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Numerical Study of Concrete Deep Beams Reinforced by Inclined Web Stirrups

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Deep beam, shear, inclined stirrups, FEM.

Abstract

The shear behaviour of deep beams considers as the main parameter of their load capacities. Therefore, the understanding of shear behaviour improvement by using inclined stirrups is essential to optimum design of web stirrups for deep beams. In this study, a numerical investigation is presented to describe the effect of inclined stirrups on the shear behaviour for reinforced concrete deep beams. This study was preceded by two steps as a part of verification and parametric study. In the first step, three beams were analysed with different arrangements of their stirrups. Two beams are reinforced with either vertical or inclined stirrups when there are no stirrups in the third beam. The numerically load-deflection curves, cracking patterns, and load-maximum crack opening curves were presented for all analysed beams. These numerical results were verified by the experimental study showing an excellent agreement between them. In second step, another 16 beams within 4 groups were analysed to show the parametric influence of using inclined stirrups. This parametric study presented the effect of main reinforcement ratio, shear span-to-depth ratio, space between stirrups, and bar cross-sectional area of stirrups on the deep beams reinforced with either vertical or inclined stirrups. The deep beam reinforced with inclined stirrups leads to a significant increase in the shear strength due to the contribution of these stirrups to restrict the opening of diagonal shear crack.

1. Introduction

Reinforced concrete (RC) deep beams can be classified as load distribution structural elements as shear wall and pile caps. Those ultimate load capacity should be evaluated by their shear behavior rather than their flexural strength. The reason is that their geometries as span-to-depth ratio values are usually smaller than for the regular RC beams. Therefore, the shear behavior of the deep beams were subjected to several studies [1, 2, 3]. Moreover, there are many methods to predict the shear capacity of RC deep beams such as ACI 318-19 [4] and truss model [5, 6].

One of the most effective parameter for the shear behavior of deep beams is the web reinforcement [3, 7]. Teng, Ma et al. [1] studied the effect of web reinforcement and its arrangement on the shear load capacity. Three beams were tested with different arrangement of their web reinforcement as

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without web reinforcement, with vertical web reinforcement only, and with inclined web reinforcement. The deep beam with inclined web reinforcement presented a significant improvement on its shear behavior than other beams according to the experimental study of Tan, Kong et al. [7].
The only experimental results in previous studies cannot be satisfied to understand the effect of arrangement of web reinforcement. Therefore, the numerical analysis is strongly required to present the clear explanation of the failure mechanism of RC deep beams. In this study, two-dimensional (2D) finite-element (FE) modeling of deep beams with different web reinforcement type were analyzed to investigate their failure mechanism. Therefore, this numerical study provides a good guidance to understand the effect of inclined web stirrups of deep beams on their shear capacity. This investigation is provided by the parametric study which presents the effect of main reinforcement ratio, shear span-to-depth ratio, space between stirrups, and bar cross-sectional area of stirrups on the deep beams reinforced with vertical and inclined stirrups. The commercial name of FEA software is ATENA 2D V5, which is accurate for 2D RC structures.

1. METHODOLOGY

1.1. Reinforcing bar model

Embedded reinforcing bar was used for main reinforcements and stirrups. **Fig. 1 (b)** shows a bilinear stress-strain relationship for steel, which was adopted as reinforcement in this numerical model. The initial elastic part represents the modulus of elasticity of steel, E_s. The second line shows the prefect plasticity with slope of hardening equalling zero. In the experimental study of Teng et al.[1], there is no debonding failure for reinforcing bar in all tested beams. Furthermore, the yield strength of stirrups, compression, and tension bars are 480, 575, and 670 MPa, respectively. These bars are modelled as a smeared reinforced bar in the concrete elements.

1.2. Concrete model

For concrete, nonlinear constitutive laws were presented in this study to provide the stress-strain relationships in both tension and compression zones as shown in **Fig. 1 (a)**. These constitutive laws can be listed in **Error! Reference source not found.**. Concrete elements were modelled as 4-nodes 2D smeared cracking element, which is integrated material in the proposed materials library of ATENA 2D V5 [8]. Crack propagation of concrete is adapted with multiple fixed crack concept [9]. This concept can be summarized as that the first crack propagates perpendicularly on the direction of the maximum principal strain in the concrete matrix when tensile strain is larger than the cracking strain. When the second tensile strain component preceded the cracking strain, the second crack is reproduced perpendicular to the direction of the first crack. Therefore, at the initiation of crack, the principal stress directions are used to produce the first and second crack directions. These cracks directions are assumed as fixed directions along analysis [10]. During further loading, increasing principal stress produces a shear stress on the crack face to keep the fixed crack direction. This leads to a reduction of shear modulus according to the law derived by Kolmar [11] as the following equations (1) and (2);

$$G = \frac{-\ln\left(\frac{1000\varepsilon_1}{c_1}\right)}{c_2} \cdot G_c \,, \qquad (1)$$

$$G_c = \frac{E_c}{2(1+\upsilon)}, \qquad (2)$$

where, G_c ,G is the initial and reduced shear modulus, E_c is the initial elastic modulus, υ is the Poisson's ratio, c_1 and c_2 are parameters depending on the reinforcing crossing the crack direction [11].

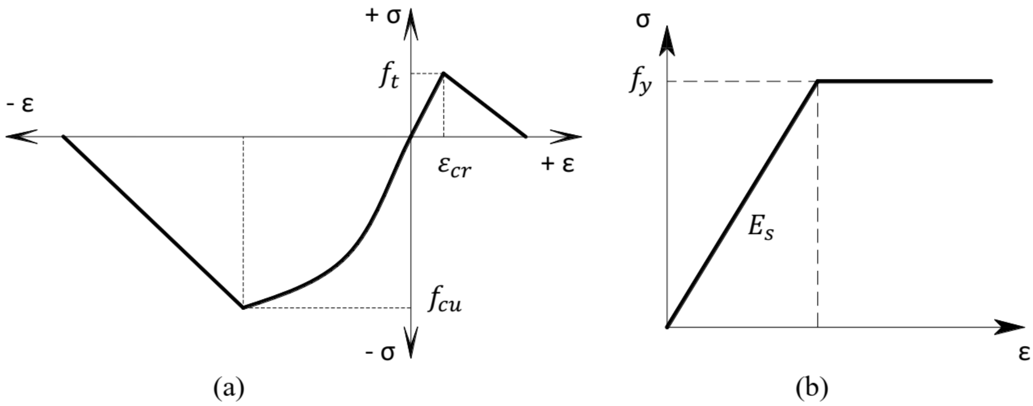


Fig. 1 Stress-strain relationship for **(a)** Concrete and **(b)** Reinforcing bar

Table 1 Constitutive laws of concrete

Compression		Tension	
$0 \geq \varepsilon \geq \varepsilon_m$	$\sigma = f_c \dfrac{\varepsilon}{\varepsilon_m}\left(2 - \dfrac{\varepsilon}{\varepsilon_m}\right)$	$\varepsilon_t \geq \varepsilon \geq 0$	$\sigma = E_c \varepsilon$
$\varepsilon_m \geq \varepsilon \geq \varepsilon_u$	$\sigma = f_c \dfrac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_m}$	$\varepsilon > \varepsilon_t$	$\sigma = f_t - E_{Soft}(\varepsilon - \varepsilon_t)$
E_c = the modulus of elasticity of concrete, f_c = concrete compressive strength, ε_m = $f_c/2E_c$ = concrete strain corresponding f_c, f_t = tensile strength, ε_t = f_t/E_c = strain at tensile strength, and E_{soft} is crack softening stiffness.			

2. VERIFICATION OF NUMERICAL RESULTS

Three deep beams were analysed using the dimension and arrangement of reinforcing bars as shown in ***Fig. 2*** **(a)**. In this figure, simply supported deep beams are shown with a cross-sectional dimensions of 150 x 800 mm. To validate these numerical results, these dimensions and reinforcement are the same as those in the tested deep beams in experimental study of Teng, Ma [1]. Two-dimensional finite-element method was employed using ATENA 2D V5 [8] considering the proposed integrated materials for concrete and reinforcement. **Fig. 2 (b, c, and d)** shows the used numerical models for beams CO-ST, CV-ST, and CI-ST, respectively. Concrete strength of these beams was assumed as the same of experimental data [1] equalling to 41, 40, and 40 MPa, respectively. Beam CO-ST was fabricated without any stirrups. However, beams CV-ST and CI-ST were reinforced with vertical and inclined stirrups, respectively, as same as those in the tested deep beams in experimental study of Teng, Ma [1]. Only half of beam is analysed due to its symmetry. For all beams, one half of the beam was analysed due to their symmetrical dimensions and loading. Loading and support plates are modelled as solid steel with linear strain-stress relationship. Furthermore, mid-span deflection, maximum crack opening, cracking pattern, and concrete strain were observed in this study.

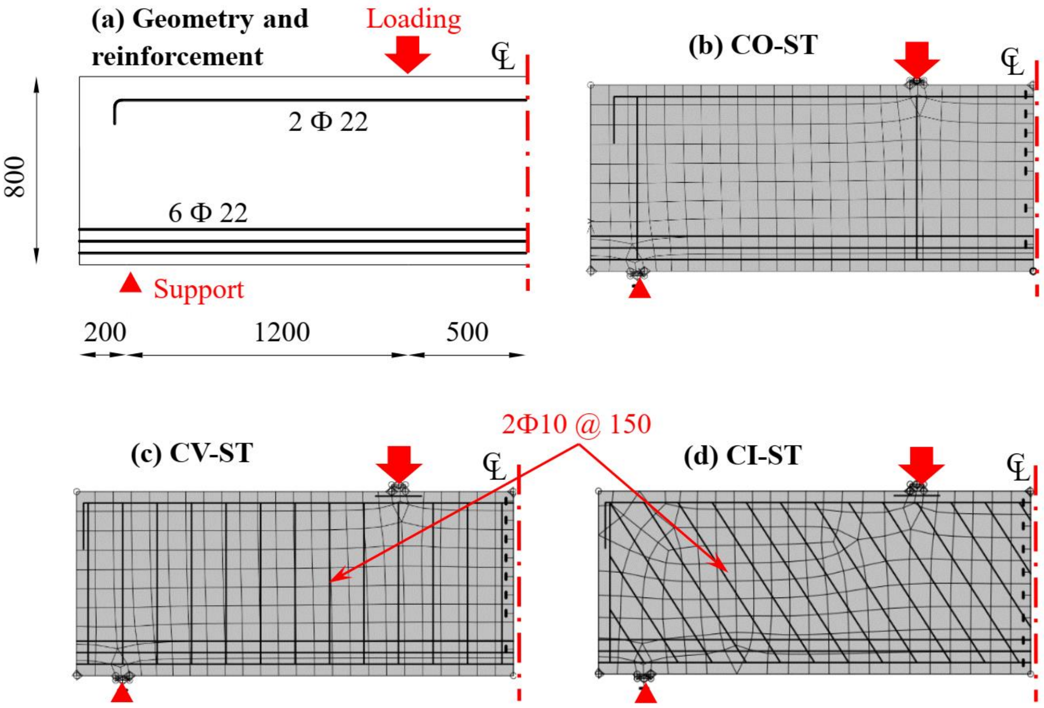


Fig. 2 (a) Arrangement of main reinforcement and numerical model of **(b)** CO-ST, **(c)** CV-ST, and **(d)** CI-ST (all dimensions in mm)

2.1. Load-deflection curves

Numerical and experimental load-midspan deflection curves are presented in **Fig. 3**. For all beams, these curves began with the same overall beam stiffness until cracking load due to the almost same concrete strength. Then, their stiffness decreases with increasing of applied load due to the propagation of cracks resulting in increase on their mid-span deflection. The overall beam stiffness can be indicated by the slope of load-deflection relationship. For applied load over the cracking load, using vertical, CV-ST, or inclined, CI-ST, stirrups show an improvement of overall-beam stiffness and its loading capacity than that without stirrups, CO-ST. This improvement is significant for RC deep beam reinforced with inclined stirrups, CI-ST, then that reinforced with vertical stirrups, CV-ST. This improvement will be fully discussed in the following sections. By comparing the numerical and experimental results, there is an acceptable agreement between them especially for CO-ST and CV-ST. The difference between experimental and numerical results is lower than 5%.

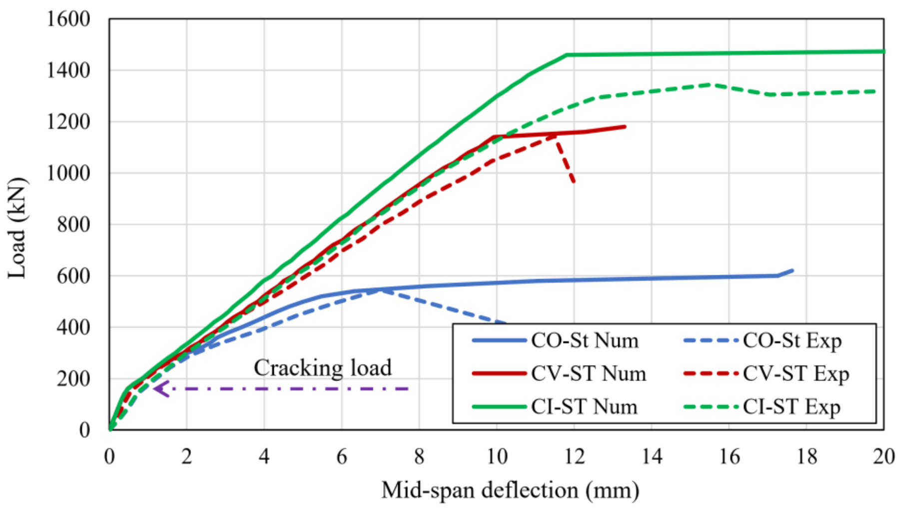


Fig. 3 Load-deflection curves for numerical and experimental beams

2.2. Cracking pattern and crack width

Fig. 4 shows the numerically and experimentally cracking pattern with the numerical principal strain distribution on concrete surface. This strain distribution can be considered as an indication of crack opening. Moreover, the bolded cracked line indicates a wider crack-opening in numerical and experimental cracking pattern. For beam CO-ST, the main diagonal crack is more localized than those in other beams with stirrups. This observation is approved by comparing the numerical and experimental cracking-pattern for all beams. Moreover, the maximum value of principal strain distribution of this beam is higher than the corresponding values in other beams. This indicates a wide crack opening for RC deep beam without stirrups. Using vertical, CV-ST, or inclined, CI-ST, stirrups lead to restrict the opening of propagated cracks resulting in improvement of overall-beam stiffness and its loading capacity. This leads to a decrease of the maximum principal strain indicating to a smaller crack opening than that in beam without stirrups. Moreover, theses beams show wide distribution of the propagated cracks in the shear zone. This scenario is more significant in the RC deep beam with inclined stirrups. In this beam, the direction of inclined stirrups is perpendicular on the direction of the propagated shear crack. This leads to a significant decreasing of the maximum principal strain than that for reinforced with vertical stirrups as shown in **Fig. 4**. Also, in this beam, there is a wide flexure crack in mid-span at further loading level. This crack is significantly wider crack in the experimentally cranking pattern of beam CI-ST. This observation agrees with its numerically principal strain distribution, which shows a higher value in its mid-span as shown in **Fig. 4 (c)**. For all beams, the numerically and experimentally cracking pattern shows almost same crack orientation indicating an acceptable agreement between them.

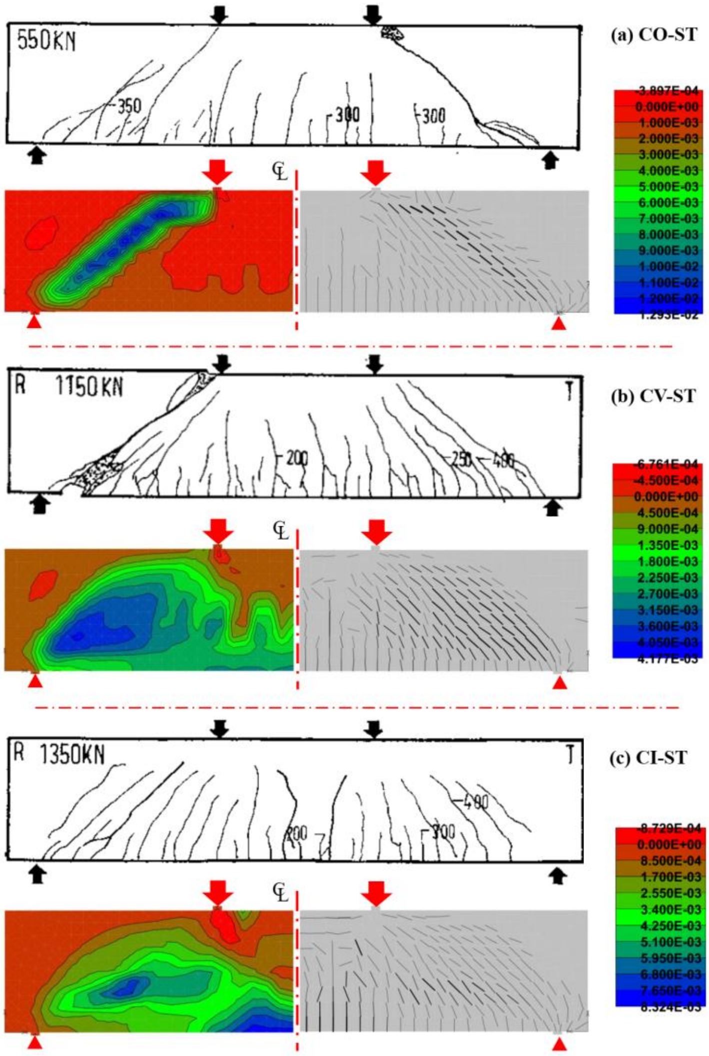


Fig. 4 Experimentally and numerically cracking pattern and principal strain distribution

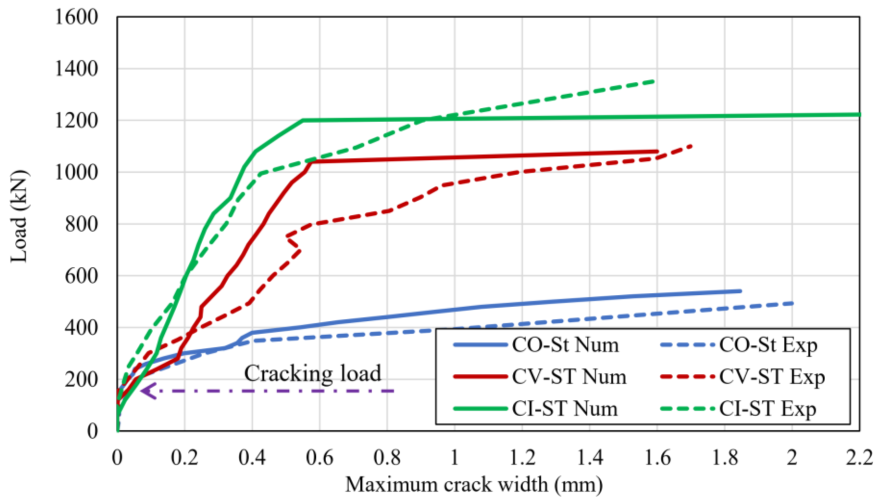


Fig. 5 Load-maximum crack width curves for experimental and numerical results

Fig. 5 shows the numerical and experimental relationship between the maximum cracking width and the applied load. Increasing load level leads to an increase of the cracking width for all beams. This increasing is significant after the beginning of failure. At the same load level, deep-beams reinforced with stirrups show smaller crack opening than that in beam of CO-ST as shown in **Fig. 5**. This observation can be approved by the numerical principal strain distribution in **Fig. 4**. Beam with inclined stirrups result in the smallest crack opening than other beams. These inclined stirrups were perpendicular to the direction of diagonal shear crack resulting a narrower crack widths and uniform cracking pattern.

2.3. Shear strain distribution

The shear strain distribution of concrete for all beams at shear failure is shown in **Fig. 6**. For RC beam without stirrups, the maximum value of shear strain is higher than beams have stirrups. Moreover, this beam shows localized shear strain values in the shear zones to establish a connected line between the loading and supporting points. Adding stirrups leads to a significant decrease in the shear strain values. The distribution of shear strain is more widening at the zone of shear failure for the beams reinforced with vertical or inclined stirrups. Beam reinforced with inclined stirrups shows the smallest values of shear strain distribution than other beams indicating the role of inclined stirrups for decreasing the shear on the concrete surface. Therefore, the failure mode of this beam is a combination of shear failure and flexural failure modes as shown in **Fig. 4 (c)**.

To understand the effect of inclined stirrups, numerically axial plastic-strain of stirrups at the failure for beams reinforced with stirrups is shown in **Fig. 7**. The plastic-strain for stirrups can be used as an indication of post-yielded bars. For the beam reinforced with vertical stirrups only one stirrup reached plastic strain in the lower third of the beam depth. On other hand, seven stirrups exceeded the plastic strain in the beam reinforced with inclined stirrups. The plastic strain on stirrups was located on the line of diagonal shear crack, which is formed between the loading and supporting points. This observation indicates to an excellent contribution of the inclined stirrups formed perpendicular on the expected diagonal shear crack. This leads a significant decreasing of shear strain on the concrete surface for beam reinforced with inclined stirrups as shown in **Fig. 6 (c)**.

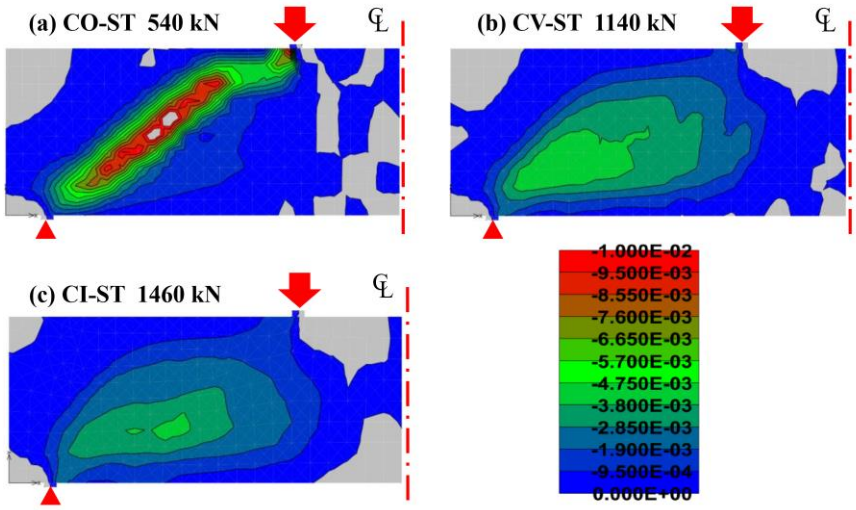


Fig. 6 Numerically shear strain distribution

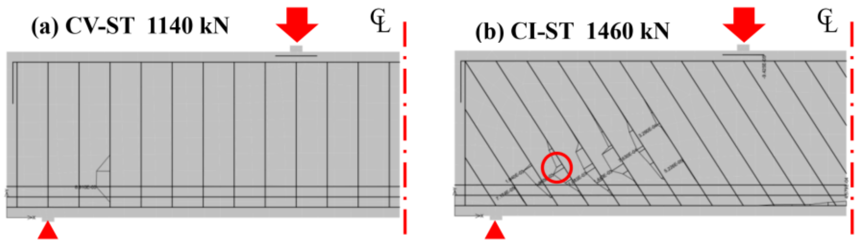


Fig. 7 Numerically axial plastic-strain of stirrups at the failure

3. PARAMETRIC STUDY

In this section, the effect of inclined stirrups will be clarified at different parameters and will be compared with the corresponding deep beams reinforced with vertical stirrups. This comparison will show the significance of the improvement for using inclined stirrups in different cases compared with those reinforced by vertical stirrups. The studied parameters can be listed as main reinforcement ratio, shear span-to-depth ratio, space between stirrups, and cross-sectional area of stirrups. Theses parameters are detailed in **Table 2**. The analysed beams of CV-ST and CI-ST are used as control beams in these comparisons.

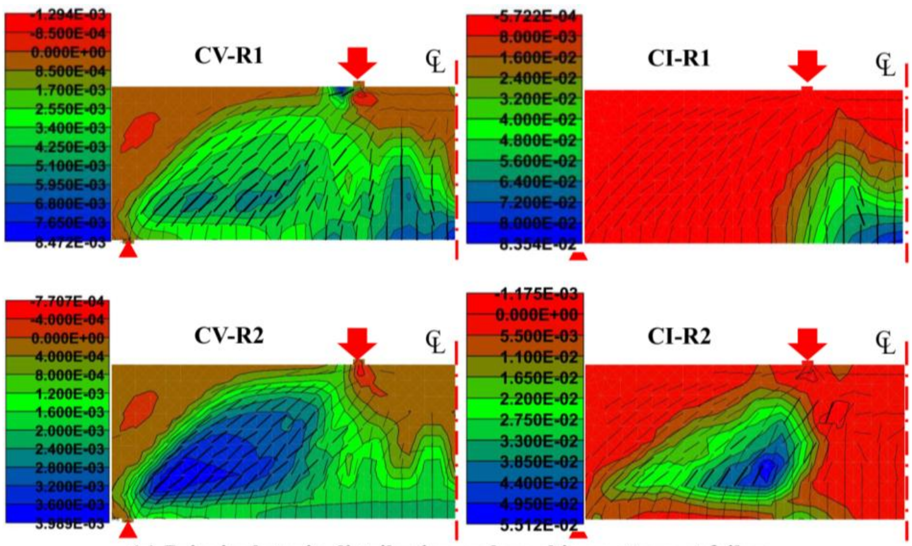
Table 2 Details of parametric studied beams

ID	Main bars	Stirrups (mm)	Shear span (mm)	f_c (MPa)	ID	Main bars	Stirrups (mm)	Shear span (mm)	f_c (MPa)
CV-R1	**6D18**	D10@150	1200	36	CV-S1	6D22	**D10@100**	1200	36
CV-R2	**6D25**	D10@150	1200	36	CV-S2	6D22	**D10@200**	1200	36
CI-R1	**6D18**	D10@150	1200	36	CI-S1	6D22	**D10@100**	1200	36
CI-R2	**6D25**	D10@150	1200	36	CI-S2	6D22	**D10@200**	1200	36
CV-A1	6D22	D10@150	**800**	36	CV-D1	6D22	**D8@150**	1200	36
CV-A2	6D22	D10@150	**1000**	36	CV-D2	6D22	**D12@150**	1200	36
CI-A1	6D22	D10@150	**800**	36	CI-D1	6D22	**D8@150**	1200	36
CI-A2	6D22	D10@150	**1000**	36	CI-D2	6D22	**D12@150**	1200	36

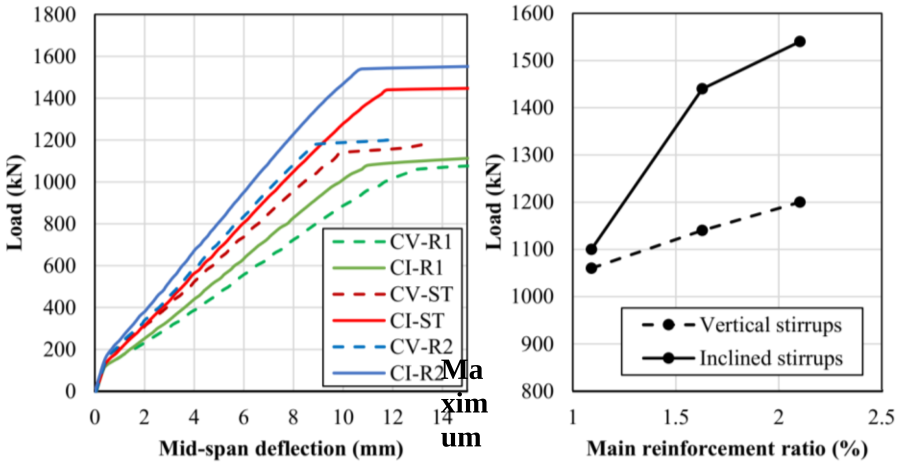
3.1. Main reinforcement ratio

Fig. 8 (a) show the principal strain distribution and cracking pattern at failure for analysed beams at different main reinforcement ratios, which can be defined as the percentage ratio of main reinforcement area to the concrete section area. At lower ratio, RC deep beam reinforced with vertical stirrups, CV-R1, shows a mix of shear and flexural cracks. Moreover, strain in flexural zone shows a slightly higher values than those values in the shear zone indicating a wider crack opening in flexural zone. This concludes a flexural failure in this beam. However, the beam reinforced with inclined stirrups, CI-R1, shows a higher strain value in flexural zone only pointing out the flexural failure in this beam. On other hand, higher main reinforcement ratio leads to higher strain values at shear zone for both beams which are reinforced with vertical and inclined stirrups. This scenario leads to shear failure in theses beams. However, beam of CI-R2 presents a localized and higher shear strain than beam reinforced with vertical stirrups. According to these behaviours shown in **Fig. 8 (a)**, using high main reinforcement ratio decreases the chances of occurring flexural failure before shear failure.

Fig. 8 (b) show the relationship between applied load and midspan deflection of beam reinforcement ratios with vertical and inclined stirrups. Increasing main reinforcement ratio for analysed beams leads to an increase of their ultimate load capacity and their overall beam stiffness. This observation was presented in both shear reinforcement techniques as vertical and inclined stirrups. However, the beams reinforced with inclined stirrups show an improvement than those beams reinforced with vertical stirrups. **Fig. 8 (c)** shows the relationship between the ultimate load and main reinforcement ratio. The improvement in the ultimate load is significant at higher main reinforcement ratio than that in lower ratio. At lower reinforcement ratio in the left side in **Fig. 8 (c)**, both beams reinforced with vertical and inclined stirrups present almost same ultimate load. The reason of this can be concluded as that the failure mode of these beams is flexural failure at their midspan bottom layers. While both beams have the same main reinforcement and cross section resulting in a same flexural stiffness.



(a) Principal strain distribution and cracking pattern at failure



(b) Load-deflection curves (c) Main reinforcement effect

Fig. 8 Parametric study of reinforcement ratio

3.2. Shear span-to-depth ratio

Fig. 9 **(a)**. At lower shear span-to-depth ratio, shear strain values are smaller than those in the beams with higher shear span-to-depth ratios. The shear strain distribution in beams reinforced with vertical stirrups is more localized indicating shear failure mode in these beams. However, the beams with inclined stirrups shows shear and flexural cracks. The crack opening in flexural zone for beam reinforced with inclined stirrups with smaller shear span-to-depth ratio, CI-A1, is wider than other beams. Therefore, for beams reinforced with inclined stirrups, increasing shear span-to-depth ratio leads to flexural failure mode. This observation is significant in beam CI-ST shown in **Fig. 4 (c)**. In other hand, other beams in this group failed in shear failure mode.

The load deflection curves and shear span-to-depth ratio effect on the ultimate load are shown in **Fig. *9* (b)** and **(c)**, respectively. The beam reinforced with inclined stirrups presents a higher ultimate load and a slight increase of the overall beam stiffness than that reinforced with vertical stirrups at same shear span-to-depth ratio. Moreover, increasing the shear span-to-depth ratio leads to a significant decrease in overall beam stiffness. However, the ultimate load of analysed beams in this group is almost constant value till the shear span-to-depth ratio equal to 1.34. Then, their ultimate loads show a decrease with the increase of shear span-to-depth ratio. This observation is significant in beams CV-ST and CI-ST which have a higher shear span. This leads to a clear indication of flexural crack propagation than those with lower shear span as shown in **Fig. *9* (a)**. Therefore, beams with lower shear span-to-depth ratio less than 1.34 failed in shear failure which is affected by the concrete strength and shear reinforcement. These similar parameters lead to an equal ultimate load values and failure type as shear mode. On the other hand, higher shear span-to-depth ratio leads to flexural failure before shear failure mode. This results in a smaller ultimate load due to flexural failure than that failed due shear failure.

3.3. Space between stirrups

In **Fig. 10 (a)**, principal strain distribution and cracking pattern were presented to show the effect of increasing shear reinforcement by changing the space between stirrups for vertical and inclined shear reinforcement. For beams reinforced with vertical stirrups, all beams are failed in shear failure mode. However, beam reinforced with narrow spacing stirrups shows wider flexural cracks opening in midspan than that reinforced with wide spacing stirrups. On other side, for beams reinforced with inclined stirrups, flexural cracking failure was presented for beam with narrow spacing stirrups. However, beam with wide spacing inclined stirrups shows a clear localized shear cracking failure. Therefore, narrow spacing stirrups lead to flexural failure mode which is significant in beams reinforced with inclined stirrups.

In **Fig. 10 (b)** and **(c)**, the load-deflection curves and the relationship between ultimate load and space between stirrups. The slope of load-deflection curve for analysed beams is mostly the same with changing of the space between stirrups. Therefore, the overall beam-stiffness is not affected by reinforcing with narrow spacing stirrups. However, increasing space between stirrups leads to a decreasing of the ultimate load for beams reinforced with vertical and inclined stirrups. The improvement of using inclined stirrups is significant at wider spacing between stirrups.

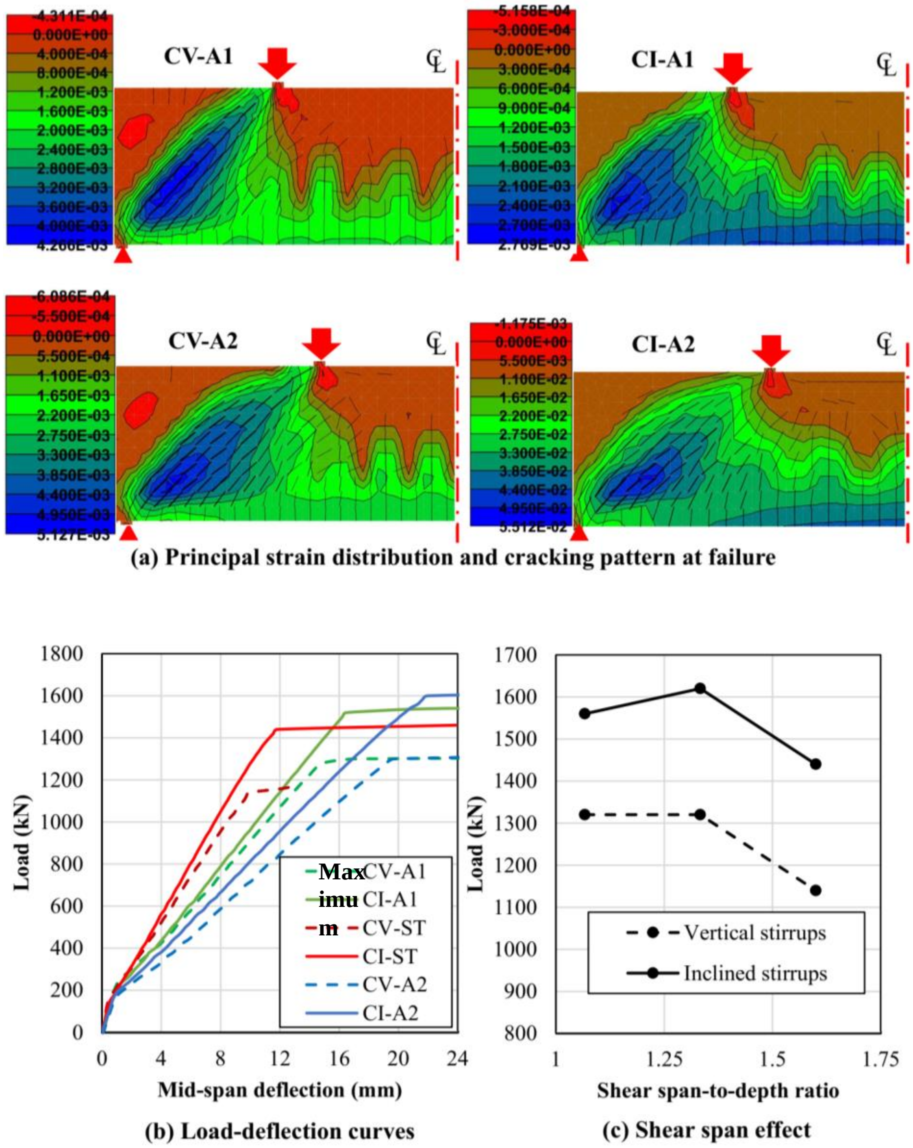


Fig. 9 Parametric study of shear span-to-depth ratio

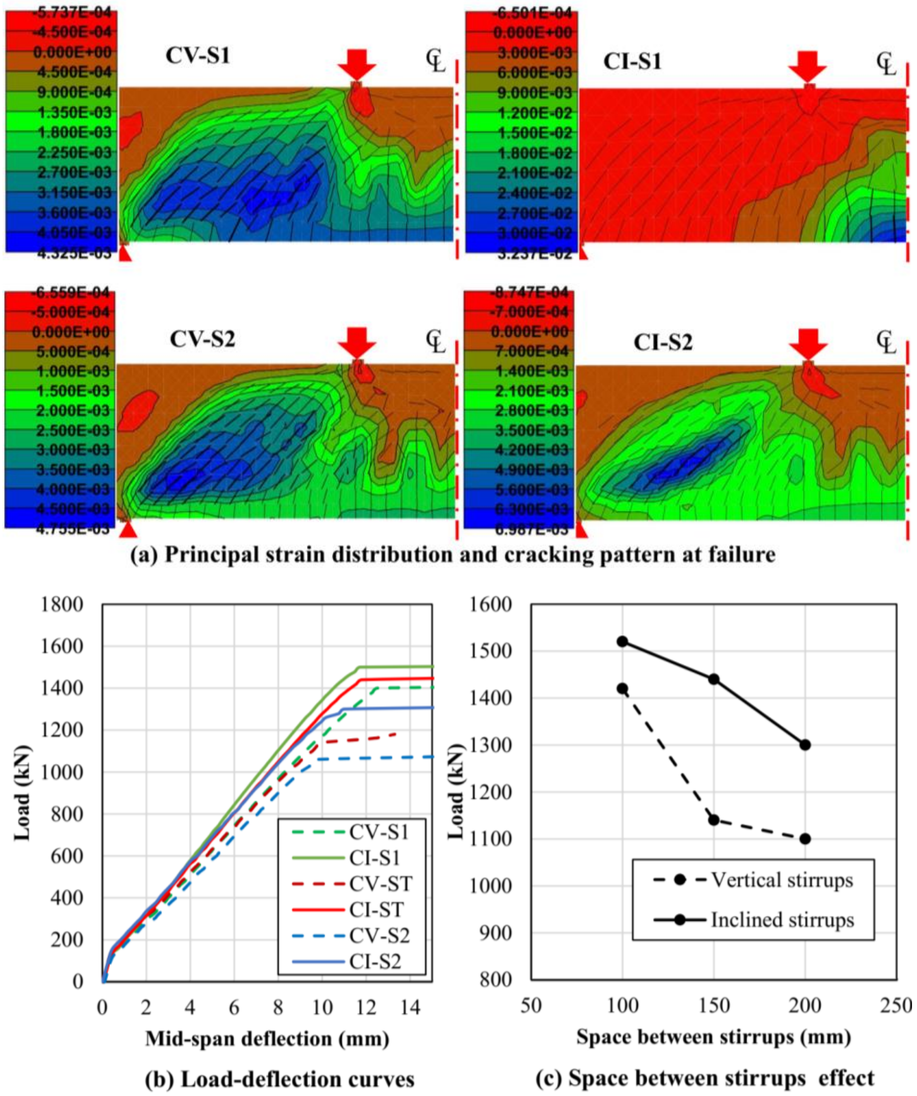


Fig. 10 Parametric study of space between stirrups

3.4. Cross-sectional area of stirrups

Fig. 11 (a) shows principal strain distribution and cracking pattern for beams reinforced by stirrups with different cross-sectional area. In the left side of beams reinforced with vertical stirrups, increasing cross-sectional area of stirrups leads to a propagation of the flexural cracks in mid-span as shown in beam of CV-D2. Finally, all beams reinforced with vertical stirrups failed by shear failure mode. However, beam reinforced with smaller cross-sectional area of stirrups shows a higher principal strain distribution indicating wide shear crack opening than that reinforced with higher cross-sectional area of stirrups. In the right side of beams reinforced with inclined stirrups, beam reinforced with higher cross-sectional area shows a clear flexural crack resulting in a flexural failure. However, a localized shear failure is presented in the beam reinforced with smaller cross-sectional area of inclined stirrups.

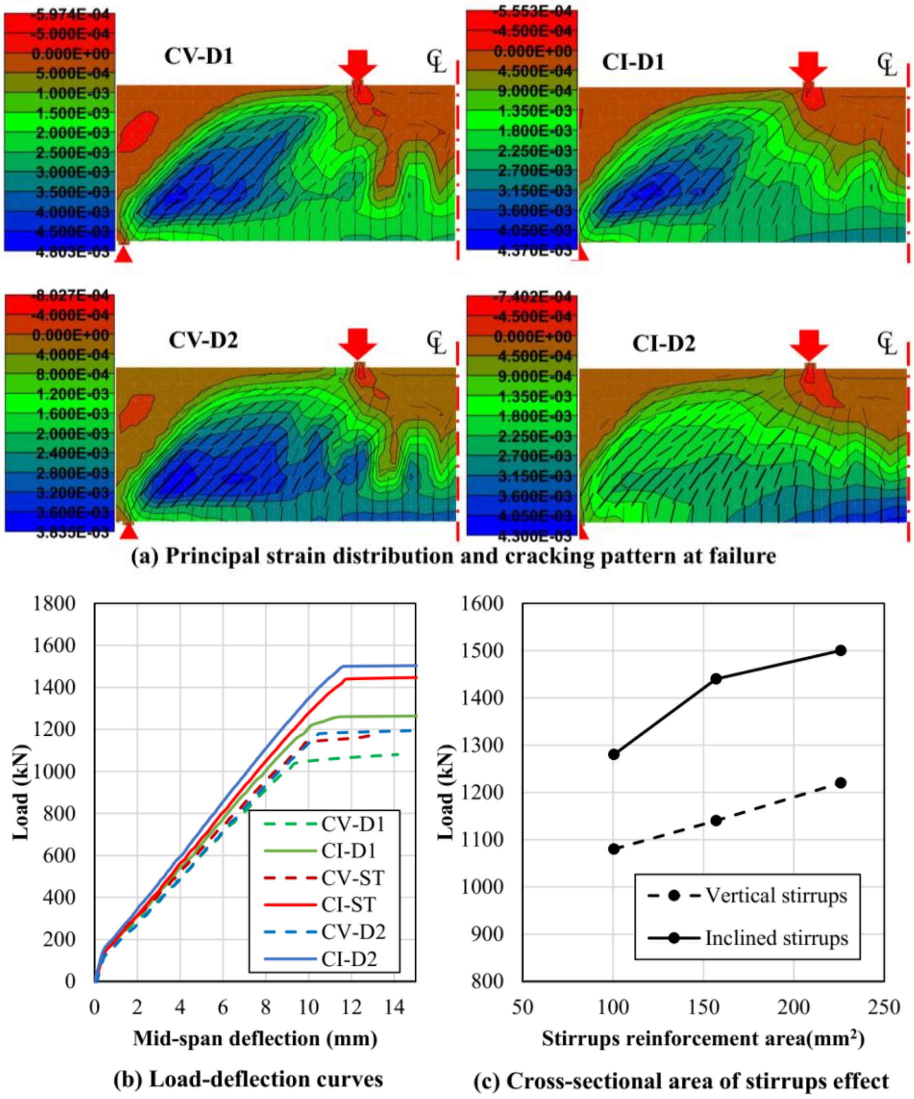


Fig. 11 Parametric study of cross-sectional area of stirrups

Fig. 11 (b) and **(c)** respectively shows the load-deflection curves and the relationship between the cross-sectional area of stirrups and the ultimate load for beams reinforced with vertical and inclined stirrups. As same as **Fig. 10 (b)**, there is no changes of the slope of load-deflection relationship for all analysed beams indicating to a similar overall beam-stiffness. In **Fig. 11 (c)**, increasing of cross-sectional area of stirrups leads to an increase of the ultimate load for beams reinforced with vertical and inclined stirrups. However, using inclined stirrups results in an approximately constant improvement than using vertical stirrups at different cross-sectional area of stirrups.

As shown in **Fig. 12**, the last two parameters can be combined as a shear reinforcement ratio which can be defined as the precent ratio between the totally cross-sectional area of stirrups to the sectional area of concrete. The effect of shear reinforcement ratio on the ultimate load and failure mode is listed in **Table 3**. In the lower shear reinforcement ratio, there is an improvement equal to 18.3% in the ultimate load for beams reinforced with inclined stirrups than those reinforced with vertical stirrups. In this stage, all beams are failed as shear failure mode. The mode of failure transfers to flexural failure by increasing shear reinforcement ratio than 0.86% and 0.45% for beams reinforced with vertical and inclined stirrups, respectively. This transformation of the failure mode is indicated in **Fig. 12** by changing in the slope of the relationship between shear reinforcement ratio and ultimate load. For higher shear reinforcement ratio, the improvement in the ultimate load by using inclined stirrups was found be very small as 7% only. The mode of failure in this stage is considered as a flexural failure for both shear reinforcement types. By respecting the same main

flexural reinforcement in this stage, there is no significant improvement in the ultimate load for using inclined stirrups.
In the **Table 3**, the increasing of ultimate load percentage compared by deep beam without any stirrups are listed.

Table 3 The effect of shear reinforcement ratio on the ultimate load and failure mode

ID	Shear reinf. ratio (%)	Ultimate load (kN)	Failure mode	Inc.%	ID	Shear reinf. ratio (%)	Ultimate load (kN)	Failure mode	Inc.%
CV-D1	0.382781	1080	Shear	196%	CI-D1	0.382781	1280	Shear	233%
CV-S2	0.448571	1100	Shear	200%	CI-S2	0.448571	1300	Shear	236%
CV-ST	0.598095	1140	Shear	207%	CI-ST	0.598095	1440	Flexural	262%
CV-D2	0.860952	1220	Shear	222%	CI-D2	0.860952	1500	Flexural	273%
CV-S1	0.897143	1420	Flexural	258%	CI-S1	0.897143	1520	Flexural	276%

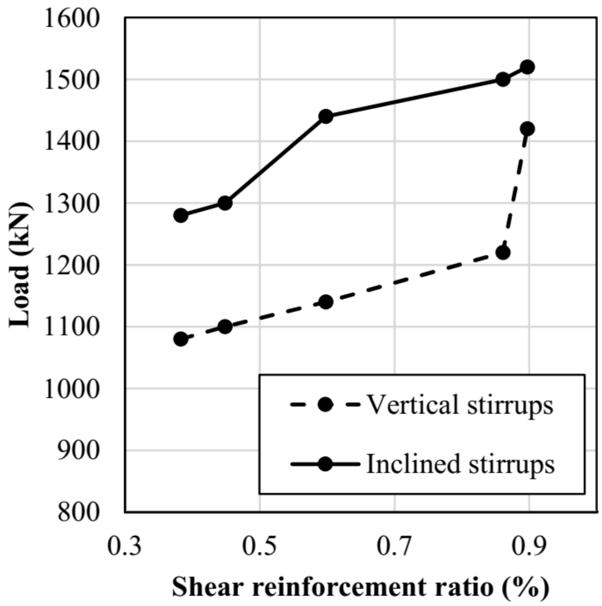


Fig. 12 Relationship between shear reinforcement ratio and ultimate load

4. CONCLUSIONS

This study proposed a numerical investigation to present the effect of the inclined stirrups on shear behaviour of deep beams and other parameters. The numerical results were verified with the experimental results showing a good agreement between them.
Using inclined stirrups, which is perpendicular on the direction of the main diagonal shear crack, restrict the opening of this crack and decrease the principal and shear strain on concrete surface. This observation can be also proved by the distribution of plastic strain on stirrups at failure showing a prefect contribution for the stirrups located in shear failure zone.
According to the part of parametric study in this paper, the improvement of using inclined stirrups as shear reinforcement is significant at higher main reinforcement ratio and lower shear reinforcement ratio.
Theses parameters leads to shear failure mode which magnifies the role of inclined stirrups.

REFERENCES

1. Teng, S., W. Ma, and F. Wang, *Shear strength of concrete deep beams under fatigue loading.* Structural Journal, 2000. **97**(4): p. 572-580.
2. Russo, G. and G. Zingone, *Flexure-shear interaction model for longitudinally reinforced beams.* Structural Journal, 1991. **88**(1): p. 60-68.
3. Russo, G. and G. Puleri, *Stirrup effectiveness in reinforced concrete beams under flexure and shear.* Structural Journal, 1997. **94**(3): p. 227-238.
4. Institute, A.C., *Building Code Requirements for Structural Concrete (ACI 318-19): Commentary on Building Code Requirements for Structural Concrete (ACI 318R-19): an ACI Report*. 2020: American Concrete Institute. ACI.
5. Singh, B., et al., *Design Of A Continuous Deep Beam Using The Strut and Tie Method.* 2006.
6. Wight, J.K. and G. Parra-Montesinos, *Strut and Tie Model for Deep Beam Design.* Concrete international, 2003. **25**(5): p. 63-70.
7. Tan, K.-H., et al., *Effect of web reinforcement on high-strength concrete deep beams.* Structural Journal, 1997. **94**(5): p. 572-582.
8. Červenka, V., L. Jendele, and J. Červenka, *ATENA theory manual*. Cervenka Consulting, 2009.
9. Rots, J.G. and J. Blaauwendraad, *Crack models for concrete, discrete or smeared? Fixed, multi-directional or rotating?* HERON, 34 (1), 1989, 1989.
10. Cervenka, V., *Constitutive Model for Cracked Reinforced Concrete.* Journal Proceedings, 1985. **82**(6).
11. Kolmar, W., *An Approach of the Force Transfer throughout Cracks by the Nonlinear Finite Element Analysis of the Reinforced Concrete Items*. 1986, PhD thesis. Darmstadt: TH, 1986. 94 p.(in German).