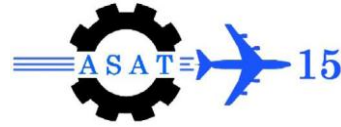


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## A Comparison Between Magnetic Control Methods Commonly Used in Satellite Attitude Control

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**Comment [DA2]:** Check references beyond 11

**Abstract:** The problem of attitude control of remote sensing satellite using magnetic actuators is considered in this paper. Magnetic actuator was used because it is low power consumption, small mass, low cost and reliable attitude actuator. The attitude control problem of the satellite involves angular velocity suppression, attitude acquisition and finally attitude stabilization will be solved by magnetic actuator only. A comparison between the commonly used controllers for satellite attitude control is presented. The comparison parameters are the total consumed power, the time required to accomplish the angular velocity suppression and attitude acquisition, calculation time of the control algorithm and steady state error in angles and angular velocity. The simulation is done using the complete nonlinear model of satellite. Based on results, a new combined control algorithm was developed to assemble the advantages of these commonly used controllers. Simulation results showed the validity of the developed combined algorithm.

**Keywords:** Attitude control, magnetic control, angular velocity damping, 3-axis sterilization

### Acronyms

ADCS	-	attitude determination and control subsystem
GCS	-	Greenwich coordinates system
OCS	-	Orbit coordinates system
MM	-	Magnetometer
AVM	-	Angular velocity meter
MT	-	Magnetorquer
LV	-	Lunch vehicle
RCS	-	Reference coordinate system
BCS	-	Body coordinate system
DM	-	Detumbling mode

### List of symbols

$T_{gg}$	Gravitational torque
$J$	Moment of inertia tensor for the satellite
$B$	Earth geomagnetic field vector
$T_{dis}$	Total external disturbance torque
$T_c$	Control torque

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$\omega$	Satellite absolute angular velocity
$y$	Satellite relative angular velocity
$\omega_o$	The orbit rate
$er$	The 3rd column in the rotation matrix from OCS to BCS
$\Lambda$	Quaternion describes the orientation of BCS with respect to OCS
$\lambda_0$	Scalar part of quaternion
$\lambda$	Vector part of quaternion
$Y$	Instantaneous angular velocity of satellite in quaternion form
$F$	Input matrix for control
$U$	Vector of input control torque
$X$	State vector
$L$	Dipole moment
$\omega_e$	Earth rotation velocity
$\psi$	Way angle that describes the rotation about z axis
$\phi$	Roll angle that describe the rotation about x axis
$\theta$	Pitch angle that describe the rotation about y axis
$\circ$	Quaternion multiplication

## 1. Introduction

The main tasks of ADCS are to control the angular rotation of satellite starting from separation from launcher then attitude acquisition and then keep satellite stabilization at nadir pointing. Recently magnetic actuator become one of the most used actuators in spacecraft attitude control. Generally magnetic control algorithms used in attitude control are divide to angular suppression algorithms, in addition to attitude acquisition and stabilization algorithms [3, 4].

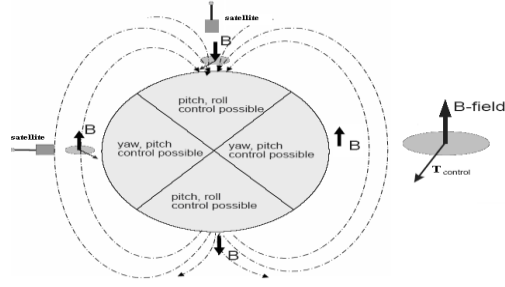
### 1.1 Attitude Magnetic Control Concept

The main concept of magnetic attitude control of satellite is to generate dipole moment  $L$ . This dipole moment reacts with the earth magnetic field  $B$  generating torque  $T_c$  used to control the satellite rotation.

$$T_c = L \times B \quad (1)$$

The satellite actuated by a set of magnetorqure (MT) has a serious limitation [9]. The mechanical torque, produced by the interaction of the geomagnetic field and dipole moment generated by the MT, is always perpendicular to the geomagnetic field vector. Thus, the direction parallel to the geomagnetic field vector is not controllable. The geomagnetic field changes its orientation in the OCS when the satellite moves in orbit. This implies that yaw is not controllable over the poles but controllable over the equator and roll is not controllable over the equator but controllable over the poles, see Figure (1)

Therefore beside the magnetic control it is required another source of torque to control the satellite, when it is magnetically uncontrollable. This source of torque can be expensively achieved by so-called momentum bias configuration [1,2] or cheaply achieved by using gravity gradient torque [3]. This paper will focus in using gravity gradient torque beside the magnetic torque to control the satellite



**Figure (1) Control torque is always perpendicular to the geomagnetic field vector.**

## 2. Mathematical Model of Satellite

The attitude dynamics of a rigid spacecraft can be expressed by the well-known Euler's equations, as follows

$$\dot{\omega} = J^{-1} \left[ -\omega \times (J \cdot \omega) + 3 \cdot \omega_o^2 \cdot er \times (J \cdot er) + T_{dis} + T_c \right] \& er = \begin{bmatrix} 2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) \\ 2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1) \\ \lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2 \end{bmatrix} \quad (1)$$

And the kinematic equation of satellite model can be expressed as

$$2\dot{\Lambda} = \Lambda \circ \begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \omega_o \\ 0 \end{bmatrix} \circ \Lambda \quad (2)$$

### 2.1 Linearized Equations of Motion

According to [20] the linearized model of magnetic actuated satellite can be represent as

$$\begin{bmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \\ \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = A \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} + FL \quad (3)$$

where

re

$$\sigma_1 = \frac{J_2 - J_3}{J_1} \& \sigma_2 = \frac{J_1 - J_3}{J_2} \& \sigma_3 = \frac{J_2 - J_1}{J_3}$$

$$F = \frac{J^{-1}}{\|B\|^2} \begin{bmatrix} -B_2^2 - B_3^2 & B_1 B_2 & B_1 B_3 \\ B_1 B_2 & -B_1^2 - B_3^2 & B_2 B_3 \\ B_1 B_3 & B_2 B_3 & -B_1^2 - B_2^2 \end{bmatrix} \& en = \begin{bmatrix} 2(\lambda_1 \lambda_2 + \lambda_3 \lambda_0) \\ \lambda_0^2 - \lambda_1^2 + \lambda_2^2 - \lambda_3^2 \\ 2(\lambda_2 \lambda_3 - \lambda_1 \lambda_0) \end{bmatrix} \quad (4)$$

$$A = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ -8\sigma_1\omega_o^2 & 0 & 0 & 0 & 0 & \omega_o(\sigma_1-1) \\ 0 & -6\sigma_2\omega_o^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2\sigma_3\omega_o^2 & \omega_o(1-\sigma_3) & 0 & 0 \end{bmatrix} \quad \& \quad y = \omega - \omega_o \cdot en \quad (5)$$

where

$B_i, i = 1, 2, 3$  are the components of measured Earth magnetic field.

$y_i, i = 1, 2, 3$  are the components of satellite relative angular velocity

### 3. Algorithms Used For Angular Velocity Suppression by Magnetic Actuators

The objective of the angular velocity suppression or detumbling controller is to suppress the high angular velocity of satellite obtained due to separation from launcher. Commonly there are two methods used for satellite angular suppression, angular velocity feedback and B-dot technique.

#### 3.1 Angular Suppression Using Velocity Feed Back

The main idea here is to use controller able to dissipate the satellite high energy gained during separation from launcher. A very simple controller is suggested in [5] which use angular velocity measurements from gyro, the stability of velocity feedback is examined using Lyapunov Stability Lyapunov Stability.

Consider the following Lyapunov candidate function expressing the total energy of the satellite

$$V = \frac{1}{2} y^T \cdot J \cdot y + \frac{3}{2} \omega_o^2 (er^T \cdot J \cdot er - J_{zz}) + \frac{1}{2} \omega_o^2 (J_{yy} - en^T \cdot J \cdot en) \quad (1)$$

This candidate function satisfies

$$\begin{aligned} V(0) &= 0 \\ V(x) &> 0 \quad \forall x \neq 0 \end{aligned} \quad (2)$$

And has the equilibrium for  $V(0)$

$$\{(y, en, er) : \left( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \pm 1 \end{bmatrix}, \begin{bmatrix} 0 \\ \pm 1 \\ 0 \end{bmatrix} \right)\} \quad (3)$$

The time derivate of  $V$

$$\dot{V} = y^T \cdot J \cdot \dot{y} + 3\omega_o^2 \cdot er^T \cdot J \cdot \dot{er} - \omega_o^2 en^T \cdot J \cdot \dot{en} = y^T \cdot T_c \quad (4)$$

So if the controlling torque is chosen as angular velocity feedback as shown below in (5)

$$T_c = -K_v \cdot y \quad (5)$$

where

$K_v$  is the positive constant

Then,

$$\dot{V} = -y^T \cdot K_v \cdot y \quad (6)$$

Therefore, the controller (5) guarantees damping of the relative angular velocity to zero. Hence the required dipole moment to generate the control torque (5) can be calculated using (13) [1].

$$L = \frac{B \times T_c}{\|B\|^2} \quad (7)$$

where

L is the dipole moment

B is the earth magnetic field

### 3.2 Angular Suppression Using B-Dot Technique

The main idea for the B-dot technique based on the fact that the variation of earth magnetic field is slow (i.e. 3.34e-4 Hz), so the difference between two successive measurements from magnetometer depends on the rotation of satellite around its center of mass rather than the center of mass motion in orbit. Commonly there are three methods used to express the magnetic field derivative.

Since the condition (4) required to guarantee  $\dot{V} < 0$  could be rewritten in another form as follows

$$y^T T_c < 0 \quad (8)$$

Since the generated torque,  $T_c$  due to earth magnetic field, B and MT dipole moment, L is given by

$$T_c = L \times B \quad (9)$$

Condition(8) can be rewritten as

$$L^T (y \times B) < 0 \quad (10)$$

This inequality dictates that the magnetic moment L needs to have a component, which is anti-parallel to the direction of  $y \times B$ . Maximum efficiency, is provided by ensuring that the entire vector is anti-parallel. In other words, the inequality can be solved by expanding (10) with a positive scalar gain K

$$L = -K(y \times B) \quad (11)$$

Since derivative of earth magnetic field in BCS as given [11]

$$\dot{B} = y \times B \quad (12)$$

Finally becomes the B-dot detumbling control law

$$L = -K\dot{B} \quad (13)$$

Therefore, the dipole moment calculated by the form (11) guarantees suppression of satellite angular velocity.

The time derivative of measured magnetic field can be calculated by three methods as follows

#### 3.2.1 Angular suppression using B-dot technique No. 1

Since the derivative of earth magnetic field B can be calculated using magnetometer measurements and satellite relative angular velocity using (12). However, the relative angular velocity  $y$  is calculated from gyro measurements  $\omega$  and orbital rate  $\omega_o$  by the following form

$$y = \omega - \omega_o e_n \quad (14)$$

Since the orbital rate is very small compared with satellite angular velocity during detumbling mode and it can be neglected so equation (11) can be rewritten as

$$L = -K(\omega \times B) \quad (15)$$

### 3.2.2 Angular suppression using B-dot technique No. 2

Here a simple way used to calculate the derivative of earth magnetic field  $B$  done by a simple backwards difference method (i.e. calculation of the difference between two successive magnetometer measurement  $B(t), B(t+dT)$  divided by the difference between the times measurements  $dT$  [23].

$$\dot{B} = \frac{B(t+dT) - B(t)}{dT} \quad (16)$$

Then the required dipole moment is calculated from (13)

### 3.2.3 Angular suppression using B-dot technique No. 3

This technique was developed by M Guelman and used in the Israeli Guerwin-Techsat This satellite was a 50kg cube, launched in July 1998, and successfully functioned for over 4.5 years [6]. The derivative of the magnetic field is calculated as

$$\dot{B} = \frac{\Delta B(t+dT) - \Delta B(t)}{dT} \quad \& \quad \Delta B(t) = B(t) - B_o(t) \quad (17)$$

where

$B_o$  is the earth magnetic field in OCS

Then the required dipole moment is calculated from (13)

## 4. Simulation Results

The following data are used for judge the performance of the above mentioned angular velocity suppression controllers

**Table 1 Initial data used for satellite simulation**

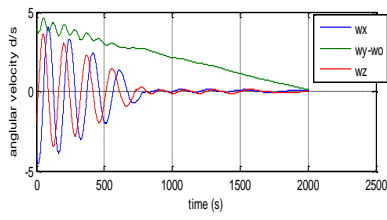
Parameter	Value
Satellite moment of inertia tensor	$\begin{bmatrix} 6 & 0.05 & -0.08 \\ 0.05 & 8 & 0.06 \\ 0.08 & -0.06 & 4 \end{bmatrix} \text{ kgm}^2$
Initial satellite angular velocity	$[4.5 \ -4.5 \ 4.5]^T$
Maximum dipole moment generated from MT	$10 \text{ Am}^2$
Orbit altitude	660 Km
Orbit eccentricity	0
Orbit inclination	$98^\circ$
Orbit argument of perige	$0^\circ$
Local time of ascending node	10:00 am
True anomaly	0 deg
Satellite residual dipole moment	$[0.5 \ 0.3 \ -0.2]^T \text{ Am}^2$
The desired angular velocity to be reached aster angular suppression $y_d$	0.13 °/s

#### 4.1 Simulations Results for Angular Velocity Feedback

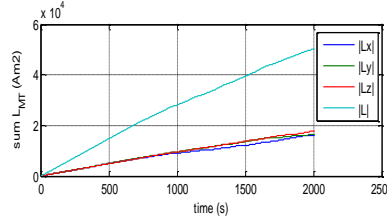
Simulations results for the controller (5) are shown in Fig. 2 and Fig. 3 and it shows that the satellite angular is damped from its initial value and reached  $|y_d| < 0.13^\circ/s$ .

#### 4.2 Simulation results for B-dot techniques

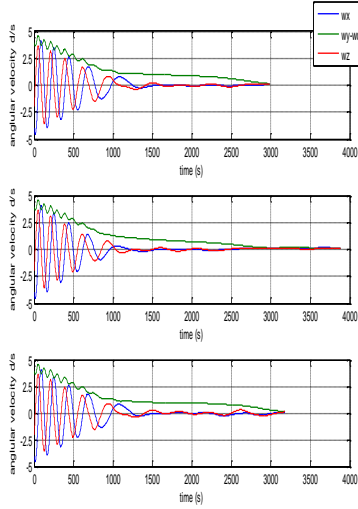
The data presented in Table 1 used for verification of used control laws (15)-(17) and simulations results shown in Fig. 4 and Fig. 5 and it shows that the satellite angular is damped from its initial value and reached  $|y_d| < 0.13^\circ/s$ .



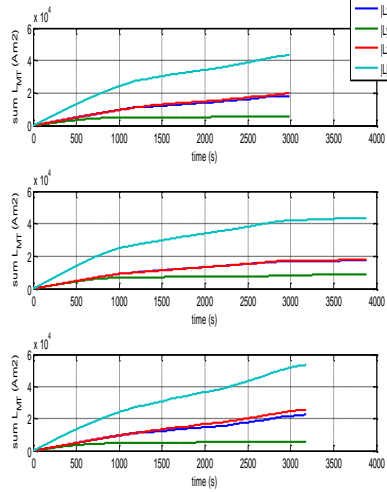
**Fig. 2 Satellite angular velocity suppression using angular velocity feedback**



**Fig. 3 Required dipole moment to suppress the satellite angular velocity using angular velocity feedback**



**Fig. 4 Satellite angular velocity suppression**



**Fig. 5 Required dipole moment to suppress the satellite angular velocity**

(a) B-dot technique No.1 (b) B-dot technique No.3 (c) B-dot technique No. 2

#### 4.3 Comparison between Angular Velocity Suppression Algorithms

The following parameters are considered as comparison parameters to judge performance of the above mentioned algorithms:

1. The required time to accomplish the angular velocity suppression (i.e.  $|y| < 0.13^\circ/s$ ),  $T_{sup}$ .
2. The total dipole moment required to accomplish the angular velocity suppression,  $L_{sup}$ .
3. The calculation time for one cycle of the algorithm  $T_{cal}$

#### 4. Qualitative the estimation cost.

For the calculation time for one cycle of the algorithm the following environments was used for the evaluation.

- Algorithm was developed and run using Borlandc 3.11.
- Run on PC with Pentium (D) CPU 3.40 GHz. With 1 GB RAM
- Used operating system XP SP2

The following table summarizes the comparison results

**Table 2 The comparison result between angular velocity suppression algorithms**

Parameter	Angular velocity feedback	B-dot		
		Technique No1	Technique No2	Technique No3
$T_{sup}$	2004 s	2988 s	3885 s	3181 s
$L_{sup}$	5.002e4 Am <sup>2</sup>	4.363e4 Am <sup>2</sup>	4.336e4 Am <sup>2</sup>	5.344e4 Am <sup>2</sup>
$T_{cal}$	4.396e-6 s	3.605e-6 s	3.287e-6 s	1.4305e-4 s
Cost	Need MM and Gyros	Need MM and Gyros	Need MM only	Need MM only

#### 4.4 Results Conclusion

The above comparison table showed that

1. Angular velocity feedback algorithms could achieve angular suppression very fast but it consumes high power compared with B-dot technique No.1 and No. 2
2. B-dot technique No. 1 comes in the middle in calculation resources and power consumption.
3. B-dot technique No. 2 is the simplest (i.e. calculation time is the smallest one) , lowest power consumption and need MM only but it need largest time to achieve the angular velocity suppression.
4. B-dot technique No. 3 comes in the middle in the required time to achieve angular suppression and low cost but it is the highest power and calculation time

### 5. Algorithms Used For Attitude Acquisition and Stabilization

After angular velocity suppression, satellite may have arbitrary orientation (i.e. BCS of the satellite may not co-aside with the OCS). Therefore, it is required to make attitude acquisition or reorient the satellite in order to make the satellite nadir pointing (i.e. to make BCS of the satellite co-aside with the OCS), after that it is required to make attitude stabilization or keeping the satellite at nadir pointing. To make attitude acquisition and stabilization, the used control algorithm must contain information about the satellite attitude and angular velocity. Commonly there are three methods used for attitude acquisition and stabilization

1. PD-Like Controller
2. Sliding Mode Controller
3. Linear Quadratic Regulator

In the next subsections, adscription for the above mentioned controllers and simulation results will be presented then comparison between them will be introduced.



### 5.1 PD-Like Controller

Three axes attitude control requires accurate attitude knowledge to function correctly. So PD-like controller [1] will be used, here the feedback from relative angular velocity is used instead of the derivative of quaternion [7]. Therefore, the algorithm itself becomes just a simple rate and position feedback; hence the required magnetic torque can be calculated as follows

$$T_c = -k_v \cdot y - k_p \cdot \lambda \quad (1)$$

where

$k_v$  Positive rate gain constant

$k_p$  Positive position gain constant

And the required dipole moment will be

$$L = \frac{B \times T_c}{\|B\|^2} \quad (2)$$

### Checking the stability of PD-Like Controller

To check the stability of the PD like controller (1), let us reconsider the candidate Lyapunov function

$$V = \frac{1}{2} y^T \cdot J \cdot y + \frac{3}{2} \omega_o^2 (er^T \cdot J \cdot er - J_{zz}) + \frac{1}{2} \omega_o^2 (J_{yy} - en^T \cdot J \cdot en) + K_p (\lambda^T \lambda + (1 - \lambda_o)^2) \quad (3)$$

Therefore, the derivative of Lyapunov function can be easily calculated as

$$\dot{V} = y^T \cdot T_c + k_p y^T \lambda \quad (4)$$

$$\dot{V} = y^T \cdot (-k_v \cdot y - k_p \cdot \lambda) + k_p y^T \lambda = -y^T k_v \cdot y \quad (5)$$

$$\therefore \dot{V} < 0$$

Thus the satellite with control law (1) would be globally asymptotically stable at nadir pointing (i.e. at the reference  $\{(y, \lambda) : (0, 0)\}$  )

### 5.2 Sliding Mode Controller

A sliding mode controller is implemented for the attitude corrections using magnetic actuator . Full attitude information in the form of the attitude quaternion  $\Lambda$  and the satellite relative angular velocity  $y$  are used as feedback signals. The objective of the attitude control is to get nadir pointing; the design strategy of the sliding mode controller consists of two steps,

- Sliding manifold design.
- Sliding condition design.

The description and design of sliding mode controller are based on [5]

#### Sliding Manifold Design

Consider manifold, a 3 dimensional hyper plane, in the state space of a 6th order system  $[y \lambda]^T$ . The sliding manifold is designed in such a way that the satellite trajectory, if on the hyper plane, converges to the reference. However, the satellite motion is not confined to the 3 dimensional hyper plane in general. Therefore, a control law forcing the satellite motion toward the manifold is necessary for achieving stable satellite motion

Let a sliding variable  $s$  be defined as in

$$s = J \cdot y + A_q \cdot \lambda \quad (6)$$

where  $A_q$  is a positive definite matrix.

The sliding manifold is the subspace of the state space, where the sliding variable equals

$$S = \{\lambda, y : s = 0\} \quad (7)$$

The definition of the sliding variable,  $s$ , guarantees convergence of  $\lambda$  to zero with an exponential rate. To prove this statement, consider a Lyapunov candidate function

$$\begin{aligned} V &= \lambda^T \cdot \lambda + (1 - \lambda_o)^2 \\ &= 2(1 - \lambda_o) \end{aligned} \quad (8)$$

The time derivative of the Lyapunov candidate function is calculated applying the kinematics

$$\dot{V} = \lambda^T \dot{y} \quad (9)$$

But  $y = -A_q \lambda$  thus

$$\dot{V} = -\lambda^T A_q \dot{\lambda} \quad (10)$$

The time derivative of the Lyapunov function is negative definite, since  $A_q$  is positive definite matrix. According to Lyapunov's direct method the equilibrium  $y = 0, \lambda = 0, \lambda_o = 1$  is asymptotically stable if the satellite is on the sliding manifold  $S$ .

### Sliding Condition Development

The sliding condition keeps decreasing the distance from the state to the sliding manifold, such that every solution  $y, \lambda$  originating outside the sliding manifold tends to it. Now the manifold is an invariant set of the satellite motion and the trajectory of the system converges to the reference.

The objective Sliding Condition Development is to derive the desired control torque  $T_d$  turning the satellite trajectory towards the sliding manifold. The representation of the satellite motion in the space of the sliding variable is calculated by differentiation of the sliding variable,  $s(t)$  w.r.t. time, which describes projection of the satellite motion on the space of the sliding variable (the  $s$ -space)

$$\dot{s} = J\dot{\omega} - \omega_o J \dot{e}n + A_q \dot{\lambda} \quad (11)$$

The derivatives of the satellite angular velocity and the attitude quaternion are calculated according to the equations of kinematics and dynamics (1) and (2),

$$\dot{s} = -\omega \times J\omega + 3\omega^2 e_r \times J e_r - \omega_o J(e_n \times y) + \frac{1}{2} A_q (y \lambda_o + y \times \lambda) + T_c \quad (12)$$

Assume that the satellite trajectory is on the sliding manifold. An equivalent torque  $T_{eq}$  is a control torque necessary to keep the satellite on the sliding manifold. In other words, if the control torque  $T_c$  is equal to the equivalent torque  $T_{eq}$  then the time derivative of the sliding variable equals zero. If the satellite is not on the sliding manifold, a desired control torque  $T_d$  equals the sum of the equivalent torque  $T_{eq}$  and a part making the sliding variable converge zero [5]

$$T_d = T_{eq} - A_s \text{sign}(s) \quad (13)$$

where

$A_s$  is a positive definite matrix and

$$T_{eq} = \omega \times J\omega - 3\omega^2 e_r \times J e_r + \omega_o J(e_n \times y) - \frac{1}{2} A_q (y \lambda_o + y \times \lambda) \quad (14)$$

Finally, the required dipole moment to generate the desired control torque  $T_d$  according to ref [5] will be

$$L = \frac{B \times T_d}{\|B\|^2} \quad (15)$$

### 5.3 Linear Quadratic Regulator

The satellite dynamics are quite nonlinear; therefore, linear control approach will be attractive if the system could be linearized about the desired reference states. Traditionally, LQR (Linear Quadratic Regulator) has been used on magnetic actuated satellites because of their reliability and robustness. The LQR strategy is based on linearizing the systems dynamics, defining an object function which shall be minimized and generate a gain matrix which is used for feedback. For more details on LQ-control problems see [5, 10].

The linearization of the satellites attitude was presented in (0), recalling the system of differential equation that describe the linear mode of the satellite (3)

$$\dot{x}(t) = A \cdot x(t) + F(t) \cdot L(t) \quad (16)$$

where matrices  $A$  and  $F(t)$  are defined in (5) and (6). The cost function which should be to minimized according to [8]:

$$J_{op} = \int_{t_0}^T [x \cdot Q \cdot x + u \cdot P \cdot u] dt \quad (17)$$

The matrices  $Q$  and  $P$  are positive semidefinite and are used to weight state and actuator usage. The solution to the LQ-problem is given by the Ricatti equation

$$\dot{R}_{ric}(t) = -R_{ric}(t) \cdot A - A^T \cdot R_{ric}(t) + R_{ric}(t) \cdot F(t) \cdot P^{-1} \cdot F(t)^T \cdot R_{ric}(t) - Q \quad (18)$$

The solution of the LQR problem yields the time varying controller

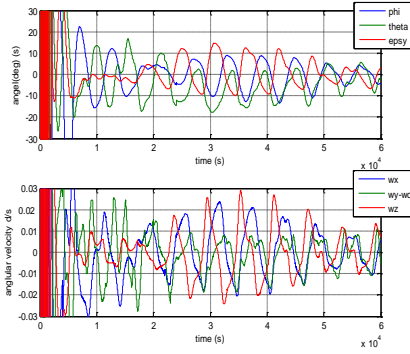
$$L(t) = -P^{-1} \cdot F(t)^T \cdot R_{ric}(t) x(t) \quad (19)$$

### Simulation results

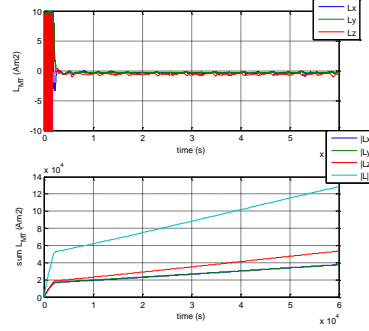
To judge the performance of attitude acquisition and stabilization algorithms it was considered the initial value is the same in Table 1 and the angular velocity suppression will be done using angular velocity feedback algorithm. It is also considered that the attitude acquisition phase is finished when  $|\Phi| < 10^\circ, |y| < 0.02^\circ / s$  where  $\Phi$  is the satellite attitude with respect OCS and the simulation time will be 10 orbital period or 60,000 sec in order to investigate the satellite steady state behavior under the effect of external disturbance .

### Simulations results for PD-like controller

Simulations results for the controller (1) are shown in Fig. 6 and Fig. 7 and it shows that the satellite achieved the attitude acquisition and kept nadir pointing against the external disturbances.



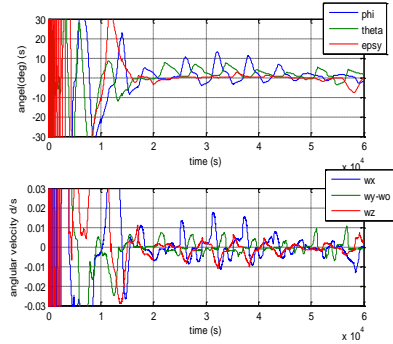
**Fig. 6** Satellite response during angular velocity suppression using angular velocity feedback, followed by, attitude acquisition and stabilization using PD-like controller



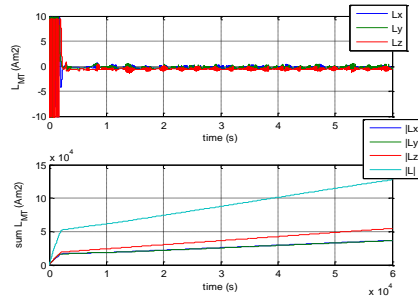
**Fig. 7** Required dipole moment during angular velocity suppression using angular velocity feedback, followed by, attitude acquisition and stabilization using PD-like controller

### Simulations results for sliding mode controller

Simulations results for the controller (19) are shown in Fig. 8 and Fig. 9 and it shows that the satellite achieved the attitude acquisition and kept nadir pointing against the external disturbances.



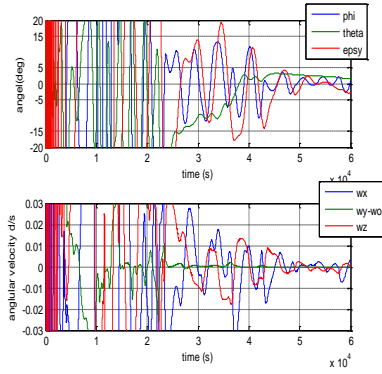
**Fig. 8** Satellite response during angular velocity suppression using angular velocity feedback, followed by, attitude acquisition and stabilization using sliding mode controller



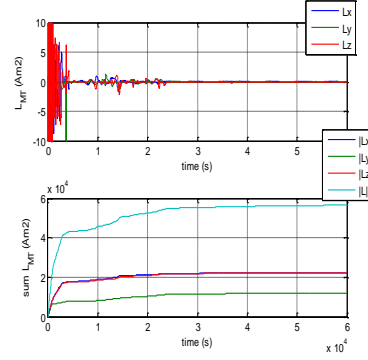
**Fig. 9** Required dipole moment during angular velocity suppression using angular velocity feedback, followed by, attitude acquisition and stabilization using sliding mode controller

### Simulations results for LQR

Simulations results for the controller (19) are shown in Fig. 10 and Fig. 11 and it shows that the satellite achieved the attitude acquisition and kept nadir pointing against the external disturbances.



**Fig. 10** Satellite response during angular velocity suppression using angular velocity feedback, followed by, attitude acquisition and stabilization using LQR



**Fig. 11)** Required dipole moment during angular velocity suppression using angular velocity feedback, followed by, attitude acquisition and stabilization using LQR

The following parameters are considered as comparison parameters to judge performance of the above mention attitude acquisition and stabilization algorithms.

The required time to finish the attitude acquisition (i.e.  $|\lambda| > 0.1, |y| < 0.02^\circ$ ),  $T_{acq}$ .

Required dipole moment until finishing the attitude acquisition,  $L_{acq}$

The total dipole moment required during 10 orbital period  $L_{tot}$

The steady stat error for angels and angular velocity  $E_{ss\alpha}$ ,  $E_{ss\omega}$ .

The calculation time for one cycle of the algorithm  $T_{cal}$

For the calculation time for one cycle of the algorithm the following environments was used for the evaluation.

Algorithm was developed and run using Borlandc 3.11.

Run on PC with Pentium (D) CPU 3.40 GHz. With 1 GB RAM

Used operating system XP SP2

The following table summarizes the comparison results

**Table 3** The comparison result between attitude acquisition and stabilization algorithms

Parameter	PD-like controller	Sliding Mode controller	LQR
$T_{acq}$	5,754 s	9,609 s	25,340 s
$L_{acq}$	5.658e4 Am <sup>2</sup>	6.259e4 Am <sup>2</sup>	5.480e4 Am <sup>2</sup>
$T_{cal}$	4.451e-6 s	6.091e-6 s	3.539e-3 s
$L_{tot}$	12.84e4 Am <sup>2</sup>	12.99e4 Am <sup>2</sup>	5.652e4 Am <sup>2</sup>
$E_{ss\alpha}$	$\pm 10^\circ$	$\pm 7^\circ$	$\pm 4^\circ$
$E_{ss\omega}$	$\pm 0.013^\circ/\text{s}$	$\pm 0.011^\circ/\text{s}$	$\pm 0.008^\circ/\text{s}$

### Results conclusion

The above comparison table showed that

PD-like controller is the simplest (i.e. calculation time is the smallest one ) algorithm and could achieve attitude acquisition very fast but needs large dipole moments during stabilization and give low accuracy

The LQR is very useful to save the consumed power by MT specially during the stabilization period and give very high accuracy but needs high calculation resources and it is not suitable for attitude acquisition.

the sliding mode comes in the middle in calculation resources and achieved accuracy but need the largest power during stabilization

Combined Attitude Control

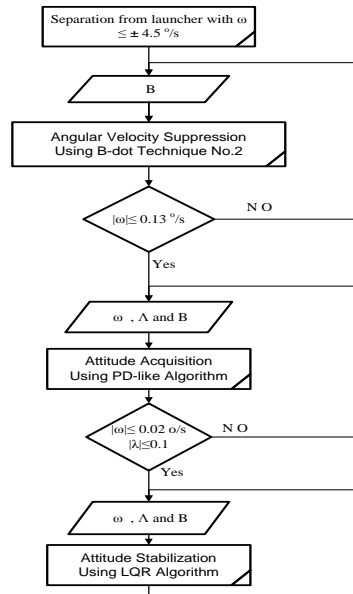
As conclusion from the above comparisons between the different magnetic attitude control algorithms used during angular suppression, attitude acquisition and attitude stabilization, it can be concluded the following facts

if it is considered that the comparison parameters in Table 2 are equally- weighted, then the B-dot technique No2 will be the best one for angular velocity suppression

if it is considered, that the above comparison parameters in Table 3 are equally- weighted, beside  $T_{acq}$ ,  $L_{acq}$ ,  $T_{cal}$  as evaluation parameters for attitude acquisition, Then, the PD-like is the best one for attitude acquisition

if it is considered, that the above comparison parameters in Table 3 are equally- weighted, beside  $L_{lat}$ ,  $E_{ss\alpha}$ ,  $E_{ss\omega}$ ,  $T_{cal}$  as evaluation parameters for attitude stabilization Then, LQR is the best one for attitude stabilization

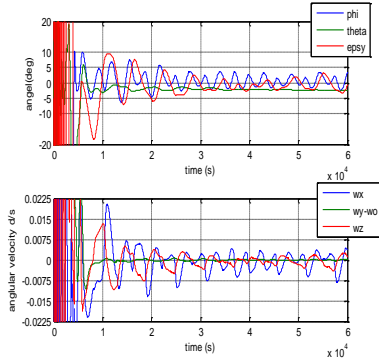
As a result, a combined algorithm was developed as a combination from the best algorithms (i.e. according to mentioned comparison parameter) in the corresponding phase.. This algorithm will switch between the selected control algorithms according to the operation phase (i.e. angular suppression, attitude acquisition and attitude stabilization), based on the values of relative angular velocity and quaternion. The following flowchart shows the cyclogram for the combined algorithm



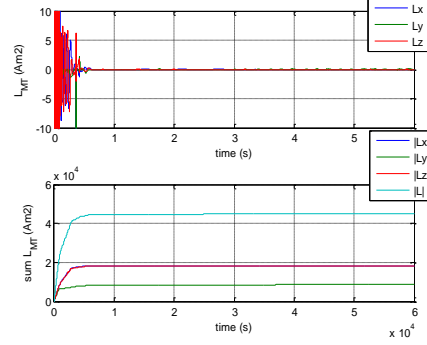
**Fig. 12 S Cyclogram of the developed combined algorithm**

### Simulations results for combined algorithm

Simulations results for the combined algorithm and it shows that the satellite achieved the attitude acquisition and kept nadir pointing against the external disturbances.



**Fig. 13** Satellite response during angular velocity suppression using angular velocity feedback, followed by, attitude acquisition and stabilization using combined algorithm



**Fig. 14** Required dipole moment during angular velocity suppression using angular velocity feedback, followed by, attitude acquisition and stabilization using combined algorithm

The above simulation results could be surmised in the following table.

**Table 4** The combined algorithms results

Comparison Parameter	Value
$T_{acq}$	5,807 s
$L_{acq}$	4.443e4 Am <sup>2</sup>
$L_{tot}$	4.49e4 Am <sup>2</sup> /s
$T_{cal}$	4.99e-6 / 3.539e-3 S
$E_{ss\alpha}$	$\pm 3^\circ$
$E_{ss\omega}$	$\pm 0.0075^\circ/\text{s}$

## 6. Conclusion

Commonly used controllers for magnetic attitude control of satellite, had been studied and compared in this thesis. The comparison was done during angular velocity suppression, attitude acquisition and attitude stabilization. The simulation results showed that. Angular velocity suppression done by B-dot technique which depends on the difference between two successive readings from magnetometer divided by period between the two readings is optimum algorithm. Also, during attitude acquisition simulation results showed that PD-like algorithm is good choice. Finally LQR algorithm was very suitable to keep stabilization of satellite at nadir pointing.

Based on the simulation results a combined algorithm was developed in to serve all over the satellite modes by switching between the suitable controllers depending on the satellite stats , the simulation results showed the effectiveness of using the developed algorithm in transient and steady state of satellite attitude beside saving the overall consumed power.

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