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Transient Generalized Couette Flow with Heat Transfer of a Dusty Conducting Fluid with Ion Slip

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Abstract: In this study, the transient generalized Couette flow with heat transfer of a dusty viscous incompressible electrically conducting fluid is studied under the influence of a constant pressure gradient and considering the ion slip. An external uniform magnetic field is applied perpendicular to the plates which are kept at constant temperatures. A numerical solution for the governing momentum and energy equations are obtained using the method of finite differences. The influence of various magnetic field parameters (magnetic field intensity, Hall parameter, ion slip parameter) on the velocity and temperature fields of both the fluid and dust particles phases is demonstrated.

Keywords: Couette flow, heat transfer, dusty fluid, conducting fluid, ion slip effect

Introduction

The flow and heat transfer of conducting fluids through channels of different geometries in the presence of an external magnetic field have interesting applications in magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters and has possible applcations in nuclear reactors, filtration, geothermal systems [1,2]. The existence of a dusty phase in the form of solid particles has important practical applications such as ash or soot in combustion MHD generators and plasma MHD accelerators [1,2].

The flow of dusty fluids was investigated by many authors [3-7]. The MHD flow of dusty fluids was also handled [8-12]. The Hall and ion slip effects were taken into considerations in applying generalized Ohm's law only in the case of strong magnetic fields [13,14]. The influence of the Hall current or the ion slip on the unsteady flow between parallel non-conducting plates, known as Hartmann flow, of a non-dusty fluid was examined by a number of researchers [15-19]. The influence of the Hall current on unsteady Couette flow of a dusty conducting fluid with heat transfer was studied by Attia [20].

In the present work, unsteady Couette flow with heat transfer of an electrically conducting, viscous, incompressible, dusty fluid is studied including both the Hall current and ion slip effects. The upper plate is moving with a constant velocity and the lower plate is fixed where the mass conservation is assumed. The fluid is flowing between two parallel infinite plates maintained at two constant but different temperatures. The fluid is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The governing momentum and energy equations including the viscous and Joule dissipation terms are solved numerically using the method of finite differences to determine the velocity and temperature distributions for both the fluid and dust particles for various values of the parameters associated with the electromagnetic effect.

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Description of the Problem

The dusty fluid is flowing between two parallel infinite horizontal plates located at the $y=\pm h$ planes. The upper plate is given with a constant velocity U_o in the x-direction while the lower plate is fixed. The dust particles are assumed to be electrically non-conducting spherical in shape and uniformly distributed throughout the fluid. The two plates are assumed to be electrically non-conducting and kept at two constant temperatures T_1 for the lower plate and T_2 for the upper plate where $T_2>T_1$. A uniform and constant pressure gradient is applied in the x-direction and a uniform magnetic field B_o is applied in the positive y-direction. Due to the inclusion of the Hall current term, z-components of the velocity for the fluid and dust particle phases arise. Due to the infinite dimensions in the x- and z-directions, the physical quantities do not change in these directions and therefore we have $\partial/\partial x=\partial/\partial z=0$ and the problem becomes one-dimensional.

The Velocity Distribution

The momentum equation for fluid phase is given by

$$\rho \frac{Dv}{Dt} = -\nabla P + \mu \nabla^2 v + JxB_o - KN(v - v_p) \tag{1}$$

where ρ is the density of the fluid, μ is the viscosity of the fluid, $\boldsymbol{v}$ is the velocity vector of the fluid, $\boldsymbol{v}=u(y,t)\boldsymbol{i}+w(y,t)\boldsymbol{k}$, $\boldsymbol{v}_p$ is the velocity vector of dust particles, $\boldsymbol{v_p}=u_p(y,t)\boldsymbol{i}+w_p(y,t)\boldsymbol{k}$, $\boldsymbol{J}$ is the current density, N is the number of dust particles per unit volume, K is the Stokes constant $=6\pi\mu a$, and a is the average radius of dust particles.

The last two terms in the right side of Eq. (1) are, respectively, the Lorentz force and the force associated with the relative motion between the fluid and dust particles. If the Hall and ion slip terms are retained, the current density $\boldsymbol{J}$ from the generalized Ohm's law is given by [13,14]

$$J = \sigma(E + VxB_o - \beta(JxB_o) + \frac{\beta Bi}{B_o}(JxB_o)xB_o) \tag{2}$$

where σ is the electric conductivity of the fluid, β is the Hall factor and Bi is the ion slip parameter [13,14]. Solving Eq. (2) for $\boldsymbol{J}$ yields

$$JxB_o = \frac{\sigma B_o^2}{(1+BiBe)^2 + Be^2}\{((1+BiBe)u + Bew)i + ((1+BiBe)w - Beu)k\} \tag{3}$$

where $Be=\sigma\beta B_o$, is the Hall parameter [13,14]. In terms of Eq. (3), the two components of Eq. (1) read

$$\rho \frac{\partial u}{\partial t} = -\frac{dP}{dx} + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{(1+BiBe)^2 + Be^2}((1+BiBe)u + Bew) - KN(u - u_p) \tag{4}$$

$$\rho \frac{\partial w}{\partial t} = \mu \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_o^2}{(1+BiBe)^2 + Be^2}((1+BiBe)w - Beu) - KN(w - w_p) \tag{5}$$

Applying Newton's second law applied in the x and z-directions, gives the motion of the dust particles in the form

$$m_p \frac{\partial u_p}{\partial t} = KN(u - u_p) \tag{6}$$

$$m_p \frac{\partial w_p}{\partial t} = KN(w - w_p) \tag{7}$$

where m_p is the average mass of dust particles. It is assumed that the pressure gradient is impulsively applied at t=0 and the fluid starts its motion from rest. Thus,

$$t \le 0 : u = u_p = w = w_p = 0 \tag{8a}$$

For t>0, the no-slip condition at the plates results in

$$t > 0,\ \ y = -h:\ u = u_p = w = w_p = 0 \tag{8b}$$

$$t > 0,\ \ y = h:\ u = U_o, u_p = w = w_p = 0 \tag{8c}$$

The Temperature Distribution

Heat transfer is from the upper hot plate to the lower cold plate by conduction through the fluid. There is no natural convection as the hot plate is above, however there is a forced convection due to the presence of suction and injection. Also there is a heat generation due to Joule and viscous dissipations. The dust particles gain heat from the fluid by conduction through their spherical surface [12,21]. The energy equations describing the temperature distributions for both the fluid and dust particles are respectively

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right) + \frac{\sigma(1+BiBe)B_o^2}{(1+BiBe)^2 + Be^2}(u^2 + w^2) + \frac{\rho_p c_s}{\gamma_T}(T_p - T), \tag{9}$$

$$\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma_T}(T_p - T), \tag{10}$$

where T is the temperature of the fluid, T_p is the temperature of the dust particles, c is the specific heat capacity of the fluid at constant volume, k is the thermal conductivity of the fluid, ρ_p is the mass of dust particles per unit volume of the fluid, γ_T is the temperature relaxation time, and c_s is the specific heat capacity of the particles.

The last three terms on the right side of Eq. (9) represent the viscous dissipation, the Joule dissipation, and the heat conduction between the fluid and dust particles respectively. The temperature relaxation time depends on the geometry and since the dust particles are assumed to be spherical in shape, the last term in Eq. (9) is equal to $4\pi aNk(T_p\text{-}T)$. Then

$$\gamma_T = \frac{3\Pr \gamma_p c_s}{2c}$$

where γ_p is the velocity relaxation time=$2\rho_s a^2/9\mu$, Pr is the Prandtl number=$\mu c/k$, and ρ_s is the material density of dust particles=$3\rho_p/4\pi a^3 N$.

T and T_p must satisfy the initial and boundary conditions

$$t \le 0 : T = T_p = T_1, \tag{11a}$$

$$t > 0, y = -h : T = T_p = T_1, \tag{11b}$$

$$t > 0, y = h : T = T_p = T_2. \tag{11c}$$

Introducing the following dimensionless variables and parameters

$$(\hat{x},\hat{y}) = \frac{(x,y)}{h}, \hat{t} = \frac{tU_o}{h}, (\hat{u},\hat{w}) = \frac{(u,w)}{U_o}, (\hat{u}_p,\hat{w}_p) = \frac{(u_p,w_p)}{U_o}, \hat{P} = \frac{P}{\rho U_o^2}, \hat{T} = \frac{T - T_1}{T_2 - T_1}, \hat{T}_p = \frac{T_p - T_1}{T_2 - T_1}$$

$\mathrm{Re} = U_o \rho h/\mu$, is the Reynolds number,

$Ha = B_o h\sqrt{\sigma/\mu}$, the Hartmann number,

$E_c = U_o^2/c(T_2 - T_1)$, the Eckert number,

$G = m_p \mu/\rho h^2 K$, is the particle mass parameter,

$R = KNh^2/\mu$ is the particle concentration parameter.

$L_o = \rho h^2/\mu\gamma_T$ is the temperature relaxation time parameter.

Equations (4)-(11) take the form (the hats are dropped for convenience)

$$\frac{\partial u}{\partial t}=-\frac{1}{\mathrm{Re}}\frac{dP}{dx}+\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2}-\frac{H_a^2}{\mathrm{Re}((1+BiBe)^2+Be^2)}((1+BiBe)u+Bew)-\frac{R}{\mathrm{Re}}(u-u_p) \tag{12}$$

$$\frac{\partial w}{\partial t}=\frac{1}{\mathrm{Re}}\frac{\partial^2 w}{\partial y^2}-\frac{H_a^2}{\mathrm{Re}((1+BiBe)^2+Be^2)}((1+BiBe)w-Beu)-\frac{R}{\mathrm{Re}}(w-w_p) \tag{13}$$

$$G\frac{\partial u_p}{\partial t}=u-u_p \tag{14}$$

$$G\frac{\partial w_p}{\partial t}=w-w_p \tag{15}$$

$$t\le 0: u=u_p=w=w_p=0 \tag{16a}$$

$$t>0, y=-1: u=u_p=w=w_p=0 \tag{16b}$$

$$t>0, y=1: u=1, u_p=w=w_p=0 \tag{16c}$$

$$\frac{\partial T}{\partial t}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}+\frac{E_c}{\mathrm{Re}}((\frac{\partial u}{\partial y})^2+(\frac{\partial w}{\partial y})^2)+\frac{(1+BiBe)H_a^2E_c}{\mathrm{Re}((1+BiBe)^2+Be^2)}(u^2+w^2)+\frac{2R}{3\,\mathrm{Pr}}(T_p-T), \tag{17}$$

$$\frac{\partial T_p}{\partial t}=-L_o(T_p-T), \tag{18}$$

$$t\le 0: T=T_p=0, \tag{19a}$$

$$t>0, y=-1: T=T_p=0, \tag{19b}$$

$$t>0, y=1: T=T_p=1. \tag{19c}$$

Equations (12)-(19) represent a system of partial differential equations which is solved numerically using finite differences. The Crank-Nicolson implicit method [22] is used at two successive time levels where the finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximation in the *y*-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. Finally, the resulting block tri-diagonal system is obtained using generalized Thomas-algorithm [22]. Calculations are made for dP/dx=5, Re=1, R=0.5, G=0.8, L_o=0.7, Pr=1, and E_c=0.2.

Results and Discussion

Figures 1-3 present the progression with time of the velocity components and temperature at the centre of the channel (y=0), respectively, for the fluid and particle phases for various values of the Hall parameter *Be* and the ion slip parameter *Bi* and for Ha=1. It is apparent from Figs. 1a and 1b that increasing the parameter *Be* or *Bi* increases both u and u_p which can be attributed to the act that the effective conductivity ($\sigma/((1+BeBi)^2+Be^2)$) decreases with increasing *Be* or *Bi* which decreases the magnetic resistive force on u. In Figs. 2a and 2b, the velocity components w and w_p decrease with increasing *Be* or *Bi* due to decreasing the source term of w and increasing its damping term.

Figures 3a and 3b show that, unless *Be* and *Bi* are large, increasing *Be* or *Bi* decreases T and T_p with times since an increase in *Be* or *Bi* decreases the Joule dissipation which is proportional to ($1/((1+BeBi)^2+Be^2)$). On the other hand, for large values of *Be* and *Bi*, increasing *Bi* decreases T and T_p at small times and increases it at large times. This is because, for small times, u and w are small and an increment in *Be* or *Bi* decreases the Joule dissipation which is also proportional to $(1/((1+BeBi)^2+Be^2))$. For large times, increasing Be increases both u and w and, consequently increases the Joule and viscous dissipations. For large times, increasing Bi, although it decreases w, it increases the velocity u of the main flow and, in turn increases the viscous and Joule dissipations. This results in the crossing of the curves of T with time for higher values of Be and Bi. The same observations are clear in Fig. 3b. Comparing Figs. 6a and 6b emphasizes that the temperature of the fluid approaches the steady state faster than the temperature of the particles.

Figures 4-6 present the time development of the velocity components and temperature at the centre of the channel, respectively, for the fluid and particle phases for various values of the Hartmann number Ha and the ion slip parameter Bi and for Be=3. Figures 4a and 4b indicate that the effect of Bi on u and u_p depends on Ha. For small values of Ha, a small increment of Bi decreases u and u_p as a result of increasing the resistive force on u which is proportional to Bi. Increasing Bi more increases the effective conductivity and, consequently reduces the resistive force on u which increases both u and u_p. However, for larger values of Ha, u becomes small, and therefore increasing Bi always decreases the effective conductivity which increases u and u_p. It is also apparent that the effect of Bi on u and u_p becomes more pronounced for higher values of Ha. Figures 5a and 5b emphasizes that increasing the ion slip parameter Bi decreases w and w_p for all values of Ha and its effect is more clear for higher values of Ha. It is interesting to detect an overshooting in the velocity components u and w for higher values of Ha and Bi. Figures 6a and 6b show that the effect of Bi on T and T_p depends on Ha. For small values of Ha increasing Bi reduces T and T_p as a result of decreasing u which decreases the dissipations. Increasing Bi more increases T and T_p as a result of increasing u which increases the dissipations. However, for larger values of Ha, u and w are small and increasing Bi decreases T and T_p as a result of decreasing the Joule dissipation. It is also detected that the effect of Bi on T and T_p becomes more pronounced for higher values of Ha.

Conclusion

The transient generalized Couette flow with heat transfer of a dusty conducting fluid was studied considering the Hall and ion slip effects in the presence of uniform suction and injection. The influence of the magnetic field, the Hall parameter, and the ion slip parameter on the velocity and temperature distributions for both the fluid and particle phases was demonstrated. Increasing the ion slip parameter increases the velocity components for fluid and particle phases u and u_p, but decreases the velocity components w and w_p and temperatures T and T_p. For large values of the Hall parameter, the variation of T and T_p with the ion slip parameter was shown to depend on time. The influence of the Hall current on the velocity and temperature distributions decreases with increasing the ion slip. It is detected that the effect of the ion slip on the velocity components and temperatures of the fluid and dust particles becomes more apparent for higher values of Ha and the variation of the velocities u and u_p and temperatures T and T_p with the ion slip parameter depends upon the magnetic field.

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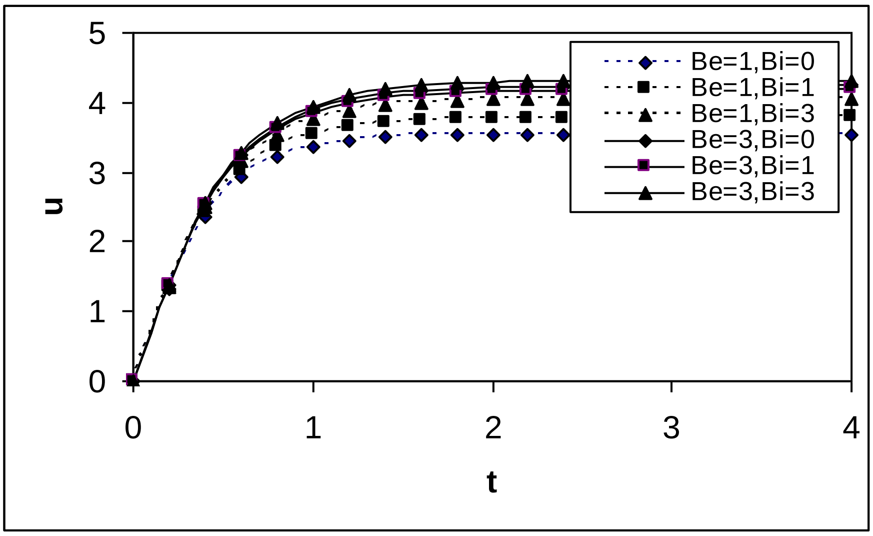

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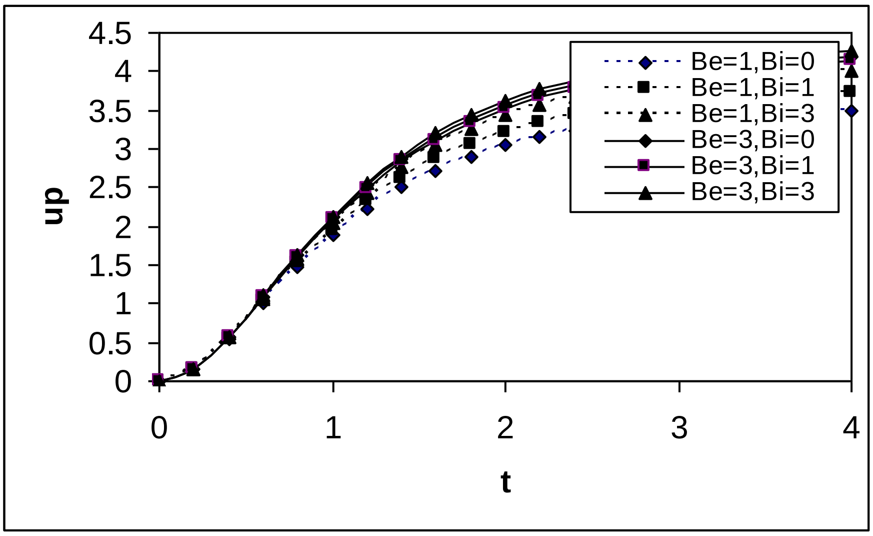
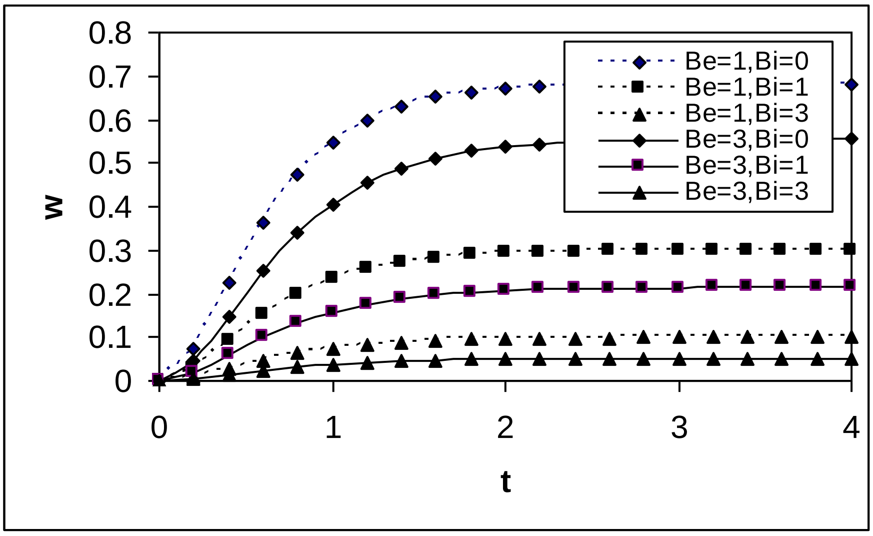


(b)

Fig. 1 Effect of the parameters Be and Bi on the time variation of: (a) u at y=0 and (b) u_p at y=0. (Ha=1)



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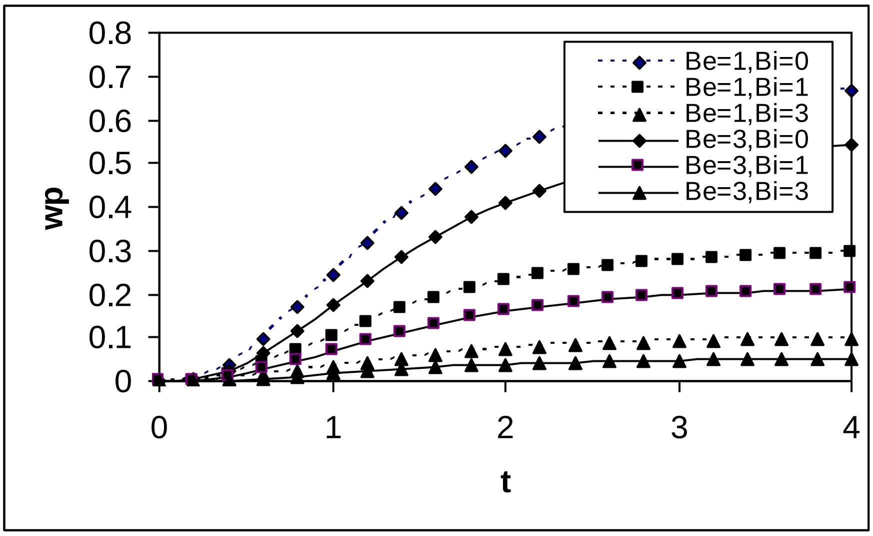
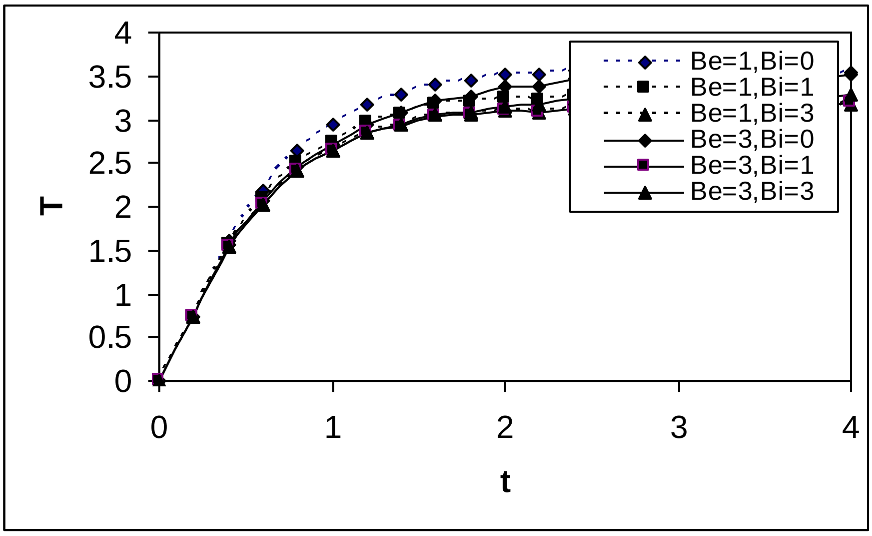


(b)

Fig. 2 Effect of the parameters Be and Bi on the time variation of: (a) w at y=0 and (b) w_p at y=0. (Ha=1)



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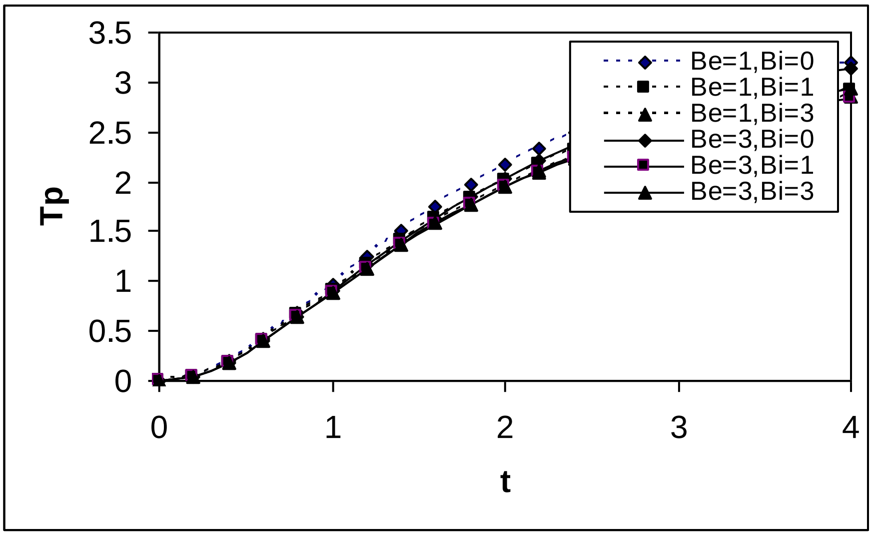

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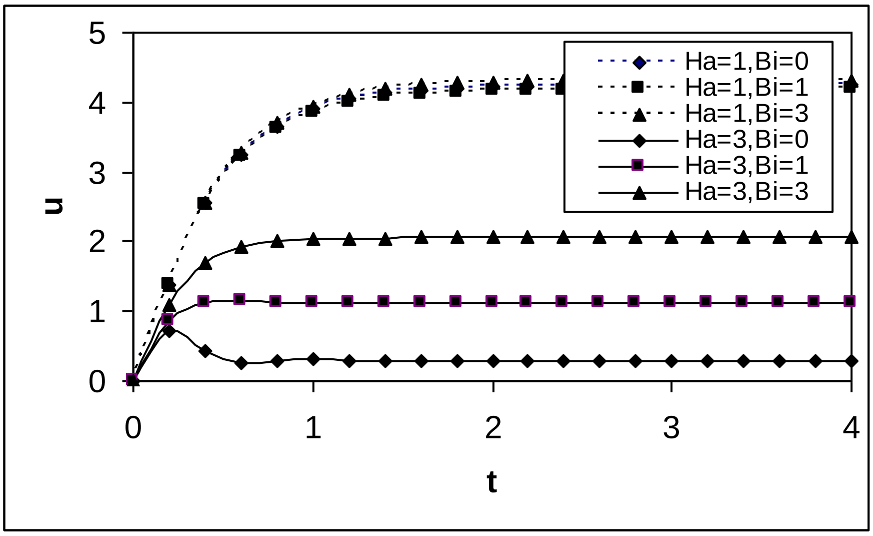
Fig. 3 Effect of the parameters Be and Bi on the time variation of: (a) T at y=0 and (b) T_p at y=0. (Ha=1)



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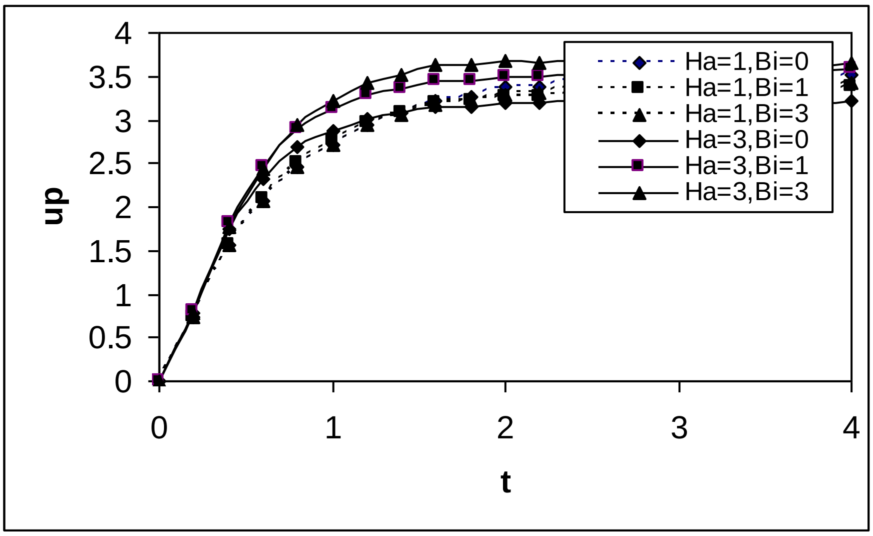
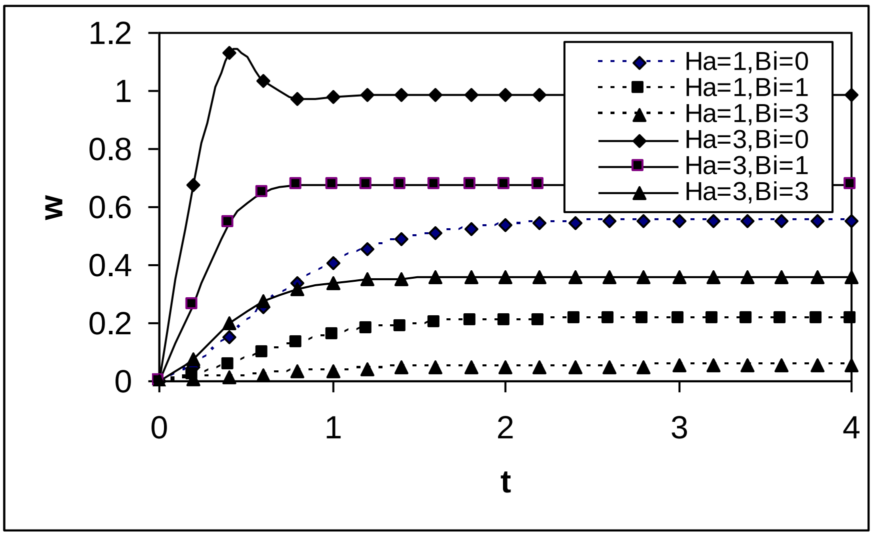


(b)

Fig. 4 Effect of the parameters Ha and Bi on the time variation of: (a) u at y=0 and (b) u_p at y=0. (Be=3)



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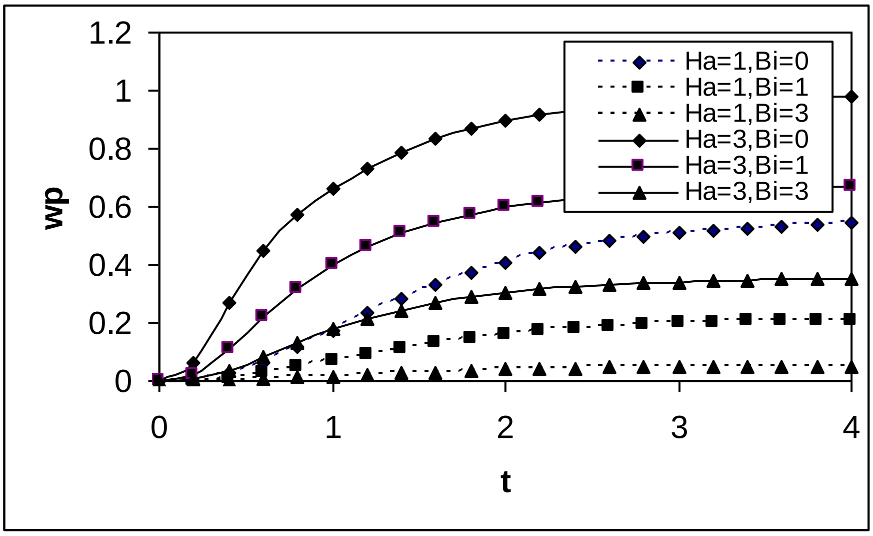

(b)

Fig. 5 Effect of the parameters Ha and Bi on the time variation of:
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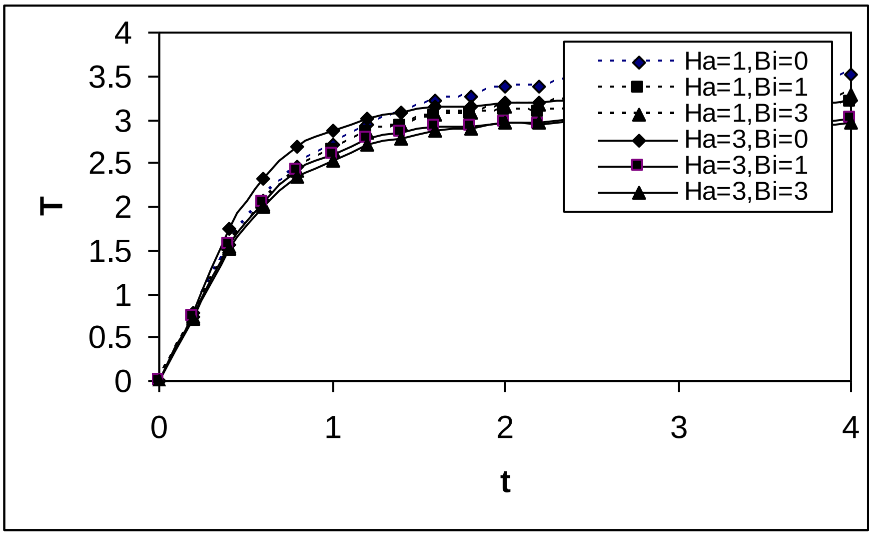

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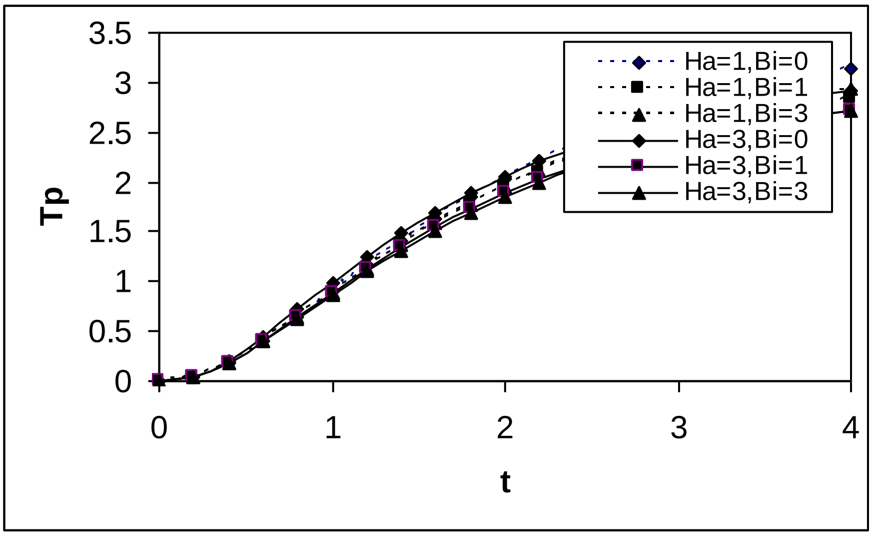

(b)

Fig. 6 Effect of the parameters Ha and Bi on the time variation of: (a) T at y=0 and (b) T_p at y=0. (Be=3)