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# GENERALIZED $\delta - s \wedge_{ij}$ −SETS IN BITOPOLOGICAL SPACES

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**In this paper, we introduce and study $ij-\delta-$semi closed and $ij-\delta-$semi open sets in bitopological spaces. Furthermore, we introduce and study the notions of $\delta-s\wedge_{ij}-$sets and $g\delta-s\wedge_{ij}-$sets in bitopological spaces. Also, we define a new closure operator $Cl_{\delta}^{s\wedge_{ij}}$ and associated topology $\tau_{\delta}^{s\wedge_{ij}}$ on the bitopological space $(X,\tau_1,\tau_2)$.**

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## INTRODUCTION

A triple $(X,\tau_1,\tau_2)$, where *X* is nonempty set $\tau_1$ and $\tau_2$ are topologies on *X*, is called a bitopological space and Kelly [4] in 1963 initiated the systematic study of such spaces. After the work of Kelly various authors have contributed to the development of this theory. In 1981, Bose [2] introduced the notion of $ij-$semi open sets in bitopological spaces. In 1987, Banerjee [1], introduced the notion of $ij-\delta-$open sets in such spaces. Further investigations of $ij-\delta-$open sets were found in [5,6]. In this paper we introduce and study $ij-\delta-$semi closed and $ij-\delta-$semi open sets in bitopological spaces and then we introduce and study the notions of $\delta-s\wedge_{ij}-$sets and $g\delta-s\wedge_{ij}-$sets in bitopological spaces by generalizing the results obtained in [3] to the bitopological setting. Also, we define a new closure operator $Cl_{\delta}^{s\wedge_{ij}}$ and associated topology $\tau_{\delta}^{s\wedge_{ij}}$ on the bitopological space $(X,\tau_1,\tau_2)$.

Throughout this paper $(X,\tau_1,\tau_2)$ ( or briefly $X$ ) always means a bitopological space on which no separation axioms are assumed unless explicitly stated. Let $A$ be a subset of $X$, by $i-Cl(A)$ and $i-Int(A)$ we denote the closure and the interior of $A$ in the topological space $(X,\tau_i)$. By $i$-open (or $\tau_i-$open) and $i$-closed (or $\tau_i-$closed) we mean open and closed in the topological space $(X,\tau_i)$. $X\setminus A=A^c$ will denote the complement of $A$ and $I$ denote for an index set. Also $i,j=1,2$ and $i\neq j$ .

Let $A$ be a subset of a bitopological space $(X,\tau_1,\tau_2)$ . A point $x$ of $X$ is called an $ij-\delta-$cluster point of $A$ [1] if $i-Int(j-Cl(U))\cap A\neq\phi$ for every $\tau_i$-open set $U$ containing $x$. The set of all $ij-\delta-$cluster points of $A$ is called the $ij-\delta-$closure of $A$ and is denoted by $ij-Cl_\delta(A)$ . A subset $A$ is said to be $ij-\delta-$closed if $ij-Cl_\delta(A)=A$. The complement of an $ij-\delta-$closed set is called $ij-\delta-$open. A subset $A$ of $X$ is called $ij-$semi open [2] if $A\subset j-Cl(i-Int(A))$.

## 2. $ij-\delta-$semi open sets.

**Definition 2.1** A subset A of bitopological space $(X,\tau_1,\tau_2)$ is called $ij-\delta-$semi open if there exists $ij-\delta-$open set U such that $U\subset A\subset j-Cl(U)$. The complement of an $ij-\delta-$semi open set is called $ij-\delta-$semi closed.

A point $x\in X$ is called an $ij-\delta-$semi cluster point of $A$ if $A\cap U\neq\phi$ for every $ij-\delta-$semi open set $U$ of $X$ containing $x$. The set of all $ij-\delta-$semi cluster points of $A$ is called the $ij-\delta-$semi closure of $A$ and is denoted by $ij-\delta sCl(A)$. The collection of all $ij-\delta-$semi open (resp. $ij-\delta-$semi closed) sets of $X$ will be denoted by $ij-\delta SO(X)$ (resp. $ij-\delta SC(X)$).

A subset $U$ of $X$ is called $ij-\delta$-semi neighborhood (briefly, $ij-\delta$-semi nbd) of a point $x$ if there exists an $ij-\delta$-semi open set $V$ such that $x\in V\subseteq U$ .

**Lemma 2.1**. The union of arbitrary collection of $ij-\delta-$semi open sets in $(X,\tau_1,\tau_2)$ is $ij-\delta-$semi open.

**Proof**: Since arbitrary union of $ij-\delta-$open sets is $ij-\delta-$open [ 4 Lemma 2.2], the result follows directly.

**Lemma 2.2**. The intersection of arbitrary collection of $ij-\delta-$semi closed sets in $(X,\tau_1,\tau_2)$ is $ij-\delta-$semi closed.

Proof: Follows directly by Lemma 2.1.

**Corollary 2.3**. Let $A \subset X$, $ij-\delta sCl(A) = \cap\{F : A \subseteq F,\ F \in ij-\delta SC(X)\}$.

**Corollary 2.4**. $ij-\delta sCl(A)$ is $ij-\delta-$semi closed, that is $ij-\delta sCl(ij-\delta sCl(A)) = ij-\delta sCl(A)$.

**Lemma 2.5.** For subsets *A*, *B* and $A_k\ (k \in I)$ of a bitopological space $(X, \tau_1, \tau_2)$, the following hold

(1) $A \subseteq ij-\delta sCl(A)$.

(2) $A \subseteq B \Rightarrow ij-\delta sCl(A) \subseteq ij-\delta sCl(B)$.

(3) $ij-\delta sCl(\cap_k A_k) \subseteq \cap_k \{ij-\delta sCl(A_k)\}$.

(4) $ij-\delta sCl(\cup_k A_k) = \cup_k \{ij-\delta sCl(A_k)\}$.

(5) A is $ij-\delta-$semi closed if and only if $A = ij-\delta sCl(A)$

## 3. $\delta - s \wedge_{ij}$ –sets and $g\delta - s \wedge_{ij}$ –sets.

**Definition 3.1**. For a subset *B* of a bitopological space $(X, \tau_1, \tau_2)$, we define

$B_{\delta}^{s\wedge_{ij}} = \cap\{O \in ij-\delta SO(X) : B \subseteq O\}$

$B_{\delta}^{s\vee_{ij}} = \cup\{F \in ij-\delta SC(X) : F \subseteq B\}$.

**Definition 3.2**. A subset *B* of a bitopological space $(X, \tau_1, \tau_2)$ is called $\delta - s \wedge_{ij}$ –set (resp. $\delta - s \vee_{ij}$ –set) if $B = B_{\delta}^{s\wedge_{ij}}$ (resp. $B = B_{\delta}^{s\vee_{ij}}$ ).

**Definition 3.3**. A subset *B* of a bitopological space $(X, \tau_1, \tau_2)$ is called

(1) generalized $\delta - s \wedge_{ij}$ –set (briefly, $g\delta - s \wedge_{ij}$ -set ) if $B_{\delta}^{s\wedge_{ij}} \subseteq F$ whenever $B \subseteq F$ and $F \in ji-\delta SC(X)$.

(2) generalized $\delta - s \vee_{ij}$ –set (briefly, $g\delta - s \vee_{ij}$ -set ) if $B^c$ is $g\delta - s \wedge_{ij}$.

By $G_{\delta}^{s\wedge_{ij}}$ ( resp. $G_{\delta}^{s\vee_{ij}}$ ) we will denote the family of all $g\delta - s \wedge_{ij}$ -sets (resp. $g\delta - s \vee_{ij}$ -sets).

**Proposition 3.4**. Let *A*, *B* and $\{B_k : k \in I\}$ be subsets of a bitopological space $(X, \tau_1, \tau_2)$. The following properties hold:

(1) $B \subseteq B_{\delta}^{s\wedge ij}$.

(2) If $A \subseteq B$, then $A_{\delta}^{s\wedge ij} \subseteq B_{\delta}^{s\wedge ij}$.

(3) $\left(B_{\delta}^{s\wedge ij}\right)_{\delta}^{s\wedge ij} = B_{\delta}^{s\wedge ij}$.

(4) $\left(\bigcup_{k\in I} B_{\lambda}\right)_{\delta}^{s\wedge ij} = \bigcup_{k\in I}(B_k)_{\delta}^{s\wedge ij}$.

(5) If $A \in ij-\delta SO(X)$, then $A = A_{\delta}^{s\wedge ij}$.

(6) $\left(B^c\right)_{\delta}^{s\wedge ij} = \left(B_{\delta}^{s\vee ij}\right)^C$.

(7) $B_{\delta}^{s\vee ij} \subseteq B$.

(8) If $B \in ij-\delta SC(X)$, then $B = B_{\delta}^{s\vee ij}$.

(9) $\left(\bigcap_{k\in I} B_k\right)_{\delta}^{s\wedge ij} \subseteq \bigcap_{k\in I}(B_k)_{\delta}^{s\wedge ij}$.

(10) $\left(\bigcup_{k\in I} B_{\lambda}\right)_{\delta}^{s\vee ij} \supseteq \bigcup_{k\in I}(B_k)_{\delta}^{s\vee ij}$.

**Proof**: (1) Clear by Definition 3.1.

(2) Suppose $x \notin B_{\delta}^{s\wedge ij}$. Then there exists an $ij-\delta-$semi open set U such that $B \subseteq U$ and $x \notin U$. Since $A \subseteq B$, then $x \notin A_{\delta}^{s\wedge ij}$ and therefore $A_{\delta}^{s\wedge ij} \subseteq B_{\delta}^{s\wedge ij}$.

(3) Follows from (1) and Definition 3.2.

(4) Let $x \notin \left(\bigcup_{k\in I} B_k\right)_{\delta}^{s\wedge ij}$. Then there exists an $ij-\delta-$semi open set U such that $\bigcup_{k\in I} B_k \subseteq U$ and $x \notin U$. Thus for each $k \in I$ we have $x \notin (B_k)_{\delta}^{s\wedge ij}$. This implies that $x \notin \bigcup_{k\in I}(B_k)_{\delta}^{s\wedge ij}$.

Conversely, let $x \notin \bigcup_{k\in I}(B_k)_{\delta}^{s\wedge ij}$. Then there exists an $ij-\delta-$semi open set $U_k$ (for each $k \in I$) such that $x \notin U_k$, $B_k \subseteq U_k$. Let $U = \bigcup_{k\in I} U_k$. Then we have $x \notin U = \bigcup_{k\in I} U_k$, $\bigcup_{k\in I} B_k \subseteq U$ and $U$ is $ij-\delta-$semi open. This implies that $x \notin \left(\bigcup_{k\in I} B_k\right)_{\delta}^{s\wedge ij}$. This completes the proof of (4).

(5) By definition and since A is an $ij-\delta-$semi open set, we have $A_{\delta}^{s\wedge ij} \subseteq A$. By (1), we have $A_{\delta}^{s\wedge ij} = A$.

(6) $\left(B_{\delta}^{s\vee ij}\right)^C = \cap\{F^c : F^c \supseteq B^c, F^c \in ij-\delta SO(X)\} = \left(B^c\right)_{\delta}^{s\wedge ij}$.

(7) Clear by definition.

(8) If $B \in ij-\delta SC(X)$, then $B^c \in ij-\delta SO(X)$. By (5) and (6) $B^c = (B^c)_\delta^{s\wedge_{ij}} = \left(B_\delta^{s\vee_{ij}}\right)^c$. Hence $B = B_\delta^{s\vee_{ij}}$.

(9) Let $x \notin \bigcap_{k\in I}(B_k)_\delta^{s\wedge_{ij}}$. Then there exists $k \in I$ such that $x \notin (B_k)_\delta^{s\wedge_{ij}}$. Hence there exists $U \in ij-\delta SO(X)$ such that $B_k \subseteq U$ and $x \notin U$. Thus $x \notin \left(\bigcap_{k\in I} B_k\right)_\delta^{s\wedge_{ij}}$.

(10) $\left(\bigcup_{k\in I} B_k\right)_\delta^{s\vee_{ij}} = \left(\left(\left(\bigcup_{k\in I} B_k\right)^c\right)_\delta^{s\vee_{ij}}\right)^c = \left(\left(\bigcap_{k\in I} B_k^c\right)_\delta^{s\vee_{ij}}\right)^c \supseteq \left(\bigcap_{k\in I}\left((B_k)_\delta^{s\vee_{ij}}\right)^c\right)^c = \bigcup_{k\in I}(B_k)_\delta^{s\vee_{ij}}$.

**Theorem 3.6**. Let *B* be a subset of a bitopological space $(X,\tau_1,\tau_2)$. Then

(1) $\phi$ and *X* are $\delta - s\wedge_{ij}$ − sets and $\delta - s\vee_{ij}$ − sets.

(2) Every union of $\delta - s\wedge_{ij}$ − sets (resp. $\delta - s\vee_{ij}$ − sets) is $\delta - s\wedge_{ij}$ − sets (resp. $\delta - s\vee_{ij}$ − sets).

(3) Every intersection of $\delta - s\wedge_{ij}$ − sets (resp. $\delta - s\vee_{ij}$ − sets) is $\delta - s\wedge_{ij}$ − sets (resp. $\delta - s\vee_{ij}$ − sets).

(4) *B* is a $\delta - s\wedge_{ij}$ − set if and only if $B^c$ is a $\delta - s\vee_{ij}$ − set.

**Proof**. (1) and (4) are obvious.

(2) Let $\{B_k : k \in I\}$ be a family of $\delta - s\wedge_{ij}$ − sets in $(X,\tau_1,\tau_2)$. Then by definition and Proposition 3.4 (4) we have $\bigcup_{k\in I} B_k = \bigcup_{k\in I}(B_k)_\delta^{s\wedge_{ij}} = \left(\bigcup_{k\in I} B_k\right)_\delta^{s\wedge_{ij}}$.

(3) Let $\{B_k : k \in I\}$ be a family of $\delta - s\wedge_{ij}$ − sets in $(X,\tau_1,\tau_2)$. Then by definition and Proposition 3.4(9) we have $\left(\bigcap_{k\in I} B_k\right)_\delta^{s\wedge_{ij}} \subseteq \bigcap_{k\in I}(B_k)_\delta^{s\wedge_{ij}} = \bigcap_{k\in I} B_k$.

Hence by Proposition 3.4(1), $\bigcap_{k\in I} B_k = \left(\bigcap_{k\in I} B_k\right)_\delta^{s\wedge_{ij}}$.

**Remark 3.7**. By Theorem 3.6, the family of all $\delta - s\wedge_{ij}$ − sets (resp. $\delta - s\vee_{ij}$ − sets), denoted by $\lambda_\delta^{s\wedge_{ij}}$ (resp. $\lambda_\delta^{s\vee_{ij}}$) in $(X,\tau_1,\tau_2)$ is a topology on *X* containing all $ij-\delta-$semi open (resp. $ij-\delta-$semi closed) sets. Clearly $(X,\lambda_\delta^{s\wedge_{ij}})$ and $(X,\lambda_\delta^{s\vee_{ij}})$ are Alexandroff spaces, i.e., arbitrary intersection of open sets are open.

**Proposition 3.8**. Let $(X,\tau_1,\tau_2)$ be a bitopological space. Then

(1) Every $\delta - s\wedge_{ij} -$set is a $g\delta - s\wedge_{ij} -$set.

(2) Every $\delta - s\vee_{ij} -$set is a $g\delta - s\vee_{ij} -$set.

(3) If $B_k$ is a $g\delta - s\wedge_{ij} -$set for all $k\in I$, then $\bigcup_{k\in I} B_k$ is a $g\delta - s\wedge_{ij} -$set.

(4) If $B_k$ is a $g\delta - s\vee_{ij} -$set for all $k\in I$, then $\bigcap_{k\in I} B_k$ is a $g\delta - s\vee_{ij} -$set.

Proof. (1) Follows directly by definition.

(2) Let *B* be a $\delta - s\vee_{ij} -$subset of *X*. Then $B = B_\delta^{s\vee_{ij}}$. By Proposition 3.4(6) $(B^c)_\delta^{s\wedge_{ij}} = \left(B_\delta^{s\vee_{ij}}\right)^c = B^c$. Therefore, by (1) and definition, *B* is a $g\delta - s\vee_{ij} -$set.

(3) Let $B_k$ is a $g\delta - s\wedge_{ij} -$subset of $X$ for all $k\in I$. Then by Proposition 3.4(4) $\left(\bigcup_{k\in I} B_k\right)_\delta^{s\wedge_{ij}} = \bigcup_{k\in I} (B_k)_\delta^{s\wedge_{ij}}$. Hence, by hypothesis and definition, $\bigcup_{k\in I} B_k$ is a $g\delta - s\wedge_{ij} -$set.

(4) Follows from (3) and definition.

**Proposition 3.9**. A subset *B* of a bitopological space $(X,\tau_1,\tau_2)$ is a $g\delta - s\vee_{ij} -$set if and only if $U\subseteq B_\delta^{s\vee_{ij}}$, whenever $U\subseteq B$ and U is an $ij-\delta-$semi open subset of *X*.

**Proof**. Let *U* be an $ij-\delta-$semi open subset of *X* such that $U\subseteq B$. Then, since $U^c$ is $ij-\delta-$semi closed and $B^c\subseteq U^c$, we have $(B^c)_\delta^{s\wedge_{ij}}\subseteq U^c$ by definition. Hence by Proposition 3.4(6) $\left(B_\delta^{s\vee_{ij}}\right)^c\subseteq U^c$. Thus $U\subseteq B_\delta^{s\vee_{ij}}$. On the other hand, let *F* be an $ij-\delta-$semi closed subset of X such that $B^c\subseteq F$. Since $F^c$ is $ij-\delta-$semi open and $F^c\subseteq B$, by assumption we have $F^c\subseteq B_\delta^{s\vee_{ij}}$. Then $F\supseteq\left(B_\delta^{s\vee_{ij}}\right)^c = (B^c)_\delta^{s\vee_{ij}}$. Thus $B^c$ is a $g\delta - s\wedge_{ij} -$set, i.e., *B* is a $g\delta - s\vee_{ij} -$set.

## 4. $ij-Cl^{s\wedge\delta}$ − closure operator and associated $\tau_{\delta}^{s\wedge ij}$ .

In this section, we define a closure operator $Cl_{\delta}^{s\wedge ij}$ and the associated topology $\tau_{\delta}^{s\wedge ij}$ on the bitopological space $(X,\tau_1,\tau_2)$ using the family of $g\delta - s\wedge_{ij}$ − sets.

**Definition 4.1**. For any subset $B$ of a bitopological space $(X,\tau_1,\tau_2)$, define $Cl_{\delta}^{s\wedge ij}(B)=\cap\{U : B\subseteq U, U\in G_{\delta}^{s\wedge ij}\}$ and $Int_{\delta}^{s\wedge ij}(B)=\cup\{F : B\supseteq F, F^{c}\in G_{\delta}^{s\wedge ij}\}$.

**Proposition 4.2**. Let $A$, $B$ and $\{B_{\lambda}:\lambda\in I\}$ be subsets of a bitopological space $(X,\tau_1,\tau_2)$. Then the following properties are hold:

(1) $B\subseteq Cl_{\delta}^{s\wedge ij}(B)$.

(2) $Cl_{\delta}^{s\wedge ij}(B^{c})=\left(Int_{\delta}^{s\wedge ij}(B)\right)^{c}$.

(3) $Cl_{\delta}^{s\wedge ij}(\phi)=\phi$.

(4) $\bigcup_{k\in I} Cl_{\delta}^{s\wedge ij}(B_k)=Cl_{\delta}^{s\wedge ij}(\underset{k\in I}{\cup} B_k)$.

(5) $Cl_{\delta}^{s\wedge ij}\left(Cl_{\delta}^{s\wedge ij}(B)\right)=Cl_{\delta}^{s\wedge ij}(B)$.

(6) If $A\subseteq B$, then $Cl_{\delta}^{s\wedge ij}(A)\subseteq Cl_{\delta}^{s\wedge ij}(B)$

(7) If $B$ is s $g\delta - s\wedge_{ij}$ −set, then $Cl_{\delta}^{s\wedge ij}(B)=B$.

(8) If $B$ is s $g\delta - s\vee_{ij}$ −set, then $Int_{\delta}^{s\wedge ij}(B)=B$.

Proof: (1), (2), (3) and (6) are clear.

(4) Let $x\notin Cl_{\delta}^{s\wedge ij}(\underset{k\in I}{\cup} B_k)$. Then, there exists $U\in G_{\delta}^{s\wedge ij}$ such that $\underset{k\in I}{\cup} B_k\subseteq U$ and $x\notin U$. Thus for each $k\in I$ we have $x\notin Cl_{\delta}^{s\wedge ij}(B_k)$. This implies that $x\notin \bigcup_{k\in I} Cl_{\delta}^{s\wedge ij}(B_k)$. Conversely, let $x\notin \bigcup_{k\in I} Cl_{\delta}^{s\wedge ij}(B_k)$. Then there exist subsets $U_k\in G_{\delta}^{s\wedge ij}$ for all $k\in I$ such that $x\notin U_k$ and $B_k\subseteq U_k$. Let $U=\underset{k\in I}{\cup} U_k$. Then $x\notin U$, $\bigcup_{k\in I} B_k\subseteq U$ and $U\in G_{\delta}^{s\wedge ij}$. Thus $x\notin Cl_{\delta}^{s\wedge ij}(\bigcup_{k\in I} B_k)$.

(5) Suppose that $x \notin Cl_{\delta}^{s\wedge ij}(B)$. Then there exists a subset $U \in G_{\delta}^{s\wedge ij}$ such that $x \notin U$ and $B \subseteq U$ . Since $U \in G_{\delta}^{s\wedge ij}$ we have $Cl_{\delta}^{s\wedge ij}(B) \subseteq U$ . Thus we have $x \notin Cl_{\delta}^{s\wedge ij}(Cl_{\delta}^{s\wedge ij}(B))$. Therefore $Cl_{\delta}^{s\wedge ij}\left(Cl_{\delta}^{s\wedge ij}(B)\right) \subseteq Cl_{\delta}^{s\wedge ij}(B)$. But by (1) $Cl_{\delta}^{s\wedge ij}(B) \subseteq Cl_{\delta}^{s\wedge ij}\left(Cl_{\delta}^{s\wedge ij}(B)\right)$. Then the result follows.

(7) Follows directly by (1) and definition.

(8) Follows directly by (7) and (2) and definition.

Then we have the following

**Theorem 4.3**. $Cl_{\delta}^{s\wedge ij}$ is a Kuratowski closure operator on *X*.

**Definition 4.4**. Let $\tau_{\delta}^{s\wedge ij}$ be the topology on *X* generated by $Cl_{\delta}^{s\wedge ij}$ in the usual manner, i.e., $\tau_{\delta}^{s\wedge ij} = \{B \subseteq X : Cl_{\delta}^{s\wedge ij}(B^{c}) = B^{c}\}$.

We define a family $\rho_{\delta}^{s\wedge ij}$ by $\rho_{\delta}^{s\wedge ij} = \{B \subseteq X : Cl_{\delta}^{s\wedge ij}(B) = B\}$, equivalently $\rho_{\delta}^{s\wedge ij} = \{B \subseteq X : B^{c} \in \tau_{\delta}^{s\wedge ij}\}$.

**Proposition 4.5**. Let $(X, \tau_1, \tau_2)$ be a bitopological space. Then

(1) $\tau_{\delta}^{s\wedge ij} = \{B \subseteq X : Int_{\delta}^{s\vee ij}(B) = B\}$.

(2) $ij-\delta SO(X) \subseteq G_{\delta}^{s\wedge ij} \subseteq \rho_{\delta}^{s\wedge ij}$ .

(3) $ij-\delta SC(X) \subseteq G_{\delta}^{s\vee ij} \subseteq \tau_{\delta}^{s\wedge ij}$ .

(4) If $ij-\delta SC(X) = \tau_{\delta}^{s\wedge ij}$, then every $g\delta - s\wedge_{ij}-$set of *X* is $ij-\delta-$semi open.

(5) If every $g\delta - s\wedge_{ij}-$set of *X* is $ij-\delta-$semi open (i.e., if $G_{\delta}^{s\vee ij} \subseteq ij-\delta SO(X)$), then $\tau_{\delta}^{s\wedge ij} = \{B \subseteq X : B = B_{\delta}^{s\vee ij}\}$.

(6) If every $g\delta - s\wedge_{ij}-$set of *X* is $ij-\delta-$semi closed (i.e., if $G_{\delta}^{s\wedge ij} \subseteq ij-\delta SC(X)$), then $ij-\delta SO(X) = \tau_{\delta}^{s\wedge ij}$ .

Proof. (1) By Definition 4.1 and Proposition 4.2(2), we have: If $A \subseteq X$ , then $A \in \tau_{\delta}^{s\wedge ij}$ if and only if $Cl_{\delta}^{s\wedge ij}(A^{c}) = A^{c}$ if and only if $\left(Int_{\delta}^{s\vee ij}(A)\right)^{c} = A^{c}$ if and only if $Int_{\delta}^{s\vee ij}(A) = A$ . Thus, $\tau_{\delta}^{s\wedge ij} = \{B \subseteq X : Int_{\delta}^{s\vee ij}(B) = B\}$.

(2) Let $B \in ij - \delta SO(X)$. By Proposition 3.4(5) $B$ is a $\delta - s\wedge_{ij}$ −set. By Proposition 3.8 (1), $B$ is a $g\delta - s\wedge_{ij}$ −set, i.e., $B \in G_{\delta}^{s\wedge_{ij}}$. Let now, $B$ any element of $G_{\delta}^{s\wedge_{ij}}$. By Proposition 3.4(7) $B = Cl_{\delta}^{s\wedge_{ij}}(B)$, i.e., $B \in \rho_{\delta}^{s\wedge_{ij}}$. Therefore $ij - \delta SO(X) \subseteq G_{\delta}^{s\wedge_{ij}} \subseteq \rho_{\delta}^{s\wedge_{ij}}$.

(3) Let $B \in ij - \delta SC(X)$. By Proposition 3.4(8) $B = B_{\delta}^{s\vee_{ij}}$. Thus $B$ is a $\delta - s\vee_{ij}$ −set. By Proposition 3.8(2), $B$ is a $g\delta - s\vee_{ij}$ −set. Hence $B \in G_{\delta}^{s\vee_{ij}}$. Now, if $B \in G_{\delta}^{s\vee_{ij}}$, then by (1) and Proposition 3.4(8), $B \in \tau_{\delta}^{s\wedge_{ij}}$.

(4) Let $B$ be any $g\delta - s\wedge_{ij}$ −set, i.e., $B \in G_{\delta}^{s\wedge_{ij}}$. By (2), $B \in \rho_{\delta}^{s\wedge_{ij}}$. Thus, $B^c \in \tau_{\delta}^{s\wedge_{ij}}$. From assumption, we have $B^c \in ij - \delta SC(X)$. Hence $B \in ij - \delta SO(X)$.

(5) Let $A \subseteq X$ and $A \in \tau_{\delta}^{s\wedge_{ij}}$. Then by Definitions 4.1 and 4.4,

$$A^c = Cl_{\delta}^{s\wedge_{ij}}(A^c) = \cap\{U : U \supseteq A, U \in G_{\delta}^{s\wedge_{ij}}\}$$
$$= \cap\{U : U \supseteq A^c, U \in ij - \delta SO(X)\} = \left(A^c\right)_{\delta}^{s\wedge_{ij}}.$$

Using Proposition 3.4(6), we have $A = A_{\delta}^{s\vee_{ij}}$, i.e., $A \in \{B \subseteq X : B = B_{\delta}^{s\vee_{ij}}\}$. Conversely, if $A \in \{B \subseteq X : B = B_{\delta}^{s\vee_{ij}}\}$, then by Proposition 3.8(2), $A$ is a $g\delta - s\vee_{ij}$ −set. Thus $A \in G_{\delta}^{s\vee_{ij}}$. By using (3), $A \in \tau_{\delta}^{s\wedge_{ij}}$.

(6) Let $A \subseteq X$ and $A \in \tau_{\delta}^{s\wedge_{ij}}$. Then $A = \left(Cl_{\delta}^{s\wedge_{ij}}(A^c)\right)^c = \left(\cap\{U : A^c \subseteq U, U \in G_{\delta}^{s\wedge_{ij}}\}\right)^c = \cup\{U^c : U^c \in ij - \delta SO(X)\} \in ij - \delta SO(X)$.

Conversely, if $A \in ij - \delta SO(X)$, then by Proposition 3.4(5) and Proposition 3.8(1), $A \in G_{\delta}^{s\wedge_{ij}}$. By assumption $A \in ij - \delta SC(X)$. Using (3), $A \in \tau_{\delta}^{s\wedge_{ij}}$.

**Lemma 4.6**. Let $(X,\tau_1,\tau_2)$ be a bitopological space.

(1) For each $x\in X$, $\{x\}$ is an $ij-\delta-$semi open set or $\{x\}^c$ is a $g\delta-s\wedge_{ij}-$set of *X*.

(2) For each $x\in X$, $\{x\}$ is an $ij-\delta-$semi open set or $\{x\}$ is a $g\delta-s\vee_{ij}-$set of *X*.

Proof. (1) Suppose that $\{x\}$ is not $ij-\delta-$semi open. Then the only $ij-\delta-$semi closed set *F* containing $\{x\}^c$ is *X*. Thus $\left(\{x\}^c\right)_\delta^{s\wedge_{ij}}\subseteq F=X$ and $\{x\}^c$ is a $g\delta-s\wedge_{ij}-$set of *X*.

(2) Follows from (1) and Definition 3.3.

**Proposition 3.7**. If $ij-\delta SO(X)=\tau_\delta^{s\wedge_{ij}}$, then every singleton $\{x\}$ is $\tau_\delta^{s\wedge_{ij}}-$open.

**Proof.** Suppose that $\{x\}$ is not $ij-\delta-$semi open. Then by Lemma 4.6, $\{x\}^c$ is a $g\delta-s\wedge_{ij}-$set. Thus $\{x\}\in\tau_\delta^{s\wedge_{ij}}$ by Definition 4.4. Suppose that $\{x\}$ is $ij-\delta-$semi open. Then $\{x\}\in ij-\delta SO(X)=\tau_\delta^{s\wedge_{ij}}$. Therefore, every singleton $\{x\}$ is $\tau_\delta^{s\wedge_{ij}}-$open.

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## المجموعات المعممة من النوع $\delta - s \wedge_{ij} -$
## في الفضاءات ثنائية التوبولوجي


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في هذا البحث نقوم بتقديم ودراسة المجموعات شبه المغلقة من النوع دلتا و المجموعات شبه المفتوحة من النوع دلتا في الفضاءات ثنائية التوبولوجي ومن ثم نقدم مفهوم المجموعات من النوع $\delta - s \wedge_{ij} -$ و $g\delta - s \wedge_{ij} -$. أيضا نعرف مؤثر إغلاق جديد $Cl_\delta^{s\wedge_{ij}}$ والتوبولوجي المصاحب $\tau_\delta^{s\wedge_{ij}}$ على الفضاء ثنائي التوبولوجي $(X, \tau_1, \tau_2)$.