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SPθ-TOPOLOGICAL GROUPS IN NONSTANDARD ANALYSIS

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The aim of this paper is to introduce and study a new class of topological groups called SPθ-topological group. By using some nonstandard techniques given by A., Robison and axiomatized by E., Nelson. We investigate some nonstandard properties to distinguish βθ-monads groups.

Key Words: *Nonstandard analysis, Monads, Pθ-monad, βθ-monads, topological group.*

1. INTRODUCTION

In 1982, Mashhour et al [1], defined a new version of *nearly open* sets, which is a significant notion to the field of general topology, which are called *preopen* sets. In 1983, Abd.El-Monsef [2], introduced the notion of *β-open* sets in topological space. In 1986, Andrijevic [3], defined and investigated *semipre-open* sets which are equivalent to *β-open* sets. In 2003, Noiri [12] introduced the concept of *sp-θ-open* sets.

In this work, by using nonstandard concepts a new class of topological groups had been studded and called **SPθ-Topological Groups**. There are wide classical study about topological groups, some newest can be found in [4, 5, 6, 8, 9], although using nonstandard tools covers several research lines in mathematics, there are a few nonstandard study about topological group [7, 13, 14].

In this work, we use a *weakly β-irresolute* mapping, to define and study a mentioned class of topological groups, then to present some properties of a new type of quotient topological groups. Finally, some properties of βθ-monads have been obtained in βθ -topological groups. For this investigation, we need the following basic background in general topology and nonstandard analysis.

2. BASIC BACKROUNDS IN GENERAL TOPOLOGY

Throughout this work, (X,τ) or (simply X) denotes a standard topological space on which no separation axioms are assumed unless explicitly stated. We recall the following definitions, notational conventions and characterizations. The ***closure*** and ***interior*** of a subset A of X are denoted respectively by **cl(A)** and **int(A)**.

Definition 2.1 [11] A subset A of X is said to be

i) *β-open* set if and only if $A \subseteq cl(int(cl(A)))$. The family of all *β-open* sets of X is denoted by $\beta O(X)$ and the family of all *β-closed* sets of X containing a point $x \in A$ is denoted by $\beta O(X,x)$. [12]
ii) *pre-θ-open* set if for each $x \in A$, there is a preopen subset G of X such that $x \in G$ and $G \subseteq pcl(G) \subseteq A$.
iii) *sp-θ-open* set if for each $x \in A$, there is a β-open subset G of X such that $x \in G$ and $G \subseteq \beta\ cl(G) \subseteq A$. The family of all *sp-θ-open* sets of X is denoted by $SP\theta O(X)$.
iv) *β-closed* set if and only if $X \backslash A$ is β-open. Equivalently $int(cl(int(A))) \subseteq A$.

Definition 2.2 [12] A mapping $f: (X,\tau) \to (Y,\rho)$ is said to be ***weakly β-irresolute***, if for each $x \in X$ and each $V \in \beta O(Y, f(x))$, there exists $U \in \beta O(X, x)$ such that $f(\beta cl(U)) \subseteq V$.

Definition 2.3 [14] Let (X,τ) be a topological space. Then the ***βθ-monad*** at a point $a \in X$ is denoted by $\mu_{\beta\theta}(a)$ and defined as follows
$\mu_{\beta\theta}(a) = \cap\{\beta cl(G);\ G \in G\beta(a)\}$, where $G\beta(a) = \{A;\ a \in A \in \beta O(X)\}$.

Theorem 2.4. [14] Let (X,τ) be a nonstandard topological space, and a be any element of X. Then, there exists an infinitesimal β-open set H containing a such that $\beta cl(H) \subseteq \mu_{\beta\theta}(a)$.

Theorem 2.5. [14] Let A be a subset of a nonstandard space X. Then A is sp-θ-open if and only if $\mu_{\beta\theta}(a) \subseteq A$. for each $a \in A$.

Theorem 2.6. [14] Let (X,τ) be a standard topological space, then the following statements hold:

i) For each $a \in X$, $a \in \mu_{\beta\theta}(a)$.
ii) For each $a \in X$, $b \in \mu_{\beta\theta}(a)$ implies that $\mu_{\beta\theta}(b) \subseteq \mu_{\beta\theta}(a)$.

Theorem 2.7. [14] Let A be a subset of a nonstandard space X. Then

i) Any union of pre-θ-open sets is a pre-θ-open set.
ii) Any union of sp-θ-open sets is a sp-θ-open set.

3. BASIC BACKGROUNDS IN NONSTANDARD ANALYSIS [10, 14]

In this work, we use E.Nelson's nonstandard analysis construction, based on a theory called internal set theory IST [10]. Every set or element defined in a classical mathematics is called ***standard***. Any set or formula which does not involve new predicates "*standard*, infinitesimal, *limited*, *unlimited* is called ***internal***, otherwise it is called ***external***. The axioms of IST is the axiom of Zermelo-Frankel with the axiom of choice (briefly ZFC) together with three axioms which are the transfer axiom (T), the idealization axiom (I), and the standardization axiom (S), are stated as follow

Transfer Axiom (T)

Let $A(x, t_1, t_2, \dots, t_n)$ be an internal formula with free variables $x, t_1, t_2, \dots, t_n$ only then

$$\forall^{st} t_1, t_2, \dots, t_n (\forall^{st} x\, A(x, t_1, t_2, \dots, t_n) \Longrightarrow (\forall x\, A(x, t_1, t_2, \dots, t_n)$$

Idealization Axiom (I)

Let B(x,y) be an internal formula with free variables x ,y and with possibly other free variables then

$$\forall^{st\, fin} z \exists x \forall y \in Z \wedge B(x, y) \Leftrightarrow \exists x \forall^{st} y B(x, y).$$

Standardization Axiom (S)

Let F(Z) be a formula, internal or external with free variables z and with possibly other free variables. Then $\forall^{st} x\, \exists^{st} y\, \forall^{st} z (z \in Y) \Leftrightarrow z \in X \wedge F(z)$.

A real number x is called ***unlimited*** if $|x| > r$ for all positive standard real numbers r, otherwise it is called ***limited***.

A real number x is called ***infinitesimal*** if $|x| < r$ for all positive standard real numbers r.

Two real numbers x and y are said to be ***infinitely close*** if $x - y$ is infinitesimal and denoted by $x \cong y$.

If x is a limited number in R, then it is infinitely close to a unique standard real number. This unique number is called the ***standard part*** of x or (***shadow*** of x) denoted by st(x).or ^{0}x

The external set of all infinitesimal real numbers is called the ***monad of* 0**, and denoted by ***m*(0)**, in general, the set of real numbers which are infinitely close to a real number x is called the ***monad of x***, denoted by ***m*(*x*)**.

Definition 3.1. [14] Let (X,τ) be a topological space. Then the ***Pθ-monad*** at a standard point $a \in X$ is defined as:

Pθ-monad = ∩{pcl(A) : $A \in PO(X)$ and $a \in A$ }, and is denoted by $\mu_{p\theta}(a)$.

Theorem 3.2. [14] Let (X,τ) be a nonstandard topological space and let $a\in X$ be any element. Then there exists an infinitesimal preopen H such that pcl(H) $\subseteq \mu_{p\theta}(a)$.

Theorem 3.3. [14] Let A be a standard subset of a nonstandard space X. Then A is pre-θ-open set if and only if $\mu p\theta(a) \subseteq A$. for each $a\in A$.

4. SPθ-TOPOLOGICAL GROUPS

The main idea of this section is to define a new class of topological groups by using weakly β-irresolute mapping. The following theorem is an external characterization of weakly β-irresolute mapping, by using βθ-monads.

Theorem 4.1. A mapping $f:(X,\tau) \to (Y,\rho)$ is weakly β-irresolute at a standard point a if and only if $f(\mu_{\beta\theta}(a)) \subseteq \mu_{\beta\theta}(f(a))$.

Proof:

Suppose that f is weakly β-irresolute at a standard point a, and let G be any sp-θ-open set containing $f(a)$. Then, there exists an sp-θ-open set U containing a such that $f(U) \subseteq G$, since $f(\mu_{\beta\theta}(a)) \subseteq f(U) \subseteq G$, therefore $f(\mu_{\beta\theta}(a)) \subseteq \mu_{\beta\theta}(f(a))$.

Conversely, let U be a standard sp-θ-open set containing $f(a)$, and V be a standard sp-θ-open set containing a. Then $V \in \beta O(X)$, and by Theorem 2.4, we get $\beta cl(V) \subseteq \mu_{\beta\theta}(a)$, which implies that $f(\beta cl(V)) \subseteq f(\mu_{\beta\theta}(a)) \subseteq \mu_{\beta\theta}(f(a))$. Now, Theorem 2.5 implies that $f(\beta cl(V)) \subseteq U$. Hence f is weakly β-irresolute.

Definition 4.2. Let G be a standard group and (G,τ) be a standard topological group. Then (G,τ) is said to be **SPθ-topological group** if the mappings $g: G\times G \to G$, $g(x,y) = xy$ and $h: G \to G$, $h(x) = x^{-1}$ are weakly β-irresolute.

Example 4.3. Consider the group G={-1,1,i,-i}. If τ is the indiscrete topology on G, then the mappings g and h are weakly β-irresolute, where $g: G\times G \to G, g(x,y) = xy,\ h: G \to G, h(x) = x^{-1}$.

Theorem 4.4. Let G be a standard group with a standard topology τ. Then, (G,τ) is a standard SPθ-topological group, if and only if the internal mapping $f: G\times G \to G,\ f(x,y) = xy^{-1}$ is weakly β-irresolute.

Proof:

Assume that (G,τ) is a standard SPθ-topological group, then the mappings $g: G \times G \to G,$ defined by $g(x,y) = xy,$ a and $h: G \to G,$ defined by $h(x) = x^{-1}$, are βθ-irresolute for each $x, y \in G$

If we take $\mathrm{f}(x,y) = g(x,y^{-1}) = xy^{-1}$, therefore f is βθ-irresolute.

Conversely assume that $\mathrm{f}(x,y) = xy^{-1}$ is βθ-irresolute function and we must show that (G,τ) is a standard SPθ-topological group,

If we take $g(x,y) = f(x,y^{-1}) = xy$ and $h(x) = f(x^{-1},e) = x^{-1}$

Then g and h are βθ-irresolute functions.

Theorem 4.5. (G,τ) is a standard SPθ-topological group, if the following conditions are satisfied:

i) For every standard points $x, y \in G$ and each standard sp-θ-open set H containing $x.y$, there exists a standard sp-θ-open sets U and V of x and y respectively such that $U.V \subseteq H$.

ii) For every standard point $x \in G$ and each standard sp-θ-open set V containing x^{-1} there exists a standard sp-θ-open set U of x such that $U^{-1} \subseteq H$.

Proof:

Let (G,τ) be a standard SPθ-topological group, and $x, y \in G$ be standard points. Let H sp-θ-open set containing $x.y$. Since the mapping $h: G \times G \to G$, defined by $h(x,y) = xy$ is weakly β-irresolute then

$$h^{-1}(H) = \{(x,y) \in G \times G; h(x,y) \in H\},$$
$$= \{(x,y) \in G \times G; x.y \in H\} \text{ is sp-θ-open subset of } G \times G .$$

Thus, there exists sp-θ-open sets U and V of x and y, respectively, in G such that $h^{-1}(H) = U \times V$.

Now, $U.V = \{x.y; x \in U \wedge y \in V\} = \{x.y; (x,y) \in U \times V\}$

$$= \{x.y; (x,y) \in f^{-1}(H)\}$$
$$= \{x.y; f(x,y) \in H\}$$
$$= \{x.y; x.y \in H\} \subseteq H$$

Let V be a sp-θ-open set containing x^{-1}. Since, the mapping $g: G \to G,$ defined by $g(x) = x^{-1}$ is weakly β-irresolute, then $g^{-1}(V)$ is sp-θ-open set. Therefore, there exists sp-θ-open set U of x such that $g\text{^}(-1)\ (V) = U$.

Now, we have $U^{-1} = \{x^{-1}, x \in U\}$

$$= \{x^{-1}, x \in g^{-1}(V)\}$$
$$= \{x^{-1}, g(x) \in V\}$$
$$= \{x^{-1}, x^{-1} \in V\} \subseteq V.$$

Definition 4.6. Let (G,τ) and (G^*,τ^*) be SPθ-topological groups. A bijective mapping

$f:(G,\tau)\to(G^*,\tau^*)$ is called ***SPθ-homeomorphism*** if f and f^{-1} are weakly β-irresolute.

Theorem 4.7. Let (G,τ) be a standard βθ-topological group. Then the following mappings

i) $r_a:(G,\tau)\to(G,\tau)$, defined by $r_a(x)=xa$,
ii) $l_a:(G,\tau)\to(G,\tau)$, defined by $l_a(x)=ax$,
iii) $f:(G,\tau)\to(G,\tau)$, defined by $f(x)=x^{-1}$,
iv) $g:(G,\tau)\to(G,\tau)$, defined by $g(x)=axa^{-1}$, are βθ-homeomorphisms, for a fixed $a\in G$ and for all $x\in G$.

Proof:
Only we prove (ii)
It is clear that $l_a:(G,\tau)\to(G,\tau)$, defined by $l_a(x)=ax$, for all $x\in G$ is a bijective mapping. Let $x\in G$ and H be a sp-θ-open set which contains $a.x$. Since (G,τ) is a βθ-topological group, then by Theorem 4.5 there exists an sp-θ-open sets U and V of x and a, respectively, such that $U.V\subseteq H$.
Therefore, $l_a(U)\subseteq H$. Hence l_a is weakly β-irresolute.
Now, let $y=l_a(x)$, then $y=ax$, $x=a^{-1}y$, which implies that $l_a^{-1}(x)=l_{a^{-1}}(x)=a^{-1}x$.
By a similar way one can prove that r_a^{-1} is weakly β-irresolute.

Theorem 4.8. Let (G,τ) be a SPθ-topological group, U and V be subsets of G, and $\in$ G . Then,

i) If V is an sp-θ-open set, then Vg, gV, gVg^{-1} and V^{-1} are sp-θ-open sets.

ii) If U is an sp-θ-closed set, then Ug, gU, gUg^{-1} and V^{-1} are sp-θ-closed sets.

iii) If V is an sp-θ-open set and A is any subset of G, then VA and AV are sp-θ-open sets.

Proof:
i) and **ii)** follows directly from Theorem 4.7
iii) Since, $\mathrm{AV}=\bigcup_{a\in A} aV$, then by part (i),$aV$ is an sp-θ-open set, and by Theorem 2.6 (ii) , we have aV is an sp-θ-open set and, by a similar way, we can prove that VA is an sp-θ-open set.

5. QUOTIENT SPθ-TOPOLOGICAL GROUPS

In this section, we introduce and study a new type of quotient topological groups called quotient SPθ-topological group. Let (G,τ) be a SPθ-topological group, and H be a normal subgroup of G. Let $\varphi: \mathrm{G} \rightarrow \mathrm{G}/\mathrm{H}$ be the canonical homomorphism on G.

Definition 5.1. Let G be a group. A subset B of G/H. is said to be ***sp-θ-open*** if $\varphi^{-1}(\mathrm{B})$ is an sp-θ-open subset of G.

Definition 5.2. The intersection of all βθ-closed sets containing A is called **βθ-closure** and denoted by $\beta cl_\theta(A)$.

Theorem 5.3. Let (G, τ) be a SPθ-topological group, and H be a normal subgroup of G, then the canonical mapping φ has the following properties

- **i)** φ is weakly β-irresolute, and
- **ii)** φ is an open mapping with respect to the sp-θ-open set.

Proof:

- **i)** Follows directly from the Definition 5.1.
- **ii)** Follows directly from Theorem 4.8.

Theorem 5.4. Let (G, τ) be a SPθ-topological group, and H be a normal subgroup of G. Then $\mathrm{SP\theta O}(\mathrm{G}/\mathrm{H})$ is a discrete space if and only if H is an sp-θ-open set.

Proof:

Let $\mathrm{SP\theta O}(\mathrm{G}/\mathrm{H})$ be a discrete space. Then each element of G/H is an sp-θ-open set.
Now, $eH = H \in G/H$ is an sp-θ-open set, where e is the identity element of G.
Conversely, suppose that H is an sp-θ-open set, then by Theorem 4.1.8 for each $x \in G$, xH is an sp-θ-open set, which implies that $\mathrm{SP\theta O}(\mathrm{G}/\mathrm{H})$ is a discrete space.

Theorem 5.5 Every sp-θ-open subgroup of a SPθ-topological group is an sp-θ-closed set.

Proof:

Let (G, τ) be a SPθ-topological group, and H be a sp-θ-open subgroup of G. Then $H = G \setminus \bigcup xH$, for $x \notin H$, and by Theorem 4.8 we have xH is an sp-θ-open set, and by Theorem 1.7(ii) $\bigcup xH$ is an sp-θ-open set. Hence H is an sp-θ-closed set.

Theorem 5.6. Let (G, τ) be a SPθ-topological group, and A, B be two subsets of G. Then

- **i)** $\beta cl_\theta(aAa^{-1}) = a.\beta cl_\theta(A).a^{-1}$
- **ii)** If $\beta\mathrm{cl}_\theta(\mathrm{A}) \times \beta\mathrm{cl}_\theta(\mathrm{B}) \subseteq \beta\mathrm{cl}_\theta(\mathrm{A} \times \mathrm{B})$, then
  $\beta\mathrm{cl}_\theta(\mathrm{A}).\beta\mathrm{cl}_\theta(\mathrm{B}) \subseteq \beta\mathrm{cl}_\theta(\mathrm{A}.\mathrm{B})$ and $\beta\mathrm{cl}_\theta(\mathrm{A}).\beta\mathrm{cl}_\theta(\mathrm{B}^{-1}) \subseteq \beta\mathrm{cl}_\theta(\mathrm{A}.\mathrm{B}^{-1})$

Proof:

i) Since $\beta cl_\theta(A)$ is an sp-θ-closed subset of G, then by Theorem 4.8(ii) we have:

$a.\beta cl_\theta(A).a^{-1}$ is also an sp-θ-closed set, since $\beta Cl_\theta(aAa^{-1})$ is the smallest sp-θ-closed set containing aAa^{-1} then we have $\beta cl_\theta(aAa^{-1}) \subseteq a.\beta cl_\theta(A).a^{-1}$, and by Theorem 4.7 a mapping $f:(G,\tau) \to (G,\tau)$ defined by $f(x) = axa^{-1}$ is $\beta\theta$-homeomorphism, therefore f 〖(βcl〗_θ (A)) ⊆ 〖(βcl〗_θ (f(A)).

Hence $a.\beta cl_\theta(A).a^{-1} \subseteq \beta cl_\theta(aAa^{-1})$, which implies that $\beta cl_\theta(aAa^{-1}) = a.\beta cl_\theta(A).a^{-1}$.

Theorem 5.7 Let (G,τ) be a SPθ-topological group, and H be a subset of G, and

$\beta cl_\theta(H) \times \beta cl_\theta(H) \subseteq \beta cl_\theta(H \times H)$. Then,

- **i)** If H is a subgroup of G, then $\beta cl_\theta(H)$ is a subgroup of G.
- **ii)** If H is a normal subgroup of G, then $\beta cl_\theta(H)$ is a normal subgroup of G.

Proof:

i) Let H be a subgroup of G. Then $HH \subseteq H$, thus $\beta cl_\theta(HH) \subseteq \beta cl_\theta(H)$.
By Theorem 5.5 we have $\beta cl_\theta(H).\beta cl_\theta(H) \subseteq \beta cl_\theta(H)$.
Since H is a subgroup of G, then $H \subseteq H^{-1}$ and so $\beta cl_\theta(H) \subseteq \beta cl_\theta(H^{-1})$.
Also, by Theorem 5.5 we have $\beta cl_\theta(H) \subseteq \beta cl_\theta(H)^{-1}$.
Hence $\beta cl_\theta(H)$ is a normal subgroup of G.

6. SOME PROPERTIES OF βθ-MONADS IN SPθ-TOPOLOGICAL GROUPS

In this section, by using nonstandard techniques, we give some properties of βθ-monads in βθ-topological groups.

Theorem 6.1 Let a and b be two standard points in a Spθ-topological group (G,τ). Then μ_$\beta\theta$ $(a).\mu$_$\beta\theta$ $(b) \subseteq \mu$_$\beta\theta$ $(a.b)$.

Proof:

Let $x \in \mu_{\beta\theta}(a)$ and $y \in \mu_{\beta\theta}(b)$. By Theorem 2.5 there exists two standard sp-θ-open sets U and V of a and b, respectively. Therefore, $a.b \in U.V$
Since (G,τ) is βθ-topological group, then by Theorem 4.1.5, for any standard sp-θ-open set W containing $a.b$, we have $U^{st}.V^{st} \subseteq W^{st}$. By transfer axiom we have $U.V \subseteq W$ for all sp-θ-open sets U,V and W. Hence $\mu_{\beta\theta}(a).\mu_{\beta\theta}(b) \subseteq \mu_{\beta\theta}(a.b)$.

Theorem 6.2. Let a be a standard point in a standard SPθ-topological group (G, τ), then $\mu_{\beta\theta}(a^{-1}) = (\mu_{\beta\theta}(a))^{-1}$.

Proof:
Let V be an sp-θ-open set containing a^{-1}. Since, (G, τ) is a standard SPθ-topological group, then by Theorem 4.5 there exists a standard sp-θ-open set U of x such that $a^{-1} \in U^{-1} \subseteq V$.
Then by Theorem 2.5 we have $\mu_{\beta\theta}(a^{-1}) \subseteq V$, $\mu_{\beta\theta}(a) \subseteq U$, and $U^{-1} \subseteq V$.
Therefore $(\mu_{\beta\theta}(a))^{-1} \subseteq U^{-1} \subseteq V$.
Since V is an arbitrary standard sp-θ-open set and SPθO(X) ⊆ βO(X), $V \subseteq \beta cl\,(V)$,then we have $(\mu_{\beta\theta}(a))^{-1} \subseteq \cap \{\beta cl(V)\ ;\ V \in \beta O(X, a^{-1})\} = \mu_{\beta\theta}(a^{-1})$.
If we replace a by a^{-1} we get $\mu_{\beta\theta}(a^{-1}) \subseteq (\mu_{\beta\theta}(a))^{-1}$. Hence $\mu_{\beta\theta}(a^{-1}) = (\mu_{\beta\theta}(a))^{-1}$.

Theorem 6.3. If a and b be any two standard points in a standard SPθ-topological group (G, τ), then $\mu_{\beta\theta}(a).\mu_{\beta\theta}(b) = \mu_{\beta\theta}(a.b)$.

Proof:
The proof is similar to that of Theorem 6.1.
Note that we use a nonstandard technique to rewrite Theorem 4.8

Theorem 6.4. If U is a standard sp-θ-open subset of G, then $U.a$ is also standard sp-θ-open subset of G.

Proof:
Let U be a standard sp-θ-open subset of a group G. and $b \in U$. Then by Theorem 2.5 we have $\mu_{\beta\theta}(b) \subseteq U^{st}$, and hence by transfer axiom we get $\mu_{\beta\theta}(b) \subseteq U$, for each U.
Now, let $c \in U.a$. Then $c=da$ for some $d \in U$, and $\mu_{\beta\theta}(d).a \subseteq \mu_{\beta\theta}(d).\mu_{\beta\theta}(a)$. By Theorem 6.1, we have

$$\mu_{\beta\theta}(d).a \subseteq \mu_{\beta\theta}(da) = \mu_{\beta\theta}(c) \quad \dots (1)$$

If $g \in \mu_{\beta\theta}(c)$, then $f = ga^{-1}$ where $f \in \mu_{\beta\theta}(c)\mu_{\beta\theta}(a^{-1}) = \mu_{\beta\theta}(c.a^{-1}) = \mu_{\beta\theta}(d)$. Thus $g = g.a^{-1}.a = f.a \in \mu_{\beta\theta}(d).a$. That is $\mu_{\beta\theta}(c) \subseteq \mu_{\beta\theta}(d).a$ … (2)

From (1) and (2) we have $\mu_{\beta\theta}(c) = \mu_{\beta\theta}(d).a$. Hence $\mu_{\beta\theta}(c) \subseteq U.a$ and by Theorem 2.5 we deduce that $U.a$ is an sp-θ-open set.

Theorem 6.5. Let (G, τ) be a standard SPθ-topological group. Then U is a standard sp-θ-open subset of G if and only if $U.a$ is a standard sp-θ-open subset of G .

Proof:

From Theorem 6.4 for any standard element a in G, if U is a standard sp-θ-open subset of G, then $U.a$ is also standard sp-θ-open subset of G. Therefore it is enough to show that if $U.a$ is a standard sp-θ-open subset of G, then U is also standard sp-θ-open subset of G.
Now, since G is a group and $a \in G$, then it follows from Theorem 4.8(i) that $(U.a).a^{-1} = U$ is also sp-θ-open set.

Theorem 6.6. Let e be the identity element of a group G. Then, $\mu_{\beta\theta}(e)$ is a subgroup of G**.**

Proof:
From Theorem 2.6 (i) it is clearly that $e \in \mu_{\beta\theta}(e)$.
Let $a, b \in \mu_{\beta\theta}(e)$. Then $a.b \in \mu_{\beta\theta}(e).\mu_{\beta\theta}(e)$, and by Theorem 6.1 we have $a.b \in \mu_{\beta\theta}(e)$. Now, let $a \in \mu_{\beta\theta}(e)$. Then $a^{-1} \in (\mu_{\beta\theta}(e))^{-1}$, and by Theorem 6.2 we have $a^{-1} \in \mu_{\beta\theta}(e)$. Hence, $\mu_{\beta\theta}(e)$ is a subgroup of a group G.

Definition 6.7. Let G be a group. An element $a \in G$ is said to be ***SPθ-near-standard*** in G if there exists a standard point $b \in G$, such that $a \in \mu_{\beta\theta}(b)$. The set of all *βθ-near-standard* points in G is denoted by $\beta\theta$-ns(G)

Theorem 6.8. $\beta\theta$-ns(G) is a subgroup of G.

Proof: Let $x \in \mu_{\beta\theta}(a)$ and $y \in \mu_{\beta\theta}(b)$ for some standard points a, b in G.
Then $x.y \in \mu_{\beta\theta}(a).\mu_{\beta\theta}(b)$, and by Theorem 6.1, we deduce that $x.y \in \mu_{\beta\theta}(a.b)$. Therefore $x.y \in ns(G)$. If $x \in \mu_{\beta\theta}(a)$ for some standard a in G, then $x^{-1} \in (\mu_{\beta\theta}(a))^{-1}$, and by Theorem 6.2, we deduce that $x^{-1} \in \mu_{\beta\theta}(a^{-1})$. Thus $x^{-1} \in$ βθ-ns(G). Hence βθ-ns(G) is a subgroup of G.

Theorem 6.9. $\mu_{\beta\theta}(e)$ is a normal subgroup of βθ-ns(G).

Proof:
Theorem 6.6 implies that $\mu_{\beta\theta}(e)$ is a subgroup. Thus to show that it is a normal it is enough to show that $g.\mu_{\beta\theta}(e)g^{-1} \subseteq \mu_{\beta\theta}(e)$ for any $g \in$ ns(G). For this let $a \in \mu_{\beta\theta}(e)$ and $g \in$ ns(G). Then there exists a standard b in G, such that $g \in \mu_{\beta\theta}(b)$.
Now, $gag^{-1} \in \mu_{\beta\theta}(b)\mu_{\beta\theta}(e)(\mu_{\beta\theta}(b))^{-1}$, then by Theorem 6.2 we have

$gag^{-1} \in \mu_{\beta\theta}(b)\mu_{\beta\theta}(e).\mu_{\beta\theta}(b^{-1})$, and by Theorem 6.1 we deduce that

$$gag^{-1} \in \mu_{\beta\theta}(b.e.b^{-1}) = \mu_{\beta\theta}(e).$$

Hence $\mu_{\beta\theta}(e)$ is a normal subgroup.

7. CONCLUSION

In nonstandard analysis, one is working with two structures. The standard universe and the nonstandard universe. It is a mathematical framework in which one extends the classical mathematical universe of standard numbers, standard sets, standard functions, etc. into a larger nonclassical universe of nonstandard numbers, nonstandard sets, nonstandard functions, etc. In this paper we conclude that the use of nonstandard tools make a price reformulating and introducing some concepts of topological groups with nice and more compact results.

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الزمرة التبولوجية من نمط اس بى ثيتا فى التحليل غير القياسى

يهتم هذا البحث بتقديم ودراسة صنف جديد من الزمرة التبولوجية يسمى الزمرة التبولوجية من النمط اس بى ثيتا، بإستخدام بعض التقنيات غير القياسية الذى اوجده وربنسون و وضعه نيلسون، حصلنا على بعض الخواص لموناد من النمط بيتا ثيتا فى الزمرة التبولوجية من النمط اس بى ثيتا.