (resp. $T_{\frac{1}{2}}$).

One cane observe that, $(X,\tau_1^{\delta},\tau_2^{\delta})$ is pairwise Hausdorff if and only if $(X,\tau_1^{\theta},\tau_2^{\theta})$ is pairwise T_1 if and only if $(X,\tau_1^{\theta},\tau_2^{\theta})$ is pairwise $T_{\frac{1}{2}}$ if and only if $(X,\tau_1^{\theta},\tau_2^{\theta})$ is pairwise T_0.

Theorem 2.7. For a bitopological space (X,τ_1,τ_2), the following are equivalent:
(a) (X,τ_1,τ_2) is pairwise Hausdorff.
(b) (X,τ_1,τ_2) satisfies A.
(c) (X,τ_1,τ_2) is pairwise almost weakly Hausdorff and $ij-\delta-$closed singletons are $ij-\theta-$closed.

Proof: (a) $\Rightarrow$ (b) If (X,τ_1,τ_2) is pairwise Hausdorff, then $(X,\tau_1^{\theta},\tau_2^{\theta})$ is pairwise T_1, i.e., singletons are $ij-\theta-$closed. Thus (X,τ_1,τ_2) satisfies A.
(b) $\Rightarrow$ (a) If (X,τ_1,τ_2) satisfies A, then by Remark 2.5., each singleton is either $ij-\delta-$clopen or $ij-\theta-$closed. Hence $(X,\tau_1^{\theta},\tau_2^{\theta})$ is pairwise T_1 and thus (X,τ_1,τ_2) is pairwise Hausdorff.
(b) $\Rightarrow$ (c) Suppose that (X,τ_1,τ_2) satisfies A. Then each singleton is clearly either $ji-\delta-$open or $ij-\delta-$closed, i.e., (X,τ_1,τ_2) is pairwise almost weakly Hausdorff. If $\{x\}$is $ij-\delta-$closed, then $\{x\}$ is either $ij-\delta-$ clopen or $ij-\theta-$closed, hence always $ij-\theta-$closed.
(c) $\Rightarrow$ (b) This is obvious.

Theorem 2.8. For a bitopological space (X,τ_1,τ_2), the following are equivalent:
(a) (X,τ_1,τ_2) is pairwise weakly Hausdorff.
(b) (X,τ_1,τ_2) satisfies B.

Proof: (a) $\Rightarrow$ (b) If (X, τ_1, τ_2) is pairwise weakly Hausdorff, then each singleton is $ij-\delta-$closed. Hence (X, τ_1, τ_2) satisfies B.

(b) $\Rightarrow$ (a) This follows from the fact that each $ij-\theta-$open singleton must be $ji-$clopen.

Observation 2.9. Let (X, τ_1, τ_2) be a bitopological space and $x \in X$. Then

(a) $\{x\}$ is either $ij-$preopen or $ji-$nowhere dense (i.e., $j-Cl(i-Int(\{x\})=\phi)$.

(b) $\{x\}$ is either $i-$open or $ji-$preclosed.

(c) $\{x\}$ is $i-$open if and only if $\{x\}$ is $ij-\alpha-$open if and only if $\{x\}$ is $ij-$semi open.

(d) $\{x\}$ is $ij-$preopen if and only if $\{x\}$ is $ij-\beta-$open.

(e) If $\{x\}$ $ji-$nowhere dense, then $\{x\}$ is $ij-\alpha-$closed and thus $ij-$semiclosed, $ij-$preclosed and $ij-\beta-$closed.

(f) $\{x\}$ is $ji-$semiclosed if and only if $\{x\}$ $ji-$nowhere dense or $ij-$regular open.

3- Characterizations of some separation axioms.

Definition 3.1. A bitopological space (X, τ_1, τ_2) is called:

(i) pairwise semi T_1 [10] (resp. pairwise pre T_1, pairwise $\beta-T_1$ if each singleton is pairwise semiclosed (resp. pairwise preclosed, pairwise $\beta-$closed) .

(ii) pairwise $T_{\frac{3}{4}}$ if each singleton is either $ij-\delta-$open or $j-$closed.

(iii) pairwise semi $T_{\frac{1}{2}}$ [10] if each singleton is either $ji-$semiopen or $ij-$semiclosed.

(iv) pairwise feebly T_1 if each singleton is either $ji-$nowhere dense or $ij-$clopen.

(v) pairwise T_{gs} if each singleton is either $ij-$preopen or $j-$closed.

As an immediate consequence of observation 2.9 we note that a bitopological space (X, τ_1, τ_2) is pairwise semi $T_{\frac{1}{2}}$ if and only if each singleton is either $j-$open or $ij-\alpha-$closed.

Proposition 3.2. For a bitopological space (X, τ_1, τ_2), the following are equivalent:

(a) X is pairwise semi T_1 .

(b) each singleton is either $ji-\theta-$open or $ij-$semiclosed.
(c) each singleton is either $ji-\delta-$open or $ij-$semiclosed.
(d) each singleton is either $ji-\delta-$open or $ij-\alpha-$closed.
Proof: (a) $\Rightarrow$ (b) $\Rightarrow$ (c): Obvious.
(c) $\Rightarrow$ (d) Follows from Observation 2.9.
(d) $\Rightarrow$ (a): Clear.

By observing that a $ji-\theta-$open set must be $ji-$clopen and by Observation 2.9 we have a space (X,τ_1,τ_2) is pairwise feebly T_1 if and only if each singleton is either $ji-\theta-$open or $ij-\alpha-$closed. By a similar argument (X,τ_1,τ_2) is pairwise pre T_1 if and only if each singleton is either $ji-\theta-$open or $ij-$preclosed. In addition, (X,τ_1,τ_2) is pairwise T_1 if and only if each singleton is either $ji-\theta-$open or $i-$closed.

Proposition 3.3. For a bitopological space (X,τ_1,τ_2), the following are equivalent:
(a) X is pairwise $\beta-T_1$.
(b) each singleton is either $ji-\theta-$open or $ij-\beta-$closed.
(c) each singleton is either $ji-\delta-$open or $ij-\beta-$closed.
(d) each singleton is either $ji-\delta-$open or $ij-$preclosed.
Proof: (a) $\Rightarrow$ (b) $\Rightarrow$ (c): Obvious.
(c) $\Rightarrow$ (d): Let $x\in X$ such that $\{x\}$ is $ij-\beta-$closed. If $j-Int(\{x\})=\phi$, then $\{x\}$ $ij-$preclosed. Otherwise $\{x\}$ is $j-$open and $ij-\beta-$closed and so is $ji-$regular open, i.e., $ji-\delta-$open.
(d) $\Rightarrow$ (a): Clear.

Definition 3.4. A bitopological space (X,τ_1,τ_2) is called
(1) pairwise R_1 [14] if for each $x,y\in X$ such that $x\notin i-Cl(\{y\})$, there is an $i-$open set U and a $j-$open set V such that $x\in U$, $y\in V$ and $U\cap V=\phi$.
(2) pairwise subweakly T_2 if $ji-\delta-Cl(\{x\})=j-Cl(\{x\})$ for each $x\in X$.
(3) pairwise pointwise semi regular if each $j-$closed singleton is $ji-\delta-$closed.
(4) pairwise pointwise regular if each $j-$closed singleton is $ji-\theta-$closed.

Lemma 3.5. Let (X,τ_1,τ_2) be a bitopological space, then

(a) X is pairwise R_1 if and only if $j-Cl(\{x\}) = ij-\theta-Cl(\{x\})$ for each $x \in X$ [15].
(b) If $A \subset X$ is an $ij-$preopen set, then $j-Cl(A) = ji-\theta-Cl(A)$.

Proof:
(b) In general, $j-Cl(A) \subset ji-\theta-Cl(A)$. Now let $x \notin j-Cl(A)$. Then there exists a $j-$open set U such that $x \in U$ and $U \bigcap A = \phi$. Then $U \bigcap j-Cl(A) = \phi$ and so $U \bigcap i-Int(j-Cl(A)) = \phi$. From this we have $i-Cl(U) \bigcap i-Int(j-Cl(A)) = \phi$. Since $A \subset i-Int(j-Cl(A))$, then $i-Cl(U) \bigcap A = \phi$. This shows that $x \in ji-\theta-Cl(A)$. Therefore $ji-\theta-Cl(A) \subset j-Cl(A)$.

Theorem 3.6. For a bitopological space (X, τ_1, τ_2), the following are equivalent:
(a) Each singleton is either $ji-\theta-$closed or $ij-$preopen.
(b) X is pairwise T_{gs} and pairwise R_1.
(c) X is pairwise T_{gs} and pairwise pointwise regular.

Proof: (a) $\Rightarrow$ (b): Suppose that each singleton is either $ji-\theta-$closed or $ij-$preopen. Then (X, τ_1, τ_2) clearly is pairwise T_{gs}. Let $x \in X$. If $\{x\}$ is $ij-$preopen, $j-Cl(\{x\}) = ij-\theta-Cl(\{x\})$ by Lemma 3.5. If $\{x\}$ is $ji-\theta-$closed then $\{x\} = ij-\theta-Cl(\{x\}) = j-Cl(\{x\})$. Hence X is pairwise R_1.
(b) $\Rightarrow$ (c): Follows immediately from Lemma 3.5.
(c) $\Rightarrow$ (a): Follows directly from the definitions.

Theorem 3.7. For a bitopological space (X, τ_1, τ_2), the following are equivalent:
(a) Each singleton is either $ji-\delta-$closed or $ij-$preopen.
(b) X is pairwise T_{gs} and pairwise subweakly T_2.
(c) X is pairwise T_{gs} and pairwise pointwise semi regular.

Proof: (a) $\Rightarrow$ (b): Suppose that each singleton is either $ji-\delta-$closed or $ij-$preopen. Then (X, τ_1, τ_2) clearly is pairwise T_{gs}. Let $x \in X$. If $\{x\}$ is $ij-$preopen, then $j-Cl(\{x\}) = j-Cl(i-Int(j-Cl(\{x\})))$, i.e., $j-Cl(\{x\})$ is $ji-$regular closed set and so $j-Cl(\{x\}) = ji-\delta-Cl(\{x\})$. If $\{x\}$ is $ji-\delta-$closed, then obviously we have $j - Cl(\{x\}) = ji - \delta - Cl(\{x\})$. Thus X is pairwise subweakly T_2.

(b) $\Rightarrow$ (c) $\Rightarrow$ (a): It is clear.

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المجموعات المغلقة المعممة من النوع دلتا والنوع ثيتا
في الفضاءات ثنائية التوبولوجي

فتحي هشام خضر
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في هذا البحث نقدم نوعين جديدين من المجموعات المغلقة المعممة في الفراغات ثنائية التوبولوجي وندرس بعض خواصها وعلاقاتها بالأنواع الأخرى من المجموعات المغلقة المعممة في . باساخدام هذه المجموعات نحصل على تشخيص لبعض مسلمات الفصل.