AN INVESTIGATION OF ^{6}HE+^{12}C ELASTIC SCATTERING USING MICROSCOPIC CLUSTER MODEL

Awad A. Ibraheem*

Physics Department, Al-Azhar University, Assiut Branch, Assiut 71524, Egypt

ABSTRACT

In the frame work of the cluster folding model, for the first time, the 6He+12C real folded potentials are derived using new forms of the nuclear matter density of 6He nucleus based on α+2n or di-triton configurations. The angular distributions of the elastic scattering differential cross section at energies 38.3,41.6 and 82.3 MeV/nucleon are reexamined using the generated potentials. Comparisons between the extracted and measured angular distributions of the differential cross sections are presented. Satisfactory reproduction of the data is extracted by using the square Woods-Saxon (WS2) form instead of the usual WS shape without need to renormalize the derived potentials. The resulted reaction cross sections are also investigated and compared with the optical limit Glauber model predictions.

Keywords: Optical model; Elastic scattering; Halo nuclei; folding model, Glauber model.

I. INTRODUCTION

The study of light systems with very weakly-bound and neutron-rich exotic nuclei has been of particular interest. In order to obtain sufficient information about their weakly-bound nature, internal structure, the large radial extent in their densities and the dynamics of its interactions, most of studies have performed both experimentally and theoretically [1-18].

Elastic scattering of ^{6}He from a nucleus is considered as the door way reaction which has been studied to find characteristics that would be typical for weakly bound light nuclei and would help understanding their structure [5]. The structure of ^{6}He nucleus is a prototypical example of the Borromean structure (a tightly bound ^{4}He core and two valence neutrons). The two valence neutrons extend well beyond the ^{4}He core with a separation energy S_{2n}= 0.975 MeV. The elastic scattering data at 38.3 and 41.6 MeV/nucleon from ^{12}C were analyzed within the framework of the optical model. Initial analyses [16] were done using double-folded real potential using DDM3Y interactions [3] and a Woods-Saxon (WS) imaginary potential. From those analyses, it was concluded that a satisfactory description of the whole angular range of the data could not be obtained by adjusting the imaginary potential parameters and a simple renormalization of the real potential. A systematic analysis in conjunction with ^{6}Li data at similar energies was able to provide a good description of the data when a repulsive empirical dynamic polarized potential DPP of a surface form was added to the real part of the potential and an absorptive surface form DPP to the imaginary part [16]. Another analysis for ^{6}He and ^{6}Li elastic scattering at about 35 MeV/nucleon revealed that the data could be described by optical model potentials which has a deeper imaginary potential well for ^{6}Li[3].

The main objective of the present study is (i) to explore another structure of the ^{6}He nucleus within Triton-Triton (t+t) approximation beside three-particle approximation (α+2n). This was recently investigated by means of a 2n transfer reaction and the result shows that the spectroscopic factor obtained for the α+2n configuration is as good as expected, but the spectroscopic factor obtained for the t+t configuration is much smaller than the theoretical predictions [19,20].(ii) to check the ability of the imaginary part W(R) of square Woods Saxon(WS2) form instead of the usual Woods Saxon (WS) shape to describe the experimental data of ^{6}H+ ^{12}C elastic scattering with minimal number of fitting parameters as possible.

However, the need for a renormalization of the ^{6}He+^{12}C potential has been shown by several authors [21,22]. A previous folding analysis performed for of ^{6}He + ^{12}C system has clearly indicated the need for an appreciate the dynamic polarized potential DPP to be added to the folded potential as shown, for example, in equation (3) of Ref.[16].

At the intermediate energies considered, the

polarization potential is meant to arise mainly from the strong coupling to the breakup channels of ^{6}He. This has been in fact shown very clearly in the four-body CDCC analysis done by the Kyushu group[23], who found that the polarization potential due to breakup channels is repulsive, and this should lead to a normalization constant less than 1 in the double folding DF analysis.

For this purpose, we re-examine the elastic scattering of ^{6}He from ^{12}C target at three different energies 38.3, 41.6 and recently 82.3 MeV/ nucleon using several density distributions of ^{6}He nucleus. Firstly, we perform the calculations with a phenomenological volume WS real and imaginary potential. Second, we execute the same calculations using the microscopic double folding (DF) cluster model using ^{4}He +2n and Triton-Triton models (denoted as D1 and D2, respectively) as well as the few body Faddeev calculation (denoted as Q3) for the density of ^{6}He nucleus [6,18] to calculate the central real part of the DF optical potential. We show in this paper the ^{6}He+^{12}C data can be well reproduced and interoperated within the frame work of DF real potential , without renormalization factor, supplemented with an imaginary part W(R) of square Woods Saxon(WS2) form). Finally, all the results are compared with each other as well as with the experimental data obtained from EXFOR database [24].

In the next section the theoretical formalism is described while results on the analysis and discussion are presented in sec. III. Section IV summarizes the conclusions extracted from the present study.

II. FORMALISM

II-1 Optical potential

The optical model potential involved in this work has the standard form,

$$U(R) = V_C(R) + V_r(R) + iW_i(R), \quad (1)$$

$V_r(R)$ and $W_i(R)$ are the attractive real and imaginary parts of the nuclear potential, respectively.

$V_C(R)$ is the Coulomb potential due to a uniform distribution of appropriate size, radius $R_c = 1.13\left(A_P^{1/3} + A_T^{1/3}\right)$, then

$$V_C(R) = \begin{cases} \frac{Z_P Z_T e^2}{R}, & R > R_c \\ \frac{Z_P Z_T e^2}{2R_c}\left[3 - \left(\frac{R}{R_c}\right)^2\right], & R \le R_c \end{cases} \quad (2)$$

A_p and A_T are the mass numbers of the projectile (P) and the target (T) nuclei while, Z_p and $Z_T$denote their corresponding charges, respectively.

In the phenomenological analysis the attractive real and imaginary potentials are treated phenomenologically using conventional forms like WS potentials or any other forms. Alternative analyses replace the phenomenological real part of equation (1) by a microscopic one based on the DF approach. The DF potential may be written as the double-convolution integral

$$V(R) = \int \rho_p(\vec{r_1})\rho_T(\vec{r_2})\upsilon_{NN}(|\vec{s}|)d^3r, \vec{s} = \vec{R} - \vec{r_1} + \vec{r_2} \quad (3)$$

Where ρ_p and ρ_T are the ground state density distribution of projectile (^{6}He) and target (^{12}C) nuclei. The effective nucleon–nucleon (NN) interaction, υ_{NN}, is integrated over both density distributions. Several NN interaction expressions can be used for the folding model potentials. We have chosen the most common one, the M3Y (Michigan 3 Yukawa) realistic interaction [25]. In the present work, we use the former form with the relevant exchange correction term due to the Pauli principle, given by

$$\mathbf{v_{eff}^{M3Y}(s,E)} = 7999\frac{\mathbf{e^{-4s}}}{\mathbf{4s}} - 2134\frac{\mathbf{e^{-2.5s}}}{\mathbf{2.5s}} - 276\left[1 - 0.005\mathbf{E}\right]\delta(\mathbf{s}), \vec{\mathbf{s}} = \vec{\mathbf{R}} - \vec{\mathbf{r}}_1 + \vec{\mathbf{r}}_2, \quad (4)$$

II.2 THE NUCLEAR DENSITY OF ^{6}He

There are some uncertainties concerning the density of ^{6}He. We have several choices to study the effect caused by the halo structure of ^{6}He[6-16]. For this reason, three different forms of the ground state density distribution are used in the folding calculation. The first one, the ^{6}He nucleus is assumed to consist of a core of ^{4}He and two halo neutrons (^{4}He +2n). Then, one may formulate the nuclear matter density of ^{6}He as[14]

$$\rho(r)=\int|\psi(R')|^2(\rho_c\left(\left|\bar{r}-\frac{1}{3}\overline{R'}\right|\right)+\rho_{2n}\left(\left|\bar{r}+\frac{2}{3}\overline{R'}\right|\right))d\overline{R'} \quad (5)$$

where the core-halo relative wave function is represented in the form[14]

$$\psi(R')=\frac{4\alpha}{\sqrt{15}}\left(\frac{2\alpha}{\pi}\right)^{3/4}R'^2\exp(-\alpha R'^2) \quad (6)$$

With α= 0.341 fm^{-2}. The core (^{4}He) and halo densities are taken in a Gaussian forms, respectively, as [20]

$$\rho_c(r)=4(\frac{\gamma_c}{\pi})^{3/2}\exp(-\gamma_c r^2) \quad (7)$$

And

$$\rho_{2n}(r)=2(\frac{\gamma_h}{\pi})^{3/2}\exp(-\gamma_h r^2) \quad (8)$$

Where γ_c = 0.6756 fm^{-2} and γ_h= 0.1305 fm^{-2}. The resulted density, denoted as D1, yields the extracted root mean square (rms) radii 1.49, 3.39 and 2.54 fm for the free ^{4}He, two-neutron and ^{6}He densities, respectively. These radii are close to the value evaluated from the four-body analysis of ^{6}He+^{12}C total reaction cross sections [6], and by the analysis of elastic scattering of ^{6}He on protons at high energies [7].

In parallel to the ^{4}He +2n cluster model, we also introduce, for the first time, another form of the nuclear matter density of ^{6}He based on two *Triton-Triton* (^{3}H$_{1}$-^{3}H$_{1}$) cluster model as,

$$\rho(r)=\int|\psi(R)|^2(\rho_{^3H1}\left(\left|\bar{r}-\frac{\overline{R}}{2}\right|\right)+\rho_{^3H1}\left(\left|\bar{r}+\frac{\overline{R}}{2}\right|\right))d\overline{R} \quad (9)$$

Where R is the ^{3}H$_{1}$-^{3}H$_{1}$ separation inside ^{6}He nucleus, $\psi(R)$ is the wave function of the relative motion of ^{3}H$_{1}$-^{3}H$_{1}$ clusters in the ground state of ^{6}He nucleus. This relative wave function is taken also in the form of equation (6) with . A Gaussian form for the density distribution of the triton (^{3}H$_{1}$) has been assumed as [26]

$$\rho_{^3H_1}(r)=3(\frac{\gamma_1}{\pi})^{3/2}e^{-\gamma_1 r^2} \quad (10)$$

with . The calculations have been performed with the code MATHEMATICA [27]. The resulted density is denoted by D2. Finally, for the sake of comparison, we considered another form for ^{6}He density which is taken from Ref. [6,7] obtained by Faddeev wave function and denoted by Q3.

Throughout the present work, this density can be expressed as a summation of thirteen Gaussian terms as

$$\rho_{^{17}F}(r)=\sum_{k=1}^{13}c_k\exp\left[-a_k r^2\right] \quad \text{fm}^{-3} \quad (11)$$

The parameters C_k and a_k are listed in Table (1). The corresponding matter rms is 2.54 fm. It should be noted that, the first term represents also, the matter density distribution of alpha particle with rms radius equals to 1.47 fm.

The obtained densities are shown in Fig.1. It is noticed that the D1 density seems substantially deeper than D2 and Q3 at the center (r = 0.0), however, D1 and D2 densities have consistent values through the radial range r = 3.0- 5.0 fm, One may notice also that all densities have the same value (~0.056 fm^{-3}) at r ≅1.78 fm ,

The ground state matter density of ^{12}C is taken as a two parameter Fermi function as [28]

$$\rho(r)=\frac{0.207}{1+e^{\left(\frac{r-2.1545}{0.425}\right)}} \quad (12)$$

This density yields a rms radius equal 2.298 fm close to those obtained from (*e*,*e*) scattering measurements. This density has a similar shape to the one obtained by shell model calculations [28].

III. RESULTS AND DISCUSSION

In this work, we checked the ability of the derived cluster folding potential based on α+2n or, for the first time, di-triton configurations to analyze the elastic scattering of ^{6}He + ^{12}C at 38.3, 41.6 and 82.3 MeV/nucleon. The analysis is performed in the framework of optical model. First a phenomenological analysis is preformed using the standard WS optical potential. Second, the analysis is done using a real part of the optical potential obtained microscopically by the DF model of equation (4). In this model, we use three different forms of the nuclear matter density distribution of ^{6}He (D1, D2 and Q3) folded with a realistic M3Y effective NN interaction. The resulted potentials with each density are de-

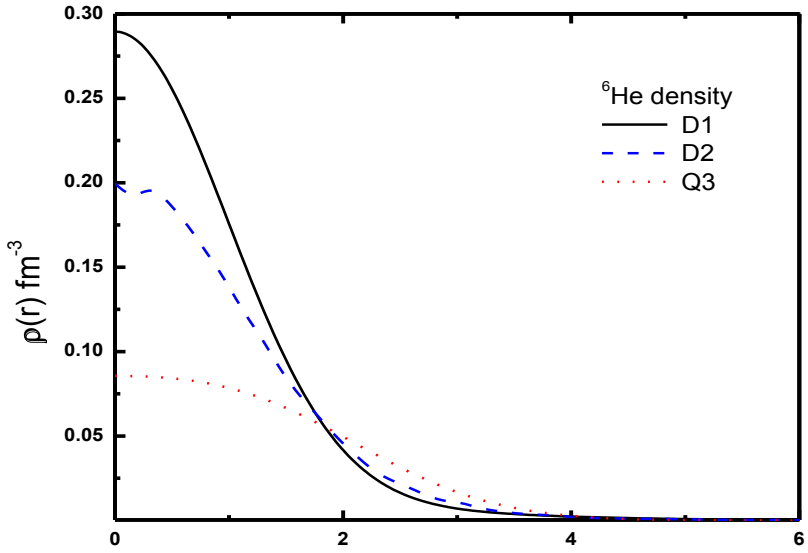


FIG. 1.The nuclear matter density distribution of ^{6}He (D1 and D2) deduced from Eqs.(5 and 9) in comparison with those of Faddeev model (Q3) in Eq.(11).

Table 1: The obtained parameters C_k and A_k of Eq.(11).

k	C_k(fm^{-3})	A_k(fm^{-2})
1	0.4154230	0.694153
2	-2.208400	0.603806
3	0.0018019	0.063792
4	0.5811340	0.522915
5	0.5696400	0.522679
6	-2.380550	0.424470
7	0.5809250	0.522914
8	0.9387690	0.355672
9	0.5797400	0.522909
10	0.5678100	0.523446
11	0.5757130	0.522877
12	-0.608940	5.640160
13	0.6015380	5.509230

noted as DFC1, DFC2 and DFC3, respectively.

This analysis are carried out using the HIOPTIM-94 program[29] fed with either phenomenological potential of equation (1) or by the calculated microscopic real potentials (DFC1 , DFC2 and DFC3) supplemented by an imaginary part W(R) of square Woods Saxon(WS2) form instead of the usual Woods Saxon (WS) shape as

$$W(R)=\frac{W_0}{\left(1+\exp\left(\frac{R-R_I}{a_I}\right)\right)^2},\quad R_I=r_I\left(A_P^{1/3}+A_T^{1/3}\right) \tag{13}$$

Where W_0, r_I and a_I are the depth, radius and diffuseness parameters, respectively. Best fits are obtained by minimizing χ^2, where

$$\chi^2=\frac{1}{N}\sum_{k=1}^{N}\left[\frac{\sigma_{th}(\theta_k)-\sigma_{\exp}(\theta_k)}{\Delta\sigma_{\exp}(\theta_k)}\right]^2 \tag{14}$$

σ_{th} (σ_{exp}) is the theoretical (experimental) cross section at angle θ_k in the center-of-mass system, $\Delta\sigma_{exp}$ is the experimental error and N is the number of data points.

All the obtained potentials have similar behavior with each other. The reaction cross-sections for the microscopic potential calculation are consistent with those obtained in Refs.[1,2,and 16]. The best fit parameters extracted from the auto search code are listed in the Table(2,3).

In a previous folding analysis performed for of ^{6}He + ^{12}C system has clearly noted the data required a strong renormalization factor N_r of the real part to be correctly described the data with standard imaginary WS potential. In our calculations, the situation is different, where we replaced the standard WS with square WS2 form of equation (13). In this case, a very satisfactory a agreement is reached with normalization factor N_r equal unity.

On the other hand, authors of Refs.[1,2] used two sets (A and B) of the imaginary parts of optical potential to fit the experimental data at 82.3 MeV/nucleon including contributions from the inelastic channels with real DF potential based on CDM3Y6 effective NN interaction[3]. These sets give total reaction cross-sections σ_R of 853 and 843 ***mb***. In our analysis, new search for the phenomenological potential corresponding parameters $V_0, r_0, a_0, W_i, r_i and\ a_i$ (Fig. 2) are determined by analyzing the angular distributions of the elastic scattered ^{6}He on ^{12}C target (full curves). The obtained total reaction cross-section value is less than that obtained in Refs. [1,2] and inconsistent with the results of DFC1, DFC2 and DFC3. The maximum and minimum observed in the experimental data around 40 MeV/nucleon are in phase problem with our theoretical results.

The total reaction cross-sections corresponding to the above microscopic calculations are also presented in the Table-3, which are consistent with those obtained at other energies [16]. The total reaction (absorption) cross section, σ_R, is considered as an important quantity in the analysis of the elastic scattering reaction. Hence, it would be interesting to investigate whether one could generate a reasonable determination of σ_R using the derived potentials. Therefore, we demonstrated in Fig. 3, the imaginary volume integral as well as the obtained energy dependence of σ_R obtained with the present analysis using the derived potential results and compared within the optical limit Glauber model approximation OLA. Within the optical limit Glauber model, the total reaction cross section, σ_R is expressed as [30-32].

$$\sigma_R = 2\pi \int_0^\infty bdb\big(1 - T(b)\big) \tag{15}$$

where T (b) is the transparency function of the collision at impact parameter b.

$$\mathbf{T(b) = \left|exp\big(i\chi(b)\big)\right|^2 = exp\big(-2Im\chi(b)\big)} \tag{16}$$

The phase χ(b) is simply related to the nucleon-nucleon profile function γ_{NN} by:

$$\boldsymbol{\chi(b) \approx \chi_0(b) = i \int\int d\vec{r}_1 d\vec{r}_2 \rho_P(r_1)\gamma_{NN}(|s_1 - s_2 - b|)\rho_T} \tag{17}$$

Calculations are carried out by evaluating Eqs.(15-17) numerically. Important information that one can obtain from the elastic scattering is the volume integral Ju . For an interaction potential U(R) between two nuclei that have nucleon numbers A_P and A_T , the volume integral per interacting nucleon pair Ju is defined as

$$J_U = \frac{1}{A_P A_T} \int U(R) d\vec{R} \tag{18}$$

This quantity is currently used as a sensitive measure of the potential strength. In the present work, we apply this definition to the real and to the imaginary parts of U(R), independently denoted as J_R and J_I , respectively. It is also obvious that the obtained reaction cross section and the imaginary volume integral have almost identical energy dependence as shown in the Table-3. This result is physically expected where both J_i and σ_R concern the absorption to nonelastic channels.

IV. CONCLUSION

This work presents an optical model analysis of several ^{6}He+^{12}C reactions at three different energies using both phenomenological as well as microscopic (double-folding) optical model potentials. For the latter, several prescriptions for the ^{6}He matter density (D1,D2 and Q3) are used and compared to study the effects of these densities on the elastic scattering observables. The double folding DF real potentials based upon the M3Y interaction successfully reproduced the scattering data all over the measured angular ranges without renormalization factors. The imaginary part is parameterized during the analysis in a squared WS2 form instead of the usual WS shape. This is an interesting result, where it was pointed out [2] the calculation included contributions from inelastic excitations of the carbon target is reasonably reproduced by microscopic JLM potential with a reduction of imaginary part. On the other hand, the normalized DF real potential with standard WS imaginary part fail to reproduce the experimental data for the whole angle range, while WS2 potential are in agreement with the considered data. It should be noted also, in the forward angle between (3◦ - 9◦) the data at 82.3 MeV/nucleon are not described in a quantitative way. Finally the comparison

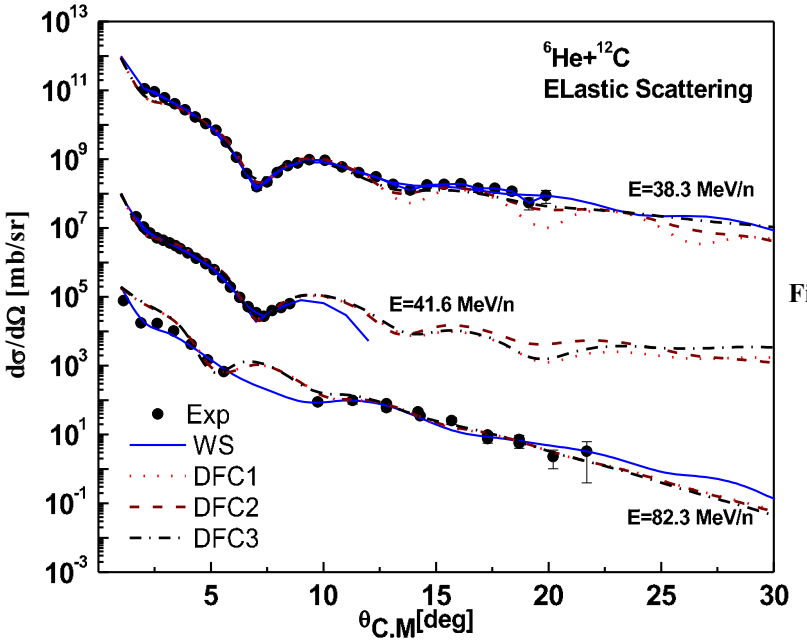
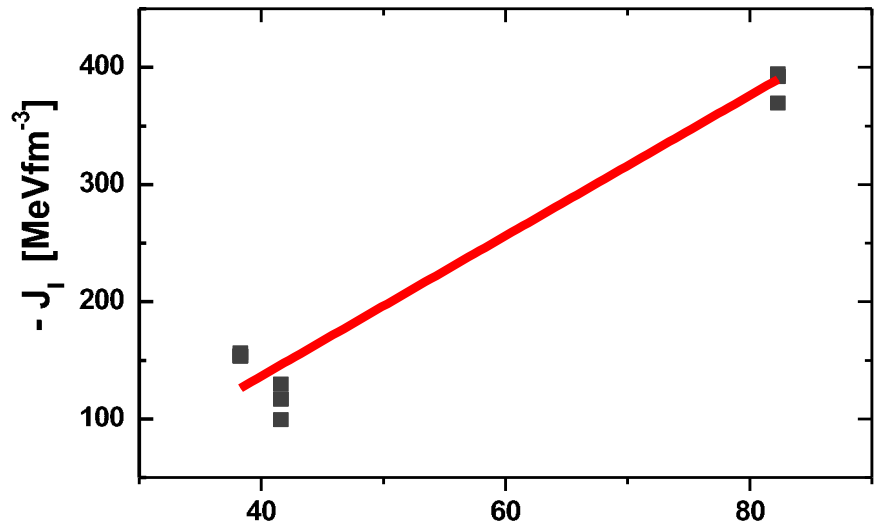


Fig 2. Elastic scattering data for 6He on 12C at 38.3, 41.6 and 82.3 MeV/nucleon in comparison with the results given by the real folded potentials (DFC1, DFC2 and DFC3 respectively) obtained with the M3Y interaction.



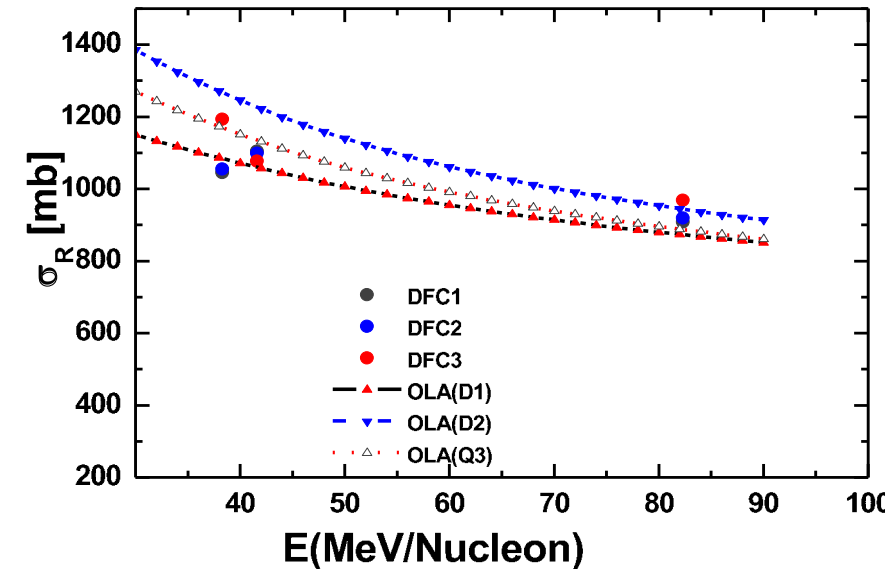


Fig 3. Energy dependence of the imaginary volume integrals and reaction cross sections obtained by the real double folding potentials include the total reaction cross sections results of optical limit phase shift calculations with D1, D2 and Q3 namely OLA(D1), OLA(D2) and OLA(Q3) respectively.

Table 2. Optical potential parameters obtained from analysis of the 12C(6He,6He)12C elastic scattering at different energies using the OMP potentials. Volume depth (W0), radius and diffuseness parameters(ri and ai), total reaction cross section (σR) and the best fit χ2.

ELab (MeV/N)	V0 (MeV)	r0 (fm)	a0 (fm)	Wi (MeV)	ri (fm)	ai (fm)	χ^2	σR (mb)
38.3	35.47	1.07	0.350	3.93	1.80	0.610	2.87	1386.0
41.6	48.17	0.940	0.350	3.07	1.730	0.840	5.68	1135.9
82.3	106.03	0.276	0.493	6.301	1.233	0.504	2.35	573.88

Table 3. Parameters of the optical potential for the 6He+12C system at 38.3,41.6 and 82.3 MeV/nucleon. The real folded potential is calculated with the M3Y interaction using D1 and D2 densities or Q3 one. Both have an rms radius equal to 2.54 fm. The normalization factor Nr is equal unity. The volume depth (W0) in MeV, radius and diffuseness parameters(ri and ai) in fm, real and imaginary volume integrals (JR and JI) in MeV.fm3, total reaction cross section (σR) in mb and the best fit χ2.

Pot	N_r	W_0 (MeV)	r_i (fm)	a_i (fm)	J_R (MeVfm^3)	J_I (MeVfm^3)	χ^2	σ_R (mb)
E=82.3 MeV/nucleon								
DFC1	1.0	320.8	0.7210	1.0183	308.20	391.94	65.9	908.6
DFC2	1.0	285.7	0.7366	1.1903	308.20	369.5	65.5	919.0
DFC3	1.0	254.8	0.7894	1.1980	308.20	394.42	65.9	968.0
E=41.6 MeV/nucleon								
DFC1	1.0	12.082	1.4679	0.7670	364.41	116.68	5.5	1104.0
DFC2	1.0	14.91	1.4191	0.7670	364.41	129.44	7.3	1099.0
DFC3	1.0	9.542	1.5028	0.7670	364.41	99.24	5.6	1077.0
E=38.3 MeV/nucleon								
DFC1	1.0	16.686	1.4119	0.4862	368.97	153.63	14.6	1045.0
DFC2	1.0	27.185	1.2926	0.7744	368.97	175.45	8.40	1055.0
DFC3	1.0	182.06	0.4208	2.0422	368.97	156.17	9.70	1193.0

between the elastic angular distributions calculated with the OMPs and those obtained from the present cluster folding model proves the latter as most reliable

ACKNOWLEDGEMENT

I would like to thank Prof. Dr. M. El-Azab Farid for reading the manuscript and his valuable comments.

REFERENCES

[1] J. L. Lou et al., Phys. Rev. C83(2001) 034612.

[2] J. L. Lou et al., Proceedings of the 14th National Conference on Nuclear Structure in China (2012) pp. 158-163.

[3] Dao T. Khoa, Phys. Rev. C 63 (2001) 034007.

[4] L. Giot, et.al, Phys. Rev. C 71(2005) 064311

[5] N. Keeley, et.al., Phys. Rev. C 77 (2008) 057601.;N. Keeley, et.al, Progress in Particle and Nuclear Physics 63 (2009) 396.

[6] J. S. Al-Khalili, et.al, Phys. Rev. C54,(1996)1843 .

[7] J.S. Al-Khalili and J.A. Tostevin, Phys. Rev. C57 (1998)1846.

[8] J. S. Al-Khalili, Phys. Lett.B378,(1996)45 .

[9] Z. M. M. Mahmoud et.al., Int. J. Mod. Phys E 23 (2014) 1450008 .

[10]Z. M. M. Mahmoud et.al., Int. J. Mod. Phys E 22 (2013) 1350086.

[11]Awad A. Ibraheem et.al., Journal of the Korean Physical Society 63 (2013)965.

[12]Awad A. Ibraheem, et.al, Physics of Atomic Nuclei 75 (2012)969.

[13]M Aygun et.al., Nucl. Phys. A 848 (2010)245.

[14]M. El-Azab Farid et al., Arab Journal of Nuclear Sciences and Applications 43 (2010)169.

[15]M. El-Azab Fari, et.al., Int. J. Mod. Phys E 17 (2008)715

[16] V. Lapoux et al., Phys. Rev C66(2002) 034608.

[17]N. Keeley et.al., Phys. Rev. C 77 (2008) 057601

[18]M. V. Zhukov et al.,Nucl. Phys. A 552 (1993), 353.

[19]L. Giot et al , Phys. Rev. C 71(2005) 064311

[20]Xin-Shuai Yan et.al.,Chinese Physics C 35(2011) 550

[21] Lapoux et.al.,Phys. Rev. C 66, (2002) 034608

[22] Lukyanov et.al.,Phys. Rev. C 82, (2010) 024604.

[23]Matsumoto et.al.,Phys. Rev. C 70, (2004)061601.

[24]Exfor database, http://www-nds.iaea.org/exfor/exfor.htm.

[25]G. R. Satchler and W. G. Love, Phys. Rep. 55 (1977)183.

[26]J. Cook et.al.,Phys. Rev. C27(1983)1536.

[27]S. Wolform, MATHEMATICA: A program for doing Mathematics by Computer, Addidson –Wesley, Reading , MA 1988.

[28] M. El-Azab Farid and G.R. Satchler, Nucl. Phys. A438, (1985) 525.

[29]N. M. Clarke (private communication).

[30]R.J. Glauber; Lectures on Theor. Phys. 1 (Interscience, New York, 1959).

[31]Awad A. Ibraheem, Int. J. Mod. Phys. E20 (2011) 721.

[32]Awad A. Ibraheem et.al., J. Phys. Scoc. Jap. 81 (2012) 124201