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Automation of Design Tool for an Axial Flow Gas Turbine Stage Used in Small Gas Turbine Engine

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Abstract: This paper describes the automation of 3D design and aerothermodynamic analysis of a high pressure, single-stage axial flow turbine driving the compressor of the small gas turbine engine Jet Cat p200 using an analytical method. The specifications of design are based on the reverse engineering of the small gas turbine. Baseline design parameters such as flow coefficient, stage loading coefficient are close to 0.5 and 1.16 respectively with maximum flow expansion in the NGV rows. In the thermal cycle and 1D analysis calculations, the total conditions of all engine stations and all design controlling parameters are determined. In 2D design analysis, the mean line approach is used to generate the turbine flow path and 2D airfoil design, and by using the free vortex law of blading, five blade sections are generated. These sections are at hub, mean and tip and two intermediate sections between hub-mean and mean-tip using an approximate method to the Ainley, Mathieson, Dunham and Came loss model to meet the design constraints. An average exit swirl angle of less than 5 degrees is achieved leading to minimum losses in the stage. Also, NGV and rotor blade numbers are chosen based on the original turbine object. Blade profile is redesigned using the results from blade-to-blade analysis and through-flow analysis using special analytical method, [1]. Aerothermodynamic parameters like pressure ratios, aerothermodynamic power, and efficiencies are computed analytically. The results are compared with the published data in the engine manual.

Keywords: Aerothermodynamics of gas turbine, Design of axial flow turbine, Axial flow turbine.

Introduction

With the great demand of turbomachinery design codes, the task of designing the turbines has become cost effective in terms of time and money[2, 3]. Today's design codes include the optimization approach to reach the most robust design[4]. This paper presents a special methodology for designing a turbine stage. Such methodology includes 1D, 2D, and 3D calculation techniques. The design proceeds as follows:

- Data input collection (from the base engine data manual) [5].
- Aerothermodynamic calculations to determine the dimensionless groups that control the design process.
- 1D and 2D throughflow calculation to determine the inlet and exit velocity triangles.
- 3D blade airfoil stacking using free vortex law of blading.

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A MATLAB code is designed to do these functions. The design code has the capability to meet the comprehensive design requirements of the turbine. There are three set of modules available in this code to carry out the 1D/2D/3D design and analysis. The Aerothermodynamic calculations and 1D module are used to determine the turbine dimensions with the given set of design requirements. The through-flow calculation, 2D/3D modules are used for blade profile design and 3D flow analysis. These modules are used in a rational manner in the process of an axial flow turbine stage effective design.
The MATLAB code is developed to obtain the 3D distribution of blade airfoil at different blade sections. This code is based on dividing the airfoil into four sections, leading edge, trailing edge, suction side and pressure side. Both pressure and suction side curves are designed using quadratic B-spline fitting equations. The 3D point distribution of the airfoil is repositioned using the mathematical transformation.
The overall design process of a gas turbine engine usually starts with a given set of specifications, which normally determined from market demands or by specific customer requirements. There are three principle steps involved in turbine aerodynamic design process: preliminary design using mean line approach, through flow design, and airfoil design which is followed by a 3D. In this paper, an attempt is made to carry out the aerothermodynamic design and analysis of a single axial flow turbine stage using the mentioned codes and redesign the blade profiles based on the analytical analysis.

Reverse Aerothermodynamic Design of Jet Cat P200 Gas Turbine Stage

In this phase of design, A try is attempted to achieve the engine official manual given data [5] using the throughflow aerothermodynamic calculations model across the different engine basic parts and determining the dimensionless parameters that control the turbine design.

Turbine Rotor Meridional Plane Verification

Thermal cycle calculation:

To start the determination of gas flow parameters at different engine sections, the needed given data is obtained from the engine manual [5].The engine is assumed to work at standard conditions.
The total flow parameters at different engine modules are obtained by solving Continuity, Energy and Momentum equation

1- Flow through intake system:

The process of the inlet system at design conditions is ploytropic compression process. The losses in the inlet system are evaluated by coefficient of conservation of total pressure σ_{iN}.

$$M_0 = \frac{C_0}{\sqrt{\gamma R T}} = 0 \quad \text{(assumption)} \tag{1}$$

$$T_{1t} = T_0 + \frac{C_0^2}{2C_p} = T_0\left(1 + \frac{\gamma - 1}{2} M_0^2\right) = T_{0t} \tag{2}$$

2- Flow through compressor:

$$\pi_{Ct} = \frac{P_{2t}}{P_{1t}} \tag{3}$$

$$T_{2t} = T_{1t}\left[1 + \frac{\pi_{Ct}^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{Ct}}\right] \xrightarrow{yields} \eta_{Ct} = \frac{T_{1t}\left(\pi_{Ct}^{\frac{\gamma-1}{\gamma}} - 1\right)}{T_{2t} - T_{1t}} \tag{4}$$

3- Flow through combustion chamber:

$$T_{3t} = constant\ at\ certain\ RPM \tag{5}$$

$$P_{3t} = P_{2t} *\ \sigma_{cc} \tag{6}$$

Where σ_{cc} is the coefficient of conservation of total pressure in the combustion chamber

4- Flow through turbine:

From the torque balance between compressor and turbine, for single spool jet engine:

$$C_{p}(T_{2t} - T_{1t}) = \eta_m \bar{C}_p\ (T_{3t} - T_{4t}) \tag{7}$$

$$T_{4t} = T_{3t} - \frac{C_p(T_{2t} - T_{1t})}{\bar{C}_p\ \eta_m} \tag{8}$$

Then the turbine efficiency:

$$\eta_{T_t} = \frac{l_{eT}}{l_{adT_t}} = \frac{T_{3t} - T_{4t}}{T_{3t} - T_{4adt}} \tag{9}$$

5- Flow through exhaust system:

$$c_6 = \sqrt{2\grave{C}_p\eta_{EN}T_{4t}\left[1 - \left(\frac{P_0}{P_{4t}}\right)^{\frac{\grave{\gamma}-1}{\grave{\gamma}}}\right]} \tag{10}$$

$$\eta_{EN}\ (assumed)$$

$$T_{cr} = \frac{2}{\grave{\gamma} + 1}T_{4t} \tag{11}$$

$$a_{cr} = \sqrt{\grave{\gamma}\grave{R}T_{cr}} \tag{12}$$

Since $c_6 > a_{cr} \xrightarrow{yields} chocking\ happens\ at\ nozzle\ exit$:

$$c_5 = a_{cr} \tag{13}$$

$$T_5 = T_{cr} \tag{14}$$

$$T_{5ad} = T_{4t} - \frac{T_{4t} - T_5}{\eta_{EN}} \tag{15}$$

$$P_5 = P_{4t}\left(\frac{T_{5adt}}{T_{4t}}\right)^{\frac{\grave{\gamma}}{\grave{\gamma}-1}} \tag{16}$$

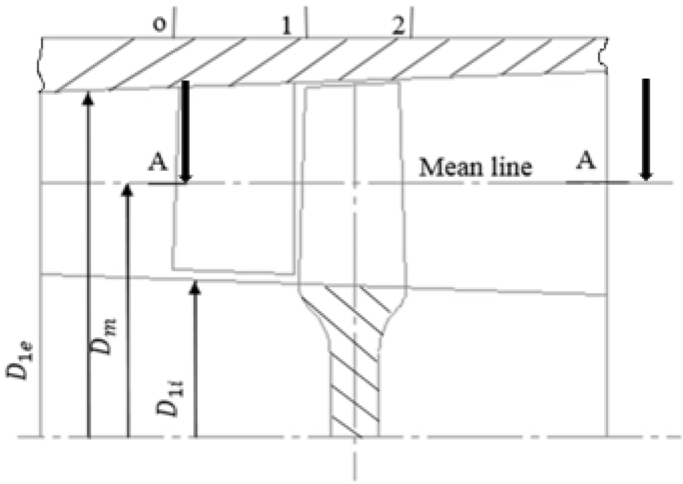
1D Mean line design

It is familiar to start the design with a mean line 1D design of the turbine using the given data from the thermal cycle calculations for the determination of the turbine stage main dimensions in meridional plane [1]. No variation in the radial or the tangential direction and the flow will only be calculated along the mean radius, Fig. 1. In this phase of the design many of the parameters come to play as preliminary guidelines. The overall purpose of the mean streamline design is to determine the basic parameters of the turbine at mid radius, and consequently the turbine stage geometry in meridional plane. Together with the loss model, a first estimation of the performance of the turbine stage can be made. These parameters are:

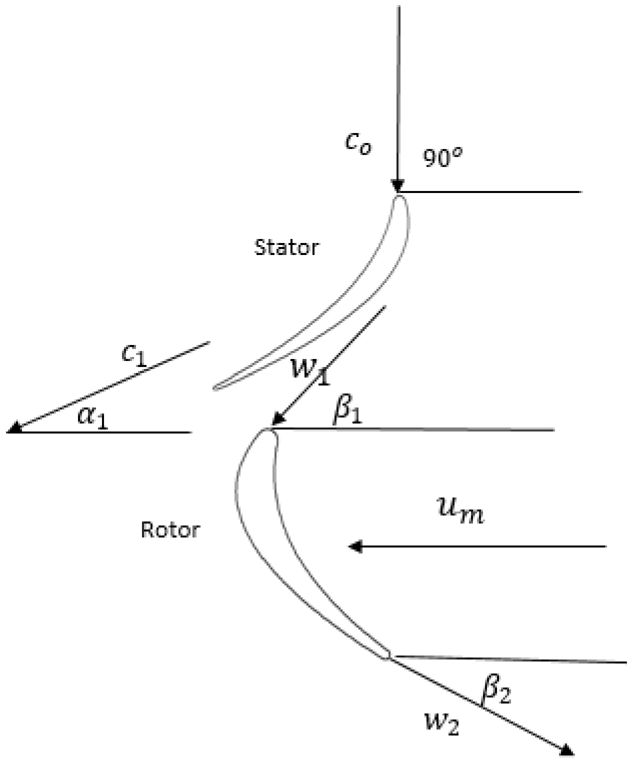
the loading factor
$$\psi = \frac{H_o}{U_m^2} \tag{17}$$

stage reaction
$$\rho = \frac{h_{o2}}{H_o} \tag{18}$$

where H_o ...is the total enthalpy change across the turbine stage.
U_m ..... is the rotor mean velocity.
h_{o2}..is the static enthalpy extracted across the rotor. Fig. 2.



a. Meridional plane.



b. Rotated section A-A
Fig. 1. Axial flow turbine stage.

flow coefficient $$\varphi = \frac{c_a}{U_m} \tag{19}$$

The values of these parameters are chosen based on the near optimum pitch-to-chord ratio. The choice of these parameters decides the velocity triangle at the meanline section and are therefore the most crucial parameters to decide initially.

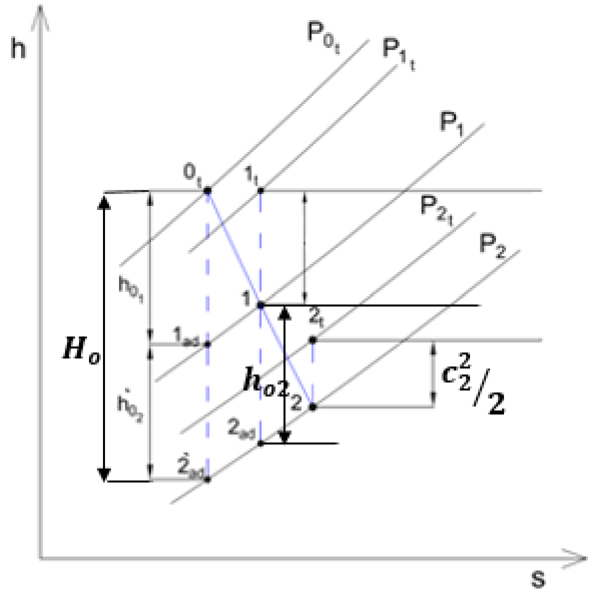


Fig. 2. Enthalpy-Entropy diagram

Starting design point

According to the exit total conditions from the combustion chamber, the flow parameters like density, velocity and other static parameters could be determined [6].

A MATLAB code was designed to solve a system of four equations representing the flow state conditions.

$$\mathrm{m}_o = c_o * \rho_o * A_o \tag{20}$$

$$T_{ot} = T_o + \frac{c_o^2}{2 * \bar{C}_p} \tag{21}$$

$$\rho_o = \frac{p_o}{R * T_o} \tag{22}$$

$$p_o = p_{ot} * \left(\frac{T_o}{T_{ot}}\right)^{\left(\frac{\dot{\gamma}}{\dot{\gamma}-1}\right)} \tag{23}$$

By solving this model of equations, the velocity entering the turbine stage is determined.

Velocity Triangles and Airfoil Design at Mean Stream Line

Mainly this phase is the stage flow path design using inlet operating conditions of the turbine to meet the given geometric limitations. The mean stream line approach serves as the base for gas path preliminary design in a turbomachinery design cycle. The design of high-pressure turbine stage is carried out using a mean line design analytical methodology [1] based on the data produced from the thermal cycle cal.& 1D analysis. This methodology gives full nomenclature data for the airfoil and the velocity triangles at both leading and trailing edges.

Assuming that the flow entering the turbine in absolute axial direction, so, the inlet stator blade angle is 90^o.

During the 1D design analysis, some parameters are assumed within certain range obtained from the previous experimental work. These values are tabulated in table 1.

Table 1. Design controlling parameters

Parameter	U/c_1	α_1	ρ_i
Value	$(0.5 - 0.7)$	$(18^o - 32^o)$	$(0 - 0.2)$

Where, the ρ_i is the stage reaction at blade hub to prevent back flow at hub radius. Knowing the engine RPM and flow coefficient $\emptyset = U/c_1$, the absolute inlet flow velocity to the rotor and all inlet velocity triangle components at mean radius are now determined.
As the Stage Reaction describes the flow expansion through the turbine cascade channel, then by applying the energy equation for relative flow, it is valid

$$T_1 + \frac{w_1^2}{2 * \bar{cp}} = T_2 + \frac{w_2^2}{2 * \bar{C}_p} \tag{24}$$

$$\bar{C}_p * (T_1 - T_2) = (\frac{w_2^2}{2} - \frac{w_1^2}{2}) \tag{25}$$

$$\Delta h = (\frac{w_2^2}{2} - \frac{w_1^2}{2}) \tag{26}$$

The static enthalpy difference can be obtained from the stage reaction as shown in h-s diagram Fig. 2. At this point, the inlet and exit velocity triangles features are determined at mean blade section Fig.3.and the mean blade airfoil can be determined graphically.

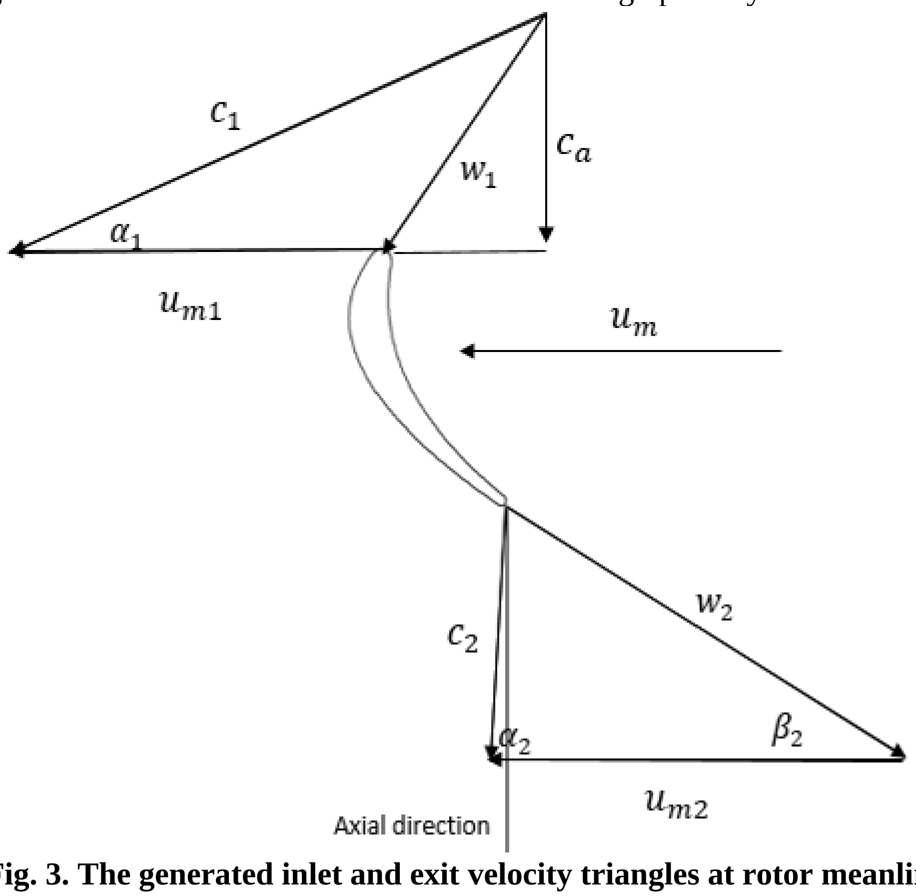


Fig. 3. The generated inlet and exit velocity triangles at rotor meanline

Blade Stacking using the Free Vortex Law of Blading

After having the mean line section airfoil from 2D design, it is needed to determine the airfoil at different blade sections.

As the velocity triangles changes along the blade span from hub to tip because of the change of the blade linear tangential velocity. Free vortex law of blading [7] describes the change of velocity triangles along the blade span, gives the twist from hub to tip. This theory can be represented mathematically as follows:

$$c_u * r = const. \tag{27}$$

$$c_a = const. \tag{28}$$

$$\tan \alpha_i = \frac{r_i}{r_m} * \tan \alpha_m \tag{29}$$

Which leads to

This model of equations gives the change in the inlet flow angle and velocity with turbine rotor radius which means that the velocity triangles at all blade sections are now determined Fig. 4.

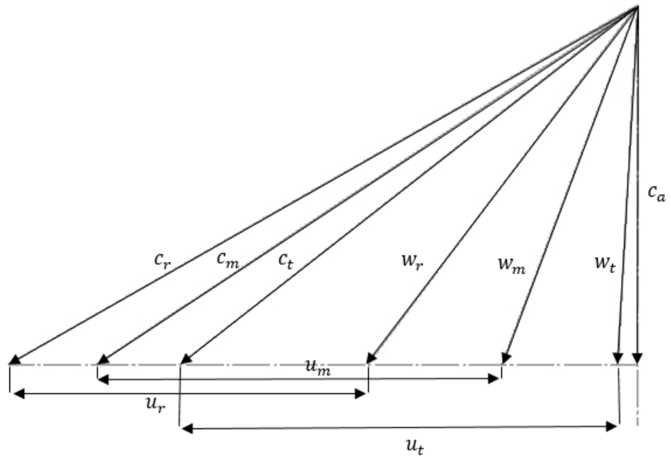


Fig. 4. Velocity triangles at hub, mean and tip sections

Table 2 includes the inlet and exit flow, blade angles and so, the main design parameters affecting factors generated by An EXCEL preliminary design code.

Table 2. Main results generated from design EXCEL sheet

SI. No.	Hub	Mean	Tip
α_1	18.33	22	25.48
α_2	94.9	93.88	93.3
β_1	37.38	68.55	109.68
β_2	38.15	33.2	28.5
ψ	1.54	1.04	0.75
ρ	0.18	0.4	0.5
φ	0.61	0.51	0.43
η	----	89%	----

2D Airfoil Design and Drawing

In this section, a graphical method is introduced to draw the blade by knowing the blade nomenclature (β_1 , β_2 , H , b). This method is mainly based on assuming certain leading and trailing edge thicknesses along the blade length. In this approach, the blade thickness is determined by assuming the values of angles γ_1 , γ_2. Fig. 5**.**

where γ_1 ………… The leading edge cone angle

γ_2 ………… The trailing edge cone angle

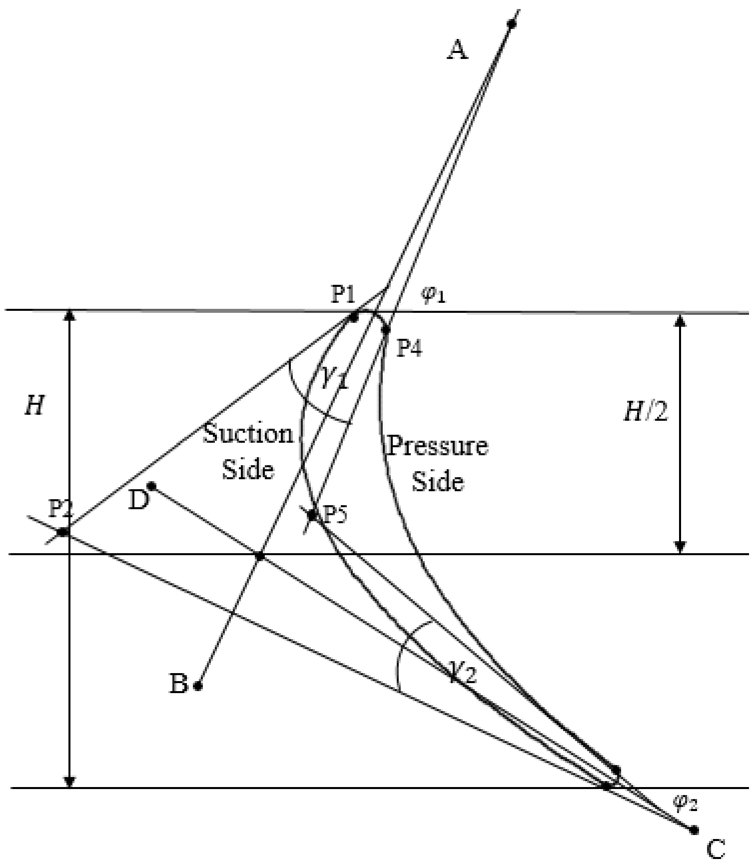


Fig. 5. The proposed cone angles (γ_1, γ_2) to control spline curves

Airfoil Parametrization

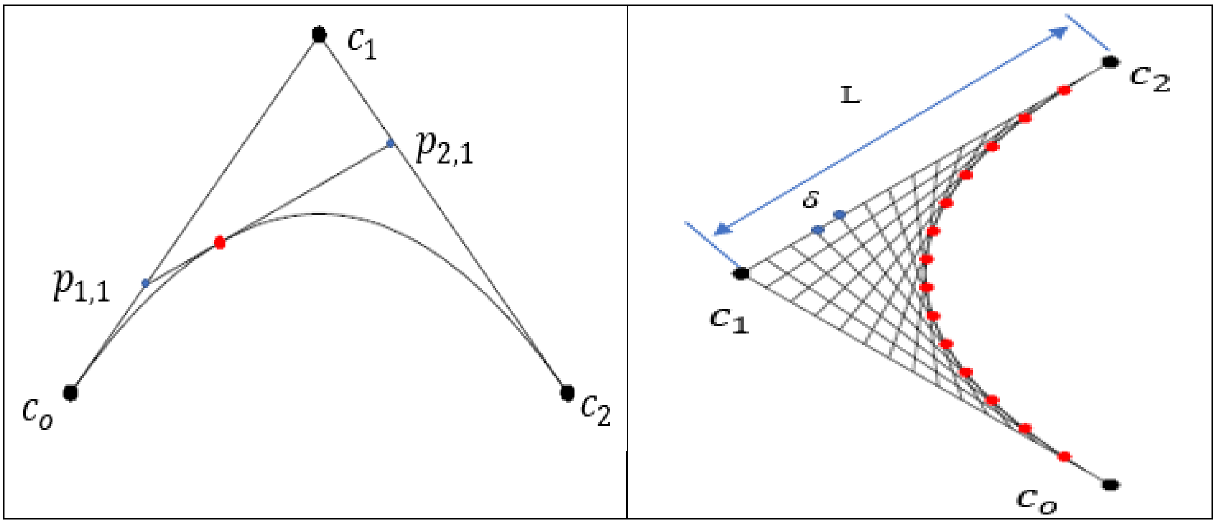
Bézier curves

The used blade airfoil curve construction method is an alternative to polynomial interpolation by applying Bézier curves. Bézier curves are polynomial curves, they avoid the problem of great change in direction of high order curves because all computations are true convex combinations. It also turns out that segments of Bézier curves can easily be joined smoothly together to form the turbine blade airfoil. This avoids the problem of using curves of high polynomial degree when many points are approximated. Bézier curves are a special case of the spline curves [8].

This method is mainly based dividing of the lines between c_o, $c_1$and c_1, $c_2$to equal segments δ. The line segment fraction $t = {}^{\delta}/_{L}$. The divided points are connected together as shown in Fig. 6. With the decrease of line segment fraction, the curve smoothness increases. The path of a curve that tangents all lines generates the target curve, as follows.

Tacking three points in the plane c_o, c_1 and c_2, and based on these points a smooth curve is constructed, by forming convex combinations of the given points. The natural solution is to start by defining the two line segments over the same interval, say [0:1] for simplicity,

$$p_{2,2}(t) = p(t\,|c_o, c_1, c_2) = (1-t)p_{1,1}\,(t) + t\,p_{2,1}(t)$$
$$p_{1,1}(t) = p(t\,|c_o, c_1) = (1-t)c_o + t\,c_1$$
$$p_{2,1}(t) = p(t\,|c_1, c_2) = (1-t)c_1 + t\,c_2$$



(a) **(b)**

Fig. 6. Bézier curves construction

Applying Bézier curves on turbine profile cascade design

In the proposed mothed the parabolic spline function order is applied, which means three control points are needed to interpolate the airfoil points between them. Fig .7. describes the airfoil graphical drawing:

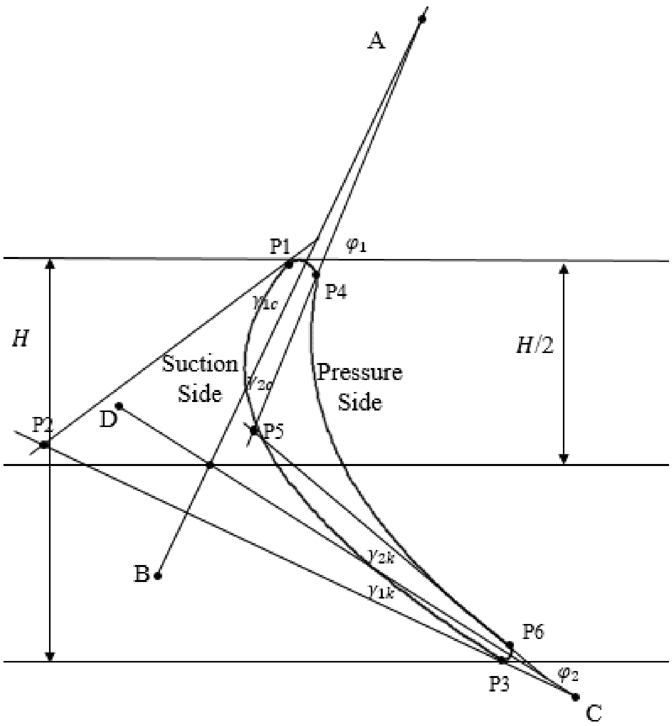


Fig. 7. Detailed figure for airfoil construction

As shown in Fig. 7. the airfoil is divided into four sectors, the leading edge and the trailing edge curves are a part of circles. Each one of suction and pressure sides curves will be drawn using parabolic B-spline fit curve with three control points that are created by the intersection the introduced cone angles lines, which means that the curve points distribution or the blade thickness distribution will be function of the proposed cone angles.

B-Splines Rotor Airfoil Curves Representation

Leading and trailing edges

To start drawing the airfoil, first it is determined the peripheral limits of the rotor by the profile width (H), then draw line AB forming angle $\varphi_1 = \beta_1 + i$ with the inlet peripheral line. Line AB extends to intersect the line H/2 in a point which will be the start point to draw line CD with angle $\varphi_2 = \beta_2$ and extends to intersect the exit peripheral line in the trailing edge point. At trailing edge point, a perpendicular segment equal to the trailing edge thickness S is drawn. To form the leading edge, a circle on line AB with radius equal to the leading edge radius r is drawn, such that the leading edge curve tangents the inlet peripheral line and for the trailing edge, draw a semicircle to form it.

Suction side curve

As mentioned before, three control points are needed to construct a parabolic B-spline fit curve are needed. So, an angle γ_{1c} is introduced by drawing line 1 tangents the leading edge curve in point P_1 and intersects line AB to form the angle γ_{1c} as shown in Fig. 8. At the trailing edge, draw line 2 to form the angle γ_{2c} with line CD. tangent to the trailing edge curve in point P_3 and extends to intersects line 1 in point P_2.

As nominal design values in consequence of experience, take $\gamma_{1c} = (10^o - 30^o)$, $\gamma_{2c} = (1.5^o - 3^o)$

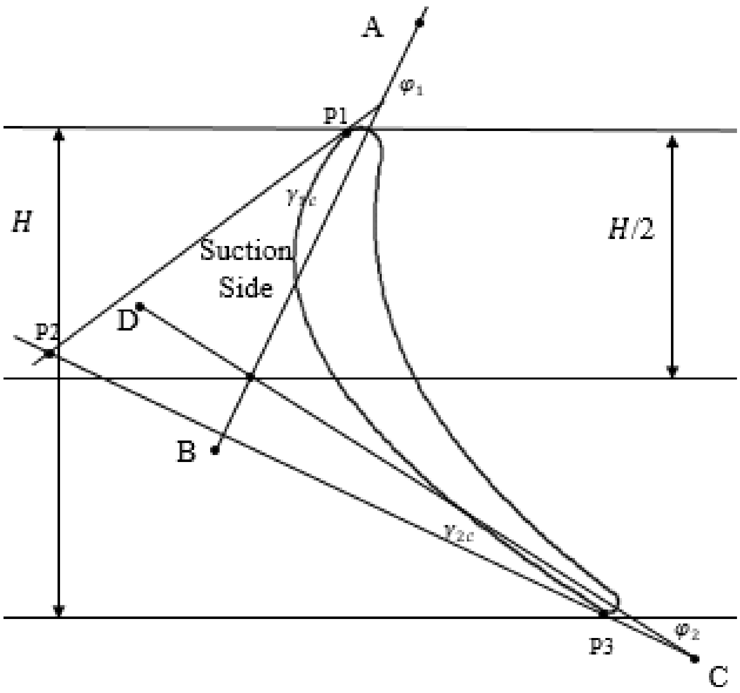


Fig. 8. Suction side curve representation

Now there are three control points to fit the blade suction side curve using Bézier spline curve by applying the following equation:

$$P_{2.2}(t) = (1-t)^2 * P1 + t^2 * (1-t) * P2 + t^2 * P3$$

where t…………….... is a line segment fraction.

P1, P2, P3…... Are the control points.

$P_{2.2}(t)$ …….... A function represents the locus of a point moving on the spline curve.

This function will form a smooth quadratic curve tangent to the leading and trailing edges.

Pressure side curve

In this side, we introduce an angle γ_{1k} by drawing line 3 tangents the leading-edge curve in point 4 and intersects line AB to form the angle as shown in Fig. 9. At the trailing edge, draw line 4 to form the angle γ_{2k} with line CD. tangent to the trailing edge curve in point 6 and extends to intersects line 3 in point 5.

According to experience, take [$\gamma_{1k} = 0.5\gamma_{1c}, \gamma_{2k} = 0.5\gamma_{2c}$] as nominal values.

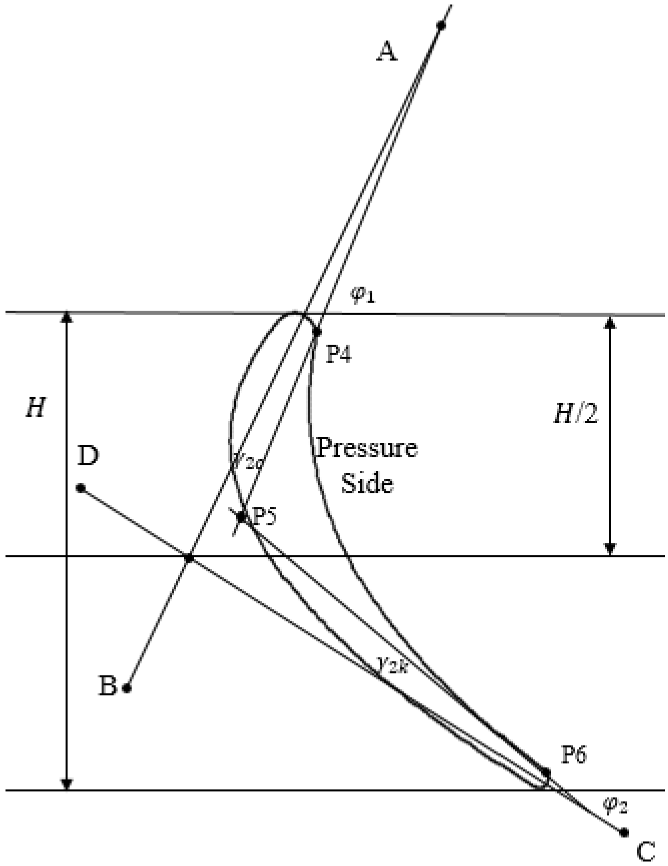


Fig. 9. Pressure side curve representation

Stator 2D airfoil design

The stator blade airfoils are dealt with the same way of the rotor, the same steps and procedure using its own nomenclature.

Automation of Rotor and Stator Blades Cascade Design

A MATLAB code is developed to perform the mentioned graphical procedure in previous sections based on the generated data from the design phases (2D,3D). The thermal cycle, 1D and 2D calculations are generated using an excel code to be the input data for the MATLAB code. The two codes are linked to facilitate the design process and further use of this code for optimization, so, by changing the engine operating conditions in the EXCEL code and running the MATLAB code, the new detailed stator and rotor blades design will be generated. This code generates several airfoils at different blade sections based on the velocity triangles calculated using the free vortex law of blading. The airfoils are generated in cylindrical coordinates to ensure the considered accuracy. Fig. 10. and .11 are generated graphs from the MATLAB code for final blade airfoil sections shape design. This data is sent automatically to the program SOLIDWORKS to draw the 3D blade model with only one run as shown in Fig. 12.

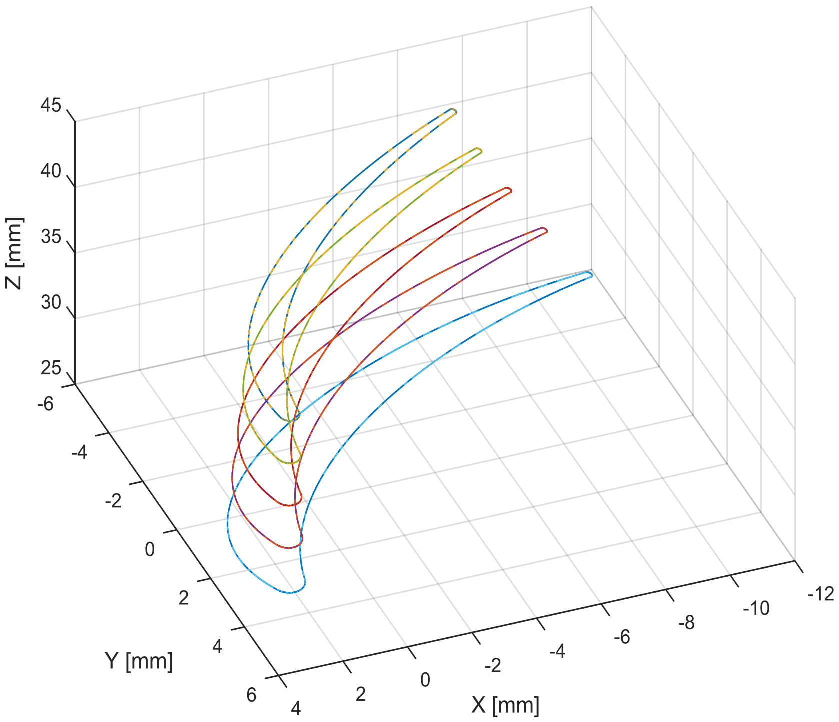


Fig. 10. Stator blade section generated from MATLAB

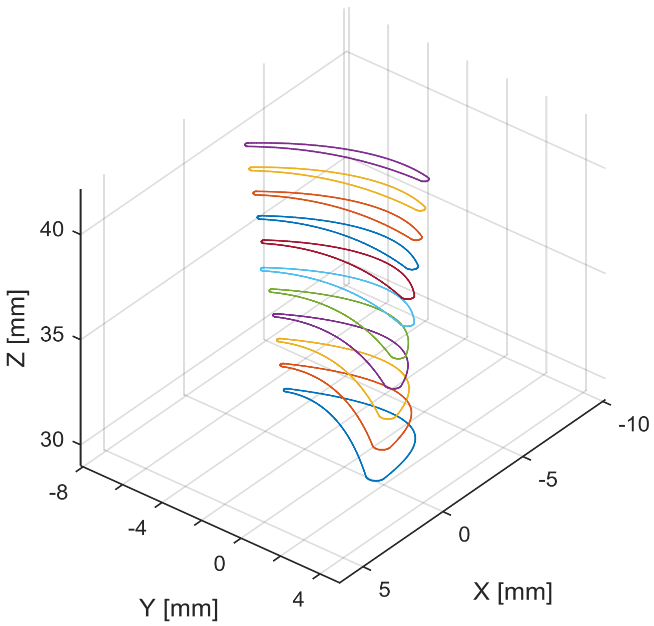


Fig. 11. Rotor blade section generated from MATLAB

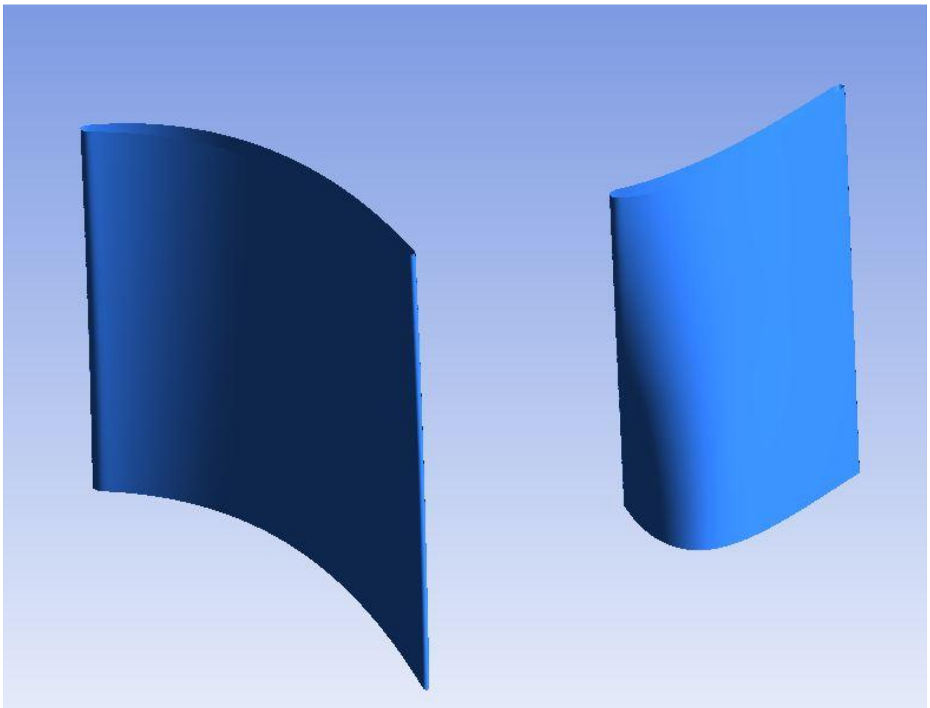


Fig. 12. The generated stator and rotor blades using SOLIDWORKS

Conclusions

The target for this work is to construct an automated implicit and robust geometric representation method of gas turbine blades that controlled by the design parameters, flexible and accurate so as to be used efficiently in automatic shape optimization. This caused the development of the proposed approximate analytical model, and a parametrization of that model using Bézier spline fit curve functions. The flexibility of the method is measured by reproducing several turbine designs with different designer requirements, the generated designs gave high both thermal and kinetic efficiencies. The final required design gave thermal efficiency of 89 % and average exit swirl velocity of $\mp 5^{o}$deviation from the axial direction which minimizes the losses at turbine stage exit. The airfoil Bézier spline curve parametrization is used in order to smoothen the curvature at the leading edge and trailing edge near the junction point between the suction side and pressure side to eliminate discontinuity at these points.

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