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ON SOFT gαb-CONTINUOUS FUNCTIONS IN SOFT TOPOLOGICAL SPACES

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The intention of this paper is to describe and study the concepts of soft gαb-continuous functions, soft gαb-irresolute functions and soft gαb-(open)closed functions on soft topological spaces. Also we have introduced the concept of soft gαb-closure and soft gαb-interior. Further relationship inter alia soft gαb-continuous functions with other soft continuous functions are established.

***Keywords and phrases*:** Soft gαb -Continuous function, Soft gαb -Closed function, Soft gαb -Open function, Soft gαb -Irresolute, soft gαb-closed, soft gα -interior, soft gαb-closure.

1. INTRODUCTION

In 1999, Molodtsov [11] presented the soft set theory as a new mathematical tool to deal with ambiguities that the known mathematical tools cannot hold. Also he has indicated a few applications in soft set theory for finding solutions to many practical problem such as economics, engineering, medical science, social science, etc. Shabir and Naz [14] introduced a concept of soft topological spaces over an initial universe with a fixed set of parameters. They also defined some concepts of soft sets on soft topological spaces such as soft interior, soft closure, soft spaces and soft separation axioms. Kharal et al. [8] introduced soft functions over classes of soft sets. Janaki and Jeyanthi [5] studied and discussed the properties of soft πgr-continuous functions in soft topological spaces. Kandil et al. [6] investegated γ-operation and decompositions of some forms of soft continuity in soft topological spaces. The purpose of this paper is to introduce and study the concepts of soft gαb-continuity, soft gαb-open functions, soft gαb-closed functions and soft gαb-irresolute functions. Also we obtain some characterization of these functions.

2. PRELIMINARIES

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1.[11]. Let X be an initial universe and E be a set of all possible parameters under consideration with respect to X, P(X) denotes the power set of X and A be a non-empty subset of E. A pair (F,A) denoted by F_A is called a soft set over X, where F is a function given by F: A $\to$P(X). In other words, a soft set over X is parameterized family of subsets of the universe X. For a particular e $\in$ A, F(e) may be considered the set of e-approximate elements of the soft set (F,A) and if e $\notin$ A, then F(e) = Φ, i.e F_A =(F,A)= {F(e) : e $\in$ A $\subseteq$ E, F:A $\to$P(X)}. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2. [9]. Let (F,A), (G,B)$\in SS(X)_A$. Then (F,A) is a soft subset of (G,B), denoted by (F,A)$\subseteq$(G,B), if A $\subseteq$ B, and F(e) $\subseteq$G(e), $\forall e \in$A. In this case, (F,A) is said to be a soft subset of (G,B) and (G,B) is said to be a soft superset of (F,A).

Definition 2.3. [9]. Two soft subsets (F,A) and (G,B) over a common universe set X are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition 2.4. [1]. The complement of a soft set (F,A), denoted by $(F,A)^c$, is defined by $(F,A)^c = (F^c,A)$, F^c: A $\to$P(X) is a function given by $F^c(e)$ = X - F(e), $\forall$ e $\in$A. Clearly $((F,A)^c)^c$ is the same as (F,A).

Definition 2.5. [14]. Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(i) X,$\Phi\in\tau$, where $\Phi(e)=\Phi$ and X(e)=X, $\forall e\in$E,
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. The members of τ are called soft open sets and the family of all soft open sets is denoted by O^s(X), the complement of the soft open sets are called soft closed sets and the family of all soft closed sets is denoted by C^s(X). The soft interior of (A,E) is the soft set int^s(A,E)=$\cup${(O,E) : (O,E) is soft open and (O,E) $\subseteq$ (F,E)}. The soft closure of (A,E) is the soft set cl^s (A,E) = $\cap${(F,E) : (F,E) is soft closed and (F,E) $\subseteq$ (A,E)}.

Definition 2.6.[13]. Let (X,τ,E) be a soft topological space and (F,E)$\in SS(X)_E$. A soft topology $\tau_{(F,E)}$={(G,E)$\cap$(F,E) : (G,E)$\in\tau$} is called a soft relative topology of τ on (F,E) and ((F,E), $\tau_{(F,E)}$) is called a soft subspace of (X,τ,E).

Definition 2.7.[15]. A soft set (F,E)∈SS(X)$_E$ is called a soft point in (X_E) (denoted by x_e) if there exist x∈X and e∈E, F(e)={x} and F(e^c)=Φ for each e^c∈E-{e}, and x_e∈(G,A) if for the element e∈A, F(e)⊆G(e).

Definition 2.8.[13]. A soft set (G,E) in a soft topological space (X,τ,E) is called

(i) a soft neighbourhood of a soft point F(e)∈ (X,τ,E) if there exists a soft open set (H,E) such that F(e) ∈(H,E) ⊆ (G,E).
(ii) a soft neighbourhood of a soft set (F,E) if there exists a soft open set (H,E) such that(F,E) ⊆ (H,E) ⊆ (G,E). The neighbourhood system of a soft point F(e) denoted by N_τ(F(e)), is the family of all its neighbourhood.

Definition 2.9.[3]. Let (X,τ,E) be a soft topological space and (F,E)∈SS(X)$_E$. Then (F,E) is said to be soft b-open set (denoted by $b^s - open$) if (F,E)⊆int^s(cl^s(F,E))∪ cl^s(int^s (F,E)) and its complement is said to be soft b-closed set (denoted by $b^s - closed$). The set of all soft b-open sets is denoted by BOS(X) and the set of all soft b-closed sets is denoted by BCS(X).

Theorem 2.10.[6]. Let (X,τ,E) be a soft topological space and (F,E)∈SS(X)$_E$. Then (F,E) is said to be,

(i) a soft preopen set(denoted by $p^s - open$)if (F,E)⊆int^s(cl^s(F,E)).
(ii) a soft semi open set (denoted by $s^s - open$)if (F,E)⊆ cl^s(int^s(F,E)).
(iii) a soft α-open set(denoted by $\alpha^s - open$) if (F,E)⊆int^s(cl^s(int^s (F,E))).
(iv) a soft regular open set(denoted by $R^s - open$) if (F,E)=int^s(cl^s(F,E))).

The collection of all soft pre open (resp. semi open, α-open, regular open,pre closed, semi closed, α-closed and regular closed) in (X,τ,E) are denoted byp^s-O(X)(resp. S^s-O(X), α^s-O(X), R^s-O(X), p^s-C(X), S^s-C(X),α^s-C(X) and R^s-C(X)

Theorem 2.11.[3]. Let (X,τ,E) be a soft topological space.Then the following properties are satisfied for the soft b-interior operators, soft b-closure operators denoted respectively by soft bint, soft bcl (in short bint^s and bcl^s)

(i) bint^s(F,E) ∪ bint^s(G,E)⊆bint^s[(F,E) ∪ (G,E)].
(ii) bint^s[(F,E) ∩ (G,E)]⊆bint^s(F,E) ∩ bint^s(G,E).
(iii) bcl^s(F,E) ∪ bcl^s(G,E)⊆bcl^s[(F,E) ∪ (G,E)].
(iv) bcl^s[(F,E) ∩ (G,E)]⊆bcl^s(F,E) ∩ bcl^s(G,E).
(v) bcl^s(F^c,E)=X- bint^s(F,E) and bint^s(F^c,E)=X- bcl^s(F,E).

Theorem 2.12.[3]. Let (X,τ,E) be a soft topological space and (F,E)∈SS(X)$_E$ Then the following properties are satisfied.

(i) bcl^s(F,E)=scl^s(F,E) ∩ pcl^s(F,E).
(ii) bint^s(F,E)=sint^s(F,E) ∪ pint^s(F,E).

Definition 2.13.[15]. Let (X,τ,E) and (Y,ρ,H) be soft topological spaces. Let μ:X→Y and p:E→H be functions. Then the function $f_{p\mu}: SS(X)_E \longrightarrow SS(Y)_H$is defined by:

(i) Let (F,E) ∈SS(X)$_E$. The image of (F,E) under $f_{p\mu}$, written $f_{p\mu}$(F,E)=

(($f_{p\mu}$F),p(E)) is a soft set in SS(Y)$_H$such that

$$f_{p\mu}(F) = \begin{cases} \cup_{x\in p^{-1}(y)\cap A}\ \mu\big(F(x)\big), p^{-1}(y) \cap A \neq \phi \\ \phi, othereise \end{cases} \text{ for all } y\in H.$$

(ii) Let (G,H) ∈SS(Y)$_H$. The inverse image of (G,H) under $f_{p\mu}$, written as $f_{p\mu}^{-1}$(G,H)=($f_{p\mu}^{-1}$ (G),p(H))is soft set in SS(X)$_E$ such that

$$f_{p\mu}^{-1}(G) = \begin{cases} \mu^{-1}\Big(G\big(p(x)\big)\Big), & p(x) \in H \\ \phi, & othereise \end{cases} \text{ for all } x \in E.$$

Definition 2.14.[6],[15]. Let (X,τ,E) and (Y,ρ,H) be soft topological spaces, μ:X→Y , p:E→H and$f_{p\mu}$: SS(X)$_E$→ SS(Y)$_H$ be functions.Then

(i) $f_{p\mu}$is a soft continuous function if $f_{p\mu}^{-1}$(G,H)∈O^s(X) ∀(G,H)∈O^s(Y).
(ii) $f_{p\mu}$ is a soft open function if $f_{p\mu}$(G,H)∈O^s(Y) ∀(G,H)∈O^s(X).
(iii) $f_{p\mu}$ is a soft closed function if $f_{p\mu}$(G,H)∈C^s(Y) ∀(G,H)∈C^s(X).
(iv) $f_{p\mu}$ is a soft pre-continuous function if $f_{p\mu}^{-1}$ (G,H)∈P^sO(X) ∀(G,H)∈O^s(Y).
(v) $f_{p\mu}$is a soft α-continuous function if $f_{p\mu}^{-1}$(G,H)∈α^sO(X) ∀(G,H)∈O^s(Y).
(vi) $f_{p\mu}$is a soft semi-continuous function if $f_{p\mu}^{-1}$(G,H)∈S^sO(X) ∀(G,H)∈O^s(Y).

Definition 2.15. A soft set (F,E) in a soft topological space (X,τ,E) is said to be:

(i) soft generalized pre closed set (in short gp^s-closed) sets [2] if cl^s(F,E) ⊆ (G,E) whenever (F,E) ⊆ (G,E) and (G,E) is soft preopen set in X.
(ii) soft generalized α-closed set (in short gα^s-closed) sets [12]if cl^s(F,E) ⊆ (G,E) whenever (F,E) ⊆ (G,E) and (G,E) is soft α-open set.
(iii) soft s*g-closed set (in short s*g^s-closed) sets [7] if cl^s(F,E) ⊆ (G,E) whenever (F,E) ⊆ (G,E) and (G,E) is soft semi-open set.

(iv) soft sg closed set (in short sg^s -closed) [4] if scl^s(A,E) $\subseteq$ (U,E) whenever (A,E) $\subseteq$ (U,E) and (U,E) is soft semiopen.
(v) soft generalized αb-closed (in short gαb^s-closed) sets [10] if bcl^s(F,E)$\subseteq$(G,E) whenever (F,E)$\subseteq$(G,E) and (G,E) is soft α-open set.
(vi) soft generalized b-closed (in short gb^s-closed) sets [10] if bcl^s(F,E)$\subseteq$(G,E) whenever (F,E)$\subseteq$(G,E) and (G,E) is soft open set.

The complement of each gp^s-closed(resp. gα^s-closed, s*g^s-closed,sg^s-closed, gαb^s-closed and gb^s-closed) sets is called gp^s-open (resp. gα^s-open, s*g^s-open, sg^s-open, gαb^s-open and gb^s-open) sets. The collection of all the gp^s-closed(resp. gα^s-closed, s*g^s-closed, sg^s-closed, gαb^s-closed, gb^s-closed and gαb^s-open) sets in (X,τ,E) are denoted by gp^s-C(X) (resp.gα^s-C(X), s*g^s-C(X), sg^s-C(X), gαb^s-C(X), gb^s-C(X) and gαb^s-O(X).

Lemma 2.16.[10]. In a soft topological space we have the following:
(i) Every soft regular open set is gαb^s-closed.
(ii) Every soft regular closed set is gαb^s-closed.
(iii) Every soft semi-closed set is gαb^s-closed.
(iv) Every soft pre-closed set is gαb^s-closed.
(v) Every soft α-closed set is gαb^s-closed.

Theorem 2.17.[10]. In a soft topological space we have the following:
(i) Every gαb^s-closed set is gb^s-closed.
(ii) Every gp^s-closed set is gαb^s-closed.
(iii) Every gα^s-closed set is gαb^s-closed.
(iv) Every s*g^s-closed set is gαb^s-closed.
(v) Every soft sg^s-closed set is gαb^s-closed.
(vi) Every soft SW^s-closed set is gαb^s-closed.

Theorem 2.18. In a soft topological space we have the following:
(i) Every gp^s-closed set is gα^s-closed.
(ii) Every s*g^s-closed set is sg^s-closed.
(iii) Every s*g^s-closed set is gα^s-closed.

Proof: Immediate.

3. Soft gαb-Continuous Functions in Soft Topological Space

The present section gives the definition of soft gαb-continuous functions and investigates some of its properties.

Definition 3.1. Let (F,E) be a soft set over X, the intersection of all soft gαb-closed sets over X containing (F,E) is called soft gαb-closure of (F,E) (denoted by gαb^s-cl(F,E)) and the union of all soft gαb-open sets over X contained in (F,E) is called soft gαb-interior of (F,E) (denoted by gαb^s-int(F,E)).

Theorem 3.2. Let (X,τ,E) be a soft topological space and (F,E) $\in$SS(X)$_E$. Then the following hold:

(i) gαb^s-cl(Φ) =Φ.
(ii) gαb^s-int(Φ) =Φ.
(iii) gαb^s-int(Φ)=Φ.
(iv) gαb^s-cl(F,E) is soft gαb-closed in X.
(v) gαb^s-int(F,E) is soft gαb-open in X.
(vi) gαb^s-cl(gαb^s-cl(F,E))= gαb^s-cl(F,E).
(vii) gαb^s-int(gαb^s-int(F,E))= gαb^s-int(F,E).

Proof: Immediate.

Theorem 3.3. Let (X,τ,E) be a soft topological space and (F,E),(B,E) $\in$SS(X)$_E$. Then the following hold:

(i) gαb^s-cl[(F,E) ∩ (B,E)]$\subseteq$ gαb^s-cl(F,E) ∩ gαb^s-cl(B,E).
(ii) gαb^s-cl(F,E) ∪ gαb^s-cl(B,E)$\subseteq$ gαb^s-cl[(F,E)∪(B,E)].
(iii) gαb^s-int(F,E) ∪ gαb^s- int(B,E)$\subseteq$ gαb^s- int [(F,E)∪(B,E)].
(iv) gαb^s-int[(F,E) ∩ (B,E)]$\subseteq$ gαb^s- int(F,E) ∩ gαb^s- int(B,E).

Proof: Immediate.

Theorem 3.4. Let (X,τ,E) be a soft topological space and (F,E) $\in$SS(X)$_E$. Then the following hold:

(i) gαb^s-int(F,E)^c = X- gαb^s-cl(F,E).
(ii) gαb^s-cl(F,E)^c = X- gαb^s-int(F,E).

Proof:

(i)X-gαb^s-cl(F,E)=(∩{(G,E):(F,E)$\subseteq$(G,E),(G,E)$\in$gαb^s-C(X)})^c=∪{ (G,E)^c: (G,E)^c$\subseteq$(F,E)^c, (G,E)^c$\in$gαb^s-O(X)}= gαb^s-int(F,E)^c.

(ii) X-gαb^s-int(F,E)=(∪{(G,E):(G,E)$\subseteq$(F,E),(G,E)$\in$gαb^s-O(X)})^c=∩{(G,E)^c:(F,E)^c$\subseteq$(G,E)^c, (G,E)^c$\in$gαb^s-C(X)}=gαb^s-cl(F,E)^c.

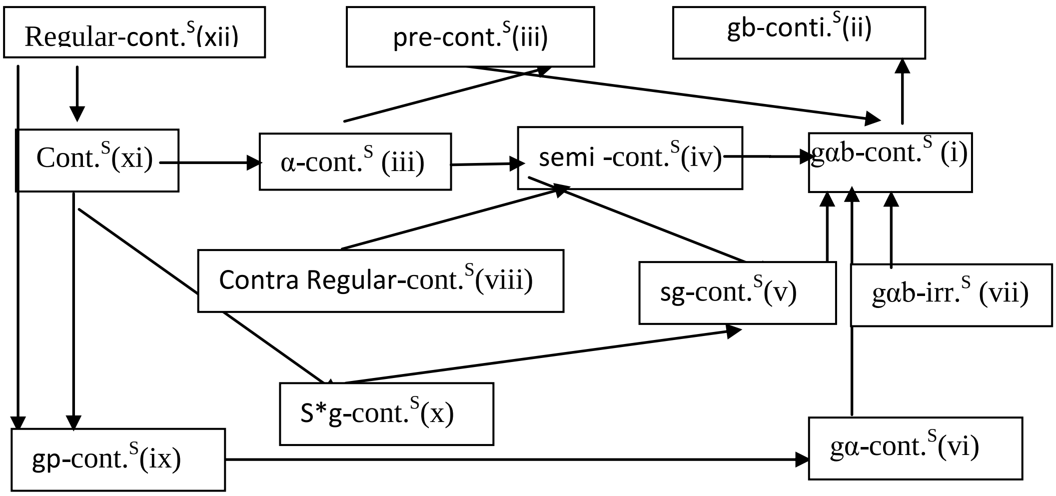
The following definitions give a modification of soft generalizedcontinuous functions.

Definition 3.5. Let (X,τ,E) and (Y,ρ,H) be soft topological spaces and letμ:X→Y, p:E→H be functions. A function $f_{p\mu}$ SS(X)$_E$→ SS(Y)$_H$ is said to be

(i) soft Regular-continuous if $f_{p\mu}^{-1}$(G,H)$\in R^s$C(X) ∀(G,H)$\in C^s$(Y).
(ii) soft Contra Regular-continuous if $f_{p\mu}^{-1}$(G,H) $\in R^s$O(X) ∀(G,H)$\in C^s$(Y).
(iii) soft gb-continuous if $f_{p\mu}^{-1}$ (G,H)$\in$gb^s-C(X) ∀(G,H)$\in C^s$(Y).
(iv) soft gαb-continuous if $f_{p\mu}^{-1}$ (G,H)$\in$gαb^s-C(X) ∀(G,H)$\in C^s$(Y).

(v) soft gp-continuous if $f_{p\mu}^{-1}$ (G,H)∈gp^s-C(X) ∀(G,H)∈C^s(Y).
(vi) soft gα-continuous if $f_{p\mu}^{-1}$ (G,H)∈gα^s-C(X) ∀(G,H)∈C^s(Y).
(vii) soft s*g-continuous if $f_{p\mu}^{-1}$ (G,H)∈s*g^s-C(X) ∀(G,H)∈C^s(Y).
(viii) soft sg-continuous if $f_{p\mu}^{-1}$ (G,H)∈sg^s-C(X) ∀(G,H)∈C^s(Y).
(ix) soft gαb-irresolute if $f_{p\mu}^{-1}$ (G,H)∈gαb^s-C(X) ∀(G,H)∈gαb^s-C(X) .

Diagram of functions



Where soft continuous (resp.soft pre-continuous, soft α-continuous, soft semi-continuous, soft Regular-continuous, soft Contra Regular-continuous,soft gb-continuous,soft gαb-continuous, soft gp-continuous, soft gα-continuous, soft s*g-continuous, soft sg-continuous, soft gαb-irresolute)functions are denoted by cont.^{S} (resp. pre-cont.^{S}, α-cont.^{S}, semi-cont.^{S}, Regular-cont.^{S}, Contra Regular-cont.^{S},gb-cont.^{S}, gαb-cont.^{S}, gp-cont.^{S}, gα-cont.^{S}, s*g-cont.^{S}, sg-cont.^{S}, gαb-irr).

Theorem 3.6. In the above diagram the following statements hold:

Proof:

(i)→(ii). Let $f_{p\mu}$: SS(X)$_E$→ SS(Y)$_H$ be a soft gαb-continuous function and (G,H) be a soft closed set over Y, then $f_{p\mu}^{-1}$ (G,H) is soft gαb-closed set over X. But every soft gαb-closed set is soft gb-closed set, $f_{p\mu}^{-1}$ (G,H) is soft gb-closed set over X.

Hence $f_{p\mu}$: SS(X)$_E$→ SS(Y)$_H$ is soft gb-continuous function.

The converse of (i)→(ii)is not be true as shown by the following example.

Example 3.7. Let X={x_1,x_2,x_3}, Y={y_1,y_2,y_3}and E=H={e_1,e_2}. Then τ={Φ,X,(F_1,E)} and ρ={Φ,Y,(G_1,E)} are soft topological spaces over X and Y respectively where (F_1,E) and (G_1,E) are soft open sets over X and Y respectively, defined by (F_1,E) ={(e_1,{x_1,x_2}),(e_2,{x_1,x_2})}, (G_1,E) ={(e_1, { y_2}),(e_2, Φ)}. Then the soft closed sets over Y are Φ,Y,$(G_1,E)^c$ and $(G_1,E)^c$ ={(e_1, { y_1,y_3}),(e_2, { y_1,y_2,y_3})}, now define the function $f_{p\mu}$: $SS(X)_E\rightarrow$ $SS(Y)_E$ by f(x_1)={y_3}, f(x_2)={y_1} and f(x_3)={y_2} then $f_{p\mu}$is a soft gb-continuous function but not soft gαb-continuous function, since $f_{p\mu}^{-1}(G_1,E)^c$= $f_{p\mu}^{-1}$({(e_1, { y_1,y_3}),(e_2, { y_1,y_2,y_3})})={(e_1, { x_1,x_2}),(e_2, {x_1,x_2,x_3})}, is a soft gb-closed set but not soft gαb-closed set over X.

(iii)→(i). The proof is directly from definition and Theorem (5:2)[6].

The converse of (iii)→(i) is not be true as shown by the following example.

Example 3.8. Let X={x_1,x_2,x_3}, Y={y_1,y_2,y_3}and E=H={e_1,e_2}. Then τ={Φ,X,(F_1,E), (F_2,E) , (F_3,E)}is a soft topology over X, where (F_1,E), (F_2,E) and (F_3,E) are soft sets over X, ρ={Φ,Y,(G_1,E)} is a soft topology over Y, where (G_1,E)is a soft set over Y, defined as follows:

(F_1,E)={(e_1,{x_1}),(e_2,{x_2})},(F_2,E)={(e_1,{x_2}),(e_2,{x_1})},(F_3,E)={(e_1,{x_1,x_2}),(e_2,{x_1,x_2})}, (G_1,E) = {(e_1,{y_2}),(e_2, {y_1})} and the soft closed sets over Y are Φ,Y,$(G_1,E)^c$, where $(G_1,E)^c$ ={(e_1,{y_1,y_3}),(e_2, {y_2,y_3})}. If we define the function $f_{p\mu}$: $SS(X)_E\rightarrow$ $SS(Y)_E$ by f(x_1)={y_2} and f(x_2)={y_1}, then $f_{p\mu}$ is a soft gαb-continuous function but not soft pre-continuous since $f_{p\mu}^{-1}(G_1,H)^c$={(e_1,{x_2}),(e_2,{x_1})} and $f_{p\mu}^{-1}(G_1,H)^c$

is soft gαb-closed sets but not soft pre-closed sets over X.

(iv)→(i). The proof is directly from definition and Theorem (5:2)[6].

The converse of (iv)→(i) is not be true as shown by the following example.

Example 3.9. Let X={x_1,x_2}, Y={y_1,y_2}and E=H={e_1,e_2}, τ={Φ,X,(F_1,E), (F_2,E), (F_3,E)} and ρ={Φ,Y,(G_1,E)} are soft topological spaces over X and Y respectively where (F_1,E), (F_2,E), (F_3,E), (G_1,E) are soft sets over X and Y respectively, defined as follows:(F_1,E) ={(e_1,{x_1,x_2}),(e_2,{x_1})},(F_2,E) ={(e_1,{x_2}),(e_2,{x_1,x_2})},(F_3,E) ={(e_1,{x_2}),(e_2,{x_1})} and (G_1,E) ={(e_1,Φ),(e_2, {y_1,y_2})}. Then the soft closed sets over Y are Φ,Y,$(G_1,E)^c$ where $(G_1,E)^c$ ={(e_1, {y_1,y_2}),(e_2, Φ)}. If we define the function $f_{p\mu}$: $SS(X)_E\rightarrow$ $SS(Y)_E$ by f(x_2)={y_1} and f(x_1)={y_2}, then $f_{p\mu}$is a soft gαb-continuous function but not soft semi-continuous, since $f_{p\mu}^{-1}(G_1,H)^c$={(e_1, { x_1,x_2}),(e_2, Φ)}, $f_{p\mu}^{-1}(G_1,H)^c$ ={(e_1, { x_1,x_2}),(e_2, Φ)} is a soft gαb-closed set but not soft semi-closed over X.

(v)→(i). Let $f_{p\mu}$: SS(X)$_E$→ SS(Y)$_H$ be a soft sg-continuous function and (G,H) be a soft closed over Y, then $f_{p\mu}^{-1}$(G,H) is soft sg-closed over X. But every soft sg-closed set is soft gαb-closed set, $f_{p\mu}^{-1}$(G,H) is soft gαb-closed set over X . Hence $f_{p\mu}$: SS(X)$_E$→ SS(Y)$_H$ is soft gαb-continuous function.

The converse of (v)→(i) is not be true as shown by the following example.

Example 3.10. Let X={x_1,x_2,x_3}, Y={y_1,y_2,y_3}and E=H={e_1,e_2}. Then τ={Φ,X,(F_1,E)} and ρ={Φ,Y,(G_1,H)} are soft topology over X and Y respectively where (F_1,E), (G_1,H) are soft sets over X and Y respectively, defined as follows:

(F_1,E) ={(e_1,{x_1,x_2}),(e_2,{x_1,x_2})},(G_1,H) ={(e_1, {y_2,y_3}),(e_2, {y_1,y_3})}. Then the set of all soft closed sets over Y are Φ,Y,$(G_1,H)^c$ where$(G_1,H)^c$ ={(e_1, {y_1}),(e_2, {y_2})}. If we define the function $f_{p\mu}$: SS(X)$_E$→ SS(Y)$_E$ by f(x_1)={y_2}, f(x_2)={y_1} and f(x_3)={y_3} then $f_{p\mu}$is a soft gαb-continuous function but not soft continuous function , since $f_{p\mu}^{-1}(G_1,H)^c$= $f_{p\mu}^{-1}${(e_1, {y_1}),(e_2, {y_2})}={(e_1, {x_2}),(e_2, {x_1})}, is a soft gαb-closed set but not soft sg-closed over X.

(vi)→(i). Let $f_{p\mu}$: SS(X)$_E$→ SS(Y)$_H$ be a soft gα-continuous function and (G,H) be soft closed set over Y, then $f_{p\mu}^{-1}$(G,H) is soft gα-closed over X. But every soft gα-closed set is soft gαb-closed set. Then $f_{p\mu}^{-1}$(G,H) is soft gαb-closed set over X . Hence $f_{p\mu}$: SS(X)$_E$→ SS(Y)$_H$ is soft gαb-continuous function.

The converse of (vi)→(i) is not be true as shown by the following example.

Example 3.11. Let X={x_1,x_2,x_3}, Y={y_1,y_2,y_3}and E=H={e_1,e_2}. Then τ={Φ,X,(F_1,E) ,(F_2,E) ,(F_3,E)} and ρ={Φ,Y,(G_1,E)} are soft topological spaces over X and Y respectively where (F_1,E), (F_2,E),(F_3,E) are soft open sets over X and (G_1,E) is soft open set over Y, where (F_1,E) ={(e_1,{x_1}),(e_2,{x_1})},(F_2,E) ={(e_1,{x_2}),(e_2,{x_2})}, (F_3,E) ={(e_1,{x_1,x_2}),(e_2,{x_1,x_2})}, (G_1,E) ={(e_1, {y_1,y_3}),(e_2,{y_1,y_3})}. Then the soft closed

sets over Y are Φ,Y,$(G_1,E)^c$ where $(G_1,E)^c$ ={(e_1, {y_2}),(e_2, {y_2})}. If we define the function $f_{p\mu}$: SS(X)$_E$→ SS(Y)$_E$ by f(x_1)={y_2}, f(x_2)={y_1} and f(x_3)={y_1} then $f_{p\mu}$is a soft gαb-continuous function but not soft gα-continuous function , since $f_{p\mu}^{-1}(G_1,H)^c$= $f_{p\mu}^{-1}${(e_1, {y_2}),(e_2, {y_2})}={(e_1, {x_1}),(e_2, {x_1})} is a soft gαb-closed sets but not soft gα-closed set over X.

(vii)→(i).Immediate.

The converse of (vii)→(i) is not be true as shown by the following example.

Example 3.12. Let X={x_1,x_2,x_3}, Y={y_1,y_2,y_3} and E=H={e_1,e_2}. Then τ={Φ,X,(F_1,E)} and ρ={Φ,Y,(G_1,E)} are soft topological spaces over X and Y respectively where (F_1,E) and (G_1,E) are soft open sets over X and Y respectively, defined by (F_1,E) ={(e_1,{x_1,x_2}),(e_2,{x_1,x_2})}, (G_1,E) ={(e_1, {y_1,y_2}),(e_2,{y_1,y_2})}. Then the soft closed sets over Y are Φ,Y,$(G_1,E)^c$ and $(G_1,E)^c$ ={(e_1, {y_3}),(e_2, {y_3})}now define the function $f_{p\mu}$: $SS(X)_E \to SS(Y)_E$ by f(x_1)={y_3}, f(x_2)={y_1} and f(x_3)={y_2} then $f_{p\mu}$ is a soft gαb-continuous function but not soft gαb-irresolute function, since (G_2,E) ={(e_1, {y_1,y_3}),(e_2,{y_1,y_3})} is a soft gαb-closed sets over Y, $f_{p\mu}^{-1}$(G_2,E) = $f_{p\mu}^{-1}$({(e_1, {y_1,y_3}),(e_2,{y_1,y_3})}) ={(e_1, {x_1, x_2}),(e_2, {x_1, x_2})} is not soft gαb-closed set over X.

(iv)→(v).

The proof is directly from definition and Theorem (5:2)[6].

The converse of (iv)→(v) is not be true as shown by the following example.

Example 3.13. In Example(3.12) if we define the function $f_{p\mu}$: $SS(X)_E \to SS(Y)_E$ by f(x_1)={y_3}, f(x_2)={y_1} and f(x_3)={y_3} then $f_{p\mu}$is a softsg-continuous function but not soft semi-continuous, since $(G_1,E)^c$ ={(e_1, {y_3}),(e_2, {y_3})} is a soft closed sets over Y, $f_{p\mu}^{-1}(G_1,E)^c$ = $f_{p\mu}^{-1}$({(e_1, {y_3}),(e_2,{y_3})}) ={(e_1, {x_1, x_3}),(e_2, {x_1, x_3})} is not soft semi closed set over X.

(viii)→(iv). Let $f_{p\mu}$: $SS(X)_E \to SS(Y)_H$ be a soft Contra Regular-continuous function and (G,H) be soft closed set over Y, then $f_{p\mu}^{-1}$(G,H) is soft regular open set over X. But every soft regular open set is soft semi closed set $f_{p\mu}^{-1}$(G,H) is soft semi closed set over X . Hence $f_{p\mu}$: $SS(X)_E \to SS(Y)_H$ is soft semi-continuous function.

The converse of (viii)→(iv) is not be true as shown by the following example.

Example 3.14. In Example(3.12) if we define the function $f_{p\mu}$: $SS(X)_E \to SS(Y)_E$ by f(x_1)={y_1}, f(x_2)={y_2} and f(x_3)={y_3} then $f_{p\mu}$is a softsemi-continuous function but not soft Contra Regular-continuous, since $(G_1,E)^c$ ={(e_1, {y_3}),(e_2, {y_3})}is a soft closed sets over Y, $f_{p\mu}^{-1}(G_1,E)^c$ = $f_{p\mu}^{-1}$({(e_1, {y_3}),(e_2,{y_3})}) ={(e_1, {x_3}),(e_2, {x_3})} is not soft regular open set over X.

(ix)→(vi). The proof is directly from Definition (3.5) and Theorem (2.18).

The converse of (ix)→(vi) is not be true as shown by the following example.

Example 3.15. Let X={x_1,x_2,x_3,x_4}, Y={y_1,y_2,y_3,y_4}and E=H={e_1,e_2}is the set of all Parameter . Then τ={Φ,X,(F_1,E), (F_2,E)}is a soft topology over X, where (F_1,E), (F_2,E) are soft sets over X and$\rho=\{\Phi,Y,(G_1,E),(G_2,E)\}$ is a soft topology over Y, where (G_1,E), (G_2,E) are soft sets over Y, defined as follows:

$(F_1,E)=\{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$,$(F_2,E)=\{(e_1,\{x_3,x_4\}),(e_2,\{x_3,x_4\})\}$ and $(G_1,E)=\{(e_1,\{y_1,y_2\}),(e_2,\{y_1,y_2\})\}$,$(G_2,E)=\{(e_1,\{y_3,y_4\}),(e_2,\{y_3,y_4\})\}$ the set of all soft closed sets over Y are $\Phi,Y,(G_1,E)^c$, $(G_2,E)^c$ where$(G_1,E)^c=\{(e_1,\{y_3,y_4\}),(e_2,\{y_3,y_4\})\}$,$(G_2,E)^c=\{(e_1,\{y_1,y_2\}),(e_2,\{y_1,y_2\})\}$. If we define the function $f_{p\mu}$: $SS(X)_E\rightarrow SS(Y)_E$ by $f(x_1)=\{y_3\}$, $f(x_2)=\{y_4\}$, $f(x_3)=\{y_4\}$ and $f(x_4)=\{y_2\}$, then $f_{p\mu}$ is a soft gα-continuous function but not soft gp-continuous function , since $f_{p\mu}^{-1}(G_1,H)^c=\{(e_1,\{x_1,x_2,x_3\}),(e_2\{x_1,x_2,x_3\})\}$ and $f_{p\mu}^{-1}(G_1,H)^c=\{(e_1,\{x_1,x_2,x_3\}),(e_2\{x_1,x_2,x_3\})\}$ is soft gα-closed sets but not soft gp-closed sets over X.

(x)→(v). The proof is directly from Definition (3.5) and Theorem (2.18).

The converse of (x)→(v) is not be true as shown by the following example.

Example 3.16. In Example (3.11) $f_{p\mu}$ is a softs*g-continuous function but not soft sg-continuous function , since $f_{p\mu}^{-1}(G_1,H)^c=f_{p\mu}^{-1}\{(e_1,\{y_2\}),(e_2,\{y_2\})\}=\{(e_1,\{x_1\}),(e_2,\{x_1\})\}$ is a soft sg-closed sets but not soft s*g-closed sets over X.

(xi)→(x).Immediate.

The converse of (xi)→(x) is not be true as shown by the following example.

Example 3.17. In Example(3.15) $f_{p\mu}$ is a soft s*g-continuous function but not soft continuous function, since $f_{p\mu}^{-1}(G_1,H)^c=\{(e_1,\{x_1,x_2,x_3\}),(e_2\{x_1,x_2,x_3\})\}$ and $f_{p\mu}^{-1}(G_1,H)^c=\{(e_1,\{x_1,x_2,x_3\}),(e_2\{x_1,x_2,x_3\})\}$ is soft s*g-closed sets but not soft closed sets over X.

(xii)→(ix).Immediate.

The converse of (xi)→(ix) is not be true as shown by the following example.

Example 3.18. Let $X=\{x_1,x_2,x_3,x_4\}$, $Y=\{y_1,y_2,y_3,y_4\}$and $E=H=\{e_1,e_2\}$is the set of all Parameter . Then $\tau=\{\Phi,X,(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E)\}$is a soft topology over X, where (F_1,E), (F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E) are soft sets over X and $\rho=\{\Phi,Y,(G_1,E)\}$ is a soft topology over Y, where (G_1,E) is a soft sets over Y, defined as follows:

$(F_1,E)=\{(e_1,\{x_1\}),(e_2,\{x_1\})\}$,$(F_2,E)=\{(e_1,\{x_2\}),(e_2,\{x_2\})\}$,$(F_3,E)=\{(e_1,\{x_3\}),(e_2,\{x_3\})\}$, $(F_4,E)=\{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$,$(F_5,E)=\{(e_1,\{x_1,x_3\}),(e_2,\{x_1,x_3\})\}$,$(F_6,E)=\{(e_1,\{x_2,x_3\}),(e_2,\{x_2,x_3\})\}$,$(F_7,E)=\{(e_1,\{x_1,x_2,x_3\}),(e_2,\{x_1,x_2,x_3\})\}$and $(G_1,E)=\{(e_1,\{y_1,y_2,y_3\}),(e_2,\{y_1,y_2,y_3\})\}$ the set of all soft closed sets over Y are $\Phi,Y,(G_1,E)^c$ where $(G_1,E)^c=\{(e_1,$

$\{y_4\}$),(e_2, $\{y_4\}$)}. If we define the function $f_{p\mu}$: $SS(X)_E \to SS(Y)_E$ by $f(x_1)=\{y_1\}$, $f(x_2)=\{y_2\}$, $f(x_3)=\{y_3\}$ and $f(x_4)=\{y_4\}$, then $f_{p\mu}$ is a soft gp-continuous function but not soft regular-continuous function, since $f_{p\mu}^{-1}(G_1,H)^c=\{(e_1, \{x_4\}), (e_2\{x_4\})\}$ and $f_{p\mu}^{-1}(G_1,H)^c=\{(e_1, \{x_4\}), (e_2\{x_4\})\}$ is soft gp-closed sets but not soft regular closed over X.

(xii)→(xi).Immediate.

The converse of (xii)→(xi) is not be true as shown by the following example.

Example 3.19. In Example(3.18) $f_{p\mu}$ is a soft continuous function but not soft regular continuous function, since $f_{p\mu}^{-1}(G_1,H)^c=\{(e_1, \{x_4\}), (e_2\{x_4\})\}$ and $f_{p\mu}^{-1}(G_1,H)^c=\{(e_1, \{x_4\}), (e_2\{x_4\})\}$ is soft closed sets but not soft regular closed over X.

(xi)→(ix).Immediate.

Theorem 3.20. Let (X,τ,E) and (Y,ρ,H) be soft topological spaces, μ:X→Y and p:E→H be a functions. If$f_{p\mu}$: $SS(X)_E \to SS(Y)_H$ is afunction, then the following statements are equivalent.

(i) $f_{p\mu}^{-1}(F,H)) \in$ gαb^s-O(X) $\forall$(F,H) $\in O^s$(Y).

(ii) $f_{p\mu}$is soft gαb-continuous function.

(iii) $f_{p\mu}$(gαb^s-cl (A,E))$\subseteq cl^s(f_{p\mu}$(A,E)) $\forall$(A,E)$\in SS(X)_E$.

(iv) gαb^s-cl($f_{p\mu}^{-1}$(F,H))$\subseteq$ ($f_{p\mu}^{-1} cl^s$(F,H)) $\forall$(F,H)$\in SS(Y)_H$.

(v) $f_{p\mu}^{-1}(\mathrm{int}^s$ (F,H))$\subseteq$ gαb^s-int($f_{p\mu}^{-1}$(F,H)) $\forall$(F,H)$\in SS(Y)_H$.

Proof:(i)⇒(ii) Let (F,H) be a closed soft set over Y. Then $(F,H)^c \in O^s(Y)$ and $f_{p\mu}^{-1}(F,H)^c \in$ gαb^s-O(X). Then from (i) and since $f_{p\mu}^{-1}(F,H)^c=(f_{p\mu}^{-1}(F,H))^c$ from Theorem 3.14[15]. Thus $f_{p\mu}^{-1}$(F,H)$\in$ gαb^s-C(X). Hence $f_{p\mu}$ is soft gαb-continuous function.

(ii)⇒(iii) Let (A,E)$\in SS(X)_E$, since $f_{p\mu}$(A,E)$\subseteq cl^s(f_{p\mu}$(A,E)) and (A,E) $\subseteq f_{p\mu}^{-1}(f_{p\mu}$(A,E)) $\subseteq f_{p\mu}^{-1}$(c$l^s(f_{p\mu}$(A,E))). Then from (ii) $f_{p\mu}^{-1}$(c$l^s(f_{p\mu}$(A,E)))$\in$ gαb^s-C(X) then (A,E)$\subseteq$gαb^s-cl (A,E)$\subseteq f_{p\mu}^{-1}$(c$l^s(f_{p\mu}$(A,E))). Hence $f_{p\mu}$(gαb^s-cl (A,E))$\subseteq cl^s(f_{p\mu}$(A,E)) $\forall$(A,E)$\in SS(X)_E$.

(iii)⇒(iv) Let (F,H)$\in SS(Y)_H$ and (A,E)= $f_{p\mu}^{-1}$(F,H). Then $f_{p\mu}$(gαb^s-cl ($f_{p\mu}^{-1}$(F,H))$\subseteq$c$l^s(f_{p\mu}(f_{p\mu}^{-1}$(F,H))). Then from(iii).Hence gαb^s-cl $f_{p\mu}^{-1}$(F,H)$\subseteq f_{p\mu}^{-1}(f_{p\mu}$(gα$b^s$-cl($f_{p\mu}^{-1}$(F,H)))) $\subseteq f_{p\mu}^{-1}$(c$l^s(f_{p\mu}{}^{-1}$(F,H)))) from Theorem3.14[15]. Thus gαb^s-cl($f_{p\mu}^{-1}$(F,H))$\subseteq$ $f_{p\mu}^{-1}$(cl^S(F,H)) $\forall$(F,H)$\in SS(Y)_H$

(iv)⇒(v) let (U,H)∈SS(Y)$_H$, (F,H)^c =(U,H). Then from(v) we obtain that gαb^s-cl ($f_{p\mu}^{-1}$(F,H)^c)⊆ $f_{p\mu}^{-1}$cl^s(F,H)^c, gαb^s-cl $f_{p\mu}^{-1}$(F,H)^c⊆ $f_{p\mu}^{-1}$(int^s(F,H)) ^{c} from Theorem 3.14 [15]. Thus gαb^s-cl($f_{p\mu}^{-1}$(F,H))^c⊆($f_{p\mu}^{-1}$(int^s(F,H))^c,(gαb^s-int ($f_{p\mu}^{-1}$(F,H)))^c⊆ ($f_{p\mu}^{-1}$(int^s(F,H))^c and hence $f_{p\mu}^{-1}$(int^s(F,H))⊆ gαb^s-int($f_{p\mu}^{-1}$ (F,H)) ∀(F,H)∈SS(Y)$_H$.

(v)⇒(i) Let (F,H) be a soft open set over Y. then int^s(F,H)=(F,H) and $f_{p\mu}^{-1}$(int^s(F,H))= $f_{p\mu}^{-1}$(F,H). Then from(v) $f_{p\mu}^{-1}$(int^s(F,H))⊆ gαb^s-int($f_{p\mu}^{-1}$(F,H)) ∀(F,H)∈SS(Y)$_H$, $f_{p\mu}^{-1}$((F,H))⊆ gαb^s-int($f_{p\mu}^{-1}$(F,H)). This means that gαb^s-int($f_{p\mu}^{-1}$(F,H))= $f_{p\mu}^{-1}$(F,H). Hence $f_{p\mu}^{-1}$(F,H)∈ gαb^s-O(X).

Theorem 3.21. Let (X,τ,E), (Y,ρ,H) and (Z,σ,K) be soft topological spaces, μ:X→Y,μ_1:Y→Z, p:E→H andp_1:H→K be functions. If $f_{p\,\mu}$: SS(X)$_E$→ SS(Y)$_H$ and
$g_{p_1\mu_1}$: SS(Y)$_H$→ SS(Z)$_K$ be two soft functions. Then

(i) $g_{p_1\mu_1} \circ f_{p\,\mu}$: SS(X)$_E$→ SS(Z)$_K$ is soft gαb-continuous function, if $f_{p\,\mu}$ is soft gαb-continuous function and $g_{p_1\mu_1}$ is soft continuous function.

(ii) $g_{p_1\mu_1} \circ f_{p\,\mu}$: SS(X)$_E$→ SS(Z)$_K$ is softgαb-irresolute function, if $f_{p\,\mu}$ and $g_{p_1\mu_1}$are soft gαb-irresolute function.

(iii) $g_{p_1\mu_1} \circ f_{p\,\mu}$SS(X)$_E$→ SS(Z)$_K$ is soft gαb-continuous function, if $f_{p\,\mu}$ is soft gαb-irresolute function and $g_{p_1\mu_1}$ is soft gαb-continuous function

Proof:

(i) Let(G,K) be a soft closed set over Z since $g_{p_1\mu_1}$:SS(Y)$_H$→SS(Z)$_K$is soft continuous function then $g^{-1}_{p_1\mu_1}$(G,K) is a soft closed set over Y. Now $f_{p\,\mu}$: SS(X)$_E$→ SS(Y)$_H$ is a soft gαb-continuous function then $f_{p\,\mu}^{-1}$($g^{-1}_{p_1\mu_1}$(G,K) =($g_{p_1\mu_1} \circ f_{p\,\mu}$)$^{-1}$(G,K) is soft gαb-closed over X. Hence $g_{p_1\mu_1} \circ f_{p\,\mu}$: SS(X)$_E$→ SS(Z)$_K$ is soft gαb-continuous function.

(ii) Let $g_{p_1\mu_1}$: SS(Y)$_H$→ SS(Z)$_K$ be a soft gαb-irresolute function and (G,K) be a soft gαb-closed set over Z, then $g^{-1}_{p_1\mu_1}$(G,K) is soft gαb-closed set over Y. Also $f_{p\,\mu}$: SS(X)$_E$→ SS(Y)$_H$ is soft gαb-irresolute function so $f_{p\,\mu}^{-1}$($g^{-1}_{p_1\mu_1}$(G,K) =($g_{p_1\mu_1} \circ f_{p\,\mu}$)$^{-1}$(G,K) is soft gαb-closed over X. Thus $g_{p_1\mu_1} \circ f_{p\,\mu}$: SS(X)$_E$→ SS(Z)$_K$ is soft gαb-irresolute function.

(iii) Let (G,K) be soft closed set over Z since $g_{p_1\mu_1}$: $SS(Y)_H \rightarrow SS(Z)_K$ is soft gαb-continuous function, $g^{-1}_{p_1\mu_1}$(G,K) is soft gαb-closed set over Y. Also $f_{p\mu}$: $SS(X)_E \rightarrow SS(Y)_H$ is soft gαb-irresolute function so every soft gαb-closed set over Y is soft gαb-closed set over X. Therefore $f^{-1}_{p\mu}(g^{-1}_{p_1\mu_1}$(G,K) =($g_{p_1\mu_1} \circ f_{p\mu})^{-1}$(G,K) is soft gαb-closed over X. Hence $g_{p_1\mu_1} \circ f_{p\mu}$: $SS(X)_E \rightarrow SS(Z)_K$ is soft gαb-continuous function.

4. Soft generalized αb-closed functions and Soft generalized αb-open functions

Definition 4.1. Let (X,τ,E) and (Y,ρ,H) be soft topological spaces, μ:X→Y and p:E→H be functions, the function $f_{p\mu}$: $SS(X)_E \rightarrow SS(Y)_H$ is said be soft gαb-closed (gαb-open)if the image of every soft closed (open) set in X is a soft gαb-closed (gαb-open) set in Y.

Theorem 4.2. Let (X,τ,E) and (Y,ρ,H) be soft topological spaces, μ:X→Y and p:E→H be functions. If $f_{p\mu}$: $SS(X)_E \rightarrow SS(Y)_H$ is a soft b-irresolute, image of each soft α-open is soft α-open set and bijective function. Then $f_{p\mu}$ is a soft gαb-irresolute function.

Proof: Let (F,E) be any soft gαb-closed set over Y and $f^{-1}_{p\mu}$ (F,E)⊆ (U,H), (U,H) be any soft α-open set over X. Then (F,E)⊆$f_{p\mu}$(U,H), bcl(F,E)⊆$f_{p\mu}$(U,H)⇒$f^{-1}_{p\mu}$(bcl(F,E))⊆(U,H) since $f_{p\mu}$ soft b-irresolute function ⇒bcl($f^{-1}_{p\mu}$(F,E)) ⊆bcl ($f^{-1}_{p\mu}$(bcl(F,E)))⊆ (U,H). Hence $f_{p\mu}$ is soft gαb-irresolute function.

Theorem 4.3. Let (X,τ,E), (Y,ρ,H) and (Z,σ,K) be soft topological spaces, μ:X→Y,μ_1:Y→Z, p:E→H and p_1:H→K be functions. If $f_{p\mu}$: $SS(X)_E \rightarrow SS(Y)_H$ is a closed function and $g_{p_1\mu_1}$: $SS(Y)_H \rightarrow SS(Z)_K$ is a gαb-closed function. Then $g_{p_1\mu_1} \circ f_{p\mu}$: $SS(X)_E \rightarrow SS(Z)_K$ is a soft gαb-closed function.

Proof: For any soft closed (F,E) over X, $f_{p\mu}$(F,E) is a soft closed over Y. Since $g_{p_1\mu_1}$: $SS(Y)_H \rightarrow SS(Z)_K$ is gαb-closed function. Then $g_{p_1\mu_1}$($f_{p\mu}$(F,E)) is soft gαb-closed set over Z,$g_{p_1\mu_1}$($f_{p\mu}$(F,E)) =($g_{p_1\mu_1} \circ f_{p\mu}$)(F,E) is soft gαb-closed set over Z. Therefore, ($g_{p_1\mu_1} \circ f_{p\mu}$) is a soft gαb-closed function.

Theorem 4.4. Let (X,τ,E) and (Y,ρ,H) be soft topological spaces, μ:X→Y and p:E→H be a functions, then $f_{p\mu}$: $SS(X)_E \rightarrow SS(Y)_H$ is a soft gαb-closed function if and only if for each soft set (G,H) in $SS(Y)_H$ and for each soft open set (F,E) in $SS(X)_E$ such that $f^{-1}_{p\mu}$(G,H)⊆(F,E), there is a soft gαb-open set (N,H) over Y such that (G,H)⊆(N,H) and $f^{-1}_{p\mu}$(N,H) ⊆(F,E).

Proof: Assume that $f_{p\,\mu}$is a soft gαb-closed function, (G,H) is a soft set in $SS(Y)_H$ and (F,E) be a soft open set in $SS(X)_E$ such that$f_{p\,\mu}^{-1}$(G,H)⊆(F,E), then (N,H) =($f_{p\,\mu}$((F,E)^c))^{c}is a soft gαb-open set over Y such that (G,H)⊆(N,H) and $f_{p\,\mu}^{-1}$(N,H)⊆(F,E).
Conversely: Assume that (L,E) be a soft closed over X.Then$f_{p\,\mu}^{-1}$($f_{p\,\mu}$(L,E))^c⊆(L,E)^c, (L,E)^c is soft open set. By the hypothesis, there is a
soft gαb-open set over Y such that ($f_{p\,\mu}$(L,E))^c⊆ (N,H) and $f_{p\,\mu}^{-1}$(N,H) ⊆(L,E)^c. Thus (N,H)^c⊆$f_{p\,\mu}$(L,E) and$f_{p\,\mu}^{-1}$ ((N,H)) ⊆(L,E)^c from Theorem 3.14 [15], $f_{p\,\mu}$(L,E) ⊆(N,H)^c, which implies ($f_{p\,\mu}$(L,E)= (N,H)^{c}since (N,H)^c is soft gαb-closed set, $f_{p\,\mu}$(L,E) is a soft gαb-closed set over Y. So $f_{p\,\mu}$ is a soft gαb-closed function.

Theorem 4.5. Let (X,τ,E), (Y,ρ,H) and (Z,σ,K) be soft topological spaces, μ:X→Y,μ_1:Y→Z, p:E→H andp_1:H→K be functions. If$f_{p\,\mu}$: $SS(X)_E$→ $SS(Y)_H$ and $g_{p_1\mu_1}$: $SS(Y)_H$→ $SS(Z)_K$ be two soft functions such that $g_{p_1\mu_1}\circ f_{p\,\mu}$: $SS(X)_E$→ $SS(Z)_K$ is soft gαb-closed function. If $f_{p\,\mu}$ is soft continuous and surjective, then $g_{p_1\mu_1}$soft gαb-closed function.

(i) If $f_{p\mu}$ is soft continuous and surjective, then $g_{p_1\mu_1}$ soft gαb-closed function.
(ii) If $g_{p_1\mu_1}$ is soft gαb-irresolute and injective function, then $f_{p\,\mu}$ is soft gαb-closed function.

Proof:
(i) Let (G,H) be soft closed set over Y, then $f_{p\,\mu}^{-1}$ (G,H) is soft closed set over X, since$f_{p\,\mu}$is soft continuous function.Since $g_{p_1\mu_1}\circ f_{p\,\mu}$ is soft gαb-closed function, then ($g_{p_1\mu_1}\circ f_{p\,\mu}$) ($f_{p\,\mu}^{-1}$(G,H))= $g_{p_1\mu_1}$(G,H) is soft gαb-closed over Z. Hence $g_{p_1\mu_1}$a soft gαb-closed function.

(ii) Let (F,E) be a soft closed set over X.Then ($g_{p_1\mu_1}\circ f_{p\,\mu}$) (F,E) is a soft closed set over Z, and $g^{-1}_{p_1\mu_1}$(($g_{p_1\mu_1}\circ f_{p\,\mu}$) (F,E)) =$f_{p\,\mu}$(F,E) is soft gαb-closed over Y. Since $g_{p_1\mu_1}$ is a soft gαb-irresolute and injective function hence $f_{p\,\mu}$ is a soft gαb-closed function.

4. CONCLUSION

In mathematics, topology is considered as an important and major area that can give many relationships between mathematical models and other scientific areas. Currently, the soft set theory that was initiated by Molodtsov [11] have been studied by many scientists, and easily applied to many problems having uncertainties from social life. In this paper, we introduce the concept of soft gαb-closure and soft gαb-interior of a soft set in soft topological spaces and study some of their properties. We also introduce the concept of soft gαb-continuous functions and soft gαb-closed, gαb-open functions in soft topological spaces and some of their properties have been established. This idea can be extended to soft bitopological spaces and soft fuzzifying topological spaces.

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الدوال المتصلة الناعمة من نمط – جي الفا بي في الفضاءات التوبولوجية الناعم

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يهتم هذا البحث بتوصيف ودراسة الدوال المتصلة الناعمة من نمط جي الفا بي والدوال الناعمة المفتوحة والمغلقة من نمط جي الفا بي في الفضاءات التوبولوجية الناعمة . كما تم تعريف الإغلاق الناعم من نفس النمط مع توضيح العلاقة بين الدوال المتصلة الناعمة من نمط جي الفا بي وبعض الأنواع الأخرى من الدوال المتصلة .