Paper: ASAT-17-017-MO

17th International Conference on
AEROSPACE SCIENCES & AVIATION TECHNOLOGY,
ASAT - 17 – April 11 - 13, 2017, E-Mail: asat@mtc.edu.eg
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Inverse Simulation of a Full-Scale Helicopter Using Finite Difference Technique

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Abstract: Inverse simulation is a computational method that determines the control inputs required for a dynamic system to achieve a desired output. In case of helicopter dynamics, inverse simulation is used to obtain the pilot control inputs required for the helicopter to accomplish a desired maneuver. Complex configuration of helicopter makes its model inversion is significantly more difficult than fixed wing aircrafts. In this paper, a general method, used to define any helicopter maneuver in earth axes system, is introduced. An algorithm, for solving the helicopter inverse simulation problem at a given maneuver by using the differentiation approach, is presented. This method is based on converting the model nonlinear differential equations to algebraic difference equations which can be solved at each time step by an iterative scheme. The accuracy of this technique is improved by increasing the order of the finite difference scheme and decreasing the time step. The verification of the inverse simulation results is achieved by supplying the resultant control inputs to the direct simulation code and the helicopter flies in the desired maneuver.

Keywords: Helicopter, Inverse Dynamics, Simulation, Finite Difference Technique.

1. Introduction

The conventional approach for aircraft simulation is to develop a mathematical model and compute its response to a set of a control inputs [1]. This kind of simulation is denoted as s direct simulation and can be used in pilot training, vehicle evaluation, investigation of flying handling qualities, and examining the aircraft stability characteristics. The direct simulation is applied to fixed wing aircrafts and helicopter and the results are compared with flight test data [2]. The simulation single main rotor and tail rotor helicopter at a specific flight conditions after simplifying the equations of motion is introduced in [3] and [4]. The direct simulation is used in control system design [3]. In the other hand, inverse simulation is the problem of determining the control inputs required to force the vehicle to achieve a desired maneuver. The helicopter complex configuration and coupling between its control inputs make its model inversion more difficult than fixed wing aircraft. The inverse simulation was successful in design controller in fixed wing aircrafts [5] and [6]. The system dynamic inversion is a powerful tool in control system design. In [7], a controller based on the inverse simulation of linearized system is designed. One of the main reason for inverse simulation development is that it can be used in feed-forward controller design to obtain a precision tracking for a desired trajectory [8]. Dynamic inversion is also used to avoid the need for the implementation of gain scheduling to ensure the stability of control system over the operational envelop [8]. This approach attracted significant attention in the aerospace engineering

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because it generates the forward control inputs to the mathematical model to follow a required trajectory. Many contributions were published in the development of numerical stability of inverse simulation schemes [9],[10] and[11]. Inverse simulation is used in the investigation of helicopter handling qualities and maneuverability at the stage of conceptual design [11]. Inverse simulation was used in investigation of control inputs required when engine failed through take-off from offshore platform [12]. Many publications used inverse simulation techniques to design the inversion based controllers such as [13]. Ref. [14] and [15] introduce the global optimization method in solving inverse simulation problem. In this paper the helicopter inverse simulation problem is solved by using finite difference method which is based on converting the model non-linear differential equations to algebraic equations which can be solved at each time step by any numerical technique such as Newton Raphson method.
The prouty example helicopter [17] is used in calculations in this study. Its characteristics could be summarized in Table 1.

Table 1: Prouty example helicopter characteristics

Property	Value	
	British units	SI units
Design gross weight	20000 lb.	9070 Kg
Main rotor radius	30 ft.	9.14 m
Main rotor tip speed	650 ft./sec.	198.12 m/s
Tail rotor radius	6.5 ft.	1.98 m.
Tail rotor tip speed	650 ft./sec.	198.12 m/s
Fuselage length	57 ft.	17.37 m
Main rotor height above CG	6 ft.	1.8 m.
Tail rotor arm	37 ft.	11.28 m

2. Helicopter Mathematical Model

Helicopter is treated as a rigid body of six degrees of freedom. Newton's second law can be applied to get six dynamic equations: three equations in the translational motion and three equations in the rotational motion. There are six kinematic equations relating the angular velocities of helicopter to the rate of change of its orientation or attitude w.r.t the fixed earth axis. The six kinematic equations are obtained from Euler angle transformation [16]. The system of equations that presents helicopter model is as:

$$\begin{aligned}
\dot{u} &= -(qw - rv) + X/m - g\sin\theta \\
\dot{v} &= -(ru - pw) + Y/m + g\sin\phi\cos\theta \\
\dot{w} &= -(pv - qu) + Z/m + g\cos\phi\cos\theta \\
\dot{p} &= (I_{zz}L^* + I_{xz}N^*)/(I_{xx}I_{zz} - I_{xz}^2) \\
\dot{q} &= (M + (I_{zz} - I_{xx})rp + I_{xz}(r^2 - p^2))/(I_{yy}) \\
\dot{r} &= (I_{xz}L^* + I_{xx}N^*)/(I_{xx}I_{zz} - I_{xz}^2) \\
\dot{\phi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\
\dot{\theta} &= q\cos\phi - r\sin\phi \\
\dot{\psi} &= q sin\phi\sec\theta + r\cos\phi\sec\theta \\
\dot{x} &= u\, C\theta C\psi + v(S\phi S\theta C\psi - C\phi S\psi) + w(C\phi S\theta C\psi + S\phi S\psi) \\
\dot{y} &= u\, C\theta S\psi + v(S\phi S\theta S\psi + C\phi C\psi) + w(C\phi S\theta S\psi - S\psi C\psi) \\
\dot{z} &= -u\, S\theta + v\, s\phi C\theta + w\, C\phi C\theta
\end{aligned} \tag{1}$$

where:

$$\begin{aligned}
L^* &= L + (I_{yy} - I_{zz})qr + I_{xz}pq \\
N^* &= N + (I_{xx} - I_{yy})pq - I_{xz}qr
\end{aligned} \tag{2}$$

where: The quantities (X, Y, Z, L, M, N) are the forces and moments acting on helicopter are due to main rotor, tail rotor and fuselage. The forces and moments acting on the helicopter due its main and tail rotors are calculated by obtaining the aerodynamic forces on the blade element and integrating these loads along the span of the blade. The lift and drag forces on the blade element are function of:

(i) The local air velocity which can be obtained by considering the helicopter velocity components at its center of gravity (u, v, w), the angular velocities (p, q, r), the blade element position relative to the C.G. (hub position, azimuth and span wise position of the blade section) and the flapping velocity.

(ii) The blade element angle of attack which is function of the blade section velocity and the control inputs: θ_0 main rotor collective pitch, θ_{1c} main rotor lateral cyclic pitch, θ_{1s} main rotor longitudinal cyclic pitch and $\theta_{0_{tr}}$ tail rotor collective pitch.

The fuselage forces are obtained by the same method used in the fixed wing aircraft by determining the fuselage angle of attack and side slip angles and taking into account the effect of main rotor induced velocity on the fuselage [4]. The only fuselage forces is the drag. This forces is transformed from fuselage wind axes to body axes system and is summed to the contribution of main and ail rotors to get a complete expressions for the total forces and moments acting on helicopter.

3. Inverse Simulation

Inverse simulation is a computational method that determines the dynamic system control inputs required to produce a desired output. The helicopter mathematical model that determines the states as response to known time history of control inputs is expressed as:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, u), \boldsymbol{x}(0) = \boldsymbol{x}_0 \qquad (3)$$

where: The vector y is the output vector. In inverse simulation, the output vector y is used to get the time history of control input. For helicopter motion, the output vector is the flight path or aircraft trajectory, so the flight path is the input to the inverse problem and the pilot control commands, required to produce that trajectory, are the output. To get a practical control inputs, the vector y must take proper smoothness. If any discontinuities appears in velocities, it makes unrealistic accelerations which require physically unachievable control commands and forces. Hence, determination of vector y is a necessary consideration in inverse problem. It is discussed in detail in the next section.

4. Maneuver Modeling

Helicopter maneuvers can be defined in terms of motion variables specified relative to earth coordinates. Any maneuver is completely defined by using four variable $(\dot{x}, \dot{y}, \dot{z}, \psi)$ as function of time[11]. The variable ψ is the helicopter heading and the other three variables are the helicopter velocity components as viewed by a stationary observer on the earth. These velocity components are related to (u, v, w) as following:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \psi \end{bmatrix} = \begin{bmatrix} \boldsymbol{C\theta C\psi} & \boldsymbol{S\phi S\theta C\psi - C\phi S\psi} & \boldsymbol{C\phi S\theta C\psi + S\phi S\psi} & \boldsymbol{0} \\ \boldsymbol{C\theta S\psi} & \boldsymbol{S\phi S\theta S\psi + C\phi C\psi} & \boldsymbol{C\phi S\theta S\psi - S\psi C\psi} & \boldsymbol{0} \\ \boldsymbol{-S\theta} & \boldsymbol{s\phi C\theta} & \boldsymbol{C\phi C\theta} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \\ \boldsymbol{w} \\ \boldsymbol{\psi} \end{bmatrix} \qquad (4)$$

In inverse simulation problem, $(\dot{x}, \dot{y}, \dot{z}, \psi)$ are given as function of time and are used in solving equations of motion to get the four control input required to fly the given maneuver.

Pop-Up Maneuver

Pop-up maneuver is longitudinal maneuver which means that the helicopter flight path is in the vertical plane. This maneuver is used to avoid obstacles such as mountain, buildings and trees by a rapid change in altitude during level flight as shown in Fig. 1

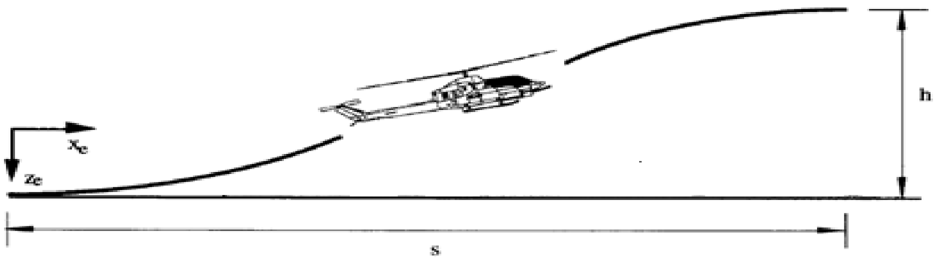


Fig. 1 Pop-up maneuver

Pop-up maneuver takes place in (x, z) plane , so ψ and $\dot{y}$ are zero during the flight. The altitude changes smoothly from zero to distance h during time interval t_f, the required to complete the maneuvre. The flight velocity V_{fe} is constant during the maneuver. The smoothness is achieved by a polynomial function as follows:

$$z(t) = -h\left[6\left(\frac{t}{t_f}\right)^5 - 15\left(\frac{t}{t_f}\right)^4 + 10\left(\frac{t}{t_f}\right)^3\right] \tag{5}$$

The horizontal velocity components is given by:

$$\dot{x}(t) = \sqrt{V_{fe}^2 - \dot{z}^2} \tag{6}$$

The main requirement is to guarantee a sufficient degree of smoothness at the entry and exit of the maneuver.

Side Step Maneuver

Side step maneuver is a lateral maneuver which means there is no change in altitude and the helicopter heading angle is assumed to be constant. The initial state of this maneuver can be hovering or any forward velocity. If the initial flight velocity $V_{fe}$is constant during flight, the helicopter will make S-shape maneuver in the horizontal plane. The time required to complete the maneuver is $t_f$and the distance in Y direction is y_f. To achieve an appropriate degree of smoothness, the following wave form is taken:

$$y(t) = \frac{y_f}{16}\left(\cos 3\pi \frac{t}{t_f} - 9\cos \pi \frac{t}{t_f} + 8\right) \tag{7}$$

Take- off from Hovering Maneuver

Take-off from hovering maneuver is also called maximum performance take-off. This maneuver is used to climb with a steep angle to avoid obstacles while operating in small areas. The velocity component in X direction changes smoothly from zero to the required value V_f at the required altitude.

$$\dot{x}(t) = V_{fe}\left[6\left(\frac{t}{t_f}\right)^5 - 15\left(\frac{t}{t_f}\right)^4 + 10\left(\frac{t}{t_f}\right)^3\right] \tag{8}$$

5. Solution of Inverse Simulation Problem by Finite Difference (First Order)

The inverse simulation problem is solved by using first order implicit scheme applied on helicopter mathematical model:

$$\frac{\boldsymbol{x}_n - \boldsymbol{x}_{n-1}}{\Delta t} = f(\boldsymbol{x}_n, u_n) \tag{9}$$

The solution procedure is

1- Given the previous state $\boldsymbol{x}_{n-1}$ and knowing $(\dot{x}_n, \dot{y}_n, \dot{z}_n, \psi_n)$from the helicopter flight path.
2- Apply numerical integration to get (x_n, y_n, z_n)
3- Solve the **twelve** equations in **twelve** unknowns: $(\theta_{0_n}, \theta_{1c_n}, \theta_{1s_n}, \theta_{0_{tr_n}}, u_n, v_n, w_n, p_n, q_n, r_n, \phi_n, \theta_n)$ at each time step by Newton Raphson method to get values for the next time step in a time marching manner.

Fig. 2 shows the block used in the solution of the inverse simulation problem.

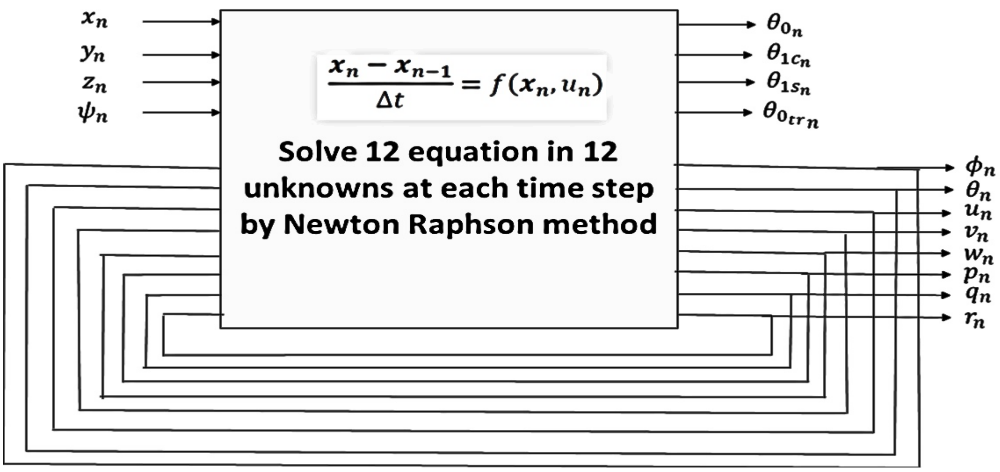


Fig. 2 The block used in the inverse simulation problem solution

Solution of twelve nonlinear algebraic equations in each time step by Newton Raphson method takes long time. The performance of simulation program can be improved by reducing the number of nonlinear algebraic equations required to be solved in each time step. This reduction is accomplished by sequencing the procedure of calculation in a certain way. The number of nonlinear algebraic equations is reduced to **six** equations in **six** unknowns $(\theta_{0_n}, \theta_{1c_n}, \theta_{1s_n}, \theta_{0_{tr_n}}, \phi_n, \theta_n)$. The procedure is:

1- Given the previous state $\boldsymbol{x}_{n-1}$ and knowing $(x_n, y_n, z_n, \psi_n)$from the helicopter flight path.
2- (u_n, v_n, w_n) are functions of (ϕ_n, θ_n) and given values as:

$$\begin{aligned}
u_n\Delta t &= C\theta_n C\psi_n(x_n - x_{n-1}) + C\theta_n S\psi_n(y_n - y_{n-1}) - S\theta_n(z_n - z_{n-1}) \\
v_n\Delta t &= (S\phi_n S\theta_n S\psi_n - C\phi_n S\psi_n)(x_n - x_{n-1}) \\
&\quad + (S\phi_n S\theta_n S\psi_n + C\phi_n C\psi_n)(y_n - y_{n-1}) + S\phi_n C\theta_n(z_n - z_{n-1}) \\
w_n\Delta t &= (C\phi_n S\theta_n C\psi_n + S\phi_n S\psi_n)(x_n - x_{n-1}) \\
&\quad + (C\phi_n S\theta_n S\psi_n - S\psi_n C\psi_n)(y_n - y_{n-1}) + C\phi_n C\theta_n(z_n - z_{n-1})
\end{aligned} \tag{10}$$

3- (p_n, q_n, r_n) are function of (ϕ_n, θ_n) are given values by kinematic relations as:

$$\begin{aligned}
p_n &= \frac{(\phi_n - \phi_{n-1}) - (\psi_n - \psi_{n-1})\sin\theta_n}{\Delta t} \\
q_n &= \frac{(\theta_n - \theta_{n-1})\cos\phi_n + (\psi_n - \psi_{n-1})\cos\theta_n \sin\phi_n}{\Delta t}
\end{aligned} \tag{11}$$

$$r_n = \frac{(\psi_n - \psi_{n-1})\cos\phi_n\cos\theta_n - (\theta_n - \theta_{n-1})\sin\phi_n}{\Delta t}$$

4- Solve the **six** dynamic equations in **six** unknowns $(\theta_{0_n}, \theta_{1c_n}, \theta_{1s_n}, \theta_{0_{tr_n}}, \phi_n, \theta_n)$by Newton Raphson method at each time step to get values for the next step.

The simulation results are obtained more rapid by using this reduction in number of nonlinear algebric equations. Fig. 3 shows the modified block used for solving the inverse simulation problem.

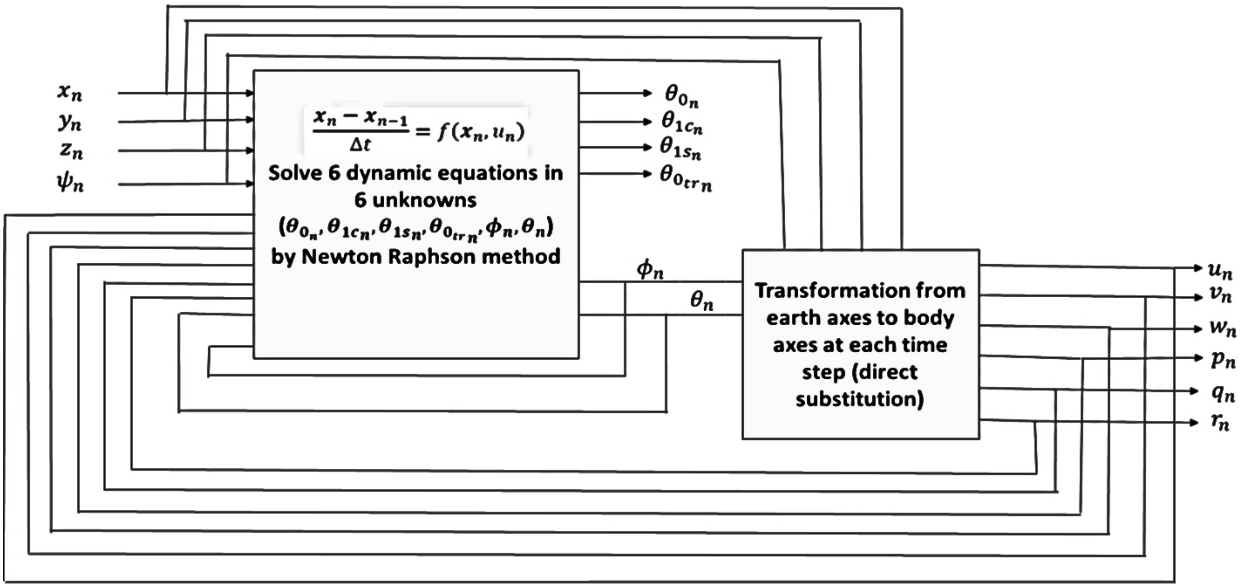


Fig. 3 The modified block used for solving the inverse simulation problem

6. Solution of Inverse Simulation Problem by Finite Difference (Second Order)

The second order scheme is used to improve the accuracy of inverse simulation solution. The truncation error is of order $(\Delta t)^2$. The second order implicit scheme is applied on helicopter mathematical model as follows:

$$\frac{3\boldsymbol{x}_n - 4\boldsymbol{x}_{n-1} + \boldsymbol{x}_{n-2}}{2\Delta t} = f(\boldsymbol{x}_n, u_n) \tag{12}$$

The first time step is solved by using first order scheme.
Solution procedure:

1- At first time step, given the previous state $\boldsymbol{x}_0$ from initial conditions and. apply the first order scheme and solve six dynamic equations to get unknowns variables $(\theta_{0_1}, \theta_{1c_1}, \theta_{1s_1}, \theta_{0_{tr_1}}, \phi_1, \theta_1)$.

2- From second time step to the end of time, given $\boldsymbol{x}_{n-1}, \boldsymbol{x}_{n-2}$, apply the second order scheme to solve the six dynamic equations to get the six unknowns $(\theta_{0_n}, \theta_{1c_n}, \theta_{1s_n}, \theta_{0_{tr_n}}, \phi_n, \theta_n)$ at n^{th} time step.

7. Results and Discussions

The solution of inverse simulation is introduced by determining the control inputs required to achieve any maneuver and the history of helicopter states $(u, v, w, p, q, r, \phi, \theta)$. The verification of the results is achieved by supplying the resultant control inputs to the direct simulation code and check if the helicopter flies on the required path or not and compare the states resulted from direct and inverse simulations. Comparison between the results of first and second order techniques is presented.

Pop-Up Maneuver

Helicopter need to avoid an obstacle of height 15 ft. during 5 sec and a distance on the earth 250 ft. as shown in Fig. 4.

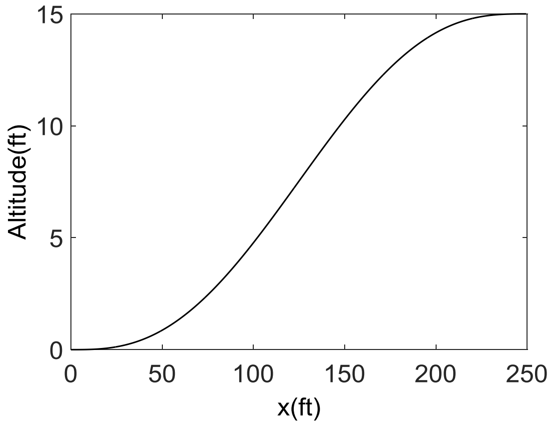
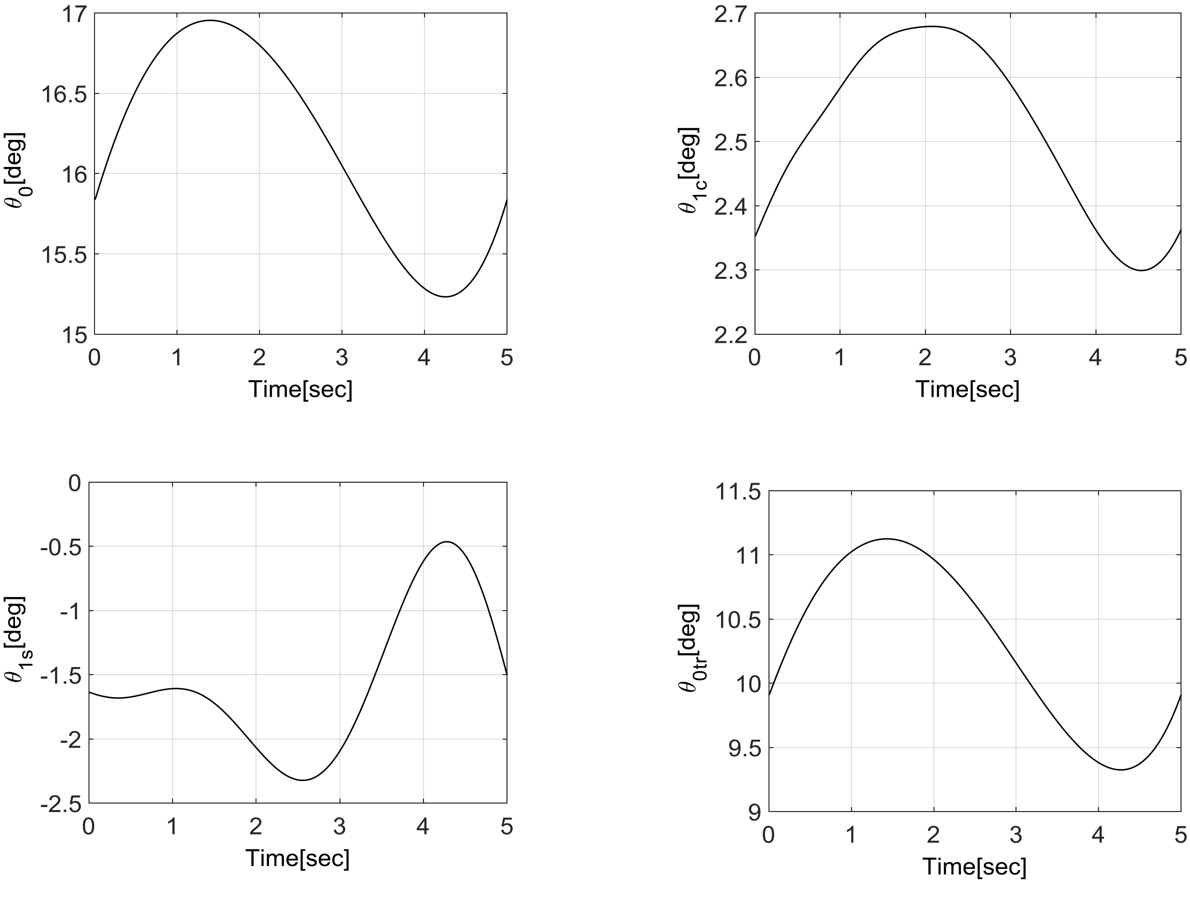


Fig. 4 Required pop-up maneuver

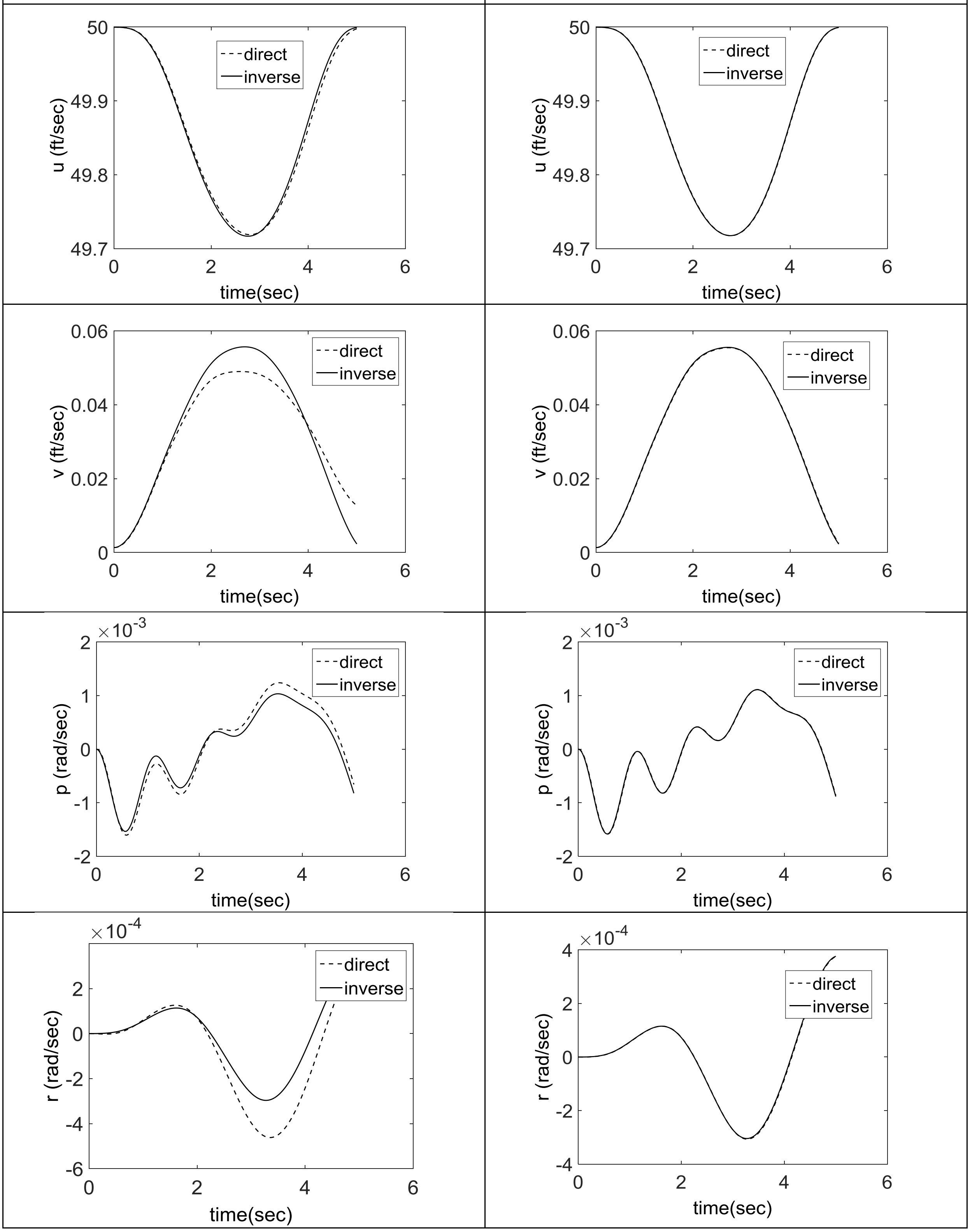
The inverse simulation technique is applied to get the control inputs required to achieve this path, then the four control inputs are as follows:

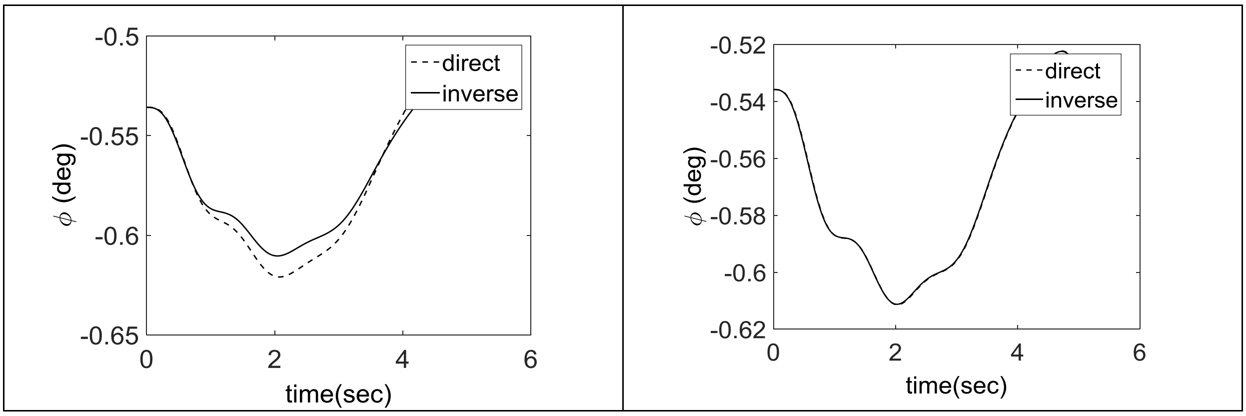


These control inputs are supplied to the direct simulation code, then the comparison between resultant states from direct simulation and inverse simulation is accomplished in the two cases of first order and second order schemes. The verification of the inverse simulation techniques is also obtained by comparison between the required maneuver and resultant maneuver from direct simulation under these inputs.

Verification of inverse simulation (first order – second order)($\Delta t = 0.01$sec)

First order	Second order





Resultant trajectory

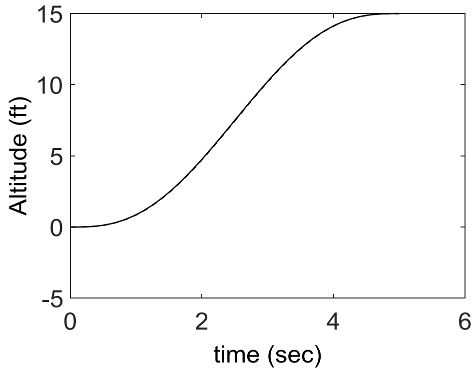


Fig. 5 Resultant trajectory from the resultant control inputs for pop-up maneuver

From the previous figures, it is obvious that the accuracy of second order scheme is higher than the first order. In the second order, the deviation of the resultant states from inverse and direct simulation is reduced. The cause of that reduction is that the direct simulation problem is solved by using fourth-order Runge-Kutta method, so as increasing the order of finite difference scheme, the difference between the results of direct and inverse simulations will reduce. The comparison between the resultant trajectory from direct simulation and the required trajectory is another tool of verification used in this paper

Side-Step Maneuver

In side step maneuver, helicopter moves at the same altitude and needs to move 15 ft. in Y direction in 5 seconds with flight velocity 30 ft. /sec.

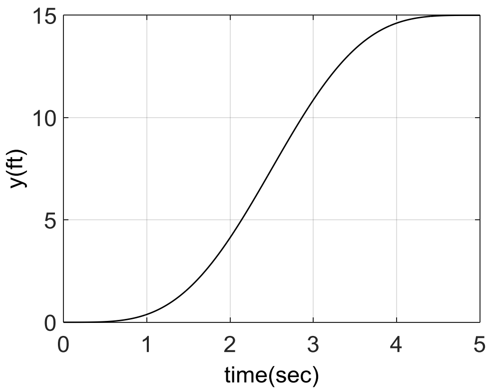
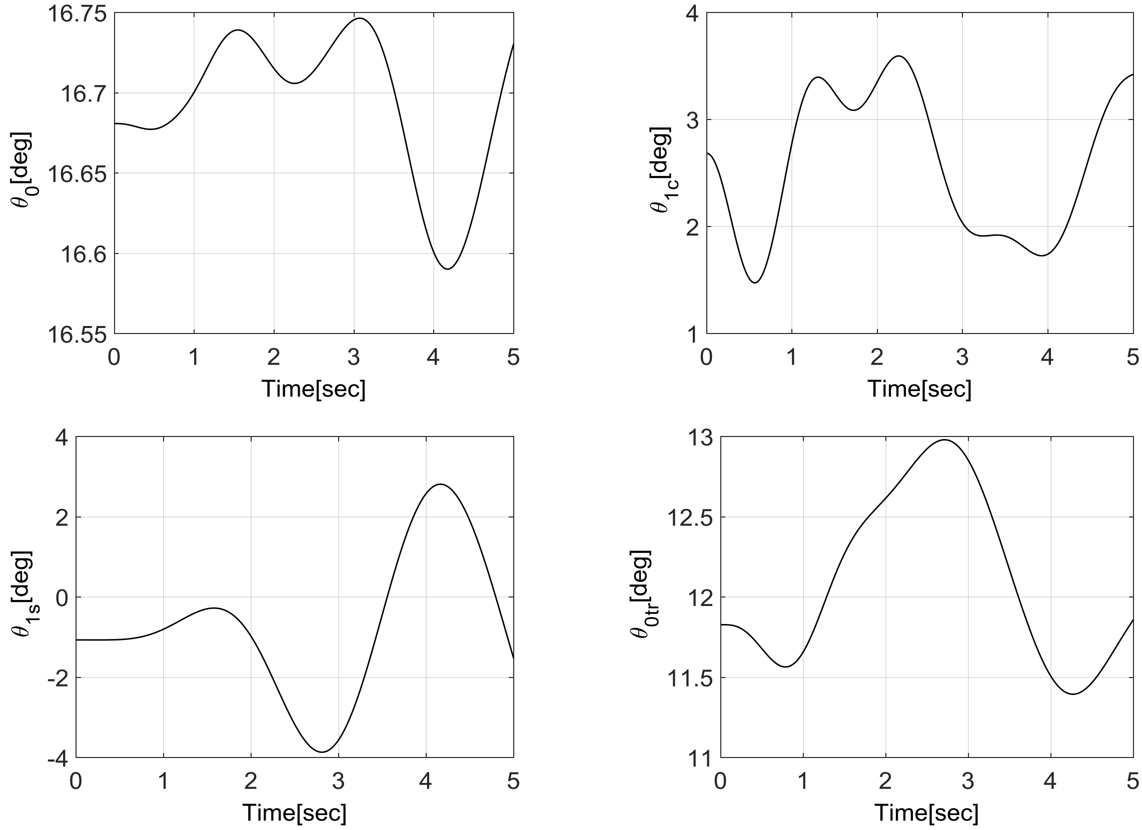
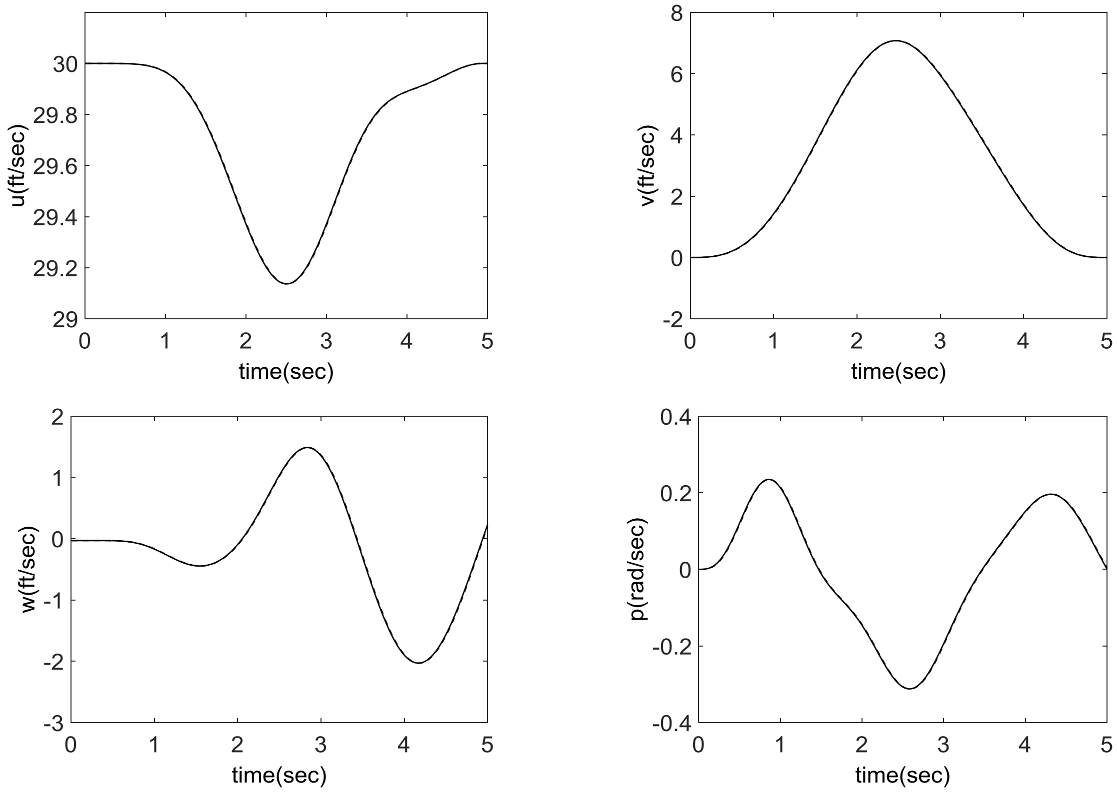


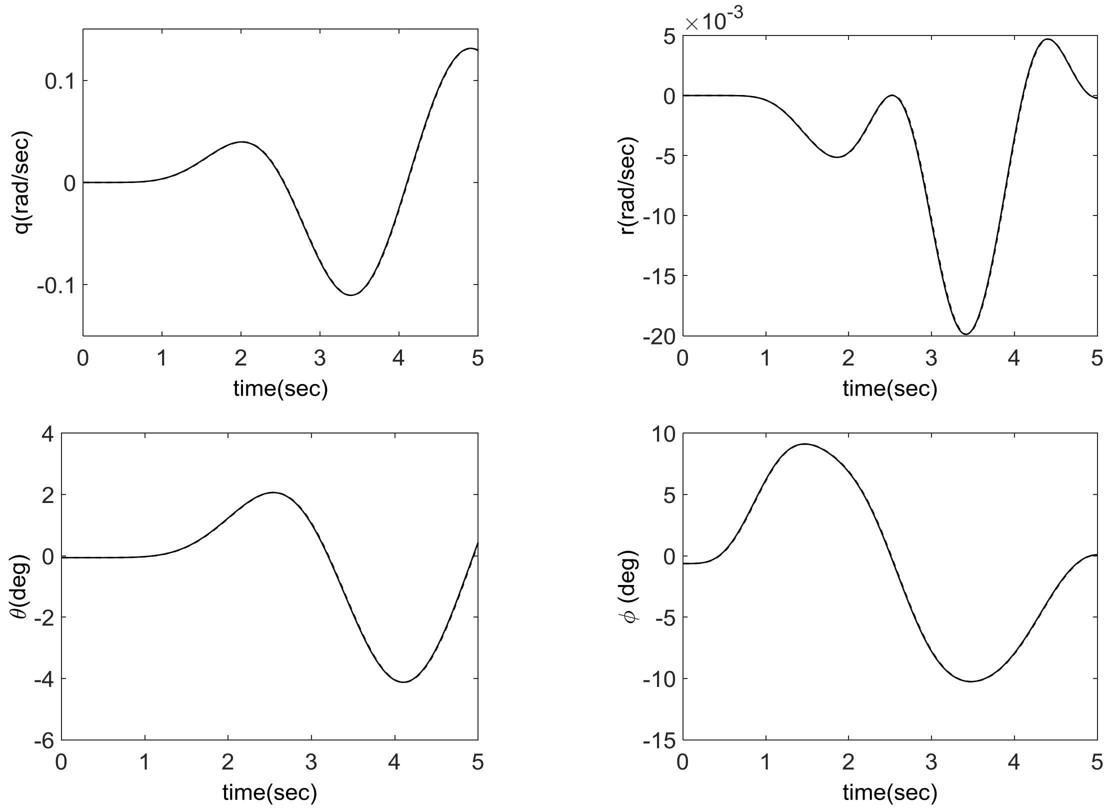
Fig. 6 Required side step maneuver

Required control inputs



Resultant states for side step maneuver ($\Delta t = 0.01$sec)





Resultant trajectory

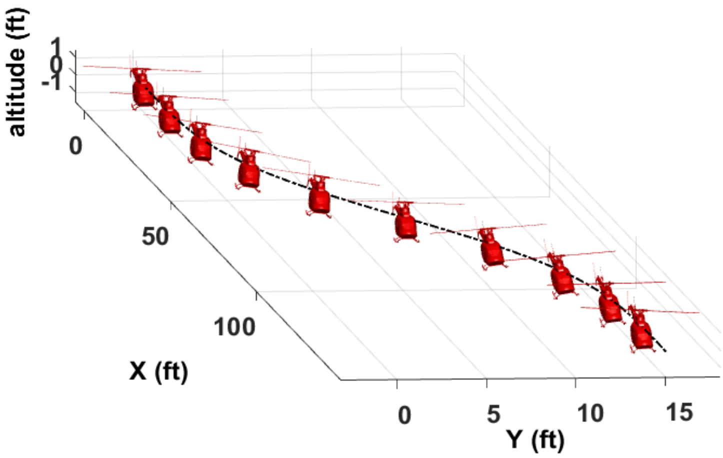


Fig. 7 Resultant trajectory from the resultant control inputs
for side step maneuver

Take-off from Hovering Maneuver

In take-off from maneuver, helicopter starts from hover flight and climb to a desired altitude with desired velocity. The required maneuver is that the helicopter climbs to altitude 50 ft. in 15 sec and starts from hover to velocity 30 ft. /sec as shown in Fig. 8

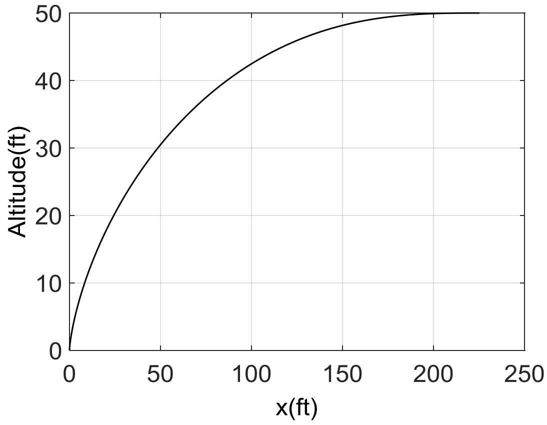
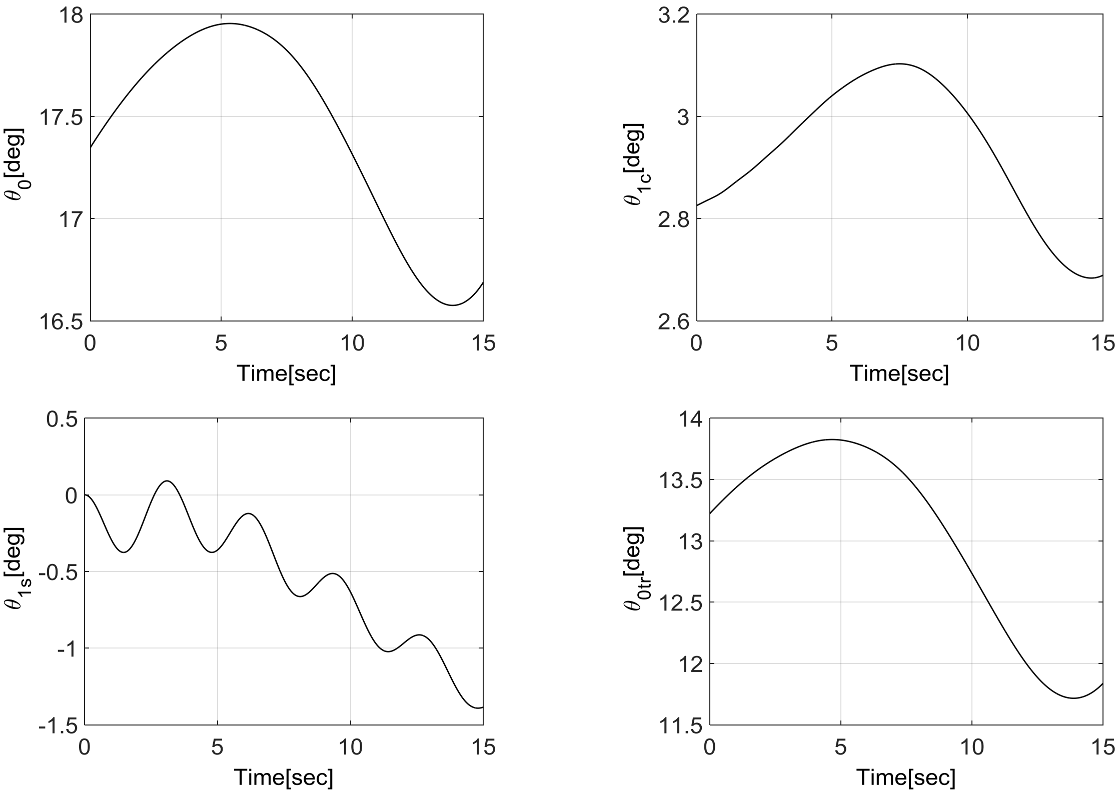
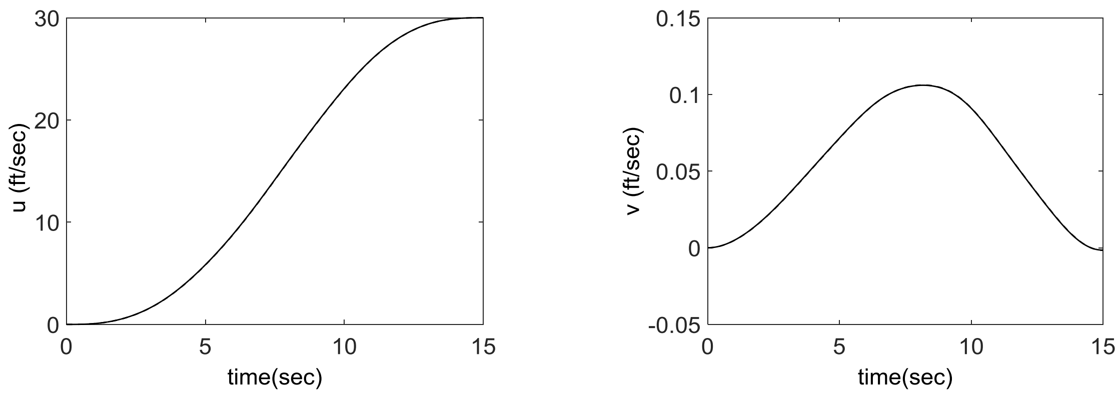


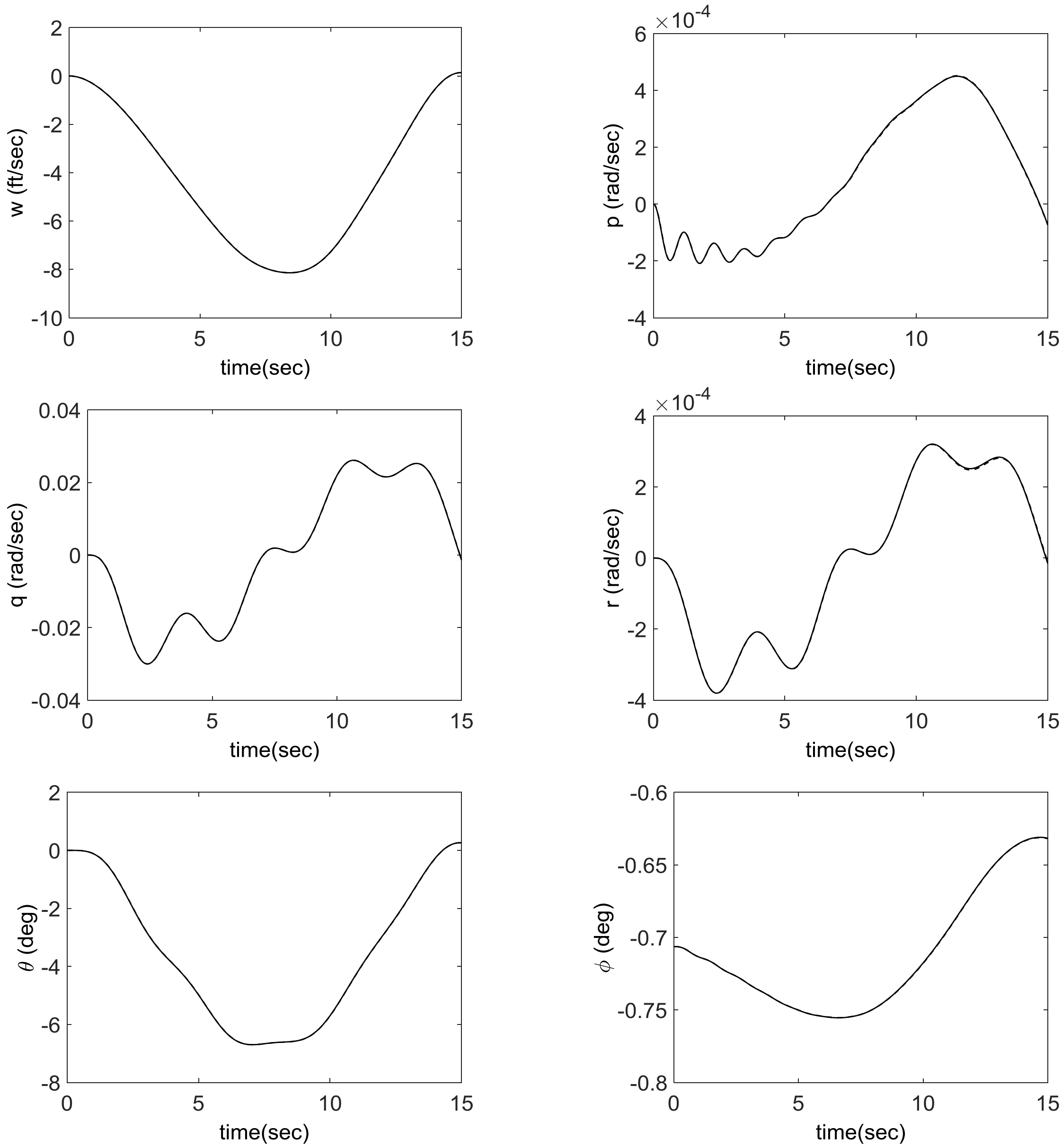
Fig. 8 Required take-off from hovering maneuver

Required control inputs for take-off from hovering maneuver ($\Delta t = 0.01$sec)



Resultant states for take-off from hovering maneuver





Resultant trajectory for take-off from hovering maneuver

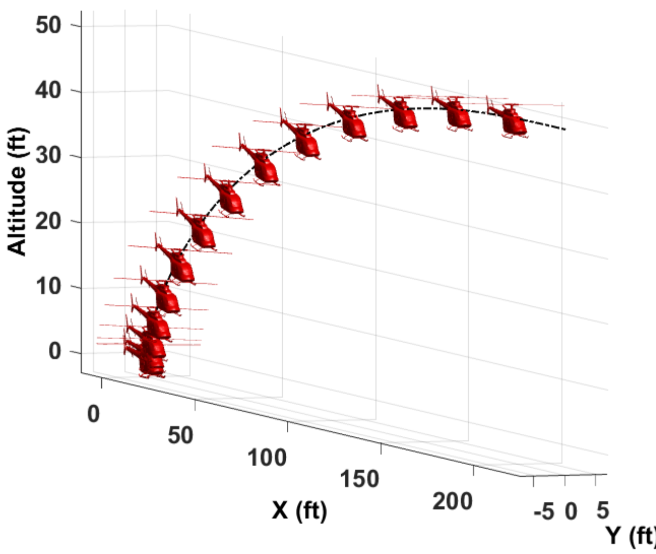


Fig. 9 Resultant trajectory from the resultant control inputs for pop-up maneuver

8. Conclusion and Discussion

This study presents a solution for the problem of helicopter inverse simulation. The time history of the change of control inputs required to achieve a specific maneuver is determined by the finite difference method. This method is based on converting the differential equations to algebraic equations by applying any differentiation scheme on the helicopter model.

The first order scheme is applied and the output controls are supplied to the direct simulation code to get the helicopter response and the flight trajectory due to these controls as a kind of verification and validation of inverse simulation. In case the first order scheme ($\Delta t = 0.01\ sec$) there is a small change in results from direct and inverse simulation. The reason of this change is that the direct simulation code is based on the integration of the equations of motion by fourth order Runge-kutta method not first order. The results of inverse simulation are improved by increasing the order of differentiation from first order to second order. The results are verified by the direct simulation code and the accuracy improved.

The acceleration of the input maneuvers to the inverse simulation should have a smoothness at least of second order to get a realist continuous change in the control inputs required by the pilot. The inverse simulation is considered to be an analytical flight test at which any mission is specified and the aircraft is forced to fly it. The results are the time history of the states and controls and performance information such as the required torque or power during this mission. The inverse simulation is used to study the effect of changing any configuration parameter on the performance. The inverse simulation techniques can be applied on systems other than helicopter because any dynamic system can be presented in a set of equations of motion.

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