17th International Conference on
AEROSPACE SCIENCES & AVIATION TECHNOLOGY,
ASAT - 17 – April 11 - 13, 2017, E-Mail: asat@mtc.edu.eg
Military Technical College, Kobry Elkobbah, Cairo, Egypt
Tel: +(202) 24025292 – 24036138, Fax: +(202) 22621908

Performance Analysis of FWM in 2-D Time-Spreading Wavelength-Hopping OCDMA Systems

Taher M. Bazan*

Abstract: A general formula for the bit error rate (BER) of 2-D time-spreading wavelength-hopping (TW) optical code division multiple access (OCDMA) systems under the impact of four-wave mixing (FWM) is derived. The results show the detrimental effect of FWM on the system performance at high chip power levels; resulting in a 4dB penalty at a power of 15dBm.

Keywords: Optical code division multiplexing, 2-D Wavelength-time coding, Four-wave mixing

1. Introduction

Optical code division multiple access (OCDMA) systems are promising techniques that satisfy asynchronous operations, all-optical processing, soft capacity on demand, potential security, easily quality of service control, and large user capacity in optical fiber networks [1-2]. One of favorite OCDMA approaches is the time-spreading wavelength-hopping (TW) scheme; exploiting the use of 2-D codes and providing enhanced correlation characteristics [3,4]. However, under the optical fiber nonlinearity, an optical mixing between chips may occur and generate other optical waves with new or existing operating frequencies. Such a process is called four-wave mixing (FWM) [5]. Recently, the total number of possible FWM products in TW OCDMA schemes for equally spaced wavelengths has been estimated where the FWM products are governed by five groups [6]. Up to now, no complete and clear mathematical model for the influence of FWM has been presented for 2-D TW OCDMA systems with coherent sources. Extending the work in [6], the target of this paper is to examine the impact of FWM on the bit error rate (BER) as a key figure of merit performance of 2-D TW OCDMA. Section II introduces general expressions of estimating the efficiency and power of the FWM in TW schemes. A mathematical model and related formulas for the BER of 2-D TW OCDMA are derived in Section III. Numerical results and discussion are presented in Section IV. Finally, some concluded notes are provided in Section V.

2. FWM in TW OCDMA

Consider a typical TW OCDMA system comprising *K* transmitters and receivers connected to a splitter/combiner as shown in Fig. 1.
Each user has a given 2-D TW code, and all users share the same set of wavelength *w*. After combining all the optical codes by the combiner, the FWM power is generated due to the

* Egyptian Armed Forces, Egypt; tbazan@hotmail.com

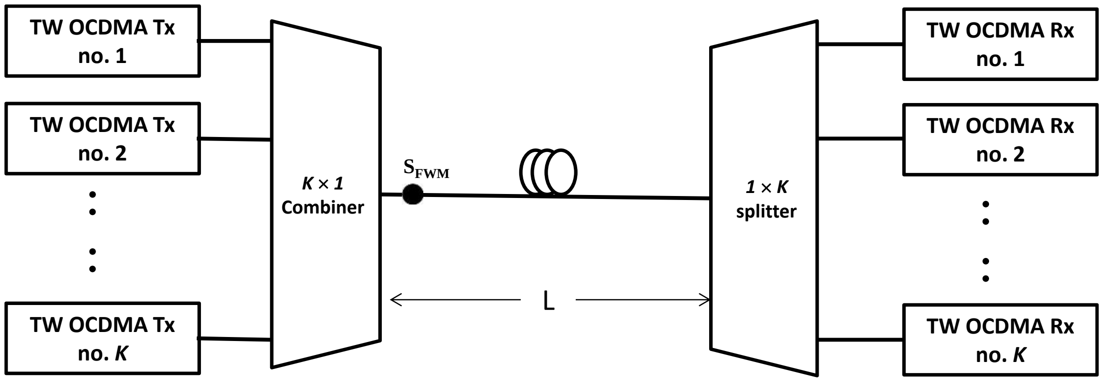

Fig. 1 Typical 2-D TW OCDMA system

nonlinear mixing of optical chips. The condition for the FWM process (starting at point S_{FWM}) when any three optical waves of different users *X*, *Y*, and *Z* with frequencies f_p^X, f_q^Y and f_r^Z interact and generate a new frequency f_l is governed by

$$f_l = f_p^X + f_q^Y - f_r^Z \quad (1)$$

If f_l coincides with one of the existing wavelengths of the employed TW codes, a coherent in-band crosstalk occurs, and FWM-induced power will be generated. Nevertheless, several frequency combinations (products) can meet the FWM condition. For TW OCDMA systems employing prime hop codes (PHCs) [3], a large number of FWM products can be generated and categorized into five groups (G_I, G_{II}, …, G_V) [6]. However, the FWM-induced power in TW OCDMA systems over fiber distance *L* can be expressed as follows:

$$P_{FWM}^{OCDMA} = \sum_{\substack{i\geq 2\\ X,Y,Z\in i}}^{K} \sum_{l=1}^{w} P_{FWM}(l, G, m_G, N_G)$$
$$= \sum_{\substack{i\geq 2\\ X,Y,Z\in i}}^{K} \sum_{l=1}^{w} \frac{\eta_{(l,G,m_G,N_G)}}{9} d_G^2\, \gamma^2\, P_p^X\, P_q^Y\, P_r^Z\, {L_{eff}}^2 \exp(-\alpha L) \quad (2)$$

Here, each FWM product can be defined by four factors: *l*, *G*, m_G, and N_G; *l* is the generated FWM frequency, *G* is the group order and *m* is the FWM product order within the group *G*, according to the set of FWM products generated at frequency *l*. N_G is the total number of FWM products at frequency *l* within group *G*. Note that all FWM products within the five groups are counted up when bit 1 is transmitted and groups I, III for bit 0. γ is the non-linear coefficient and P_p^X is the transmitted optical power per chip of frequency *p* related to the user X. Similarly, P_q^Y and P_r^Z are chip powers related to user Y and Z with frequencies *q* and *r*, respectively. L_{eff} is the effective fiber length, *d* is the degeneracy factor and α is the fiber loss coefficient. Finally, *L* is the transmission fiber length, and η is the FWM efficiency expressed. If the intended user transmit a bit 1, then, the generated FWM power is proportional to all products such that:

$$\sum_{l=1}^{w} P_{FWM}^{1}(l,G,m_G,N_G) = \sum_{l=1}^{w}\sum_{m_I=1}^{N_I} P_{FWM}(l,I,m_I,N_I) + \sum_{l=1}^{w}\sum_{m_{II}=1}^{N_{II}} P_{FWM}(l,II,m_{II},N_{II}) + \sum_{l=1}^{w}\sum_{m_{III}=1}^{N_{III}} P_{FWM}(l,III,m_{III},N_{III}) + \sum_{l=1}^{w}\sum_{m_{IV}=1}^{N_{IV}} P_{FWM}(l,IV,m_{IV},N_{IV}) + \sum_{l=1}^{w}\sum_{m_V=1}^{N_V} P_{FWM}(l,V,m_V,N_V) \quad (3)$$

When the intended user sends a bit 0, no FWM products related to G_{II}, G_{IV}, and G_V will be generated so that the total FWM power can be expressed as

$$\sum_{l=1}^{w} P_{FWM}^{0}(l,G,m_G,N_G) = \sum_{l=1}^{w}\sum_{m_I=1}^{N_I} P_{FWM}(l,I,m_I,N_I) + \sum_{l=1}^{w}\sum_{m_{III}=1}^{N_{III}} P_{FWM}(l,III,m_{III},N_{III}) \quad (4)$$

3. Performance Analysis of 2-D TW OCDMA

According to the typical 2-D TW OCDMA illustrated in Fig. 1 and the main model assumptions in [5,7-9], the received optical field at the photodetector of the desired user due to K simultaneous users (through complex notation) is written as

$$E(t) = E_d(t) + E_c(t) + E_{FWM}(t) \quad (5)$$

The first and second terms represent the electric fields of the desired and interferers, respectively, whereas the third is the total FWM electric field generated from the valid different chip frequency combinations of K users. Then,

$$\begin{aligned} E(t) \quad \propto \quad & \vec{V_d}\sum_{l=1}^{w}\sqrt{P_{d,l}(t)}\exp\left[j\left(2\pi f_{l,d}t+\theta_{l,d}(t)\right)\right] \\ & + \vec{V_c}\sum_{l=1}^{w}\sum_{i=1}^{k_l}\sqrt{P_{i,c}(t)}\exp\left[j\left(2\pi f_{l,i,c}(t-\tau_{l,i})+\theta_{l,i,c}(t-\tau_{l,i})\right)\right] \\ & + \vec{V}_{FWM}\sum_{l=1}^{w}\sum_{\substack{p,q,r\\ g=I,II,..,V}}\sqrt{P_{g,pqr,l}(t)}\exp[j\left(2\pi f_{l,g,pqr}(t-\tau_{l,g,pqr})\right. \\ & \left.+\theta_{l,g,pqr}(t-\tau_{l,g,pqr})\right)] \end{aligned} \quad (6)$$

where $\vec{V_d}, \vec{V_c}$ and $\vec{V}_{FWM}$ are the unit polarization vectors for the data, interferers, and FWM, respectively. w is the code weight, or equivalently the number of the available wavelengths assigned to all users and k_l is the number of interfering chips overlapped with the desired wavelengths λ_l. P_d(t) and $P_{i,c}$(t) are the instantaneous optical chip powers at the photodetector for desired user and ith interferer, respectively. Similarly, $P_{g,pqr,l}$(t) is the instantaneous FWM power at the photodetector due to a mixing of p, q, and r chips within the gth group; $g\in\{I,II,...,V\}$. $f_{l,d}$ is the desired data chip frequency f_l (corresponding to a chip wavelength λ_l) while $f_{l,i,c}$ is the interferer signal frequency corresponding to a chip wavelength λ_l originating from the ith interferer. $f_{l,g,pqr}$ is the FWM light of frequency f_l generated from a combination of p,q, and r frequencies in group g that satisfy the FWM condition. $\theta_{l,d}$, $\theta_{l,i,c}$,and $\theta_{l,g,pqr}$ are the laser phase noise for the desired signal, ith interferer phase corresponding to frequency f_l, and the phase of the new generated frequency f_l due to a specific FWM combination $p, q, and\ r$ frequencies in group g, respectively. $\tau_{l,i}$ is the propagation transit delay of the ith interferer relative to the data chip pulse at f_l. Similarly, let $\tau_{l,g,pqr}$ be the propagation transit delay of the new FWM signal due to the valid combination p, q, and r in gth group relative to the data pulse at f_l. Thus, according to the square law of the photodetector, and neglecting the thermal, shot, and relative intensity noise, the total output photocurrent is then expressed as:

$$i(t) = w\mathcal{R}P_d + K\mathcal{R}P_c + i_{BN}(t) + i_{FWM}(t) \tag{7}$$

where $\Re$ is the responsivity. e first, second, and third terms are the signal, MAI, and BN currents, respectively, while the fourth term represents the current due to FWM process.
After some manipulation, taking into consideration that the FWM noise components can be regarded as a mutual independent random Gaussian noise [7-9], the mean FWM current for a bit 1 can be written as

$$\langle i_{FWM}(1)\rangle = I_{FWM}^1 = \mathcal{R}\sum_{l=1}^{w} P_{FWM}^1(l, G, m_G, N_G) \tag{8}$$

where the summation in Eq. (8) can be computed according to Eq. (3). Similarly, for bit 0, it can be given by

$$\langle i_{FWM}(0)\rangle = I_{FWM}^0 = \mathcal{R}\sum_{l=1}^{w} P_{FWM}^0(l, G, m_G, N_G) \tag{9}$$

where the summation in Eq. (9) can be computed according to Eq. (3). Based on Eq. (8), the variance of the FWM current, when bit 1 is transmitted, can be expressed as

$$\sigma_{FWM}^2(1) = \sigma_{signal-FWM}^2 + \sigma_{interferer-FWM}^2(1) + \sigma_{FWM-FWM}^2(1) \tag{10}$$

$$\sigma_{FWM}^2(1) = 2\mathcal{R}^2\sum_{l=1}^{w} P_d P_{FWM}^1(l, G, m_G, N_G) + 2\mathcal{R}^2\sum_{l=1}^{w}\sum_{i=1}^{k_l} P_{i,c}P_{FWM}^1(l, G, m_G, N_G)$$
$$+ 2\mathcal{R}^2\sum_{l=1}^{w}\sum_{G=I}^{V}\sum_{m_G=1}^{N_G}\sum_{G'=I}^{V}\sum_{m'_{G'}=1}^{N'_G} P_{FWM}(l, G, m_G, N_G) . P'_{FWM}\big(l, G', m'_{G'}, l$$
$$m_G \neq m'_{G'} \quad \forall \quad G = G' \tag{11}$$

where k_l is the number of overlapped interferer chips coincides with the desired wavelength at λ_l. Similarly, for bit 0, the FWM noise is the sum of the interferer-FWM noise and FWM-FWM noise, so that

$$\sigma_{FWM}^2(0) = \sigma_{interferer-FWM}^2(0) + \sigma_{FWM-FWM}^2(0) = 2\mathcal{R}^2\sum_{l=1}^{w}\sum_{i=1}^{k_l} P_{i,c}P_{FWM}^0(l, G, m_G, N_G)$$
$$+ 2\mathcal{R}^2\sum_{l=1}^{w}\sum_{G=I,III}\sum_{m_G=1}^{N_G}\sum_{G'=I,III}\sum_{m'_{G'}=1}^{N'_G} P_{FWM}(l, G, m_G, N_G). P'_{FWM}(l, G', m'_{G'}, N'_{G'})$$
$$m_G \neq m'_{G'} \quad \forall \quad G = G' \tag{12}$$

Assuming Gaussian approximation and equal a priori probability, the error probability can be described in the following scenario; "i" users among the $(K - 1)$ possible interferers may send a data binary 1. Three possible events, obeying a multinomial distribution, may occur simultaneously at the beginning of the optical fiber after the first combiner. These events include overlapping of the desired chips and other chips originated from different users due to similar, dissimilar, and combined hits with probabilities h_s, h_d, and h_c, respectively. The similar hits arise when "j_s" interferers overlap simultaneously with the desired user chips at similar wavelengths; producing both ordinary MAI and part of FWM-induced crosstalk at the receiver end. The second process is responsible for the generation of FWM due to the mixing of "j_d" dissimilar wavelengths over the desired wavelengths within the fiber; provided the fulfillment of FWM condition. The event of combined hits may occur when any interferer has both similar and dissimilar hits with the desired chips; generating FWM-induced crosstalk. Then, the total probability of error due to sending a data bit 1 can be written as follows

$$P(E) = \frac{1}{2}\sum_{i=1}^{K-1}\binom{K-1}{i}2^{-(K-1)}\sum_{j_s=0}^{i}\sum_{j_d=0}^{i-j_s}\sum_{j_c=x}^{i-j_s-j_d}\frac{i!}{j_s!\,j_d!\,j_c!\,(i-j)!}h_s^{\,j_s}\,h_d^{\,j_d}\,h_c^{\,j_c}\,h^{(i-j)} \times\{Q(E\mid 1)+Q(E\mid 0)\} \tag{13}$$

where $j = j_s + j_d + j_c$, $x = 1$ if $j_s = j_d = 0$ and $x = 0$ if j_s and/or $j_d = 1$. $Q(E|1)$ can be defined as follows:

$$Q(E\mid 1) = \frac{w\mathcal{R}P_d + (j_s + j_c')\mathcal{R}P_c + I_{FWM}^{1} - w\mathcal{R}P_d D}{\sqrt{\sigma^2_{sig-interferer} + \sigma^2_{interferer-inter} + \sigma^2_{signal-FWM} + \sigma^2_{interferer-FWM}(1) + \sigma^2_{FWM-FWM}(1)}} \tag{14}$$

while $Q(E|0)$ is defined as

$$Q(E\mid 0) = \frac{w\mathcal{R}P_d D - (j_s + j_c')\mathcal{R}P_c - I_{FWM}^{0}}{\sqrt{\sigma^2_{interferer-interferer} + \sigma^2_{interferer-FWM}(0) + \sigma^2_{FWM-FWM}(0)}} \tag{15}$$

Here D is the threshold level. The desired signal in the nominator of Eq. (14) is the first term and the second term represents the ordinary current due to MAI while the third term is the total generated FWM-induced current. j_c' is the number of similar hits within the event of combined hits; $j_c' < j_c$. The first and second terms in the denominator represent the conventional signal-interferer and interferer-interferer beating noise, respectively, and can be defined in the same way as [9].

4. Results and Discussion

The system performance is evaluated utilizing a non-zero dispersion fiber (NZDF) with the following properties: effective area of 55μm^2, D_c = 4.5ps/Km.nm, α = 0.22 dB/Km, $dD_c/d\lambda$= 0.045ps/Km.nm^2 and γ = 1.9 W^{-1}Km^{-1}. The BER as a function of transmitted power per chip with/without including the effect of FWM at w=29 and K=20 for PHCs is illustrated in Fig. 3. The results show that the bit error probability follows approximately "U-shape" when considering the effect of FWM. When the transmitted power per chip P_t is greater than 2dBm, the effect of FWM becomes dominant and the system performance starts degradation. At P_t = 10dBm, for instance, a penalty of 1.5dB results increased to 4dB when P_t = 15dBm. Consequently, the FWM imposes an upper bound for the transmitted chip power. So, during the 2-D TW OCDMA system design, it is important to determine carefully the operational range of the transmitted power to ensure that the required system performance is always maintained.

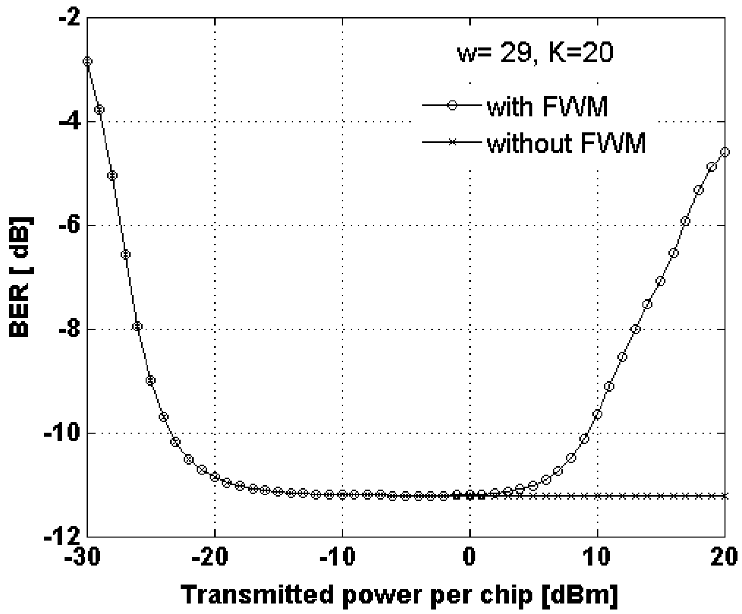

Fig. 3 BER versus the transmitted chip power with/without including FWM for transmission length *L*=20Km

5. Conclusion

The performance analysis of FWM in 2-D TW OCDMA systems has been carried out. A general formula for the error probability is derived. The mathematical results show that the FWM is a significant issue limiting the performance of TW OCDMA systems at high power levels. For example, at P_t = 10dBm, the system performance degrades under the effect of FWM with a penalty of 1.5dB; increased to 4dB at P_t=15dBm.

6. References

[1] A. Mendez, R. Gagliardi, H. Feng, J. Heritage, J.-M. Morookian, “Strategies for realizing optical CDMA for dense, high-speed, long span, optical network applications,” *J. lightwave Technol.*, vol. 18, no. 12, pp. 1685-1696, 2000.

[2] H. Mrabet and S. Mthali, “Performance enhancement of OCDMA systems for LAN consideration,” IET Optoelectronics, vol. 10, no.6, pp. 199-204, 2016.

[3] L. Tančevski and I. Andonovic, “Wavelength-hopping time-spreading code division multiple access systems,” *Electron. Lett.*, vol. 30, no. 17, pp. 1388-1390, 1994.

[4] G. Su, R. Wang, T Pu, T. Fang, J. Zheng, H. Zhu, and W. Wu, “Experimental investigation on the high chip rate of 2-D incoherent optical CDMA system,” *Proc. of SPIE*, vol. 9619, 96190J-1, 2015.

[5] K. Inoue, “Four-wave mixing in an optical fiber in the zero-Dispersion wavelength region,” *J. lightwave Technol*., vol. 10, 1553-1561, 1992.

[6] Taher M. Bazan, “FWM behavior in 2-D time-spreading wavelength-hopping OCDMA systems,” *Optical Fiber Technology*, vol. 34, pp. 23-29, Mar. 2017.

[7] X. Wang and K. Kitayama, “Analysis of beat noise in coherent and incoherent time-spreading OCDMA,” *J. Lightwave Technol*., vol. 22, no. 10, pp. 2226-2235, 2004.

[8] P. Legg, M. Tur and I. Andonovic, “Solution paths to limit interferometric noise induced performance degradation in ASK/direct detection lightwave networks, *J. Lightwave Technol*., vol. 14, no. 9, pp. 1943-1954, 1996.H. Yin and D. Richardson, “Optical code division multiple communication networks: Theory and applications,” Springer, 2007.
[9] Taher M. Bazan, D. Harle and I. Andonovic, “Performance analysis of 2-D time-wavelength OCDMA systems with coherent light sources: code design considerations,” *J. lightwave Technol*., vol. 24, no. 10, pp. 3583-3589, 2006.