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THREE DIMENSIONAL SURFACES OBTAINED BY THE EQUIFORM MOTION OF A SURFACE OF REVOLUTION

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In this paper, a three dimensional surface using equiform motion of a surface of revolution in Euclidean 3-space E^3 is generated. The main results obtained in this paper are that the surface foliated by equiform motion of sphere has a zero scaler curvature if the motion of sphere are in parallel planes. Also, the surface foliated by equiform motion of tours has a zero scaler curvature if the motion of torus are in parallel planes. Finally, for some special cases, new examples are constructed and plotted.

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INTRODUCTION

An equiform transformation in the 3-dimensional Euclidean space E^3 is an affine transformation whose linear part is composed of an orthogonal transformation and a homothetical transformation. This motion can be represented by a translation vector d and a rotation matrix A as the following.

$$\bar{x} \to \rho A x + d, \tag{1}$$

where $AA^t = A^t A = I,\ \ .\bar{x}, x \in E^3$ and ρ is the scaling factor. An equiform motion is defined if the parameters of (1) including ρ - are given as functions of a time parameter t. Then a smooth one - parameter equiform motion moves a point x via $\bar{x}(t) = \rho(t)A(t)x(t) + d(t)$. The kinematic corresponding to this transformation group is called an equiform kinematic [1, 2].

Under the assumption of the constancy of the scalar curvature, kinematic surfaces obtained by the motion of a circle have been obtained in [3]. In a similar context, one can consider hypersurfaces in space forms generated by one - parameter family of spheres and having constant curvature see [4, 5].

The purpose of this paper is to describe the kinematic surface obtained by the motion of a surface of revolution whose scalar curvature vanished.

1.1 The scalar curvature

In mathematics and physics, the Christoffel symbols, named for Elwin Bruno Christoffel, are numerical arrays of real numbers that describe, in coordinates, the effects of parallel transport in curved surfaces and, more generally, manifolds. As such, they are coordinate-space expressions for the Levi-Civita connection derived from the metric tensor. In general relativity, the Christoffel symbol plays the role of the gravitational force field with the corresponding gravitational potential being the metric tensor [6, 7, 8].

Consider the three dimensional surface M. Let $\mathbf{X} = X(x_1, x_2, x_3)$ be a local parametrization of M. The tangent vectors to the parametric curves of the surface M are:

$$\mathbf{X}_i = \frac{\partial \mathbf{X}}{\partial x_i}, \qquad i = 1,2,3. \tag{2}$$

Then, we have the components of the metric tensor as in the following form

$$g_{ij} = \langle X_i, X_j \rangle, \; g = (g_{ij}), \; (g^{ij}) = (g_{ij})^{-1}, \quad i, j = 1, 2, 3 \tag{3}$$

The Christoffel symbols of the first kind is defined as in the following form

$$\Gamma_{ijl} \equiv [ij, l] = \frac{1}{2}\left(\frac{\partial g_{il}}{\partial x_j} + \frac{\partial g_{jl}}{\partial x_i} - \frac{\partial g_{ij}}{\partial x_l}\right). \tag{4}$$

The Christoffel symbols of the second kind can be written as in the following form

$$\Gamma_{ij}^{k} = g^{kl}\Gamma_{ijl} = \frac{1}{2} g^{kl}\left(\frac{\partial g_{il}}{\partial x_j} + \frac{\partial g_{jl}}{\partial x_i} - \frac{\partial g_{ij}}{\partial x_l}\right) \tag{5}$$

In Riemannian geometry, the scalar curvature (or the Ricci scalar) is the simplest curvature invariant of a Riemannian manifold. To each point on a Riemannian manifold, it assigns a single real number determined by the intrinsic geometry of the manifold near that point. Specifically, the scalar curvature represents the amount by which the volume of a geodesic ball in a curved Riemannian manifold deviates from that of the standard ball in Euclidean space. In two dimensions, the scalar curvature is twice the Gaussian curvature, and completely characterizes the curvature of a surface. In more than two dimensions, however, the curvature of Riemannian manifolds involves more than one functionally independent quantity [7, 9, 10]. Using (5), we have the Riemannian curvature tensor as the following

$$R_{ijk}^{l} = \Gamma_{ik,j}^{l} - \Gamma_{ij,k}^{l} + \sum_{s}(\Gamma_{js}^{l}\Gamma_{ik}^{s} - \Gamma_{ks}^{l}\Gamma_{ij}^{s}). \tag{6}$$

The Riemannian tensor can be written either using its mixed components

$$R_{ijk}^{l} = -R_{ikj}^{l}, \tag{7}$$

$$R_{ijk}^{l} + R_{jki}^{l} + R_{kij}^{l} = 0, \tag{8}$$

and are hold using its covariant components:

$$R_{iklm} = \sum_j g_{ij} R^j_{klm}, \tag{9}$$

which satisfy the following properties

$$\begin{aligned} R_{ijkl} &= -R_{jikl}, \\ R_{ijkl} &= -R_{jkli}, \\ R_{ijkl} &= R_{jikl} = R_{klij}. \end{aligned} \tag{10}$$

Applying the contraction, on the Riemannian tensor, we obtain the Ricci curvature R_{ij} in the form

$$R_{ij} = g^{km} R_{mijk} = R^k_{ijk}. \tag{11}$$

The scalar curvature is given by the formula:

$$R = \sum_{i,j} g^{ij} R_{ij}, \tag{12}$$

which is invariant. The components of the Riemannian tensor satisfy the following equality identically:

$$R^m_{ijk,l} + R^m_{ikl,j} + R^m_{ilj,k} = 0. \tag{13}$$

These relations are called the Bianchi identities.

2 Some geometric invariants on an 3- dimensional surfaces obtained by the equiform motion of a surface of revolution

A surface of revolution is a surface in Euclidean space created by rotating a curve (the profile) around a straight line in its plane (the axis). Examples of surfaces of revolution generated by a straight line are cylindrical and conical surfaces depending on whether or not the line is parallel to the axis. A circle that is rotated about any diameter generates a sphere of which it is then a great circle, and if the circle is rotated about an axis that does not intersect the center of a circle, then it generates a torus which does not intersect itself (a ring torus). We can express the surface of revolution $R(x_i, x_j)$ as the following formula [6, 11, 12]

$$R(x_i, x_j) = \{F(x_i)\cos(x_j), F(x_i)\sin(x_j), H(x_i)\} \tag{14}$$

Let $\Phi = \Phi(x_1)$ be an orthogonal smooth curve to each x_1-plane of the foliation and represented by its arc length x_1. We assume that the planes of the foliation are not parallel. Let **t**, **n** and **b** be unit tangent, normal and binormal vectors, respectively, to Φ. Then, Frenet equations of the curve Φ are:

$$\begin{pmatrix} \mathbf{t}' \\ \mathbf{n}' \\ \mathbf{b}' \end{pmatrix} = \begin{pmatrix} 0 & k & 0 \\ -k & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{pmatrix}, \qquad ' = \frac{d}{dx_1}, \tag{15}$$

where k and τ are the curvature and torsion of $\Phi(x_1)$, respectively. Observe that $k \neq 0$ because $\Phi(x_1)$ is not a straight-line. The three dimensional surfaces M generated by a surface of revolution (14) is represented by:

$$M : \mathbf{X}(x_1, x_2, x_3) = \mathbf{c}(x_1) + r(F(x_3)\cos(x_2)\mathbf{t}(x_1) + F(x_3)\sin(x_2)\mathbf{n}(x_1) + H(x_3)\mathbf{b}(x_1)), \tag{16}$$

where r denote the radius and $c(x_1)$ denote the centre of each x_1-surface of revolution of the foliation, $x_2, x_3 \in [0, 2\pi]$. Also, putting

$$\frac{d\,\mathbf{c}(x_1)}{dx_1} = \mathbf{c}' = \alpha\mathbf{t} + \beta\mathbf{n} + \gamma\mathbf{b}, \tag{17}$$

where α, β, γ are smooth functions in x_1 [13].

Remark 2.1 *The surface* $R(x_i, x_j)$ *surface of revolution but the surface* $\mathbf{X}(x_1, x_2, x_3)$ *not necessary to be surface of revolution.*

Using the equation (3), after some computation we have g_{ij} as the following values

$$\begin{aligned}
g_{11} &= (krF(x_3)\cos(x_2) + r\tau H(x_3) + \beta(x_1))^2 + (\alpha(x_1) - krF(x_3)\sin(x_2))^2 \\
&\quad + (r\tau F(x_3)\sin(x_2) + \gamma(x_1))^2, \\
g_{12} = g_{21} &= rF(x_3)(krF(x_3) + r\tau H(x_3)\cos(x_2) - \alpha(x_1)\sin(x_2) + \beta(x_1)\cos(x_2)), \\
g_{13} = g_{31} &= r(r\tau H(x_3)\sin(x_2)F'(x_3) + \alpha(x_1)\cos(x_2)F'(x_3) + \beta(x_1)\sin(x_2)F'(x_3) \\
&\quad + r\tau F(x_3)\sin(x_2)H'(x_3) + \gamma(x_1)H'(x_3)), \\
g_{22} &= r^2 F(x_3)^2 \\
g_{23} = g_{32} &= 0, \\
g_{33} &= r^2(F'(x_3)^2 + H'(x_3)^2). \qquad (18)
\end{aligned}$$

From (5) and the abve equations, one can get the Christoffel symbols as the following

$$\Gamma_{11}^{1} = \frac{1}{2\lambda^2}(2(F'(x_3)^2 + H'(x_3)^2)(\beta'(x_1)(krF(x_3)\cos(x_2) + r\tau H(x_3) + \beta(x_1)) + \alpha'(x_1)(\alpha(x_1)$$

$$-krF(x_3)\sin(x_2)) + \gamma'(x_1)(r\tau F(x_3)\sin(x_2) + \gamma(x_1))) - 2(r\tau H(x_3)\sin(x_2)F'(x_3) +$$

$$\alpha(x_1)\cos(x_2)F'(x_3) + \beta(x_1)\sin(x_2)F'(x_3) + r\tau F(x_3)\sin(x_2)H'(x_3) + \gamma(x_1)H'(x_3))$$

$$(-(k\cos(x_2)F'(x_3) + \tau H'(x_3))(krF(x_3)\cos(x_2) + r\tau H(x_3) + \beta(x_1)) + k\sin(x_2)F'(x_3)$$

$$(\alpha(x_1)-krF(x_3)\sin(x_2))-\tau\sin(x_2)F'(x_3)(r\tau F(x_3)\sin(x_2)+\gamma(x_1))+F'(x_3)(\cos(x_2)$$
$$\alpha'(x_1)+\sin(x_2)\beta'(x_1))+H'(x_3)\gamma'(x_1))-2(F'(x_3)^2+H'(x_3)^2)(krF(x_3)+r\tau H(x_3)$$
$$\cos(x_2)-\alpha(x_1)\sin(x_2)+\beta(x_1)\cos(x_2))(kr\tau H(x_3)\sin(x_2)+k\alpha(x_1)\cos(x_2)+k\beta(x_1)$$
$$\sin(x_2)-r\tau^2F(x_3)\sin(x_2)\cos(x_2)-\sin(x_2)\alpha'(x_1)+\cos(x_2)\beta'(x_1)-\tau\gamma(x_1)\cos(x_2))),$$

$$\Gamma^1_{12}=\Gamma^1_{21}=\frac{1}{\lambda}(rF(x_3)(r\tau F(x_3)\sin(x_2)(kF'(x_3)H'(x_3)+\tau\cos(x_2)F'(x_3)^2+\tau\cos(x_2)$$
$$H'(x_3)^2)+\gamma(x_1)(kF'(x_3)H'(x_3)+\tau\cos(x_2)F'(x_3)^2+\tau\cos(x_2)H'(x_3)^2)-kH'(x_3)^2$$
$$(r\tau H(x_3)\sin(x_2)+\alpha(x_1)\cos(x_2)+\beta(x_1)\sin(x_2)))),$$

$$\Gamma^1_{13}=\Gamma^1_{31}=\frac{1}{\lambda}(r\tau(F'(x_3)^2+H'(x_3)^2)(rF(x_3)(k\cos(x_2)H'(x_3)+\tau\sin^2(x_2)F'(x_3))+\gamma(x_1)$$
$$\sin(x_2)F'(x_3)+H'(x_3)(r\tau H(x_3)+\beta(x_1)))),$$

$$\Gamma^1_{22}=\frac{1}{\lambda}(rF(x_3)H'(x_3)),$$

$$\Gamma^1_{23}=\Gamma^1_{32}=\frac{1}{\lambda}(r^2\tau F(x_3)\cos(x_2)H'(x_3)(F'(x_3)^2+H'(x_3)^2)),$$

$$\Gamma^1_{33}=\frac{1}{\lambda}(r(F''(x_3)H'(x_3)^2(r\tau H(x_3)\sin(x_2)+\alpha(x_1)\cos(x_2)+\beta(x_1)\sin(x_2))+F'(x_3)^2H''(x_3)$$
$$(F(x_3)r\tau\sin(x_2)+\gamma(x_1))+2r\tau\sin(x_2)F'(x_3)^3H'(x_3)+F'(x_3)H'(x_3)(-r\tau F(x_3)$$
$$\sin(x_2)F''(x_3)-\gamma(x_1)F''(x_3)-r\tau H(x_3)\sin(x_2)H''(x_3)-\alpha(x_1)\cos(x_2)H''(x_3)-$$
$$\beta(x_1)\sin(x_2)H''(x_3)+2r\tau\sin(x_2)H'(x_3)^2))),$$

where

$$\lambda=((r\tau F(x_3)\sin(x_2)F'(x_3)+\gamma(x_1)F'(x_3)-H'(x_3)(r\tau H(x_3)\sin(x_2)+\alpha(x_1)\cos(x_2)$$
$$+\beta(x_1)\sin(x_2)))^2).$$

By a computations similar to the previous results, we can find the other components of Γ^k_{ij}.

Our method depends on equations reduces (12), to an expression that can be rewrite as a linear combination of the functions $\cos(ix_j)$, $\sin(ix_j)$ whose coefficients A_i, B_i are function of the x_1, x_3- variables. Therefore, they must be vanish in some x_1, x_3-interval. By using equations (15), (17), we can expressed (12) by trigonometric polynomial on $\cos(ix_2)$, $\sin(ix_2)$, $0\le i\le 6$, these coefficients A_i, B_i, are functions on the x_1, x_3-variables.

Therefore, these coefficients must vanish in some x_1, x_3-intervals. The work is then to compute explicitly these coefficients by successive manipulations. Using the symbolic program mathematica to check our work.

$$\sum_{i=0}^{6}(A_i\cos(ix_2)+B_i\sin(ix_2))=0 \tag{19}$$

Since this is an expression on the independent trigonometric terms $\cos nx_2$ and $\sin nx_2$, all coefficients A_i , B_i vanish identically.

After some computation, the values for A_i, B_i can be write in the form

$$\sum_{j=0}^{12}(A_{i,j}\cos(jx_3)+B_{i,j}\sin(jx_3))=0 \quad i=1,...,6$$

3 Three dimensional surfaces obtained by the equiform motion of a sphere

In this section we prove that the scalar curvature of surface M obtained by the equiform motion of sphere is vanished identically. Thus, the the equation (14) written as

$$R(x_i,x_j)=\{\sin(x_i)\cos(x_j),\sin(x_i)\sin(x_j),\cos(x_i)\}. \tag{21}$$

Thus, we can rewrite the surface M as the following representation

$$M:\mathbf{X}(x_1,x_2,x_3)=\mathbf{c}(x_1)+r(\sin(x_3)\cos(x_2)\mathbf{t}(x_1)+\sin(x_3)\sin(x_2)\mathbf{n}(x_1)+\cos(x_3)\mathbf{b}(x_1)), \tag{22}$$

Using equations (22), (18), and after some computation we have A_6 of equation (19) as in the following form

$$\begin{aligned} A_6 \;=\; & \frac{1}{4}r^2\tau^2\sin^8(x_3)(-6\alpha(x_1)^2(8r^2\tau^2\cos^2(x_3)+10r\tau\beta(x_1)\cos(x_3)+3\beta(x_1)^2) \\ & +(2r\tau\cos(x_3)+\beta(x_1))^3(2r\tau\cos(x_3)+3\beta(x_1))+3\alpha(x_1)^4), \end{aligned} \tag{23}$$

we can expressed (23) by trigonometric polynomial on $\cos(jx_3)$, $\sin(jx_3)$, $0\le j\le 12$, these coefficients $A_{i,j}$, $B_{i,j}$, are functions on the x_1-variable. Therefore, these coefficients must vanish in some x_1-interval.

Since this is an expression on the independent trigonometric terms $\cos(jx_3)$ and $\sin(jx_3)$, all coefficients $A_{6,j}$, $B_{6,j}$ vanish identically.

After some computation, the coefficients for $A_{6,12}, B_{6,12}, A_{6,11}, B_{6,11}$ take the form

$$A_{6,12}=\frac{r^6\tau^6}{512},\; B_{6,12}=0,\; A_{6,11}=\frac{3}{256}r^5\tau^5\beta(x_1),\; B_{6,11}=0.$$

From $A_{6,12}$ we have one possibility, $\tau=0$, then, $A_6=0$.

Using the equation (12), it is easy to see that the scalar curvature equal to zero. Thus, we have the proof of the following theorem:

Theorem 3.1 *The three dimensional surface generated by equiform motion of a sphere has zero scalar curvature if the motions of the spheres in parallel planes.*

4 Three dimensional surfaces obtained by the equiform motion of a torus

In this section we try to prove that the scalar curvature of surface M obtained by the equiform motion of torus is vanished identically for which the torsion is zero. Thus, the the equation (14) written as

$$R(x_i, x_j) = \{(b + a\sin(x_i))\cos(x_j), (b + a\sin(x_i))\sin(x_i)\sin(x_j), a\cos(x_i)\}$$

Thus, we can rewrite the surface M as the following representation

$$M : \mathbf{X}(x_1, x_2, x_3) = \mathbf{c}(x_1) + ((b + a\sin(x_3))\cos(x_2)\mathbf{t}(x_1) + r(b + a\sin(x_3))\sin(x_2)\mathbf{n}(x_1) + a\cos(x_3)\mathbf{b}(x_1)), \quad (25)$$

Using equations (25), (18), and after some computation we have A_6 of equation (19) as the following

$$A_6 = \frac{1}{16}a^5r^2\tau^2(a\sin(x_3)+b)^2(-3\alpha(x_1)^2\sin(x_3)(r\tau\beta(x_1)\sin(2x_3)(-20a^2\cos(2x_3)+5a^2\cos(4x_3)+15a^2+40ab\sin(x_3)-4ab\sin(3x_3)+12b^2)+2r^2\tau^2\cos^2(x_3)(20a^3\sin(x_3)-10a^3\sin(3x_3)+2a^3\sin(5x_3)-24a^2b\cos(2x_3)+3a^2b\cos(4x_3)+21a^2b+23ab^2\sin(x_3)-ab^2\sin(3x_3)+4b^3)+2\beta(x_1)^2\sin^2(x_3)(9a\sin(x_3)-3a\sin(3x_3)+8b))+(r\tau\cos(x_3)(2a\sin(x_3)+b)+\beta(x_1)\sin(x_3))^3(ar\tau(\cos(x_3)(8a\sin^3(x_3)+7b)+\cos(3x_3)b)+\beta(x_1)(9a\sin(x_3)-3a\sin(3x_3)+8b))+\sin^3(x_3)\alpha(x_1)^4(9a\sin(x_3)-3a\sin(3x_3)+8b)). \quad (26)$$

We can expressed (26) by trigonometric polynomial on $\cos(jx_3)$, $\sin(jx_3)$, $0 \le j \le 12$, these coefficients $A_{i,j}$, $B_{i,j}$, are functions on the x_1-variable. Therefore, these coefficients must vanish in some x_1-interval.

Since this is an expression on the independent trigonometric terms $\cos(jx_3)$ and $\sin(jx_3)$, all coefficients $A_{6,j}$, $B_{6,j}$ vanished identically.

After some computation, the coefficients $A_{6,12}, B_{6,12}, A_{6,11}, B_{6,11}$ are give as

$$A_{6,12} = \frac{a^1 2r^6\tau^6}{512},\ B_{6,12} = 0,\ A_{6,11} = \frac{3}{256}a^1 1r^5\tau^5\beta(x_1),\ B_{6,11} = -\frac{3}{256}a^1 1br^6\tau^6.$$

From $A_{6,12}$ we have one possibility, $\tau = 0$, then, $A_6 = 0$.
Using the equation (12), one can see the scalar curvature is equal to zero. Thus, we have the proof of the following theorem:

Theorem 4.1 *The three Dimensional Surface generated by the equiform motion of a torus has zero scalar curvature if the motions of the toruses in parallel planes.*

5 Example

Example 5.1 *(For three dimensional surfaces with zero scalar curvature), consider a plane curve (unit circle) given by*

$$\boldsymbol{\Psi}(x_1) = \{\cos(kx_1), \sin(kx_1), 0\}. \tag{27}$$

where k is curvature of curve Ψ. And we consider a centers curve given by

$$\mathbf{c}(x_1) = \{\cos(x_1)t + \sin(x_1)n + x_1 b\}. \tag{28}$$

Using (15), and after some computations, we have

$$\mathbf{c}(x_1) = \{-\sin((1+k)x_1), \cos((1+k)x_1), x_1\}. \tag{29}$$

Therefore, the representation of the surface which represented by equation (16) is

$$\begin{aligned}\mathrm{M}:\mathbf{X}(x_1,x_2,x_3) \quad = \quad & \{-rF(x_3)\sin(kx_1+x_2) - \sin((k+1)x_1), rF(x_3)\cos(kx_1 + \\ & x_2) + \cos((k+1)x_1), rH(x_3) + x_1\}.\end{aligned}$$

Case 1:
We consider the sphere as a surface of revolution. Thus, we take

$$F(x_3) = a\sin(kx_3), \quad H(x_3) = a\cos(kx_3),$$

where a is radius of sphere. Therefore, the representation of the surface which represented by equation (22) is

$$\begin{aligned}\mathrm{M}:\mathbf{X}(x_1,x_2,x_3) \quad = \quad & \{-a\sin(x_3)\cos(x_2)\sin(kx_1) - a\sin(x_3)\sin(x_2)\cos(kx_1) - \sin((k+1)x_1), \\ & a\cos(kx_1+x_2)\sin(x_3) + \cos((k+1)x_1), a\cos(x_3) + x_1\}.\end{aligned} \tag{31}$$

According to theorem (3.1) this is three dimensional surface satisfying the condition of zero scalar surface. This surface plotted as in Figs. 1.

Case 2:
We consider the torus surface of revolution, Thus, we take

$$F(x_3) = B + a\sin(kx_3), \quad H(x_3) = a\cos(kx_3),$$

where a a and b are constants. Therefore, the representation of the surface which represented by equation (25) is

$$\begin{aligned}\mathrm{M}: \mathbf{X}(x_1, x_2, x_3) \;=\; \{&-\cos(x_2)\sin(kx_1)(a\sin(x_3)+B)-\sin(x_2)\cos(kx_1)(a\sin(x_3)+B)-\\ &\sin((k+1)x_1),(a\sin(x_3)+B)\cos(kx_1+x_2)+\cos((k+1)x_1)\\ &,a\cos(x_3)+x_1\}.\end{aligned} \tag{32}$$

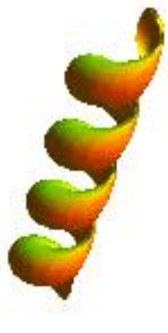
According to theorem (4.1) this is three dimensional surface satisfying the condition of zero scalar surface. This surface plotted as in Figs. 2.

$x_3 = 1,\ a = 1,\ k = 1$

$x_3 = 20,\ a = 2,\ k = 5$

$x_3 = 0.5,\ a = 5,\ k = 2$

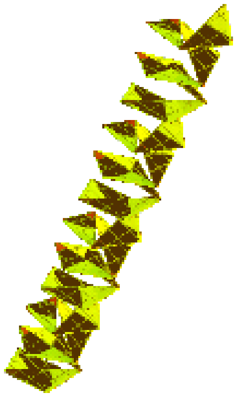
$x_2 = 10,\ a = 3,\ k = 0.25$



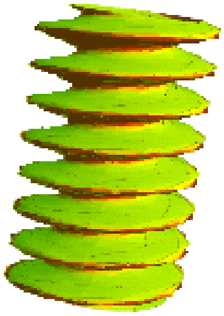
$x_2 = 1,\ a = 1,\ k = 5$

$x_2 = 20,\ a = 2,\ k = 1$

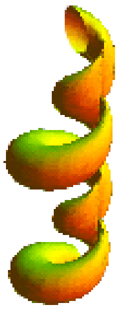
$x_3 = 0.1,\ a = 1,\ k = 2,\ B = 1$



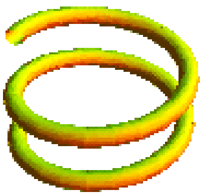
$x_3 = 20,\ a = 2,\ k = 5,\ B = 1$



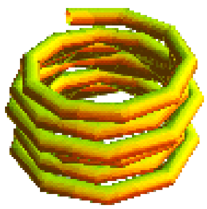
$x_3 = 0.5,\ \ a = 1.5,\ k = 3,\ B = 0.5$



$x_2 = 1,\ a = 1,\ k = 1,\ B = 1$



$x_2 = 2,\ a = 1,\ B = 10,\ k = 1$



$x_2 = 1,\ a = 1,\ B = 10,\ k = 3$

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السطوع ثلاثية البعد المولدة بالحركة شبه المتماثلة لسطوع دورانى

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فى هذا البحث تم توليد سطح ثلاثى البعد باستخدام الحركة شبة المتماثلة لسطح دورانى فى الفراغ الاقليدى. وتم استنتاج الشروط على الحركة التى تجعل السطح الناتج له انحناء قياسى منعدم وذلك فى حالة كون السطح الدورانى هو كرة sphere أو سطح قارب نجاة torus. وفى النهاية تم اختبار بعض الحالات الخاصة ورسمها باستخدام برنامج الماثيماتيكا.