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Modeling, Trimming and Simulation of a Full Scale Helicopter

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Abstract: The complex configuration of helicopter guarantees that the vehicle modeling, trim and simulation are significantly more difficult than fixed-wing aircrafts. In this paper, general expressions for aerodynamic forces and moments, acting on helicopter due to its main and tail rotors at any flight conditions, are derived by using momentum and blade element theories. These complex expressions are inserted in the rigid body equations of motion, derived from Newton second law, to build a generic nonlinear mathematical model for single main and tail rotors helicopters; in order to obtain their responses to arbitrary control inputs. This model can be used in pilot training, control system design, and studying the helicopter stability characteristics. Trimming problem is solved at general flight conditions; arbitrary turn rate, flight path and side slip angles. The power required to fly helicopter at forward flight with several flight path angles is determined. The flight path angle required for helicopter autorotation condition is calculated at any forward speed. The mathematical model is solved by numerical integration (Runge-Kutta method) in the simulation code. The resulting trim conditions are verified by supplying the trim control inputs to the simulation code and verifying that the helicopter is flying in steady-state.

Keywords: Helicopter, Mathematical Model, Trim, Simulation.

Nomenclature

a Main rotor blade section lift curve slope
a_{tr} Tail rotor blade section lift curve slope
A Main Rotor Area, πR^2
A_{tr} Tail rotor area
c Main rotor blade chord
C_T Main rotor thrust coefficient
$C_{T_{tr}}$ Tail rotor thrust coefficient
$C_{X_h}, C_{Y_h}, C_{Z_h}$ Main rotor forces coefficients in X_h, Y_h, Z_h respectively.
$C_{X_w}, C_{Y_w}, C_{Z_w}$ Main rotor forces coefficients in X_w, Y_w, Z_w respectively.
f Fuselage equivalent drag area.
h_m Distance from A.C. center of gravity and hub center in z direction
I_b Blade inertia
I_{xx}, I_{yy}, I_{zz} Moment of inertia about X_b, Y_b, Z_b respectively
l_{tr} Distance from A.C. center of gravity and tail rotor hub center in x direction
p, q, r Angular velocity about X, Y, Z body axes respectively
p_{hw}, q_{hw}, r_{hw} Angular velocity about X_{hw}, Y_{hw}, Z_{hw} hub-wind axes respectively
R Main rotor radius

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R_{tr} Tail rotor radius
T_{tr} Tail rotor thrust
T Main rotor thrust
U_T Tangential component of velocity vector at the blade section
U_p Radial component of velocity vector at the blade section
U_p Perpendicular component of velocity vector at the blade section
u, v, w Velocity components in X, Y, Z body axes respectively
u_h, v_h, w_h Velocity component in X_h, Y_h, Z_h hub axes respectively
u_{tr}, v_{tr}, w_{tr} Velocity components acting on the tail rotor
V_{fe} Flight velocity
x_{cg} Distance from A.C. center of gravity and hub center in x direction
z_{tr} Distance from A.C. center of gravity and tail rotor hub center in x direction
α_f Fuselage angle of attack
$\beta_{w_{tr}}$ Tail rotor side slip angle.
β_0 Blade conning angle
$\beta_{1c_w}, \beta_{1s_w}$ Main rotor longitudinal and lateral flapping angles in hub-win axes
β_{1c} Main rotor longitudinal flapping angle
β_{1s} Main rotor lateral flapping angle
β_f Fuselage side slip angle
γ Main rotor blade lock number
γ_{fe} Spin angle (positive downward)
θ_0 Main rotor collective pitch
$\theta_{1c_w}, \theta_{1s_w}$ Main rotor lateral and longitudinal cyclic pitch in hub-wind axes
θ_{1c} Lateral cyclic pitch
θ_{1s} Longitudinal cyclic pitch
λ Total induced flow ratio through main rotor disk
μ Forward flight advance ratio
μ_{tr} Tail rotor advance ratio
ϕ, θ, ψ Euler angles
$\dot{\Psi}$ Horizontal turn rate
σ, σ_{tr} Main rotor and tail rotor solidity respectively
Ω, Ω_{tr} Main rotor and tail rotor rotational speed

Subscripts and superscripts

b body
e equilibrium
f fuselage
h Hub
hw hub wind
mr Main rotor
tr tail rotor

1. Introduction

Helicopter has six degrees of freedom in its motion. The helicopter control is based on changing the direction and magnitude of main rotor thrust vector. Helicopter has four control inputs associated to its main rotor. These controls are the main rotor collective pitch θ_0, longitudinal cyclic pitch θ_{1s}, lateral cyclic pitch θ_{1c} and tail rotor collective pitch $\theta_{0_{tr}}$. The cross coupling between these inputs makes the vehicle unstable without the stability augmentation system [1]. To design a control system and to analyze the helicopter complicated control and dynamics, it is necessary to develop a complete dynamic model which should be linearized about an

equilibrium point. This modeling is divided into three levels according to their complexity [2]. Level 1 is the simplest model that describes the system whereas level 3 consist of set of complex models and it is more accurate in predicting the system response [2]. In level 1 helicopter is assumed rigid body and the blade dynamics are simplified to rigid blade motion, but level 3 incorporates detailed modeling of rotor blades and the modes of structure vibration. Level 2 complexity is in between level 1 and level 3. In the literature, most researchers in the field of helicopter dynamic modeling, simulation and control uses Level 1 and reasonable and accurate results [3] and [4]. The rigid body nonlinear model has helicopter orientation, position, linear and angular velocity components as states. This model is 12th order system [5]. Level 3 models are used in structure and rotor mechanical design [6]. The helicopter trim problem, determining control inputs required for steady flight, is solved in forward flight [7], [8] and [9]. All publications in the field of helicopter control solve the trim problem in steady forward flight only [10] and [11]. The conventional forward simulation or direct simulation is used to get the vehicle motion (as a function of time) as a response to a given control inputs by solving the non-linear differential equations of motion. This simulation is applied to fixed wing aircraft and helicopter and the results are compared with flight test data [4].
The main objective of this study is to build a complete non-linear model for the helicopter. This model can be used in the control system design and building direct simulation model for a full scale helicopter. In this paper the mathematical model of a single main rotor and tail rotor is presented, the total forces and moments acting in the helicopter due to its components are discussed in details, helicopter general trim problem is solved, and a simulation of helicopter motion at a steady maneuver is introduced.
The prouty example helicopter [1] is used in calculations in this study. Its characteristics could be summarized in Table.1 and Fig. 1.

Table 1. Prouty example helicopter characteristics

Property	Value	
	British units	SI units
Design gross weight	20000 lb.	9070 Kg
Main rotor radius	30 ft.	9.14 m
Main rotor disk area	2827 ft.^{2}	262.64 m^2
Main rotor chord	2 ft.	0.61 m
Main rotor tip speed	650 ft./sec.	198.12 m/s
Tail rotor radius	6.5 ft.	1.98 m.
Tail rotor disk area	133 ft.^{2}	12.36 m^2
Tail rotor tip speed	650 ft./sec.	198.12 m/s
Fuselage length	57 ft.	17.37 m
Main rotor height above CG	6 ft.	1.8 m.
Tail rotor arm	37 ft.	11.28 m

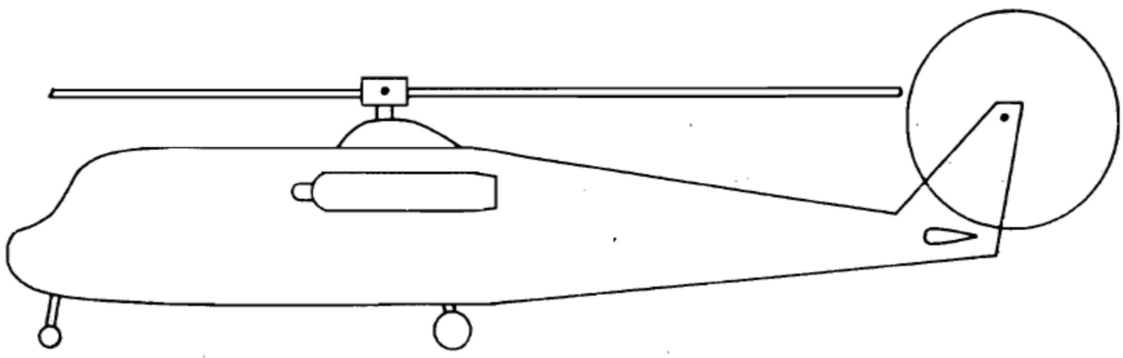

Fig. 1. Prouty example helicopter [1]

2. Helicopter Mathematical Model

Helicopter is treated as a rigid body of six degrees of freedom. Newton's second law can be applied to get six dynamic equations: three equations in the translational motion and three equations in the rotational motion. There are six kinematic equations relating the angular velocities of helicopter to the rate of change of its orientation or attitude w.r.t the fixed earth axis. The six kinematic equations are obtained from Euler angle transformation [2]. There are four control inputs $(\theta_0, \theta_{1c}, \theta_{1s}, \theta_{0_{tr}})$ that controls the force and moment components $(X, Y, Z. L, M, N)$. The system of equations that presents helicopter model is as:

$$
\begin{aligned}
\dot{u} &= -(qw - rv) + X/m - g\sin\theta \\
\dot{v} &= -(ru - pw) + Y/m + g\sin\phi\cos\theta \\
\dot{w} &= -(pv - qu) + Z/m + g\cos\phi\cos\theta \\
\dot{p} &= (I_{zz}L^* + I_{xz}N^*)/(I_{xx}I_{zz} - I_{xz}^2) \\
\dot{q} &= (M + (I_{zz} - I_{xx})rp + I_{xz}(r^2 - p^2))/(I_{yy}) \\
\dot{r} &= (I_{xz}L^* + I_{xx}N^*)/(I_{xx}I_{zz} - I_{xz}^2) \\
\dot{\phi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\
\dot{\theta} &= q\cos\phi - r\sin\phi \\
\dot{\psi} &= q sin\phi\sec\theta + r\cos\phi\sec\theta \\
\dot{x} &= u\, C\theta C\psi + v(S\phi S\theta C\psi - C\phi S\psi) + w(C\phi S\theta C\psi + S\phi S\psi) \\
\dot{y} &= u\, C\theta S\psi + v(S\phi S\theta S\psi + C\phi C\psi) + w(C\phi S\theta S\psi - S\psi C\psi) \\
\dot{z} &= -u\, S\theta + v\, s\phi C\theta + w\, C\phi C\theta
\end{aligned}
\tag{1}
$$

where:

$$
\begin{aligned}
L^* &= L + (I_{yy} - I_{zz})qr + I_{xz}pq \\
N^* &= N + (I_{xx} - I_{yy})pq - I_{xz}qr
\end{aligned}
\tag{2}
$$

The forces and moments acting on helicopter (X, Y, Z, L, M, N) are due to main rotor, tail rotor and fuselage. The forces and moments due to main rotor, tail rotor and fuselage are expressed in body axes system in detail in the following section.

3. Forces and Moments

Main Rotor Model

Main rotor is assumed to be the main source of generation forces and moments. Aerodynamically, momentum theory is used to obtain inflow ratio, the blade element theory is utilized and the forces and moments of blade section are integrated over the radius of blade. Because of ignorance of compressibility effects, reversed flow region, and stall effects, the total forces and moments are obtained by summation the contribution of each blade. A previous study showed that this type of study is valid for control and stability investigations of the helicopter for advance ratio up to 0.3 [12].

The rotor forces and moment are expressed in the hub wind axes system then transformed in the hub axes and body axes. The velocity components in (x_b, y_b, z_b)axes are (u, v, w) respectively and the angular rates are (p, q, r). Fig. 2 shows the hub, body and tail rotor axes. It also shows the horizontal distance helicopter center of gravity and tail rotor hub l_{tr} and the vertical distance between main rotor hub and CG.

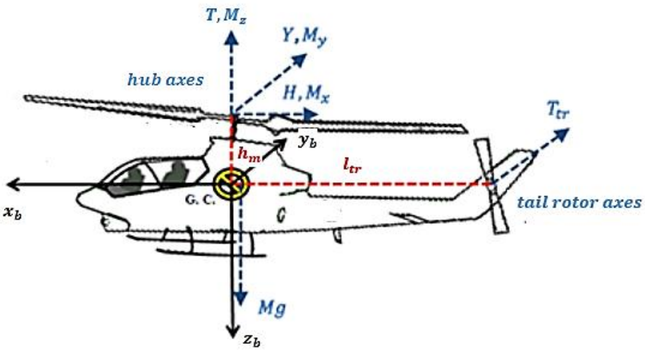

Fig. 2. Helicopter body axes, hub axes, and tail rotor axes [3]

The velocity components in body axes are transformed to the hub axes as following.

$$\begin{aligned} u_h &= u - q\,h_m \\ v_h &= v + p\,h_m + r\,x_{cg} \\ w_h &= w - q\,x_{cg} \end{aligned} \tag{3}$$

where: x_{cg} is the distance between vehicle CG and main rotor hub in x_b direction.
Side slip angle, the angle between $X_h$axis and X_{hw}axis, is defined as:

$$\beta_w = \sin^{-1}\frac{v_h}{\sqrt{(u_h^2 + v_h^2)}} \tag{4}$$

The tangential and perpendicular velocity components on the blade sections are:

$$\begin{aligned} \overline{U_T} &= \bar{r} + \mu\sin\psi + \bar{r}\,\beta\,(\overline{P_{hw}}\cos\psi - \overline{q_{hw}}\sin\psi) \\ \overline{U_p} &= \lambda + \bar{r}\beta^{\cdot} + \mu\beta\cos(\psi) - \bar{r}(\overline{P_{hw}}\sin\psi + \overline{q_{hw}}\cos\psi) \end{aligned} \tag{5}$$

where,

$$\begin{aligned} \overline{P_{hw}} &= \frac{p}{\Omega}\cos\beta_w + \frac{q}{\Omega}\sin\beta_w \\ \overline{q_{hw}} &= -\frac{p}{\Omega}\sin\beta_w + \frac{q}{\Omega}\cos\beta_w \end{aligned} \tag{6}$$

Transformation of control input from hub axes to hub wind axes is as following:

$$\begin{bmatrix} \theta_{1c_w} \\ \theta_{1s_w} \end{bmatrix} = \begin{bmatrix} \cos\beta_w & -\sin\beta_w \\ \sin\beta_w & \cos\beta_w \end{bmatrix} \begin{bmatrix} \theta_{1c} \\ \theta_{1s} \end{bmatrix} \tag{7}$$

From momentum theory, the induced flow ratio is:

$$\lambda = -\frac{w_h}{\Omega R} + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}} \tag{8}$$

Main rotor loads hub-wind axes

The hub wind axes are like the wind axes in the fixed wing aircraft. The loads are calculated in the hub-wind axes and then transformed in the hub axes. The blade element theory is applied to get loads at blade sections and integrated along the span and over the azimuth to obtain general expressions for forces and moments coefficients in hub-wind axis.

Thrust coefficient:

$$C_T = \frac{\sigma a}{2}\left[\frac{\theta_0}{3}\left(1 + \frac{3}{2}\mu^2\right) + \frac{\theta_{tw}}{4}(1 + \mu^2) + \frac{\mu}{2}\theta_{1sw} - \frac{\lambda}{2} + \frac{\mu}{4}\,\overline{p_{hw}}\right] \tag{9}$$

Drag force coefficient:

$$\begin{aligned} C_{x_w} = \frac{\sigma a}{2}\Bigg[& \theta_0\left(\frac{\lambda\mu}{2} - \frac{\beta_{1cw}}{3} - \frac{\overline{p_{hw}}}{6}\right) + \theta_{tw}\left(\frac{\lambda\mu}{4} - \frac{\beta_{1cw}}{4} - \frac{\overline{p_{hw}}}{8}\right) \\ & + \theta_{1sw}\left(\frac{\lambda}{4} - \frac{\mu\beta_{1cw}}{4} - \frac{3}{16}\mu\overline{p_{hw}}\right) + \theta_{1cw}\left(-\frac{\beta_0}{6} - \frac{1}{16}\mu\overline{q_{hw}}\right) \\ & + \frac{3}{4}\lambda\beta_{1cw} + \frac{\beta_{1sw}\beta_0}{6} + \frac{\mu}{4}(\beta_0^2 + \beta_{1cw}^2) - \frac{1}{6}\beta_0\overline{q_{hw}} + \frac{\lambda}{2}\overline{p_{hw}} \\ & + \frac{1}{16}\mu\beta_{1cw}\overline{p_{hw}} + \frac{1}{16}\mu\beta_{1sw}\overline{q_{hw}}\Bigg] + \frac{\sigma}{2}\left(\frac{\mu}{2}\delta\right) \end{aligned} \tag{10}$$

Side force coefficient:

$$
\begin{aligned}
C_{Y_w} = -\frac{\sigma a}{2}\Bigg[&\theta_0\left\{\frac{3}{4}\mu\beta_0 + \frac{\beta_{1sw}}{3}\left(1+\frac{3}{2}\mu^2\right) - \frac{1}{6}\overline{q_{hw}}\right\} + \theta_{tw}\left\{\frac{\mu\beta_0}{2} + \frac{\beta_{1sw}}{4}(1+\mu^2) - \frac{1}{8}\right. \\
&+ \theta_{1cw}\left\{\frac{\lambda}{4} + \frac{1}{4}\mu\beta_{1cw} - \frac{1}{16}\mu\overline{p_{hw}}\right\} \\
&+ \theta_{1sw}\left\{\frac{\beta_0}{6}(1+3\mu^2) + \frac{1}{2}\mu\beta_{1sw} - \frac{1}{16}\mu\overline{q_{hw}}\right\} - \frac{3}{2}\lambda\mu\beta_0 \\
&+ \beta_0\beta_{1cw}\left(\frac{1}{6} - \mu^2\right) - \frac{3}{4}\lambda\beta_{1sw} - \frac{\mu}{4}\beta_{1cw}\beta_{1sw} + \frac{1}{6}\beta_0\overline{p_{hw}} + \frac{1}{2}\lambda\overline{q_{hw}} \\
&+ \frac{5}{16}\mu\beta_{1sw}\overline{p_{hw}} + \frac{7}{16}\mu\beta_{1cw}\overline{q_{hw}}\Bigg]
\end{aligned}
\tag{11}
$$

Moment coefficient about X_{hw} axis:

$$
\begin{aligned}
C_{M_{X_w}} = \frac{\sigma a}{2}\Bigg[&\frac{\beta_{1cw}}{8} + \frac{\theta_{1sw}}{8} + \frac{1}{4}\mu\theta_{tw} + \frac{1}{3}\mu\theta_0 - \frac{1}{4}\lambda\mu - \frac{1}{16}\mu^2\beta_{1cw} + \frac{3}{16}\mu^2\theta_{1sw} \\
&+ \frac{\overline{p_{hw}}}{8}\Bigg] + \frac{N}{2}\frac{1}{\rho\pi R^2(\Omega R)^2 R}k_\beta\beta_{1sw}
\end{aligned}
\tag{12}
$$

Moment coefficient about Y_{hw} axis:

$$
\begin{aligned}
C_{M_{Y_w}} = \frac{\sigma a}{2}\Bigg[&-\frac{1}{8}\theta_{1cw} + \frac{1}{8}\beta_{1sw} - \frac{\overline{q_{hw}}}{8} + \frac{1}{6}\beta_0\mu - \frac{1}{16}\mu^2\theta_{1cw} + \frac{1}{16}\mu^2\beta_{1sw}\Bigg] \\
&- \frac{N}{2}\frac{1}{\rho\pi R^2(\Omega R)^2 R}k_\beta\beta_{1cw}
\end{aligned}
\tag{14}
$$

Moment coefficient about Z_{hw} axis:

$$
\begin{aligned}
C_{M_{Z_w}} = \frac{\sigma a}{2}\Bigg[&\theta_0\left(-\frac{1}{3}\lambda + \frac{1}{6}\mu\overline{p_{hw}}\right) \\
&+ \theta_{1cw}\left(-\frac{1}{8}\beta_{1sw} + \frac{1}{8}\overline{q_{hw}} - \frac{1}{6}\beta_0\mu - \frac{1}{16}\beta_{1sw}\mu^2\right) \\
&+ \theta_{1sw}\left(\frac{1}{8}\beta_{1cw} + \frac{1}{8}\overline{p_{hw}} - \frac{1}{4}\lambda\mu - \frac{1}{16}\beta_{1cw}\mu^2\right) \\
&+ \theta_{tw}\left(-\frac{1}{4}\lambda + \frac{1}{8}\overline{p_{hw}}\mu\right) - \frac{\delta}{4a}(1+\mu^2) + \frac{1}{8}\beta_{1cw}^2 + \frac{1}{8}\beta_{1sw}^2 \\
&+ \frac{1}{4}\beta_{1cw}\overline{p_{hw}} + \frac{1}{8}\overline{p_{hw}}^2 - \frac{1}{4}\beta_{1sw}\overline{q_{hw}} + \frac{1}{8}\overline{q_{hw}}^2 + \frac{1}{2}\lambda^2 + \frac{1}{3}\beta_0\beta_{1sw}\mu \\
&- \frac{1}{3}\beta_0\overline{q_{hw}}\mu + \frac{1}{2}\beta_{1cw}\lambda\mu + \frac{1}{4}\beta_0^2\mu^2 + \frac{3}{16}\beta_{1cw}^2\mu^2 + \frac{1}{16}\beta_{1sw}^2\mu^2\Bigg]
\end{aligned}
\tag{15}
$$

where, the flapping angles can be obtained from the flapping model as follows:

$$
\begin{aligned}
\beta_0 &= \gamma\left\{\frac{\theta_0}{8}(1+\mu^2) + \frac{\theta_{tw}}{10}\left(1+\frac{5}{6}\mu^2\right) + \frac{\mu}{6}\theta_{1sw} - \frac{\lambda}{6}\right\} \\
\beta_{1sw} - \theta_{1cw} &= \frac{-\frac{4}{3}\mu\beta_0}{1+\frac{1}{2}\mu^2} + \frac{\frac{16}{\gamma}\left(\frac{p_{hw}}{\Omega}\right) + \left(\frac{q_{hw}}{\Omega}\right)}{1+\frac{1}{2}\mu^2} \\
\beta_{1cw} + \theta_{1sw} &= \frac{-\frac{8}{3}\mu\left(\theta_0 - \frac{3}{4}\lambda + \frac{3}{4}\mu\theta_{1sw} + \frac{3}{4}\theta_{tw}\right)}{1-\frac{1}{2}\mu^2} + \frac{\frac{16}{\gamma}\left(\frac{q_{hw}}{\Omega}\right) - \left(\frac{p_{hw}}{\Omega}\right)}{1-\frac{1}{2}\mu^2}
\end{aligned}
\tag{16}
$$

Rotor profile drag coefficient δ is required in calculation of torque and drag force coefficients. Ref. [12] provides an expression for the profile drag coefficient which matches the measured section characteristics as follows:

$$
\delta = 0.009 + 0.3\left(\frac{6C_T}{\sigma a}\right)^2
\tag{17}
$$

Main rotor loads in hub axes

The aerodynamic forces and moments coefficients are derived in hub-wind axis. These expressions should be transformed to hub axis and body axis to be used in aircraft modelling and simulation. The load transformation from hub-wind axes to hub fixed axes is as follows:

$$
\begin{aligned}
C_{X_h} &= C_{X_w}\cos\beta_w + C_{Y_w}\sin\beta_w \\
C_{Y_h} &= -C_{X_w}\sin\beta_w + C_{Y_w}\cos\beta_w \\
C_{Z_h} &= C_T \\
C_{M_{X_h}} &= C_{M_{X_w}}\cos\beta_w + C_{M_{Y_w}}\sin\beta_w \\
C_{M_{Y_h}} &= -C_{M_{X_w}}\sin\beta_w + C_{M_{Y_w}}\cos\beta_w \\
C_{M_{Z_h}} &= C_{M_{Z_w}}
\end{aligned}
\tag{18}
$$

Main rotor loads in body axes

The contribution of main rotor in the total forces and moments acting on the aircraft is obtained from the transformation of its loads from hub axes to body axes as follows:

$$
\begin{aligned}
X_{mr} &= -C_{X_h}\rho A(\Omega R)^2 \\
Y_{mr} &= C_{Y_h}\rho A(\Omega R)^2 \\
Z_{mr} &= -C_{Z_h}\rho A(\Omega R)^2 \\
L_{mr} &= \left[-C_{M_{X_h}} - \frac{C_{Z_h} y_h}{R} + \frac{C_{Y_h} z_h}{R}\right]\rho R A(\Omega R)^2 \\
M_{mr} &= \left[C_{M_{Y_h}} + \frac{C_{X_h} z_h}{R} - \frac{C_{Z_h} x_h}{R}\right]\rho R A(\Omega R)^2 \\
N_{mr} &= -C_{M_{Z_h}}\rho R A(\Omega R)^2
\end{aligned}
\tag{19}
$$

Tail Rotor Model

Tail rotor is a powerful solution for torque balance in single main rotor helicopter. It is also a tool for direction stability and control Tail rotor is modeled as a teetering rotor without cyclic pitch input ($\theta_{1c}, \theta_{1s} = 0$). Flapping motion is not evident in tail rotor because the blades are rigid and shorter than main rotor blades. The velocity components acting on the tail rotor are as follows:

$$
\begin{aligned}
u_{tr} &= u \\
v_{tr} &= v + p\,z_{tr} - r\,l_{tr} \\
w_{tr} &= w + q l_{tr}
\end{aligned}
\tag{20}
$$

Tail rotor side slip angle is:

$$\beta_{w_{tr}} = \sin^{-1}\frac{w_{tr}}{u_{tr}} \tag{21}$$

Tail rotor advance ratio is:

$$\mu_{tr} = \frac{\sqrt{u_{tr}^2 + w_{tr}^2}}{\Omega_{tr} R_{tr}} \tag{22}$$

Aircraft pitch and yaw rates are transformed to the tail rotor axis as follows:

$$\overline{p_{tr}} = \frac{p\cos\beta_{w_{tr}} + r\sin\beta_{w_{tr}}}{\Omega_{tr}} \tag{23}$$

The induced inflow ratio of tail rotor is given by:

$$\lambda_{tr} = \frac{v_{tr}}{\Omega_{tr} R_{tr}} + \frac{C_{T_{tr}}}{2\sqrt{\mu_{tr}^2 + \lambda_{tr}^2}} \tag{24}$$

By applying blade element on the tail rotor blade section, the tail rotor thrust coefficient is obtained as follows:

$$C_{T_{tr}} = \frac{\sigma_{tr} a_{tr}}{2}\left[\frac{\theta_{0\,tr}}{3}\left(1 + \frac{3}{2}\mu_{tr}^2\right) + \frac{\theta_{tw_{tr}}}{4}(1 + \mu_{tr}^2) - \frac{\lambda_{tr}}{2} + \frac{\mu_{tr}}{4}\overline{p_{tr}}\right] \tag{25}$$

Tail rotor thrust is:

$$T_{tr} = C_{T_{tr}}\rho A_{tr}(\Omega_{tr} R_{tr})^2 \tag{26}$$

Tail rotor loads in body axes system are as follows:

$$Y_{tr} = T_{tr} \quad (27)$$
$$L_{tr} = T_{Tr} z_{tr}$$
$$N_{tr} = -T_{tr} l_{tr}$$

Fuselage Model

For calculation the fuselage forces and moments, it is assumed that the longitudinal forces are dependent on the angle of attack and lateral forces are dependent on the side-slip angle. The drag force is dependent on both angle of attack and side-slip angle. The velocity components on the fuselage are as follows:

$$u_f = u$$
$$v_f = v \quad (28)$$
$$w_f = w + w_{if}$$

where $w_{if}$is the induced velocity due to main rotor and can be expressed as:

$$w_{if} = \left(\frac{w_{if}}{v_i}\right) v_i \quad (29)$$

where v_i the main rotor is induced velocity and $\left(\frac{w_{if}}{v_i}\right)$ defined as the following empirical relation [12]:

$$\left(\frac{w_{if}}{v_i}\right) = 1.299 + 0.671\chi - 1.172\chi^2 + 0.35\chi^3$$
$$\chi = \tan^{-1}\frac{\mu}{-\lambda} \quad (30)$$
$$v_i = \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}(\Omega R)$$

The fuselage angle of attack is:

$$\alpha_f = \tan^{-1}\frac{w_f}{u_f} \quad (31)$$

The fuselage side slip angle is:

$$\beta_f = \sin^{-1}\frac{v_f}{V_f} \quad (32)$$

where: $V_f = \sqrt{u_f^2 + v_f^2 + w_f^2}$. The fuselage drag force is

$$D_f = \frac{1}{2}\rho V_f^2 f \quad (33)$$

The fuselage drag force coefficient is:

$$C_{D_f} = \frac{\frac{1}{2}\mu_f^2 f}{A} \quad (34)$$

where, $\mu_f = \frac{V_f}{\Omega R}$ .The fuselage load is transformed to the body axes at aircraft center of gravity as follows:

$$X_f = (-C_{D_f} \cos\alpha_f \cos\beta_f)\rho A(\Omega R)^2$$
$$Y_f = (-C_{D_f} \sin\beta_f)\rho A(\Omega R)^2 \quad (35)$$
$$Z_f = (-C_{D_f} \sin\alpha_f \cos\beta_f)\rho A(\Omega R)^2$$

4. General Trim Problem

The general trim problem is the helicopter trim at a steady turn maneuver in a spin mode (spiral climb or decent) with horizontal turn rate $\dot{\Psi}$. During this maneuver, the spin axis is vertically and there isn't any change in fuselage roll and pitch attitude (ϕ_e, θ_e), so the components of weight force in body axes are constant. In general trim problem, velocity vector magnitude V_{fe}does not change and helicopter is flying with a side slip angle β and spin angle γ_{fe}. Fig. 3 shows the general trim problem flight conditions. . By defining the following four quantities $(\gamma_f, V_f, \beta, \dot{\Psi})$, any maneuver can be defined. That is the cause of choosing these variables. For example, at steady forward flight, the maneuver is defined by putting β,γ_f, and $\dot{\Psi}$ zeros. Another example, to define horizontal turn maneuver, put $\dot{\Psi}$ by the horizontal turn rate, flight path angle is zero and side slip angle is also zero.

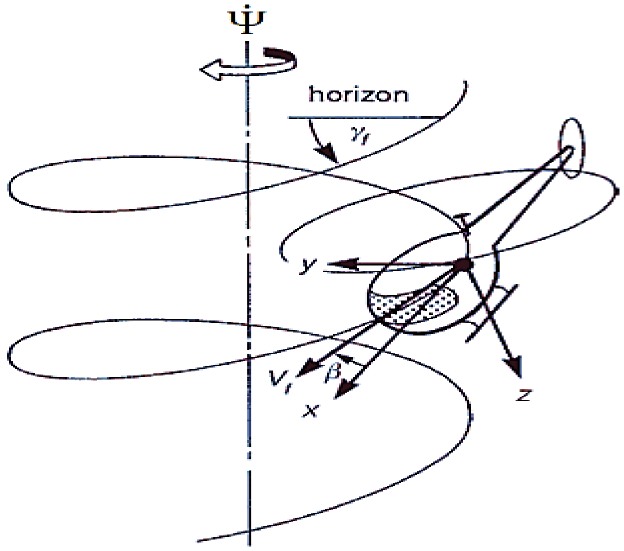

Fig. 3. General trim problem flight conditions [2]

The trim equations are non-linear algebraic equations as follows:

$$\begin{aligned}
&-(w_e q_e - v_e r_e) + \frac{X_e}{m} - g\sin\theta_e = 0\\
&-(u_e r_e - w_e p_e) + \frac{Y_e}{m} + g\cos\theta_e \sin\phi_e = 0\\
&-(v_e p_e - u_e q_e) + \frac{Z_e}{m} + g\cos\theta_e \cos\phi_e = 0\\
&(I_{yy} - I_{zz}) q_e r_e + I_{xz} p_e q_e + L_e = 0\\
&(I_{zz} - I_{xx}) r_e p_e + I_{xz}(r_e^2 - p_e^2) + M_e = 0\\
&(I_{xx} - I_{yy}) p_e q_e + I_{xz} q_e r_e + N_e = 0
\end{aligned} \tag{36}$$

where: The quantities $X_e, Y_e, Z_e, L_e, M_e, N_e$ are the aerodynamic forces acting on helicopter due to main rotor, tail rotor and fuselage. These forces and moments are functions of four control inputs $(\theta_0, \theta_{1c}, \theta_{1s}, \theta_{0_{tr}})$, translational velocities $(u_e,, v_e, w_e)$, and angular velocities (p_e, q_e, r_e). Equations (36) are six equations in twelve unknowns: translational velocities (u_e, v_e, w_e), angular velocities (p_e, q_e, r_e), four control inputs $(\theta_0, \theta_{1c}, \theta_{1s}, \theta_{0_{tr}})$, roll angle ϕ_e, and pitch angle θ_e. The four given quantities are: flight velocity V_{fe}, spin angle γ_{fe}, horizontal turn rate $\dot{\Psi}$, and side slip angle β.

The steady roll, pitch and yaw rates are related to the spin rate as:

$$\begin{aligned}
p_e &= -\dot{\Psi}\sin\theta_e\\
q_e &= \dot{\Psi}\sin\phi_e \cos\theta_e\\
r_e &= \dot{\Psi}\cos\phi_e \cos\theta_e
\end{aligned} \tag{37}$$

The velocity vector V_{fe}makes an angle $\gamma_{fe}$with the horizontal plane as shown in Figure 4.2. Then,

$$\dot{x} = V_{fe} \cos\gamma_{fe} \cos\chi$$
$$\dot{y} = V_{fe} \cos\gamma_{fe} \sin\chi \qquad (38)$$
$$\dot{z} = V_{fe} \sin\gamma_{fe}$$

where: χ is the angle between projection of velocity vector and X_0. Using the transformation matrix R between earth fixed axes and body axes.

$$u_e = V_{fe}\left[\cos\gamma_{fe} \cos\theta_e \cos\chi_e - \sin\gamma_{fe} \sin\theta_e\right] \qquad (39)$$

$$v_e = V_{fe}\left[\cos\gamma_{fe} \sin\phi_e \sin\theta_e \cos\chi_e + \cos\gamma_{fe} \cos\phi_e \sin\chi_e + \sin\gamma_{fe} \sin\phi_e \cos\theta_e\right] \qquad (40)$$

$$w_e = V_{fe}\left[\cos\gamma_{fe} \cos\phi_e \sin\theta_e \cos\chi_e - \cos\gamma_{fe} \sin\phi_e \sin\chi_e + \sin\gamma_{fe} \cos\phi_e \cos\theta_e\right] \qquad (41)$$

where: $\chi_e = \chi - \psi$ is the track angle. This angle is constant with time, but the heading angle changes with time $\psi = \dot{\Psi}t$. Track angle is the angle between velocity vector and $X_b$axis projected in the earth horizontal plane. From the definition of side slip angle $\beta = \sin^{-1}\frac{v_e}{V_{fe}}$, eq. (40) will be:

$$\sin\beta = \left[\cos\gamma_{fe} \sin\phi_e \sin\theta_e \cos\chi_e + \cos\gamma_{fe} \cos\phi_e \sin\chi_e + \sin\gamma_{fe} \sin\phi_e \cos\theta_e\right] \qquad (42)$$

Solution of General Trim Problem

From equations (37), the angular velocity components (p_e, q_e, r_e) are function of given variables and (ϕ_e, θ_e). Hence, the six equilibrium equations (36) and equations (39, 41, and 42) are nonlinear nine equations in nine unknowns $(\theta_0, \theta_{1c}, \theta_{1s}, \theta_{0_{tr}}, \phi_e, \theta_e, u_e, w_e, \chi_e)$ and can be solved by any numerical iterative method (Newton Raphson method) or fsolve matlab function.

5. Direct Simulation Procedure

Direct simulation for helicopter is a technique used to estimate the airplane motion and represent the time history of the change of the six kinematic components $(x, y, z, \phi, \theta, \psi)$, the translational and angular velocity components (u, v, w, p, q, r) for any control input $(\theta_0, \theta_{1s}, \theta_{1c}, \theta_{0_{tr}})$. This method is used to predict the helicopter response to a sequence of control inputs. It is conventionally called initial value problem.

Helicopter model consists of six dynamic equations and six kinematic equations which can be represented as

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, u), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \qquad (43)$$

where: The vectors $\boldsymbol{x}$ and u represent the helicopter states and control inputs respectively. To get the aircraft response to any control input u as function of time, these equations are integrated by Runge-kutta method [13]

Fig. 4 represents the direct simulation procedure. The control inputs and previous states are the inputs for the helicopter aerodynamic model to get the total aerodynamic forces and moments acting on the helicopter. Then, these forces and moments are inserted in the vehicle equations of motion derived by Newton's second law. Finally, the model equations are integrated by using Runge-kutta method to get the helicopter response due to any set of control inputs.

6. Results and Discussion

The complete non-linear model is solved at the trim conditions (steady flight) to get the control inputs required to trim the vehicle at any flight conditions.

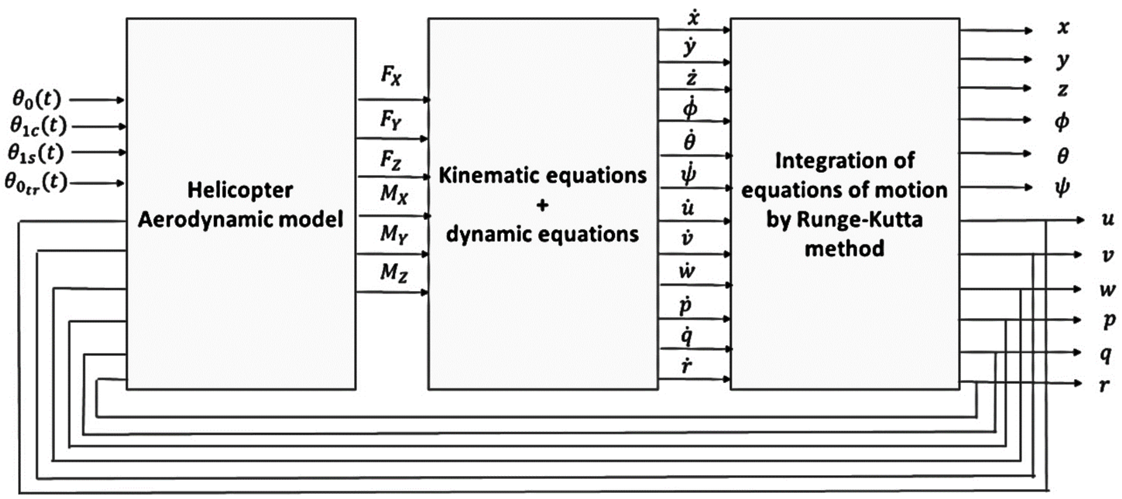

Fig. 4. Helicopter direct simulation procedure

Trim at Several Flight Path Angles

Fig.5 shows the four control inputs required to trim the helicopter at several flight path angles and different flight speeds. As shown, the main rotor collective pitch increases as the flight path angle increases which means that the vehicle needs more thrust to raise its altitude. Also, the longitudinal cyclic become more negative as increasing the flight path angle since the main rotor must tilt forward by a higher angle to support the weight of the airplane. Due to the coupling between the longitudinal and lateral cyclic, the lateral cyclic changes and the tail rotor control also changes to balance the helicopter laterally.

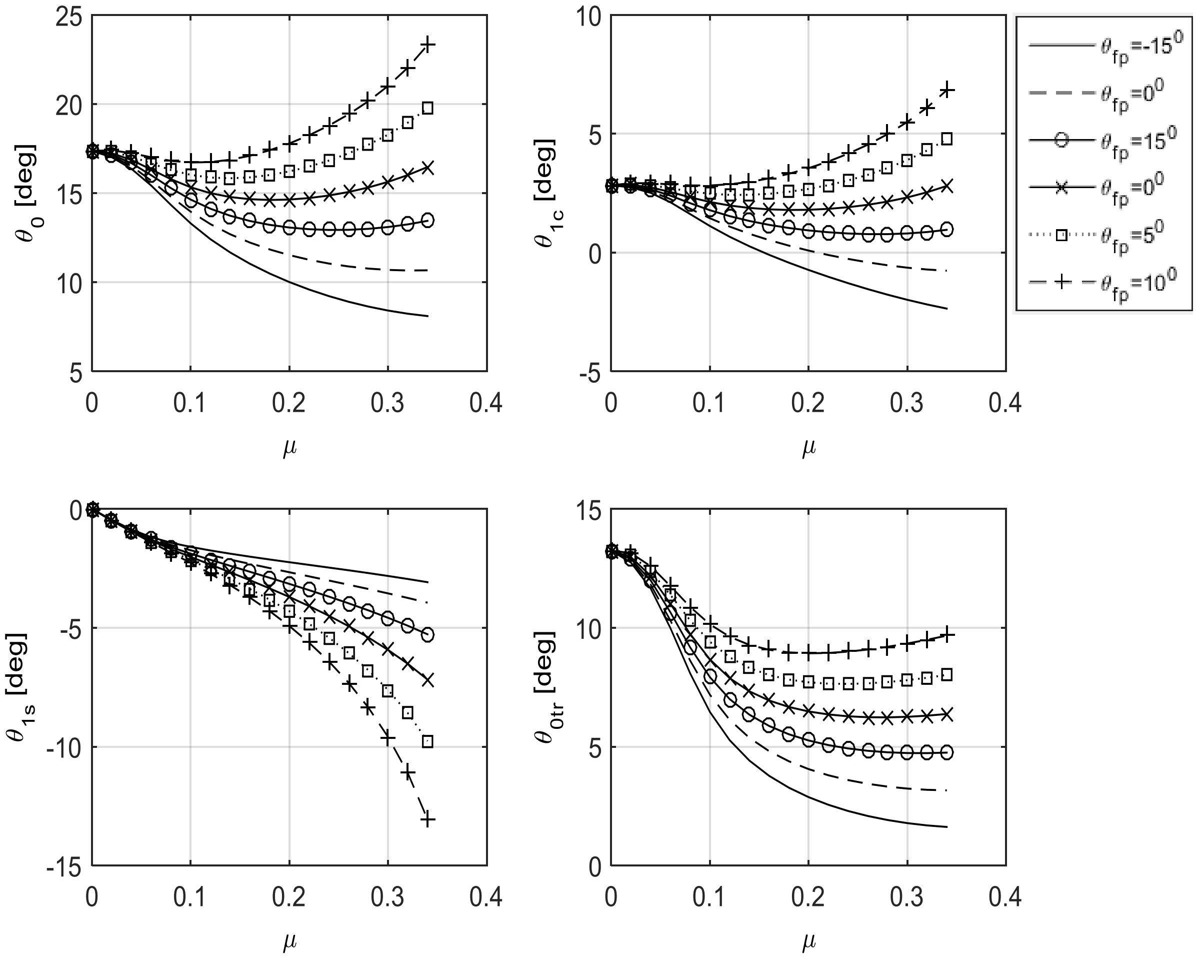

Fig. 5. Trim control inputs at several flight path angles

Trim at Several Side Slip Angles

Fig 6 presents the change of the required control inputs at various side slip angles. The main change is observed in the lateral cyclic pitch and tail rotor collective pitch.

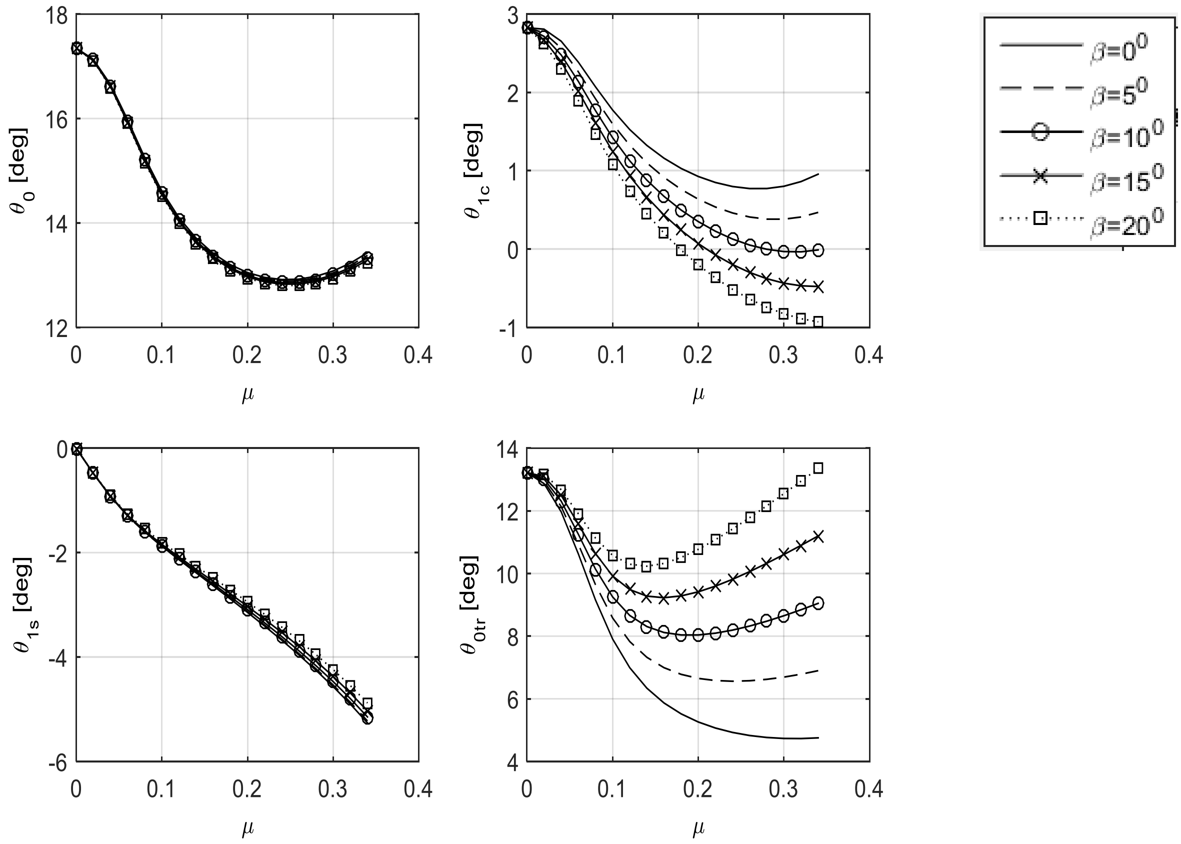

Fig. 6. Trim control inputs at several side slip angles

General Trim Results

The general trim problem is solved at turn rate 0.1 rad/sec., spin angle -5 deg., side slip angle 0 deg., and an advance ratio 0.3 then the required control inputs and helicopter attitudes at the trim condition are as follows:

Table.2 Trim control inputs and helicopter attitudes at $\boldsymbol{\mu}$ = 0.3, $\boldsymbol{\beta}$=0 deg., $\boldsymbol{\gamma_{fe}}$=-5 deg., $\dot{\boldsymbol{\Psi}}$=0.1 rad. /sec.

Main rotor collective pitch θ_0	14.3541 deg.
Longitudinal cyclic pitch θ_{1s}	-3.2058 deg.
Lateral cyclic pitch θ_{1c}	0.9255 deg.
Tail rotor collective pitch $\theta_{0_{tr}}$	12.2436 deg.
Helicopter roll angle ϕ_e	30.6468 deg.
Helicopter pitch angle θ_e	-4.9459 deg.

Verification of General Trim Results by Direct Simulation

The of general trim problem is achieved by supplying the trim control inputs as a constant function of time to direct simulation code, the simulation code that integrates the equations of motion by Runge-Kutta method. The results of this code indicates that the vehicle moves in a steady flight with the required trim conditions as shown in Fig. 6

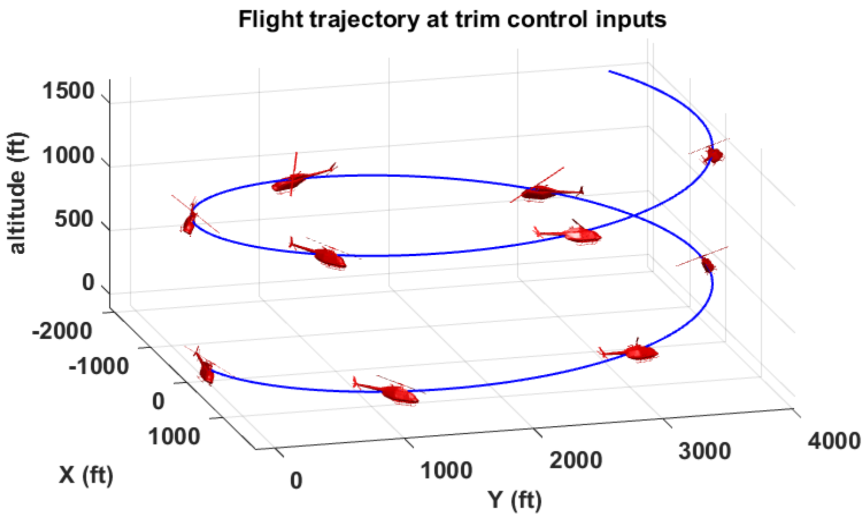

Fig. 6 Flight trajectory at trim control inputs

These results demonstrate the accuracy and correctness of the direct simulation and the trim code outputs.

7. Conclusion

In this paper, General expressions for aerodynamic forces and moments acting on helicopter due to its main components, main rotor, tail rotor and fuselage, at any flight condition are derived in detailed by using momentum theory and blade element theory. The obtained expressions from aerodynamic loads are used in the rigid body equations of motion, derived by Newton’s second law, to get a complete non-linear model for helicopter. The vehicle response to any set of control inputs is determined by this general model. This model is used in designing the control of autonomous helicopter and the automatic helicopter pilot. Many maneuvers are achieved by this model.

The first step in helicopter controller design is the linearization of vehicle model. The linearization process is about the trim point. In this study, the trim problem is solved at any flight condition; forward flight with several flight path angles, side slip angles and turn maneuvers. The solution of trim problem is determining the control inputs and helicopter pitch and roll attitudes required to fly helicopter at a steady maneuvers.

The verification of trim results is verified by direct simulation code. The resulted control input is supplied to the direct simulation code as a constant function of time. The resultant states from the simulation code are also constant with time which means that the helicopter in a steady flight and moving in the trim conditions. These results asserts that the trim results and the direct simulation code are accurate and correct.

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