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Research Article

PHYSICS

Change of Pressure for Blood Flow in a Vertical Peristaltic Colic Vein

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KEY WORDS

Peristaltic flow, Pressure gradient, Concentration of blood flow, Vertical colic vein and Modified Grashof number.

ABSTRACT

In this paper, the change of pressure in the circulation of blood and viscous blood concentration in a vertical Colic vein are studied. The mass, concentration, and Navier-Stokes equations introduce the mathematical simulation in case of long wavelength and low Reynolds number. The analytical results introduced that the gradient pressure affected with modified Grashof number, vein radius and blood viscosity. Moreover, the blood concentration, and velocity of blood is very sensitive for the small change of blood density ratio, amplitude ratio and vein length.

1. Introduction

The ascending colon, transverse colon, descending colon and sigmoid colon represent the main parts of colon. The colic venous drainage is into two veins the inferior and superior mesenteric veins [1]. Ileocolic vein drains the ileum, the lower part of the ascending colon and cecum, right colic vein drains the ascending colon and middle colic vein drains the transverse colon. These veins are tributaries of the superior mesenteric vein. This is the venous anatomy of right colon [1]. Left colic vein drains the descending colon. it has ascending and descending branches and sigmoid colic vein drains sigmoid colon and rectum It is a tributary of the inferior mesenteric vein. This is the venous anatomy of left colon. In the small veins of the mesentery such as colic veins, the blood flow is streamlined, laminar and in distinct stream. This happens because of the peristaltic movements of the wall which are small in the vein in comparison of the heart [3, 5, 9]. The blood flow in the heart is turbulent [4]. In general, the veins carry the impure, rich in wastes, gas bubbles and deoxygenated blood back from the tissues of the body to the heart except pulmonary vein. This gas bubbles in deoxygenated blood causing several healthy problems [2, 8, 11, 12]. The direction of deoxygenated blood in the vertical colic vein is up to heart against gravity.

Blood decomposes of liquid like water. This biofluid known as plasma which classified as Newtonian fluid and has properties similar to that of water. Blood is a Newtonian fluid at high shear rate [13].

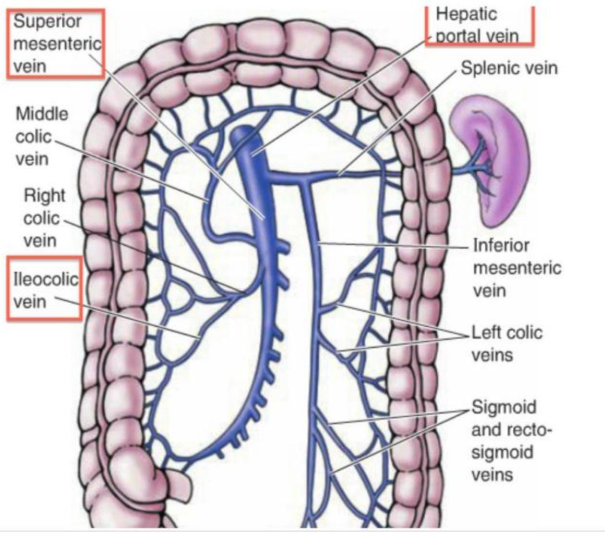

Fig.1: Sketch of colic veins

Recently the gas bubbles growth is studied by **Mohammadein *et al.***, [6, 7] and **Wen** [10]. This physical model represents the flow of blood in the vertical colic vein. In the same way, many mathematical and physical models are formulated by several authors like Mohammadein model [7, 8].

In this paper, we focus on the study for the change of pressure in the circulation of blood in a vertical colic vein. The peristaltic velocity of blood flow is obtained as a function of some physical parameters. The analytical solutions are obtained for concentration distribution of blood, blood velocity and change of pressure under the effect of modified Grashof number, initial concentration and blood viscosity.

2. Analysis

In the current problem, the viscous and incompressible Newtonian peristaltic blood flow is studied in the vertical colic vein. The mathematical simulation assume that, the blood flows in the vertical colic vein represented in the cylindrical polar coordinates with z measured along the axis of vein and r is in the radial direction with a sinusoidal wave of small amplitude traveling down its peristaltic wall as in Fig.2.

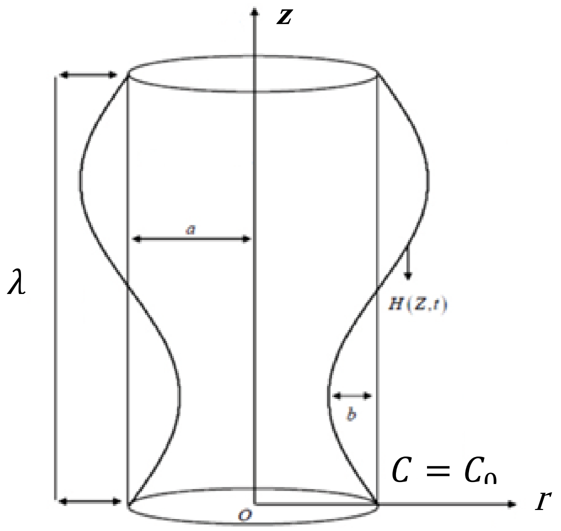

Fig.2: Sketch of mathematical problem

The peristaltic wall of vertical colic vein is considered in a following form

$$H(\bar z) = a + b\sin\frac{2\pi}{\lambda}(\bar z - \bar t) \qquad (1)$$

Where λ is the wavelength, a is the average radius of the vertical colic vein, b is the wave amplitude and c is the wall wave speed. Let the velocity components u and w is the radial and axial directions, respectively.

The conservation equations of mass, momentum and concentration distribution represent the mathematical model of the physical problem as follows:

Mass equation

$$\frac{1}{\bar r}\frac{\partial}{\partial \bar r}(\bar r\,\bar u) + \frac{\partial \bar w}{\partial \bar z} = 0 \qquad (2)$$

Navier-Stokes equations

$$\rho_b\left(\bar u\frac{\partial \bar u}{\partial \bar r} + \bar w\frac{\partial \bar u}{\partial \bar z}\right) = -\frac{\partial \bar P}{\partial \bar r} + \eta\left\{\frac{1}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\frac{\partial \bar u}{\partial \bar r}\right) - \frac{\bar u}{\bar r^2} + \frac{\partial^2 \bar u}{\partial \bar z^2}\right\} \qquad (3)$$

$$\rho_b\left(\bar u\frac{\partial \bar w}{\partial \bar r} + \bar w\frac{\partial \bar w}{\partial \bar z}\right) = -\frac{\partial \bar P}{\partial \bar z} + \eta\left\{\frac{1}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\frac{\partial \bar w}{\partial \bar r}\right) + \frac{\partial^2 \bar w}{\partial \bar z^2}\right\} + \rho g\alpha(\bar C - C_0) \qquad (4)$$

Concentration equation

$$\bar u\frac{\partial \bar C}{\partial \bar r} + \bar w\frac{\partial \bar C}{\partial \bar z} = D_T\left(\frac{\partial^2 \bar C}{\partial \bar r^2} + \frac{1}{\bar r}\frac{\partial \bar C}{\partial \bar r} + \frac{\partial^2 \bar C}{\partial \bar z^2}\right) \qquad (5)$$

Where P is the pressure, C is the concentration of blood in the bio tissues of the vertical colic vein, and $\rho_b$is the density of the blood. Introducing the dimensionless variables as follows

$$h = 1 + e\sin(2\pi(z\text{-}t)),\ \delta = \frac{a}{\lambda},\ r = \frac{\bar r}{a},$$

$$z = \frac{\bar z}{\lambda},\ u = \frac{\bar u}{c\delta},\ w = \frac{\bar w}{c},\ H = \frac{h}{a}$$

$$G_c = \frac{\rho_b g\alpha a^2 C_0}{\eta c},\ \ p = \frac{a^2}{c\lambda\eta}\bar p\ ,$$

$$\Phi = \frac{C - C_0}{C_0},\ \ R_e = \frac{\rho_b c a}{\eta} \qquad (6)$$

Where G_C is the modified Grashof number, R_e is the Reynolds number, δ is the wave number, e is the amplitude ratio and Φ is the blood concentration distribution. Substituting from relations (6) into the system (2-5), then

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u) + \frac{\partial w}{\partial z} = 0, \qquad (7)$$

$$R_e\delta^3\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial r} + \delta^2\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{u}{r^2} + \delta^2\frac{\partial^2 u}{\partial z^2}\right\} \qquad (8)$$

$$R_e\delta\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\delta^2\frac{\partial^2 w}{\partial z^2}+G_c\,\Phi \quad (9)$$

$$c\,a\,\delta\left(u\frac{\partial \Phi}{\partial r}+w\frac{\partial \Phi}{\partial z}\right)=D_T\left(\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial \Phi}{\partial r}+\delta^2\frac{\partial^2\Phi}{\partial z^2}\right) \quad (10)$$

The above system called the non-dimensional system. The physical problem is solved for the large wavelength ($\delta \ll 1$) and the Reynolds number is quite small($R_e \to 0$), then the equations (8-10) become

$$\frac{\partial P}{\partial r}=0 \quad (11)$$

$$\frac{\partial P}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+G_c\,\Phi \quad (12)$$

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial \Phi}{\partial r}=0 \quad (13)$$

The blood flow in the vertical colic vein is studied under the effect of different physical parameters like initial concentration, blood viscosity and Grashof number.

The above system (11-13) is solved under the effect of boundary conditions in case of the above physical proposals.

The blood peristaltic flow in a vertical colic vein is formulated by the system equations (11-13) in terms of pressure, blood flow velocity, and concentration. The following dimensionless boundary conditions for the system (11-13) are

$$At \quad r=0, \quad \frac{\partial w}{\partial r}=0 \quad (14)$$

$$At \quad r=h, \quad w=A_0 \quad (15)$$

$$At \quad r=-h \quad \frac{\partial \phi}{\partial r}=\Phi_1 \quad (16)$$

$$At \quad r=h, \quad \Phi=\Phi_0 \quad (17)$$

Where Φ_0 is initial blood concentration distribution at the wall of vertical colic vein. The solution of concentration equation (13) under the previous boundary conditions (16-17) becomes

$$\Phi(r,z)=C_1 Ln\, r+C_2,$$

Or $\Phi(r,z)=\Phi_0+C_1 Ln\,\frac{r}{h}$ **(18)**

where

$$C_1=-h\Phi_1, \text{ and } C_2=\Phi_0-C_1\,Ln\,h \quad (19)$$

Substituting by Eqn. (18) into the Eqn. (12) and solving Eqn. (12) under the boundary conditions (14-15), and the peristaltic blood velocity has the form

$$w(r,z)=\frac{r^2}{4}\left(\frac{dP}{dz}\right)-G_C\left(\frac{r^2}{4}\Phi_0+C_1\left(\frac{r^2}{4}Ln\,\frac{r}{h}-\frac{r^2}{4}\right)\right)+C_3 \quad (20)$$

where,

$$C_3=A_0-\frac{h^2}{4}\left(\frac{dP}{dz}\right)+G_C\left(\frac{h^2}{4}\Phi_0-\frac{h^2}{4}C_1\right) \quad (21)$$

The blood velocity in the great vertical colic vein can be rewritten as follows

$$w(r,z)=A_0+\left(\frac{dP}{dz}\right)\left(\frac{r^2-h^2}{4}\right)+G_C\left\{\frac{h^2-r^2}{4}\Phi_0+\frac{C_1}{4}\left(r^2-h^2-r^2 Ln\,\left(\frac{r}{h}\right)\right)\right\} \quad (22)$$

To find the unknown $\frac{dP}{dz}$, we need one equation more in the form

$$q=2\int_0^h wr\,dr. \quad (23)$$

The dimensionless volume flow rate in the fixed frame of reference in a vertical colic vein is defined by previous equation (23)

On the basis of equations (22-23), the pressure gradient in the vertical colic vein has the form

$$\frac{dP}{dz}=\frac{8A_0}{h^2}-\frac{8q}{h^4}+G_c\left\{\left(\Phi_0-\frac{3C_1}{4}\right)\right\} \quad (24)$$

The pressure field can be obtained by integration of the above equation (24).

3. Discussion and Results

The physical problem is described by mass, momentum, and concentration equations (2-5) respectively for a Newtonian viscous fluid flow through the vertical colic veins. The problem is solved analytically to obtain the blood velocity and concentration distribution under the effect of several physical parameters as modified Grashof number G_c , initial concentration of blood C_0, density ratio ϵ, and amplitude ratio e . The non- linear system (2-5) is transformed to another non dimensional system (7-10) with the existence of non dimensional numbers like Grashof number. The system (11-13) is obtained for the long wavelength ($\delta \to 0$). The concentration distribution of blood flow in the vertical colic vein is obtained by relation (18). The blood flow velocity is given by Eqn. (20).

Finally, the gradient pressure of blood is given by equation (24) by integration we can get blood pressure. The values of important parameters are collected in the following Table:

Symbol	G_c	e	q	A_0	Φ_0	Φ_1
Value	0.1	0.01	0.32	1	2	1

G_c: Modified Grashof number

e: Amplitude ratio

q: Blood flow rate in a colic vein.

A_0: Venous blood velocity $at\ \ r = h.$

Φ_0: Intial concentration $at\ \ r = h.$

Φ_1: Final concentration $at\ \ r = -h.$

The peristaltic blood flow and the blood Concentration Φ in terms of r for different values of amplitude ratio e and distance z along the vertical colic vein are shown in Figs. 3 and 4 respectively. It is observed that the concentration Φ is proportional with amplitude ratio e and inversely at distance z at $r < 0.2$. On contrary, when $r \geq 0.2$ the concentration Φ is proportional with distance z . and inversely with amplitude ratio e.

The blood flow velocity in the vertical colic vein w(r, t) in terms of r for different values of amplitude ratio e and modified Grashof number $G_C$shown in Figs. (5& 6) respectively. It is observed that, the blood velocity is directly proportional with modified Grashof number$G_C$and amplitude ratio at distance$r \geq 0.6$. On contrary, when $r < 0.6$ the blood velocity is inversely proportional with amplitude ratio e because the velocity affected by peristaltic vein.

The blood gradient pressure in terms of vein axis is shown in Fig. (7) It is observed that the gradient pressure $\frac{dP}{dz}$ increases by the increasing of amplitude ratio in peaks and vice versa in troughs due to the pervious duality behavior of amplitude ratio with velocity.

The blood pressure P shown in Figs. (8&9) is inversely proportional with modified Grashof number G_C because when blood viscosity increases, blood pressure also increases. Pressure in terms of amplitude ratio changes from peaks to troughs to push blood up against gravity.

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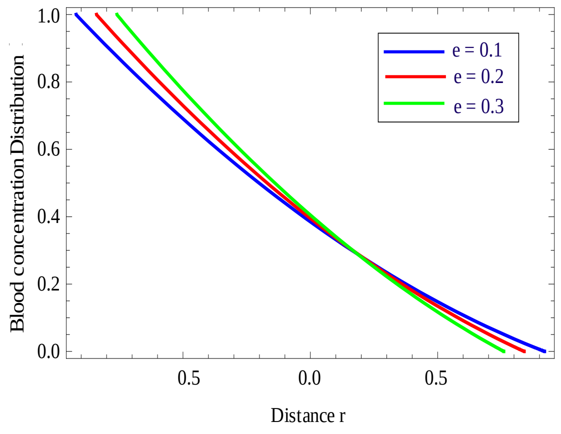

Fig.3: Blood concentration under effect of amplitude ratio

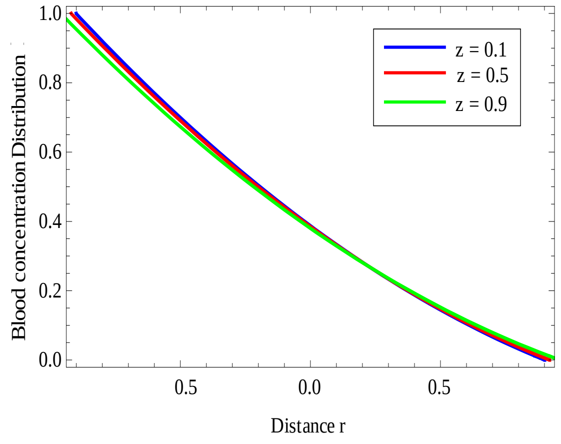

Fig.4: Blood concentration under effect of vein length

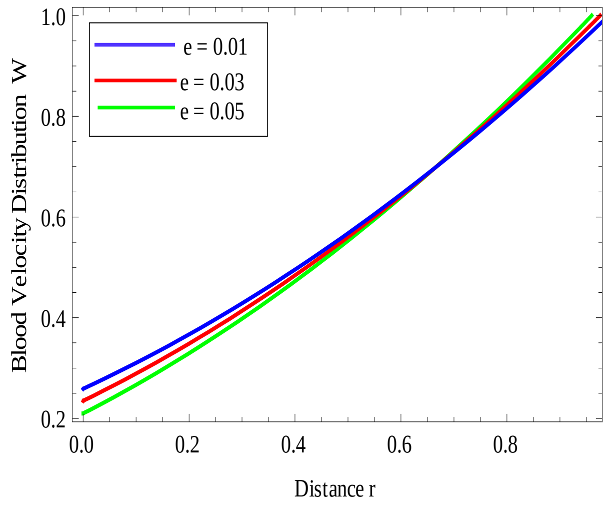

Fig.5: Blood velocity under effect of amplitude ratio

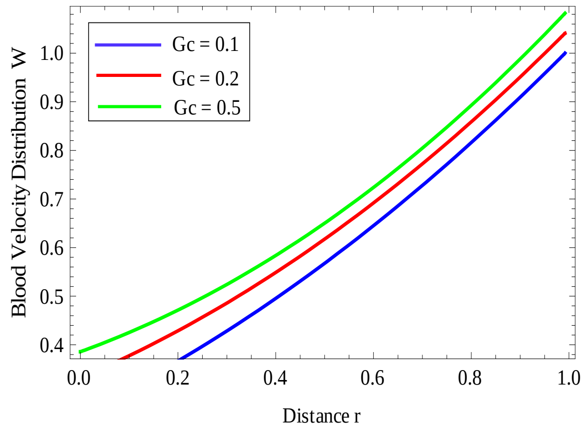

Fig.6: Blood velocity under effect of Grashof number

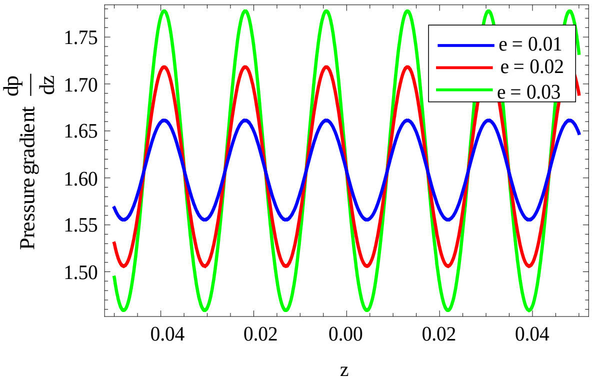

Fig.7: Pressure gradient under effect of amplitude ratio

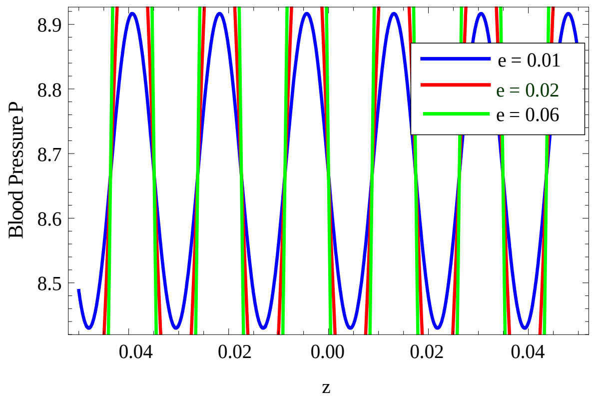

Fig.8: Blood Pressure under effect of amplitude ratio

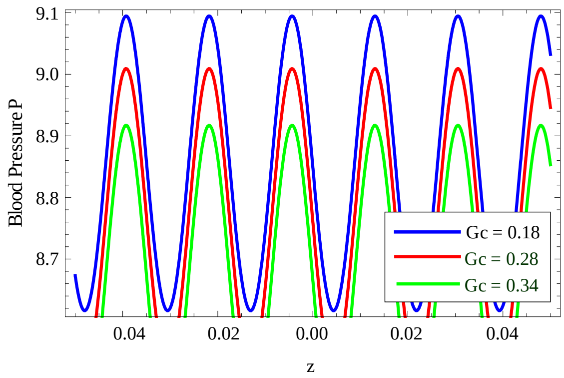

Fig.9: Blood Pressure under effect of Grashof number

4. Conclusion

Change of pressure in the circulation of blood in a vertical colic vein is solved analytically under the effect of peristaltic motion with long wavelength and low Reynolds number.

On the basis of above discussions of figures, the following remarks are concluded:

1. The concentration of blood Φ changes through the vertical colic vein according to the contraction and relaxation of the peristaltic wall of vein and affected by amplitude ratio and vein length.

2. The velocity of blood flow is directly proportional modified Grashof number G_C and with amplitude ratio $e \geq 0.6$ and inversely proportional with amplitude ratio $e < 0.6$ due to the blood velocity increase through the peristaltic vein.

3. The gradient pressure changes in peaks than troughs due to the pervious duality behavior of amplitude ratio with velocity.
The physical parameters amplitude ratio and modified Grashof number affected on pressure gradient.

4. The blood pressure increases by the decreasing of modified Grashof number G_C because of its relation with viscosity. This verified the inverse relation between blood pressure and velocity in veins. Moreover, the blood pressure is relatively increasing and decreasing in the vein and affected by amplitude ratio.
In the same concept, the blood pressure decreases through the blood flow in the direction to heart.

5. The blood concentration, blood flow velocity, pressure gradient and blood pressure are very sensitive for the small change of all physical parameters.

6. The above concluded remarks prove the validity of the proposed model, and how to extend the present model in more properties of different fluids and flows in different veins.

Conflict of interest

The authors have no conflicts of interest to disclose

5. References

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تغير الضغط لإنسياب الدم في الوريد القولونى الرأسي التمعجي

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في هذا البحث تم دراسة تغير الضغط في الوريد القولونى الرأسي التمعجي. المحاكاة الرياضية لصياغة هذة المشكلة تتمثل فى معادلات الكتلة والتركيز ونافير-ستوكس في حالة الأطوال الموجيه الطويله وإنخفاض عدد رينولدز.

حيث أظهرت النتائج التحليلية أن الإنحدار فى الضغط يتأثر بتغير رقم جراشوف ونصف قطر الوريد ولزوجة الدم. علاوة على ذلك ، فإن تركيز الدم وسرعته له حساسيه عالية للتغير الطفيف في نسبة كثافة الدم ونسبة السعة وطول الوريد.