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Perturbation effect on ground tracks of Molniya satellite orbits

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Abstract

The main concern in the present research work is to define the basic requirements study of many perturbations forces that affect ground tracks of Molniya satellite orbits.

In this we are going to design a mathematical model to calculate the footprint of any satellite taking into account the effect of disturbing forces (oblatness and solar radiation pressure) in order to get the most accurate sub-satellite point, matlab language is used to design program to calculate the mathematical models of footprint satellites with numerical application of some different Molniya satellites orbit.

Key Words: *Artificial satellites, Kozai's method, oblateness, solar radiation pressure, Ground Track, Longitude, Latitude.*

1. Introduction

Orbit computations of artificial satellites become one of the most important problems at present time. As far as the computation techniques are concerned the applications of the special perturbation methods to the equations of motion. These perturbations make a drift for the orbital elements of satellite.

Two factors influence the ground track that due to Earth's rotation include the altitude of the satellite, in turn determines the satellite's angular velocity, and the latitude at which the satellite is located, determines the component of the Earth's rotation applicable at that point. If the ground track is known that would have been there had the earth been static, modification to this track at any given point in the satellite orbit would depend on the satellite altitude at that point and also on the latitude of that point.

Some previous work which dealing with orbital perturbations. Cook (1962) used the Lagrange’s planetary equations to obtain expressions for the variation of the orbital elements during one revolution of the satellite and for the rate of variation of the same elements. Kozai's (1962) studied the problem of secular perturbations of asteroids with high inclination and eccentricity. Sehnal (1975) discussed the direct solar radiation pressure, as one of the non-gravitational forces, from all its different aspects. Blitzer (1995) study the perturbed orbit under Earth’s gravitational forces and solar radiation pressure. Baron (2010) studied the effects of the solar radiation pressure and the attraction of the sun and the moon at high Earth orbit satellite has been investigated. Saad (2010) study the joint effects of direct solar radiation pressure and the gravitation of the Earth on high-altitude Earth satellites. Wesam Taleb (2011) studied the perturbations (oblateness and solar radiation pressure [SRP]) that effects on the artificial satellites orbit and with more altitudes and inclinations. Abbas (2012) the effects of solar radiation pressure at medium Earth orbit satellite.

2. Equations of Motion with Perturbations

Knowledge of orbital motion is essential for a full understanding of space operations. Motion through space can be visualized using the laws described by Johannes Kepler and understood using the laws described by Sir Isaac Newton.

A satellite, under the influence of a perfect inverse square force field law, would have a set of constant orbital elements (a, e, I, M, Ω, ω). The general form of the equation of motion in a relative inertial coordinate system is given by

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{F} \,, \qquad (2.1)$$

where $\vec{r}$ is the position vector of the satellite, μ is gravitational constant and $\vec{F}$ is the resultant vector of all the perturbing. $\vec{F}$ may consist, in out study, of the following types of perturbation forces (Rowa, 2002):

- Gravitational potential,
- Solar radiation pressure.

2.1. The Gauss Form of Lagrange's Equations

In some cases it is useful to formulate the disturbing accelerations directly at the satellite in componential form, instead of using partial derivatives of the disturbing potential in the elements. The formulas of Lagrange's Equations are also less suitable for numerical treatment. An appropriate alternative form was developed by Gauss.

According to figure (2.1) the perturbing forces at the satellite are resolved into three mutually perpendicular components (K_1 , K_2 , K_3) (Günter, 2003).

Where

- K_1 : Perpendicular to the orbital plane, positive toward the North Pole,
- K_2 : Perpendicular to the radius vector in the orbital plane, positive in the direction of increasing longitude,
- K_3 : In the direction of the radius vector, positive in the direction of increasing radial distance, and since

$$grad(R) = \nabla R = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}. \tag{2.2}$$

The corresponding perturbed equations are:

$$\frac{\partial a}{\partial t} = \frac{2}{nx}\left[(e\sin\upsilon)k_3 + \frac{p}{r}k_2\right], \tag{2.3.1}$$

$$\frac{\partial e}{\partial t} = -\frac{\chi}{nae}\left[(e\sin\upsilon)k_3 + (\cos\upsilon + \cos E)k_2\right], \tag{2.3.2}$$

$$\frac{\partial I}{\partial t} = \frac{r\cos u}{na^3\chi}k_1, \tag{2.3.3}$$

$$\frac{\partial \Omega}{\partial t} = \frac{r}{na^2\chi}\frac{\sin u}{\sin I}k_1, \tag{2.3.4}$$

$$\frac{\partial \omega}{\partial t} = \frac{\chi}{nae}\left[(-\cos\upsilon)k_3 + \left(1+\frac{r}{p}\right)\sin\upsilon\, k_2\right] - \cos I\frac{\partial \Omega}{\partial t}, \tag{2.3.5}$$

$$\frac{\partial M}{\partial t} = n - \frac{1}{na}\left[\left(\frac{2r}{a} - \frac{\chi^2}{e}\cos\upsilon\right)k_3 - \frac{\chi^2}{nae}\left(1+\frac{r}{p}\right)\sin\upsilon\, k_2\right], \tag{2.3.6}$$

where is υ = true anomaly; $\chi\sqrt{1-e^2}$ and $u = \omega + \upsilon$.

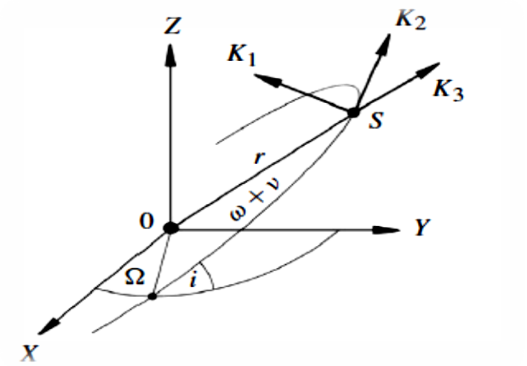

Figure: (2.1) Satellite Coordinate System (*R*, *S*, *W*).

Noting that in the literature the symbols R, S, W are also used instead of K_1 , K_2 ,K_3 .

3. Non-spherical gravity potential

The earth is not the point gravitational source assumed in Newton's gravitational law, in fact both its shape and mass distribution lead to non-inverse square law effects on an orbiting satellite. An accurate model of the earth can be obtained through the use of a series of spherical harmonics which effectively represent a gravitational body as a series of mass centers, some more dominant than others, the most dominant term being that of a perfectly uniform sphere the gravitational potential of the earth can be written as: (Wesam, 2011)

$$V = \frac{GM}{r}\left(1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n \left(C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\,P_{nm}\cos\phi\right)\right). \quad (3.1)$$

The harmonic coefficients C_{nm} , S_{nm} are integrals of the mass and describe the mass distribution within the central body, a_e the equatorial radius and P_{nm} called associated Legendre functions or Legendre polynomials.

The first term GM/r describes the potential of a homogeneous sphere and thus refers to Keplerian motion consequently it is also named the Keplerian term. The remaining expressions within the double-summation are due to the disturbing potential R, with the origin of the coordinate system transferred to the center of mass of the primary, the terms with n = 1 and m = 0, 1 become zero, hence (Günter, 2003)

$$R = \frac{GM}{r}\left(\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n \left(C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\,P_{nm}\cos\phi\right)\right). \quad (3.2)$$

With m = 0 the coefficient are named Zonal harmonics and rotationally symmetric about the pole and have n zero crossings from pole to pole. Note that S_{n0} = 0 the zonal coefficients are often represented by J's, $J_n = -C_{n0}$.

To apply the planetary equations to the non-central part of the field, write the gravity potential function as substitution of the complete disturbing function R into equations (3.2) generally has little practical value. The general approach is to divide the perturbations to the orbital elements into secular perturbations, long period perturbations, and short period perturbations. The secular variations result from averaging the equations of motion over one orbital period by assuming constant, mean values of the elements over that time. Recall the variation of parameter (Ω, ω and M) will change linearly with time (Tolson, 2005).

The oblateness perturbations, namely the rotation of the nodal and apsidal lines caused by the second order zonal harmonic, C_{20} and does not produce secular perturbations in the elements I, a, and e. However, C_{20} gives rise to secular variations of the elements, ω, Ω, and M because the numerical value of C_{20} exceeds all other potential coefficients by a factor of 10^3. These variations can be used as reference elements, they represent a secularly preceding Kepler-ellipse with the elements (a, e, I, M, Ω, ω) (Günter, 2003).

$$\Omega = \Omega_0 + \dot{\Omega}_{\sec}\,\Delta t + \dot{\Omega}_{SR}\,\Delta t\,, \quad (4.1.1)$$

$$\omega = \omega_0 + \dot{\omega}_{\sec}\,\Delta t + \dot{\omega}_{SR}\,\Delta t\,, \quad (4.1.2)$$

$$M = M_0 + \dot{M}_{\sec}\,\Delta t + \dot{M}_{SR}\,\Delta t\,, \quad (4.1.3)$$

where (M_0, Ω_0 and ω_0) are the initial value; and the suffixes (sec, SR) are the oblateness perturbations and solar radiation pressure.

4. Solution of perturbation equation (Kozai's method)

In 1959 Kozai's basic theory was used Lagrange's variation of parameters equations (VOP equations) and solved them using the averaging technique. Notice the similarity to expressions for the semi-major axis and mean motion, as well as to the first-order approximation for the mean anomaly. The elements of the orbit under perturbations can be expressed as (Vallado, 1997):

4.1 Perturbations caused by the zonal coefficients J_n

In order to estimate the effect of Earth's gravity field on particular satellite orbits, it is often sufficient to determine the accelerations caused by the first four zonal harmonics; we can obtain an expression for the secular effects on the mean anomaly, the longitude of the ascending node, and argument of perigee (Vallado, 1997).

$$\begin{aligned}\dot{M}_{\sec} = &\frac{3nR_{\oplus}^2 J_2 \chi}{4p^2}\{3\sin^2 I - 2\} + \\ &\frac{3nR_{\oplus}^3 J_2^2}{4p^4\chi}\{320e^2 - 280e^4 + (1600 - 156e^2 + 328e^4)\sin^2 I\} + \\ &\frac{3nR_{\oplus}^3 J_2^2}{4p^4\chi}\{-2096 + 1072e^2 + 79e^4 \sin^4 I\} - \\ &\frac{45nR_{\oplus}^4 J_4 e^2 \chi}{128p^4}\{-8 + 40\sin I + 35\sin^2 I\}\end{aligned} \quad (4.2.1)$$

$$\begin{aligned}\dot{\Omega}_{\sec} = &-\frac{3nR_{\oplus}^2 J_2 \cos I}{2p^2} + \\ &\frac{3nR_{\oplus}^4 J_2^2 \cos I}{32p^4}\{12 - 4e^2(80 + 15e^2)\sin^2 I\} + \\ &\frac{15nR_{\oplus}^4 J_4 \cos I}{32p^4}\{8 + 12e^2 - (14 + 21e^2)\sin^2 I\}\end{aligned} \quad (4.2.2)$$

$$\begin{aligned}\dot{\omega}_{\sec} = &\frac{3nR_{\oplus}^2 J_2}{4p^2} + \{4 - 5\sin^2 I\} + \\ &\frac{9na_e^4 J_2^2}{384p^4}\left\{\begin{matrix}56e^2 + (760 - 36e^2)\sin^2 I \\ -(890 + 45e^2)\sin^4 I\end{matrix}\right\} - \\ &\frac{15na_e^4 J_4}{128p^4}\left\{\begin{matrix}65 + 72e^2 - (248 + 252e^2)\sin^2 I \\ +(196 + 189e^2)\sin^4 I\end{matrix}\right\}\end{aligned} \quad (4.2.3)$$

4.2. The perturbing acceleration of Solar-Radiation Pressure

Solar-radiation pressure is a non-conservative perturbation, but it becomes important at higher altitudes. One of the more difficult aspects of analyzing solar radiation is accurately modeling and predicting the solar cycles and variations. During periods of intense solar storms, this effect may be much larger than all the other perturbations (depending on the altitude); at times of low activity, the effect may be negligible. A lot of literature describes the effects of solar-radiation pressure on orbits, including theoretical studies and analyses of observational data. Blitzer (1959) extracted the following equations from the work by Cook (1962) he assumed that the disturbing acceleration, F_{SR}, is given by

$$F_{SR} = P_{SR}\, C_R\, A\,, \quad (4.3)$$

The perturbing acceleration of an Earth satellite due to solar-radiation-pressure effects can be computed by means of the following equations (Chobotov, 2002):

$$\begin{Bmatrix} F_r \\ F_s \end{Bmatrix} = \cos^2(E/2)\cos^2(I/2)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(\lambda_{\mathrm{O}} - u - \Omega) -$$

$$\sin^2(E/2)\sin^2(I/2)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(\lambda_{\mathrm{O}} - u + \Omega) -$$

$$\frac{1}{2}\sin E \sin I \left[\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(\lambda_{\mathrm{O}} - u) - \begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(-\lambda_{\mathrm{O}} - u)\right] - , \quad (4.4.1)$$

$$\cos^2(E/2)\sin^2(I/2)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(-\lambda_{\mathrm{O}} - u + \Omega) -$$

$$\sin^2(E/2)\cos^2(I/2)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(-\lambda_{\mathrm{O}} - u - \Omega)$$

$$F_w = \cos^2(E/2)\sin I \sin(\lambda_{\mathrm{O}} - \Omega) -$$

$$\sin^2(E/2)\sin I \sin(\lambda_{\mathrm{O}} + \Omega) - \cos I \sin\varepsilon \sin\lambda_{\mathrm{O}} , \quad (4.4.2)$$

where F_r , F_s and F_w are components of acceleration along the satellite orbit radius vector (see Fig. 2.1), perpendicular to F_r in the orbital plane, and along the orbit normal, since $\varepsilon = 23.5°$ denotes the obliquity of the ecliptic, λ_{Θ} the ecliptic longitude of the sun, and argument of latitude.

The force components approach directly relates the perturbing force components to the rate of the orbit elements. The general form of equations of variation (the rates of the six classical elements) can be derived through the concept of perturbed variations are given by (Chobotov, 2002):

$$\dot{a}_{SR} = \frac{2e\sin v}{n\chi}F_r + \frac{2a\chi}{nr}F_s , \quad (4.5.1)$$

$$\dot{e}_{SR} = \frac{\chi\sin v}{na}F_r + \frac{\chi}{nea^2}\left(\frac{a^2\chi^2}{r} - r\right)F_s , \quad (4.5.2)$$

$$\dot{I}_{SR} = \frac{r\cos u}{2a^2\chi}F_w , \quad (4.5.3)$$

$$\dot{M}_{SR} = n - \frac{1}{na}\left(\frac{2r}{a} - \frac{\chi^2\cos v}{e}\right)F_r - \frac{\chi}{nea}\left(\frac{r}{a\chi^2} + 1\right)\sin v\, F_s , \quad (4.5.4)$$

$$\dot{\Omega}_{SR} = \frac{r\sin u}{2a^2\chi\sin I}F_w , \quad (4.5.5)$$

$$\dot{\omega}_{SR} = -\frac{\chi\cos v}{nea}F_r + \frac{p}{eh}\left[\sin v\left(\frac{1}{1 - e\cos v} + 1\right)\right]F_s - \frac{r\sin u\cot I}{na^2\chi}F_w . \quad (4.5.4)$$

5. Satellite Ground Track

A ground track is the projection of the satellite's orbit on to the surface of the Earth. We can determine the latitude and longitude of satellite (λ, φ) from the following equations:

$\lambda = \alpha -$ *G. Sideral Time*, and $\varphi = \delta - \varphi'$, (5.1)

where $$\varphi' = \tan^{-1}\left[\frac{\tan\delta}{(1-f)^2}\right],$$

and f is the flatting of the Earth.

Also,

$$\sin\delta = \frac{Z}{U}, \qquad \sin\alpha = \frac{Y}{\sqrt{X^2+Y^2}}, \qquad \cos\alpha = \frac{X}{\sqrt{X^2+Y^2}}. \quad (5.2)$$

Since

$$X = r(\cos\Omega\cos u - \sin\Omega\sin u\cos I), \quad (5.3.1)$$

$$Y = r(\sin\Omega\cos u + \cos\Omega\sin u\cos I), \quad (5.3.2)$$

$$Z = r\sin I\sin u, \quad (5.3.3)$$

and $$r = \frac{a(1-e^2)}{1+e\cos u}, \qquad U = \sqrt{X^2+Y^2+Z^2}.$$

6. Results and Conclusion

A computer program has been developed to solve the equations of orbital motion of two body problems with perturbations due to the gravitational potential and solar radiation pressure by using previous algorithms and Matlab code.

We compute the variation of latitude and longitude of satellites using for example (Molniya 1-93) satellite.

6.1. Algorithm

This algorithm to determine the position and velocity from two line orbital elements (TLE) and plot the ground track with/without perturbations of satellites.

Input:

1. TLE of satellite: (text file).
2. Set $R_\oplus$ = 6378.137 *km*, μ = 398600.4481 km^2/sec^2,
   J_2 = 0.00108263, J_4 = -0.00000161.

Computational algorithm:

1. Read the initial values Tow Line Element (TLE) for satellites then determine the six orbital elements and compute eccentric anomaly.
2. Calculate the position and velocity coordinates from Eccentric anomaly by solving Kepler's equations.
3. Calculate Greenwich sidereal time.
4. Add perturbation forces to orbital elements and repeat compute the position and velocity vectors.
5. Calculate longitude and latitude from the position vector and local sidereal time.
6. Repeat this sequence from step (1) to step (5) and used the results for drawing the ground track of satellite.
7. Algorithm is complete.

6.2 MOLNIYA1-93

Table (6.1) shows Two-line elements initial values of six orbital elements (http//:www.Celestrack.com) and table (6.2) shows a comparison between longitude and latitude with/without perturbations at many numbers of revolutions.

Figures (6.1, 6.2 and 6.3) illustrates the effect of perturbation on the ground track of this satellite at revolutions numbers (100, 1000 and1500).

Table (6.1): Two-line elements and six orbital elements for Molniya1-93 satellite.

```
MOLNIYA 1-93
1  28163U  04005A       14002.31575069  -.00000472
00000-0  00000+0 0  2796
2  28163    64.5468    98.8698  7242893  248.2772
24.7697  2.00616822 72337
```

The initial values for six orbital elements:	
(*a*): Semi major axis	26555.666741608 *km*
(*e*): Eccentricity	0.7242893
(*i*): Inclination	064.5468 *degree*
(Ω): Longitude of ascending	098.8600 *degree*

node	
(ω): Argument of perigee	248.2772 *degree*
(M): Mean anomaly	024.7697 *degree*
General satellite data from TLE	
Launched	02/01/2004
(T): Time of period	11.96 *hour*
Time at Epoch	07:34:40.8

Table (6.2): Comparison between with/without perturbations forces for Molniya1-93.

Time	Rev. No.	Longitude ($\lambda°$)			Latitude ($\phi°$)		
		Without Per.	J_2	J_2 + Solar	Without Per.	J_2	J_2 + Solar
2-1-2014 7:34:41	1	243.365	243.365	243.365	0.0384	0.0384	0.0384
20-2-2014 16:00:31	100	69.9152	63.3502	63.1108	3.49257	3.03863	3.03155
8-9-2014 1:15:02	500	94.7649	61.6785	60.4714	3.49257	4.26666	1.1685
17-5-2015 6:53:10	1000	124.079	57.8302	55.4131	6.55293	1.97443	1.90255
22-1-2016 12:31:18	1500	154.24	54.8152	51.1875	9.29076	2.4256	2.31737

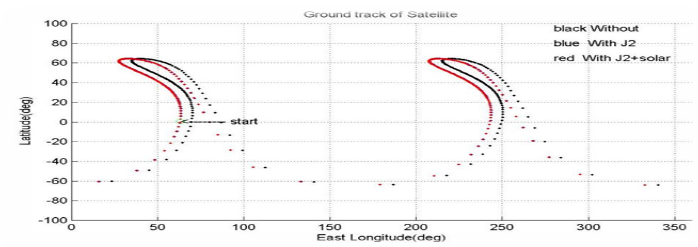

Figure (6.1): Ground track of Molniya1-93 satellite at one hundred rev.

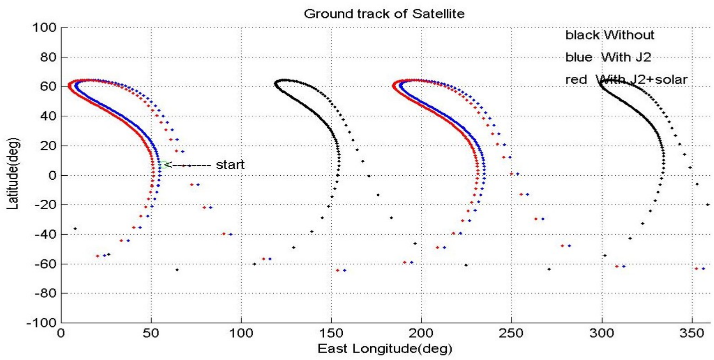

Figure (6.2): Ground track of Molniya1-93 satellite at one thousand rev.

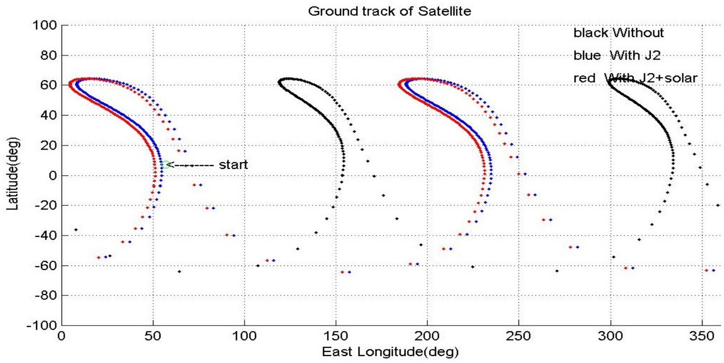

Figure (6.3): Ground track of Molniya1-93 satellite at one thousand and half rev.

6.4. Conclusions:

In this study, the mathematical model was tested to compute change in the six orbital elements for artificial satellite due to effect of the gravitation of the Earth and solar radiation pressure on the orbits of high altitude satellites (Molniya satellites) included (the zonal harmonics of the geopotential effects up to J_4 and solar radiation pressure) to compute the position and velocity.

From tables and figures we can conclude that:

- The influence of perturbation forces after short period can't be noticed; but after long period for example one hundred revaluations the effect of zonal harmonic is simple and effect of solar radiation is tiny.
- After five hundred revolutions the zonal harmonic's becomes more effect.
- In more one thousand periods we found that solar radiation pressure have important roles for getting accurate results. These results come out of working on the high satellite orbits.
- This work can be easily modified when we take the full effect of the Earth's gravitational field (zonal and Tesseral parts) and consider other forces, for example, drag on a charged satellite, Luni-solar gravity and meteorite collisions hence the accuracy of our results depend on the used forces to get more accurate foot print satellites.

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