ITERATION PROCEDURE FOR THE ANALYSIS OF PLATES IN BENDING BY FINITE STRIP METHOD

BY

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INTRODUCTION

The finite strip method as a semi analytical procedure has been recently developed and used successfully for the analysis of a certain class of two and three dimensional problems. In the analysis of bending of elastic plates, a solution technique considers the plate to be an assembly of strips joined at their longitudenal edges. The displacement function of the strip is expressed as a product of a polynomial function across the width of the strip and a series function in the longitudenal direction. The most commonly used series are the basic functions which are derived from the solution of beam vibration differential equation. These basic functions have been worked out explicitly by VLAZOV [4] for the various end conditions.

The earliest formulation of the finite strip method was developed by CHEUNG [1] who used a trigonometric series as a basic function in the analysis of elastic plates with two opposite simply supported ends. These trigonomitric series and its derivatives possess the properties of orthogonality that lead to the uncoupling of the static equilibrium equations. This means that there is orthogonality condition between the load and displacement harmonics and that each term of the series can be solved individually. Basic functions other than trigonometric series, are used by CHEUNG [2] to analyze plates with two opposite edge conditions other than simply supported. Unfortunatly, with these basic functions the uncoupling property described above cannot occur.

The object of the present paper is to develop a finite strip method with a simplified iteration solution for the basic functions other than trigonometric series. The basic idea of the iteration procedure presented herein arised from the observable dominant values of the diagonal submatrices elements in the stiffness matrix of the strip. The iteration procedure takes only into consideration the diagonal submatrices of the stiffness matrix of the strip. Accordingly, each term of the basic function can be solved individually such as that in the case of trigonometric series.

The first iteration solution considers the original load vector and results in good approximate values for the unknown nodal parameters, which can be utilized with the non diagonal submatrices of the stiffness matrix to obtain a modified load vector for each strip. The modified load vector can be used

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in the second iteration solution to give a more improved values for the unknown nodal parameters. It is concluded that two or three iterations are sufficient to achieve the accuracy required. The iteration procedure presented here was applied for the basic function of the case of clamped - clamped edge condition. The results are in very close agreement with those of the same conditions worked out by TIMOSHENKO [5]

METHOD OF ANALYSIS

In the analysis of elastic plates in bending using the finite strip method, the plate is devided into long strips Fig.1-a. The displacement function of each strip is expressed as a product of a polynomial function $f_m(x)$ across the width of the strip and a series function $Y_m(y)$ in the longitudenal direction Fig. 1-b. In order to ensure higher order nodal lines compatibility and to reduce the number of strips required for the analysis, a seventh order polynomial function is assumed to represent the deflection profile across the width of the strip.

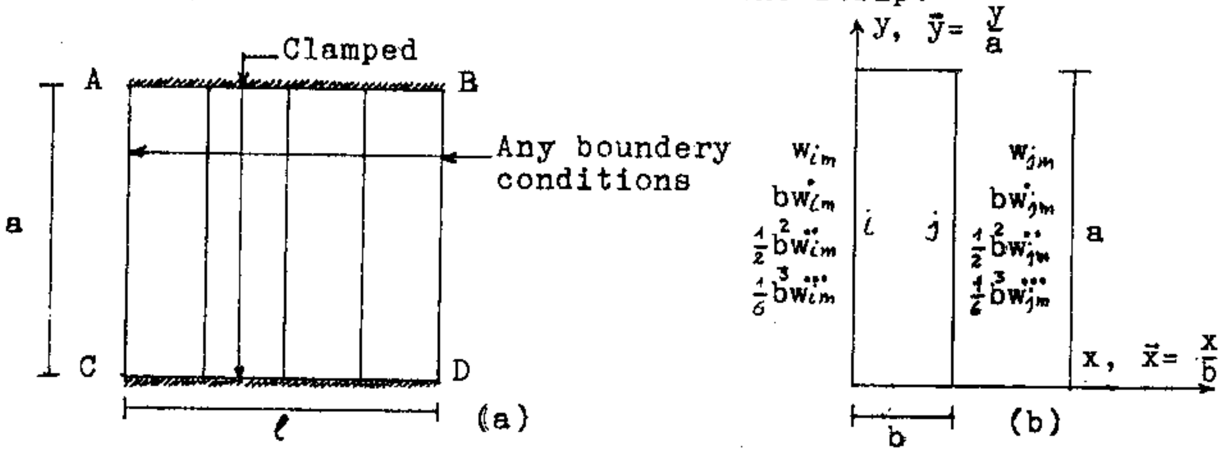

Fig. 1. Finite strip idealization

The displacement within each strip is expressed as follows

$$w = [N]\{D\} = \sum_{m=1}^{r} [N]_m \{D\}_m = \sum_{m=1}^{r} Y_m [C] \{D\}_m$$
$$= \sum_{m=1}^{r} Y_m [\ [C_i] \quad [C_j]\ ] [\ \{D_i\}_m \quad \{D_j\}_m \}^T \qquad (1)$$

where

$$[C_i] = [C_1 \ C_2 \ C_3 \ C_4] = [(1-35\bar{x}^4+84\bar{x}^5-70\bar{x}^6+20\bar{x}^7) \ (\bar{x}-20\bar{x}^4+45\bar{x}^5-36\bar{x}^6+10\bar{x}^7) \ (\bar{x}^2-10\bar{x}^4+20\bar{x}^5-15\bar{x}^6+4\bar{x}^7) \ (\bar{x}^3-4\bar{x}^4+6\bar{x}^5-4\bar{x}^6+\bar{x}^7)]$$

$$[C_j] = [C_5 \ C_6 \ C_7 \ C_8] = [(35\bar{x}^4-84\bar{x}^5+70\bar{x}^6-20\bar{x}^7) \ (-15\bar{x}^4+39\bar{x}^5-34\bar{x}^6+10\bar{x}^7) \ (5\bar{x}^4-14\bar{x}^5+13\bar{x}^6-4\bar{x}^7) \ (-\bar{x}^4+3\bar{x}^5-3\bar{x}^6+\bar{x}^7)] \qquad (2)$$

$$\{D_i\} = \{ w_{im} \quad b\dot{w}_{im} \quad \tfrac{1}{2}b^2\ddot{w}_{im} \quad \tfrac{1}{6}b^3\dddot{w}_{im} \}^T$$

$$\{D_j\} = \{ w_{jm} \quad b\dot{w}_{jm} \quad \tfrac{1}{2}b^2\ddot{w}_{jm} \quad \tfrac{1}{6}b^3\dddot{w}_{jm} \}^T \ , (\ )^{\cdot} = \frac{\partial}{\partial}$$

The basic function of clamped - clamped edge condition is given by,

$$Y_m(y) = \sin \mu_m \bar{y} - \sinh \mu_m \bar{y} - \alpha_m(\cos \mu_m \bar{y} - \cosh \mu_m \bar{y}) \qquad (3)$$

where

$$\left.\begin{aligned} \alpha_m &= \frac{\sin \mu_m - \sinh \mu_m}{\cos \mu_m - \cosh \mu_m} \\ \mu_m &= 4.73,\ 7.8532,\ 10.966,\ \ldots\ldots\ \frac{2m+1}{2}\pi \end{aligned}\right\} \qquad (4)$$

For the above mentioned basic function, the following integrals are essential to formulate the stiffness matrix

$$I_1 = {}_0\int^a Y_m Y_n\,dy\ , \qquad I_2 = {}_0\int^a \ddot{Y}_m \ddot{Y}_n\,dy\ ,$$

$$I_3 = {}_0\int^a Y'_m Y'_n\,dy\ , \quad I_4 = {}_0\int^a Y_m Y''_n\,dy\ , \quad I_5 = {}_0\int^a Y''_m Y_n\,dy$$

where $(\ )' = \frac{\partial}{\partial y}$

for m = n $\quad I_1 = \alpha_m^2 a\ , \qquad I_2 = (\frac{\mu_m}{a})^4 \alpha_m^2 a\ ,$

$$I_3 = -I_4 = -I_5 = = (\frac{\mu_m}{a})^2 (1 - \frac{2\alpha_m}{\mu_m}) a \qquad (5)$$

for m ≠ n $\quad I_1 = 0\ , \qquad I_2 = 0\ ,$

$$I_3 = -I_4 = -I_5 = = -[1+(-1)^{m+n}]\ \frac{4\ \mu_m^2\ \mu_n^2}{a^2(\mu_m^4 - \mu_n^4)}\ (\mu_m \alpha_n - \mu_n \alpha_m) a$$

The detailed derivation of the stiffness matrix of the finite strip is illustrated by CHEUNG [1-3]. The stiffness matrix of the strip $[\bar{S}]$ can be partitioned into r x r submatrices $[\bar{S}]_{mn}$ corresponding to each term of the basic function used. For each strip the stiffness matrix takes the form

$$[\bar{S}] = \begin{bmatrix} [\bar{S}]_{11} & [\bar{S}]_{12} & \cdots\cdots & [\bar{S}]_{1r} \\ [\bar{S}]_{21} & [\bar{S}]_{22} & \cdots\cdots & [\bar{S}]_{2r} \\ \vdots & \vdots & & \vdots \\ [\bar{S}]_{r1} & [\bar{S}]_{r2} & \cdots\cdots & [\bar{S}]_{rr} \end{bmatrix} \qquad (m=1 \text{ to } r,\ n=1 \text{ to } r) \quad (6)$$

Each one of the submatrices $[\bar{S}]_{mn}$ has the order of 8 x 8 (twice the number of nodal line parameters)

$$[\bar{S}_{\ell k}]_{mn} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \cdots\cdots & \bar{S}_{18} \\ \bar{S}_{21} & \bar{S}_{22} & \cdots\cdots & \bar{S}_{28} \\ \vdots & \vdots & & \vdots \\ \bar{S}_{81} & \bar{S}_{82} & \cdots\cdots & \bar{S}_{88} \end{bmatrix}_{mn} \qquad (\ell=1 \text{ to } 8,\ k=1 \text{ to } 8) \quad (7)$$

The general term of the stiffness matrix (Appendix I) can be written in the following form

$$\bar{S}_{\ell k \cdot mn} = B_o \int_o^a \int_o^b [\ddot{C}_\ell \ddot{C}_k Y_m Y_n + C_\ell C_k Y''_m Y''_n + 2(1-\gamma)\dot{C}_\ell \dot{C}_k Y'_m Y'_n + \gamma(\ddot{C}_\ell C_k Y_m Y''_n + C_\ell \ddot{C}_k Y''_m Y_n)]\,dxdy$$

$$= B_o \int_o^b [\ddot{C}_\ell \ddot{C}_k I_1 + C_\ell C_k I_2 + 2(1-\gamma)\dot{C}_\ell \dot{C}_k I_3 + \gamma(\ddot{C}_\ell C_k I_4 + C_\ell \ddot{C}_k I_5)]\,dx \quad (8)$$

Where $B = \frac{E\ t^3}{12(1-\gamma^2)}$

By substitution eqn (5) into eqn (8), we get

for m = n (general term of the diagonal submatrices)

$$\bar{S}_{\ell kmm} = B_o \int_o^b [\{\ddot{C}_\ell \ddot{C}_k + (\frac{\mu_m}{a})^4 C_\ell C_k\} I_1 + \{2(1-\gamma)\dot{C}_\ell \dot{C}_k - \gamma(\ddot{C}_\ell C_k + C_\ell \ddot{C}_k)\} I_3]\,dx \quad (9)$$

for m ≠ n (general term of the non diagonal submatrices)

$$\bar{S}_{\ell kmn} = B_o \int_o^b [\{2(1-\gamma)\dot{C}_\ell \dot{C}_k - \gamma(\ddot{C}_\ell C_k + C_\ell \ddot{C}_k)\} I_3]\,dx \quad (10)$$

From the numerical values of integrals I_1, I_2 and I_3 for different values of m and n (Appendix I), it can be observed that the elements of the diagonal submatrices of the stiffness matrix have the dominant values.

Applied loads must also be resolved into series similar to the displacement function, and a load vector $\{\bar{F}_o\}$ is then related to the nodal line parameters. For each strip this relation takes the form

$$[\bar{S}]\ \{\bar{D}\} = \{\bar{F}_o\} \quad (11)$$

For the purpose of the iteration solution, the stiffness matrix $[\bar{S}]$ of the strip is considered as a summation of two matrices $[\bar{S}d]$ and $[\bar{S}n]$. Eqn (11) can be rewritten as

$$[\bar{S}d]\ \{\bar{D}\} + [\bar{S}n]\ \{\bar{D}\} = \{\bar{F}_o\}$$

$$[\bar{S}d]\ \{\bar{D}\} = \{\bar{F}_o\} - [\bar{S}n]\ \{\bar{D}\} = \{\bar{F}_o\} - \{\Delta\bar{F}\} = \{\bar{F}_k\} \quad (12)$$

where
$[\bar{S}d]$ matrix contains the diagonal submatrices
$[\bar{S}n]$ matrix containes the non diagonal submatrices
$\{\bar{F}_o\}$ the original load vector of the strip
$\{\bar{F}_k\}$ the modified load vector of the strip

The iteration procedure considers the diagonal submatrices of the stiffness matrix which leads to an assumed uncoupling of the static equilibrium equations. Accordingly, each term of the basic function can be solved individually. For each term of the basic function, the following relation for each strip can be written as

$$[\bar{S}d]_{mm}\{\bar{D}\}_m = \{\bar{F}k\}_m \tag{13}$$

The non diagonal submatrices are not considered in the first iteration solution, i.e., $[\bar{S}n]_{mn} = [0]$ for $m \neq n$. Accordingly, eqn (13) takes the form

$$[\bar{S}d]_{mm}\{\bar{D}1\}_m = \{\bar{F}o\}_m \tag{14}$$

In which $\{\bar{D}1\}m$ is the vector of the first approximation of the unknown nodal line parameters and $\{\bar{F}o\}m$ is the original load vector

For each term of the basic function, overall stiffness matrix of the plate $[Sd]m$ of order $N \times N$ (where N is four times the number of nodal lines) is assembled from the stiffness matrices $[\bar{S}d]mm$ of the individual strips. Adding the forces on each nodal line from the two adjacent strips, a load vector $\{Fo\}_m$ can be obtained. Solution of the equation

$$[Sd]_m \{D1\}_m = \{Fo\}_m \tag{15}$$

gives an approximate values for the N unknown nodal parameters $\{D1\}m$ for each term of the basic function

The non diagonal submatrices $[\bar{S}n]mn$ are taken into condideration in the second iteration solution. The modified load vector $\{\bar{F}1\}$ is obtained for each strip according to eqn (12) as

$$\{\bar{F}1\} = \{\bar{F}o\} - \{\Delta\bar{F}\} = \{\bar{F}o\} - [\bar{S}n]\ \{\bar{D}1\} \tag{16}$$

A modified load vector $\{F1\}m$ for the assembled strips can be obtained. In the second iteration solution eqn (15) takes the form

$$[Sd]_m\{D2\}_m = \{F1\}_m \tag{17}$$

Solution of this equation gives the improved values for the N unknown nodal line parameters $\{D2\}m$ for each term of the basic function. The procedure can be repeated for the subsequent iteration solutions. The general form of eqn (17) for the iteration of order k can be written as

$$[Sd]_m\{Dk\}_m = \{F(k-1)\}_m \tag{18}$$

NUMERICAL EXAMPLES

In order to check the accuracy of the iteration procedure developed here, a constant thickness isotropic rectangular plates with two opposite clamped edges under uniformly distributed load have been analyzed. Only odd terms contribute the results, because of the symmetry of loading and edge conditions in the longitudenal direction of the strips. The analysis is carried out with ℓ devided into four strips. In case of symmetrical edge conditions Fig.(2-a,b), only half of the plate (devided into two equal strips) is used in the analysis. Poisson's ratio in all examples equals 0.3.

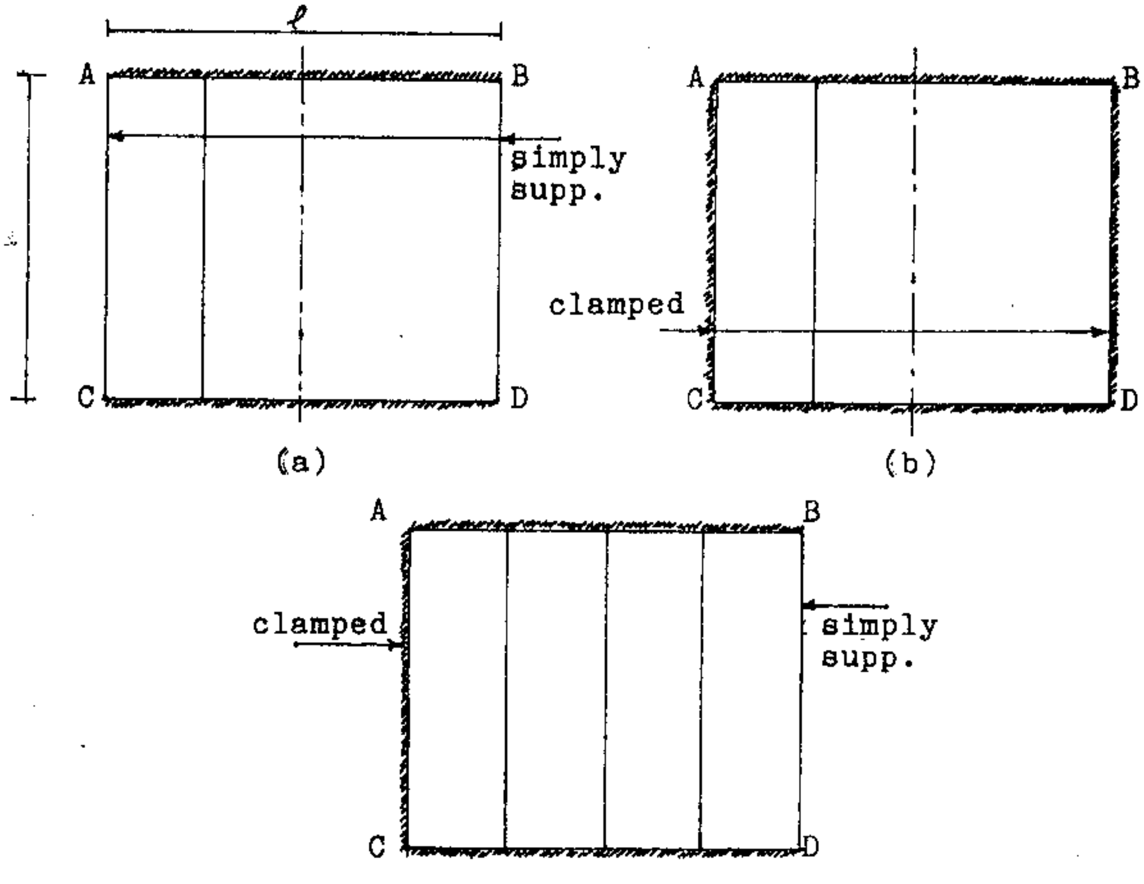

Fig. 2

Tables 1 and 3 give the first iteration solution results obtained from the analysis of symmetrical edge conditions square plates $\ell/a = 1$ Fig.(2-a,b). Greater number of terms (5 or 6) gives good results as a first approximation and emphasizes the role of the dominant diagonal submatrices of the stiffness matrix

A study of convergence of the iteration procedure is carried out for the above mentioned square plates and the results are presented in Tables 2 and 4. It should be noted that, the results of the third and fourth iteration solutions are closely identical and indicate the rapid convergence of the procedure. The results of the second iteration solution for the greater number of terms demonstrate a very close agreement with known exact solution [5]

Table 1. Analysis of square plate clamped on sides AB and CD, simply supported on the two other sides subject to uniform load of intensity q. Fig.2-a (First Iteration only) $\nu = 0.3$

NO of Terms	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB
1	19.4943	2.5939	3.8162	-5.4925
2	19.1494	2.4596	3.3919	-6.0857
3	19.1861	2.4925	3.5015	-6.2407
4	19.1783	2.4796	3.4583	-6.3018
5	19.1807	2.4859	3.4796	-6.3319
6	19.1798	2.4824	3.4676	-6.3489
Exact [5]	19.20	2.44	3.32	-6.97
Multiplier	$10^{-4}.q\ a^4/B$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$

Table 2. Study of convergence. Squar plate clamped on sides AB and CD, simply supported on the two other sides subject to uniform load of intensity q. Fig. 2-a $\nu = 0.3$

NO of Terms	Iteration Order	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB
3	1st Iter.	19.1861	2.4925	3.5015	-6.2407
	2nd "	19.2212	2.4879	3.3509	-6.8290
	3 d "	19.4196	2.5071	3.3817	-6.8929
	4th "	19.4229	2.5074	3.3804	-6.8994
4	1st Iter.	19.1783	2.4796	3.4583	-6.3018
	2nd "	19.2104	2.4666	3.2753	-6.9391
	3 d "	19.4136	2.4840	3.3075	-7.0015
	4th "	19.4179	2.4843	3.3064	-7.0077
5	1st Iter.	19.1807	2.4859	3.4796	-6.3319
	2nd "	19.2155	2.4779	3.3133	-6.9935
	3 d "	19.4198	2.4981	3.3457	-7.0545
	4th "	19.4245	2.4986	3.3446	-7.0602
6	1st Iter.	19.1798	2.4824	3.4676	-6.3489
	2nd "	19.2140	2.4714	3.2916	-7.0244
	3 d "	19.4189	2.4890	3.3233	-7.0849
	4th "	19.4238	2.4891	3.3221	-7.0905
Exact [5]		19.20	2.44	3.32	-6.97
Multiplier		$10^{-4}.q\ a^4/B$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$

Table 3. Analysis of square plate clamped on four sides subject to uniform load of intensity q. Fig.2-b (First Iteration only) $\nu = 0.3$

NO of Terms	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB	M_x at Middle of AC
1	12.9919	2.4561	2.7615	-3.6605	-5.3887
2	12.6534	2.3174	2.3431	-4.2427	-4.9660
3	12.6901	2.3504	2.4526	-4.3975	-5.0747
4	12.6823	2.3375	2.4094	-4.4586	-5.0329
5	12.6847	2.3438	2.4307	-4.4887	-5.0526
6	12.6838	2.3403	2.4187	-4.5056	-5.0422
Exact [5]	12.60	2.31	2.31	-5.13	-5.13
Multiplier	$10^{-4}.q\ a^4/B$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$

Table 4. Study of convergence. Square plate clamped on four sides subject to uniform load of intensity q. Fig. 2-b $\nu = 0.3$

NO of Terms	Iteration Order	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB	M_x at Middle of AC
3	1st Iter.	12.6901	2.3504	2.4526	-4.3975	-5.0747
	2nd "	12.6803	2.3303	2.3140	-4.9312	-5.0633
	3 d "	12.7998	2.3533	2.3369	-4.9796	-5.0800
	4th "	12.8019	2.3536	2.3356	-4.9861	-5.0832
4	1st Iter.	12.6823	2.3375	2.4094	-4.4586	-5.0329
	2nd "	12.6679	2.3084	2.2358	-5.0445	-5.0838
	3 d "	12.7902	2.3298	2.2574	-5.0960	-5.1097
	4th "	12.7930	2.3301	2.2560	-5.1028	-5.1114
5	1st Iter.	12.6847	2.3438	2.4307	-4.4887	-5.0526
	2nd "	12.6728	2.3198	2.2752	-5.1010	-5.0632
	3 d "	12.7961	2.3436	2.2982	-5.1536	-5.0821
	4th "	12.7992	2.3442	2.2970	-5.1603	-5.0829
6	1st Iter.	12.6838	2.3403	2.4187	-4.5056	-5.0422
	2nd "	12.6712	2.3133	2.2526	-5.1332	-5.0792
	3 d "	12.7947	2.3349	2.2742	-5.1868	-5.1040
	4th "	12.7981	2.3351	2.2728	-5.1934	-5.1053
Exact [5]		12.60	2.31	2.31	-5.13	-5.13
Multiplier		$10^{-4}.q\ a^4/B$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$	$10^{-2}.q\ a^2$

The analysis of rectangular plates with different ratios of rectangularity ℓ/a is achieved. The results of the second iteration solution using five terms (nine harmonics) are obtained. The results of symmetrical cases of edge conditions Fig.(2-a,b) are given in Tables 5 and 6. Table 7 gives the results for rectangular plates clamped on three sides, simply supported on the fourth side Fig.(2-c).

Table 5. Analysis of rectangular plates clamped on sides AB and CD, simply supported on the two other sides subject to uniform load of intensity q. Fig.2-a (2nd Iter.,5 Terms) $\nu = 0.3$

ℓ/a	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB	
2.0	26.1905 26.00	1.4248 1.42	4.2280 4.20	-8.4008 -8.42	F.S.Iter. Exact [5]
1.5	24.8473 24.70	1.8026 1.79	4.0841 4.06	-8.1977 -8.22	F.S.Iter. Exact
1.0	19.2155 19.20	2.4779 2.44	3.3133 3.32	-6.9935 -6.97	F.S.Iter. Exact
0.5	84.1469 84.40	8.7282 8.69	4.5891 4.74	-11.9938 -11.91	F.S.Iter. Exact
0.3333	116.6761 116.80	11.4962 11.44	4.1946 4.19	-12.0035 -12.46	F.S.Iter. Exact
Multiplier	$10^{-4}.q\ L^4/B$	$10^{-2}.q\ L^2$	$10^{-2}.q\ L^2$	$10^{-2}.q\ L^2$	

L is the smallest value of ℓ and a

Table 6. Analysis of rectangular plates clamped on four sides subject to uniform load of intensity q. Fig.2-b (2nd Iter.,5 Terms) $\nu = 0.3$

ℓ/a	Central Deflection	Central M_x	Central M_y	M_y at Middle of AB	M_x at Middle of AC	
2.0	25.3979 25.40	1.5902 1.58	4.1358 4.12	-8.2526 -8.29	-5.7121 -5.71	F.S.Iter. Exact [5]
1.5	22.0254 22.00	2.0487 2.03	3.6912 3.68	-7.5206 -7.57	-5.6820 -5.70	F.S.Iter. Exact
1.0	12.6728 12.60	2.3198 2.31	2.2752 2.31	-5.1010 -5.13	-5.0632 -5.13	F.S.Iter. Exact
0.5	25.3700 25.40	4.1613 4.12	1.5825 1.58	-5.4857 -5.71	-8.2074 -8.29	F.S.Iter. Exact
Multiplier	$10^{-4}.q\ L^4/B$	$10^{-2}.q\ L^2$	$10^{-2}.q\ L^2$	$10^{-2}.q\ L^2$	$10^{-2}.q\ L^2$	

L is the smallest value of ℓ and a

Table 7. Analysis of rectangular plates simply supported on side BD, clamped on the three other sides subject to uniform load of intensity q. Fig 2-c (2nd Iter.,5 Terms) $\nu = 0.3$

ℓ/a	Central Deflection	M_y at Middle of AB	M_x at Middle of AC	
2.0	25.7262 25.70	-8.3215 -8.37	-5.7046 -5.71	F.S.Iter. Exact (5)
1.3333	21.5867 21.50	-7.4902 -7.50	-5.6605 -5.71	F.S.Iter. Exact
1.0	15.6447 15.70	-5.9942 -6.01	-5.3890 -5.51	F.S.Iter. Exact
0.75	28.4645 28.60	-7.2245 -7.30	-8.1271 -8.38	F.S.Iter. Exact
0.5	44.6154 44.90	-7.4717 -7.86	-11.1538 -11.48	F.S.Iter. Exact
Multiplier	$10^{-4}.q\ L^4/B$	$10^{-2}.q\ L^2$	$10^{-2}.q\ L^2$	

L is the smallest value of ℓ and a

CONCLUSION

New development of the finite strip method is presented for the analysis of elastic plates in bending. In order to overcome the coupling property of the static equilibrium equations in case of basic functions other than trigonometric series, a simplified iteration procedure has been developed. The procedure is applied to the basic function of clamped - clamped end condition. The results demonstrate rapid convergence and close agreement with those of the known exact solution. The procedure has the advantage of using only relatively small overall stiffness matrix, thus requiring small core storage and short computer time for execution. Although the present application of the procedure is carried out for the basic function of clamped - clamped edge condition, it is also possible to extend the same technique for the other basic functions.

APPENDIX I

Numerical values of Integrals I_1, I_2 and I_3

$$I_1 = {}_0\int^a Y_m Y_n dy = a.({}_0\int^1 Y_m Y_n d\bar{y}) = \bar{I}_1 . a \qquad (m = 1,2,3,.. \; n = 1,2,3,..)$$

$$\bar{I}_1 = \begin{bmatrix} \underline{1.035936} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{0.998447} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{1.000067} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{0.999997} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \underline{1.000000} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{0.999999} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \underline{1.000000} \end{bmatrix}$$

$$I_2 = {}_0\int^a \ddot{Y}_m \ddot{Y}_n dy = a.({}_0\int^1 \ddot{Y}_m \ddot{Y}_n d\bar{y}) = \bar{I}_2 / a^3 \qquad (m = 1,2,3,.. \; n = 1,2,3,..)$$

$$\bar{I}_2 = \begin{bmatrix} \underline{518.5343} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{3797.622} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{14620.70} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{39943.70} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \underline{89135.41} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{173881.3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \underline{308202.4} \end{bmatrix}$$

$$I_3 = {}_0\int^a Y'_m Y'_n dy = a.({}_0\int^1 Y'_m Y'_n d\bar{y}) = \bar{I}_3 / a \qquad (m = 1,2,3,.. \; n = 1,2,3,..)$$

$$\bar{I}_3 = \begin{bmatrix} \underline{12.74442} & 0 & -9.904159 & 0 & -7.750952 & 0 & -6.216730 \\ 0 & \underline{45.9785} & 0 & -17.11558 & 0 & -15.18276 & 0 \\ -9.904159 & 0 & \underline{98.91928} & 0 & -24.35159 & 0 & -22.98607 \\ 0 & -17.11558 & 0 & \underline{171.5852} & 0 & -31.27641 & 0 \\ -7.750952 & 0 & -24.35159 & 0 & \underline{263.9980} & 0 & -38.03018 \\ 0 & -15.18276 & 0 & -31.27641 & 0 & \underline{376.1501} & 0 \\ -6.216730 & 0 & -22.98607 & 0 & -38.03018 & 0 & \underline{508.0413} \end{bmatrix}$$

$${}_0\int^a Y_m dy = [\, 1 - (-1)^m \,] \, \frac{2a}{\mu_m} \qquad (m = 1,2,3,...)$$

Stiffness Matrix

$$[\bar{S}]_{mn} = \frac{B\ b}{360360\ a^3} \begin{bmatrix} S_1 & & & & & & & \\ S_2 & S_5 & & & & & & \\ S_3 & S_6 & S_8 & & & & & \\ S_4 & S_7 & S_9 & S_{10} & & \text{Symmetrical} & & \\ S_{11} & -S_{12} & S_{13} & -S_{14} & S_1 & & & \\ S_{12} & S_{15} & -S_{16} & S_{17} & -S_2 & S_5 & & \\ S_{13} & S_{16} & S_{18} & -S_{19} & S_3 & -S_6 & S_8 & \\ S_{14} & S_{17} & S_{19} & S_{20} & -S_4 & S_7 & -S_9 & S_{10} \end{bmatrix}$$

where

$$S_1 = (\ 9172736\ \lambda^4 \bar{I}_1 + 145880\ \bar{I}_2 + 1176000\ \lambda^2 \bar{I}_3\ )$$
$$S_2 = (\ 4586368\ \lambda^4 \bar{I}_1 + 27180\ \bar{I}_2 + (227640 + 360360\nu)\ \lambda^2 \bar{I}_3\ )$$
$$S_3 = (\ 873600\ \lambda^4 \bar{I}_1 + 5480\ \bar{I}_2 + 38640\ \lambda^2 \bar{I}_3\ )$$
$$S_4 = (\ 98280\ \lambda^4 \bar{I}_1 + 766\ \bar{I}_2 + 4200\ \lambda^2 \bar{I}_3\ )$$
$$S_5 = (\ 2808000\ \lambda^4 \bar{I}_1 + 6600\ \bar{I}_2 + 216000\ \lambda^2 \bar{I}_3\ )$$
$$S_6 = (\ 591232\ \lambda^4 \bar{I}_1 + 1470\ \bar{I}_2 + 44280\ \lambda^2 \bar{I}_3\ )$$
$$S_7 = (\ 74880\ \lambda^4 \bar{I}_1 + 216\ \bar{I}_2 + 6000\ \lambda^2 \bar{I}_3\ )$$
$$S_8 = (\ 312000\ \lambda^4 \bar{I}_1 + 344\ \bar{I}_2 + 11680\ \lambda^2 \bar{I}_3\ )$$
$$S_9 = (\ 43680\ \lambda^4 \bar{I}_1 + 52\ \bar{I}_2 + 1776\ \lambda^2 \bar{I}_3\ )$$
$$S_{10} = (\ 7488\ \lambda^4 \bar{I}_1 + 8\ \bar{I}_2 + 288\ \lambda^2 \bar{I}_3\ )$$
$$S_{11} = (-9172736\ \lambda^4 \bar{I}_1 + 34300\ \bar{I}_2 - 1176000\ \lambda^2 \bar{I}_3\ )$$
$$S_{12} = (\ 4586368\ \lambda^4 \bar{I}_1 - 11430\ \bar{I}_2 + 227640\ \lambda^2 \bar{I}_3\ )$$
$$S_{13} = (-873600\ \lambda^4 \bar{I}_1 + 3100\ \bar{I}_2 - 38640\ \lambda^2 \bar{I}_3\ )$$
$$S_{14} = (\ 98280\ \lambda^4 \bar{I}_1 - 521\ \bar{I}_2 + 4200\ \lambda^2 \bar{I}_3\ )$$
$$S_{15} = (\ 1778400\ \lambda^4 \bar{I}_1 - 3730\ \bar{I}_2 + 11640\ \lambda^2 \bar{I}_3\ )$$
$$S_{16} = (-282360\ \lambda^4 \bar{I}_1 + 995\ \bar{I}_2 + 5640\ \lambda^2 \bar{I}_3\ )$$
$$S_{17} = (\ 23400\ \lambda^4 \bar{I}_1 - 165\ \bar{I}_2 - 1800\ \lambda^2 \bar{I}_3\ )$$
$$S_{18} = (-3120\ \lambda^4 \bar{I}_1 + 262\ \bar{I}_2 + 3920\ \lambda^2 \bar{I}_3\ )$$
$$S_{19} = (-7800\ \lambda^4 \bar{I}_1 - 43\ \bar{I}_2 - 876\ \lambda^2 \bar{I}_3\ )$$
$$S_{20} = (-2808\ \lambda^4 \bar{I}_1 - 7\ \bar{I}_2 - 180\ \lambda^2 \bar{I}_3\ )$$

$$\lambda = \frac{a}{b}$$

Original Load Vector (distributed Load q)

$$\{\bar{P}_o\} = q\ b\ \int_o^a Y_m\ dy\ \{\ \tfrac{1}{2} \quad \tfrac{3}{28} \quad \tfrac{1}{42} \quad \tfrac{1}{280} \quad \tfrac{1}{2} \quad \tfrac{-3}{28} \quad \tfrac{1}{42} \quad \tfrac{-1}{280}\ \}^T$$

$$= q\ \frac{ab}{\mu_m}\ [1-(-1)^m]\ \{\ 1 \quad \tfrac{3}{14} \quad \tfrac{1}{21} \quad \tfrac{1}{140} \quad 1 \quad \tfrac{-3}{14} \quad \tfrac{1}{21} \quad \tfrac{-1}{140}\ \}^T$$

NOTATION

w = transverse deflection.
a = length of the strip.
b = width of the strip.
ℓ = length of the plate.
E = modulus of elasticity.
t = thickness of the plate.
ν = poisson's ratio.
B = flexural rigidity.
Y_m = basic function.
$[C]$ = transformation matrix.
$[\bar{S}]$ = stiffness matrix of the strip.
$[\bar{S}_d]$ = matrix contains the diagonal submatrices.
$[\bar{S}_n]$ = matrix contains the non diagonal submatrices.
$\{\bar{D}\}$ = nodal line parameters of the strip.
$\{\bar{F}_o\}$ = original load vector of the strip.
$\{\bar{F}_k\}$ = modified load vector of the strip.
$[S_d]$ = overall stiffness matrix
$\{D\}$ = nodal line parameters of the assembled strips.
$\{F_o\}$ = original load vector of the assembled strips.
$\{F_k\}$ = modified load vector of the assembled strips.

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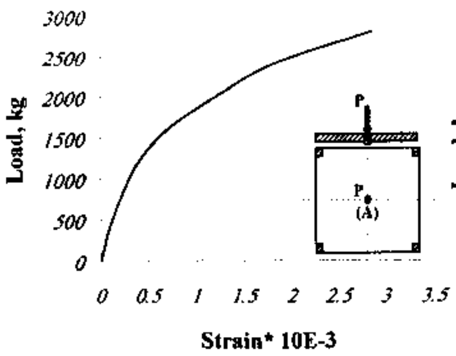

Fig.(24) Experimental Load - Max. Strain for Plate HSR1

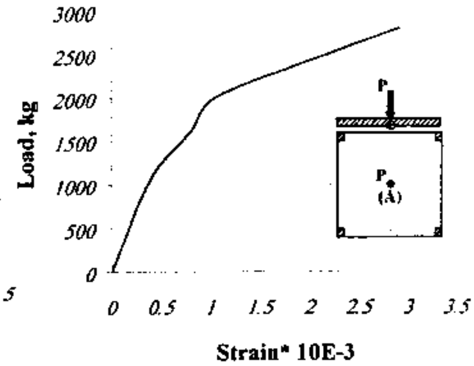

Fig. (23) Experimental Load - Max. Strain for Plate HSF3

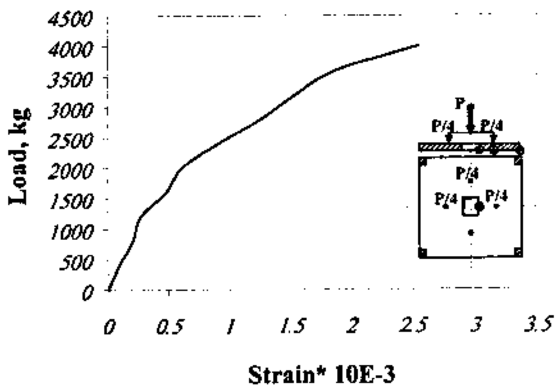

Fig.(25) Experimental Load - Max. Strain for plate HSO3

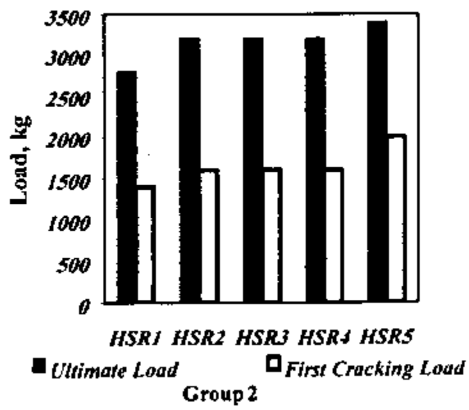

Fig. (27) Ultimate Loads And First Cracking Load For Plates Of Group (2)

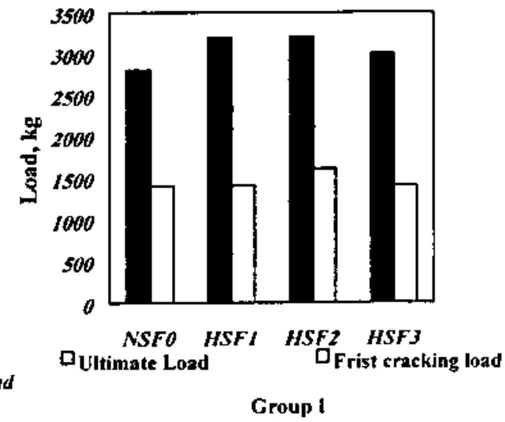

Fig. (26) Ultimate Loads and First Cracking Load For Plates Of Group (1)