A Rapid Gun Firing Correction Algorithm
Using Discrete Time Transfer Matrix Method

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Abstract: To improve the next shot hit accuracy, a firing angle correction algorithm will be developed using the distance error computed between the impact and target coordinates to determine the launch azimuth and quadrant elevation angles for artillery projectiles. This correction method will be developed using the discrete time transfer matrix method of the projectile modified point mass model DTTMM-4DOF to improve correction method performance. To validate the proposed algorithm, a spin-stabilized 155mm-HE projectile flight trajectory will be used, assuming that wind magnitude and direction with the down range are changing as function of projectile altitude.

Keywords: Firing correction; Un-guided projectile; 4-DOF; Discrete Time Transfer Matrix; 105mm-HE; 155mm M107.

1. Introduction

In case of unguided projectiles, to improve the weapon's accuracy [1], the error between the projectile impact point and target has to be estimated precisely to evaluate the weapon's accuracy and hence; an appropriate projectile firing correction is done based on the first shoot impact error to increase the next shoot accuracy. Traditionally, firing tables is used to calculate the projectile standard firing data needed to hit a desired target and also compute the firing data corrections in case of non-standard flight conditions [2]. These nonstandard flight conditions are resulted from variances in many parameters such as meteorological data, projectile mass properties, propellant temperature, and altitude difference between the launch point and the target [3]. To improve the weapon's hit accuracy, these nonstandard flight conditions have to be predicted accurately as well as possible especially the meteorological data [4], [5]. Using lookup search, all firing data can be determined automatically using a stored tabulated data for both standard and non-standard conditions. To speed up the searching process, different graphical tools such as graphical firing tables, graphical site tables, and wind cards, are used but with low accuracy due to simplifications [2], [6].

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In order to improve the performance of the fire control system, it is necessary to develop an accurate and fast correction technique for projectile external ballistics problems. Currently, firing correction methods are divided mainly into two kinds: (1) firing table approximation; and (2) numerical methods. Due to the fast developments of computer technologies, numerical methods are widely used as the calculation processes become easier and general for any kind of gun, however, firing table approximation is different for different gun type and different firing conditions.

2. Related Work

Many researchers developed different algorithms to improve the impact accuracy. In 1996, a mathematic model for determining firing data by computations considering the Coriolis effect and the firing site and target are not in the same level [7] was developed. In 1997, a weighted function model based on small perturbation assumptions for all measurable real shooting conditions was developed to estimate the correction angle required to improve impact accuracy [8]. In 2001, a firing angle iterative range correction method for tank fire system was developed based on the point mass model [9]. Another correction method used in ship-borne gun fire system has been developed to calculate the fire data against the targets on shore [10]. In 2003, a model to calculate the firing data and flight time with the target position has been developed by solving the projectile external ballistic equations once only to reduce processing time, as well as methods to correct the informal ballistic and meteorological conditions, movement of the ship and the side deviation of projectiles were presented [11]. In 2007, Harlin and Cicci developed a state transition matrix STM to make corrections to ballistic missile initial state vector to achieve predefined final state vector using a 6DOF model [12]. In 2011, a differential algorithm with two height levels was developed to accurately calculating the ahead hit position for real antiaircraft gun system [13].

In flight control systems using iterative method, the solution of azimuth and quadrant elevation is determined through iterations of trajectory simulation in nonstandard conditions instead of searching firing data on the pre-calculated tabulated data in standard condition and then, applying corrections to compensate the corresponding nonstandard conditions [3], [14]. NATO Armaments Ballistic Kernel (NABK) is one of the most notable system uses an iterative method to compute the firing data [15], where the ballistic trajectory model used during iterations is for the modified point mass model (4-DOF) [16] or five-degrees-of-freedom model [17]. Many algorithms were developed to insure the firing data convergence during iterative trajectory simulations [3], [18]. These iterative methods require an efficient algorithm to minimize the computation time.
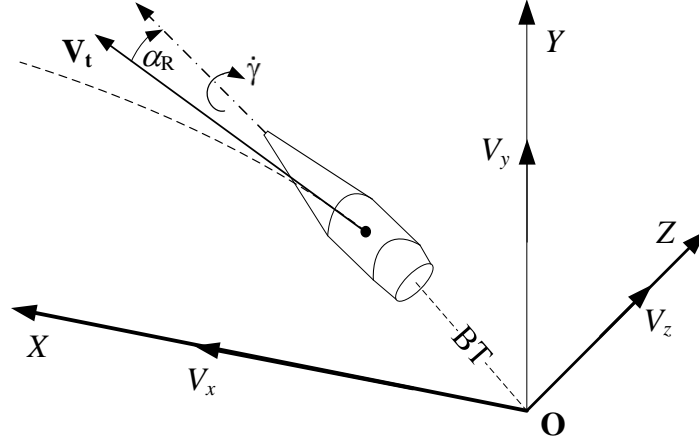
Fig. 1 Coordinate system and directions of ballistic target

3. Projectile Trajectory model

The projectile modified point mass equations of motion 4-DOF with respect to earth fixed coordinate system (X, Y, Z) as shown in Fig. 1 are given by [17, 19]

$$\dot{\mathbf{X}} = \mathbf{v} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T \tag{1}$$

$$\dot{\mathbf{v}} = -\rho |\mathbf{v}_r| a_1 C_x \mathbf{v}_r + \rho a_1 C_y' |\mathbf{v}_r|^2 \boldsymbol{\alpha}_R - \rho a_2 C_z'' \dot{\gamma} [\mathbf{v}_r \times \boldsymbol{\alpha}_R] + \mathbf{g} + \boldsymbol{\Lambda} \tag{2}$$

$$\ddot{\gamma} = -\frac{\rho . s . l^2 |\mathbf{v}_r|}{2C} \dot{\gamma} . m_{xD}' \tag{3}$$

$$\boldsymbol{\alpha}_R = \frac{2.C.\dot{\gamma}}{\rho . s . l . |\mathbf{v}_r|^4 m_z'} [\mathbf{g} \times \mathbf{v}_r] \tag{4}$$

$$\mathbf{v}_r = \mathbf{v} + \mathbf{w} \tag{5}$$

Where, C_x, C_y', C_z'', m_{xD}' and m_z' are the aerodynamic drag force, lift force, Magnus force, spin damping and pitching moments coefficients respectively; $\mathbf{v}$ is the projectile velocity vector in earth fixed coordinate; $|\mathbf{v}_r|$ is the projectile total velocity vector with respect to air; ρ is the air density; m, C, A, l, and s are the projectile mass, axial and transverse moments of inertia, reference length and reference area respectively; $\boldsymbol{\alpha}_R$ is the projectile angle of repose; $\mathbf{w}$ is the wind velocity vector in earth fixed coordinate; and the earth's gravitational $\mathbf{g}$ and Coriolis $\boldsymbol{\Lambda}$ accelerations are given by [17, 18]

$$\boldsymbol{\Lambda} = 2\Omega \begin{bmatrix} 0 & -\cos\mu\sin\lambda & -\sin\mu \\ \cos\mu\sin\lambda & 0 & \cos\mu\cos\lambda \\ \sin\mu & -\cos\mu\cos\lambda & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \tag{6}$$

$$\mathbf{g} = -g . \begin{bmatrix} \dfrac{X_1}{R_e} & 1 - \dfrac{2X_2}{R_e} & 0 \end{bmatrix}^T \tag{7}$$

Where, $g = g_o.\left[1 - 0.0026\cos(2\mu)\right]$, $g_o = 9.80665$ m/s^2; Ω is the earth's angular velocity; μ, λ are the corresponding latitude and longitude of the firing sight; and R_e is the average radius of the earth (= 6356766 m).

Due to spherical earth approximation, the instantaneous projectile altitude H is defined as [17]

$$H = X_2 + \frac{X_1^2}{2.R_e} \tag{8}$$

4. Firing Correction Algorithm

For a given projectile ballistic trajectory with impact point $\mathbf{X}_P = \{X_P, Y_P, Z_P\}$, the error between this impact point and the target $\mathbf{X}_T = \{X_T, Y_T, Z_T\}$, as mentioned in Eq. (9), has to be minimized to improve the weapon's accuracy. In case of unguided projectiles, $\Delta\mathbf{X}$ is the distance error to be iteratively compensated by correcting one or more initial flight parameters. These adjustments could be the initial position, velocity, or projectile attitude. Based on discrete time transfer matrix method DTTMM-4DOF based on projectile modified point mass model stated in ref. [21], a two-dimensional iterative method is developed to obtain the projectile launching angle for desired target distance and direction.

The proposed algorithm assumed that the only known elements of the impact state is the distance error $\Delta\mathbf{X}$, and the parameter to be corrected to compensate this error is the projectile launch angle QE and AZ, where the projectile muzzle position vector and velocity magnitude are assumed constants. This two dimensional angle is consist of quadrant elevation angle QE which is controlling the distance between launching point and the target and azimuth angle AZ which determines the firing direction. The main advantage of this method is the ability of computing the range derivative with respect to QE during computations of projectile reference trajectory including non-standard conditions, where the pre-calculated and stored derivative may become inaccurate in some non-standard conditions [3].

The projectile state error $\Delta\mathbf{S}$ is comprised of 7 elemental errors with the projectile reference trajectory, which are the projectile position, velocity, and spin rate respectively as mentioned in Eq (10).

$$\Delta\mathbf{X} = \mathbf{X}_P - \mathbf{X}_T \tag{9}$$

$$\Delta\mathbf{S}_{7\times1} = \begin{bmatrix} \Delta x & \Delta y & \Delta z & \Delta v_x & \Delta v_y & \Delta v_z & \Delta\dot{\gamma} \end{bmatrix}^T \tag{10}$$

To set a direct relation between the state errors $\Delta\mathbf{S}$ during any two successive time step, due to deviation of one or more state parameters for the reference trajectory, a state correction matrix $\mathbf{\Phi}$ is deduced using a modified Jacobian $\overline{\mathbf{J}}$ of the system mentioned in ref. [21] as

$$\Delta\mathbf{S}^{t_{i+1}} = \mathbf{\Phi}_{7\times7}^{t_i}.\Delta\mathbf{S}^{t_i} \tag{11}$$

$$\mathbf{\Phi}_{7\times7}^{t_i} = \mathbf{I}_7 + \overline{\mathbf{J}}_{7\times7}^{t_i}.\Delta t \tag{12}$$

$$\bar{\mathbf{J}} = \mathbf{J}_{7\times13} \cdot \mathbf{J}_{\Delta\boldsymbol{\alpha}} \cdot \mathbf{J}_{\Delta w} \tag{13}$$

The Jacobian matrix $\mathbf{J}_{7\times13}$, contains the partial derivatives of projectile position and velocity vectors and spin rate w.r.t. all state variables in Eq. (10), as

$$\mathbf{J}_{7\times13} = \begin{bmatrix} \mathbf{O}_{3\times3} & \mathbf{I}_3 & \mathbf{O}_{3\times1} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} \\ \mathbf{j}_{21} & \mathbf{j}_{22} & \dot{\mathbf{v}}_{\dot{\gamma}} & \mathbf{j}_{24} & \mathbf{j}_{25} \\ \mathbf{j}_{31} & \mathbf{j}_{32} & \ddot{\gamma}_{\dot{\gamma}} & \mathbf{O}_{1\times3} & -\mathbf{j}_{32} \end{bmatrix} \tag{14}$$

Where, $\mathbf{j}_{22}$, $\dot{\mathbf{v}}_{\dot{\gamma}}$, and $\mathbf{j}_{25}$ are the partial derivatives of the projectile acceleration with respect to the its velocity, spin rate and wind velocity respectively; $\mathbf{j}_{32}$ and $\ddot{\gamma}_{\dot{\gamma}}$ the partial derivatives of the rate of change of projectile spin rate with respect to its velocity and spin rate respectively [21]. The partial derivatives of the projectile velocity vector $\mathbf{j}_{21}$, spin rate $\mathbf{j}_{31}$ and the repose $\mathbf{j}_{41}$ angle with respect to position vector are given by

$$\mathbf{j}_{21} = \left[-a_1 |\mathbf{v}_r| (\rho C_x)_H \, \mathbf{v}_r + a_1 |\mathbf{v}_r|^2 (\rho C_y')_H \, \boldsymbol{\alpha}_R - a_2 \dot{\gamma} (\rho C_z'')_H [\mathbf{v}_r \times \boldsymbol{\alpha}_R] \right] \cdot \frac{\partial H}{\partial \mathbf{X}^T} \tag{15}$$

$$\mathbf{j}_{31} = -b_1 |\mathbf{v}_r| \dot{\gamma} (\rho m_{xD}')_H \frac{\partial H}{\partial \mathbf{X}^T} \tag{16}$$

$$\mathbf{j}_{41} = \frac{-b_4 \dot{\gamma} (\rho.m_z')_H}{(\rho.m_z')^2 |\mathbf{v}_r|^4} [\mathbf{g} \times \mathbf{v}_r] \cdot \frac{\partial H}{\partial \mathbf{X}^T} \tag{17}$$

Where, $b_4 = (d/b_1)$, and $(\rho C_i)_H = \partial(\rho C_i)/\partial H = \rho_H C_i + \rho(\partial C_i/\partial M) M_H$

The computations error due to the change in wind and repose angle respectively are given by

$$\mathbf{J}_{\Delta w} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{O}_{3\times3} & \mathbf{I}_3 & \mathbf{O}_{3\times1} \\ \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & 1 \\ \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} \\ \mathbf{w}_{\mathbf{X}} & \dot{\mathbf{w}}_{\mathbf{v}} \, \Delta t/2 & \mathbf{O}_{3\times1} \end{bmatrix} \tag{18}$$

Where,.

$$\mathbf{J}_{\Delta\boldsymbol{\alpha}} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} \\ \mathbf{O}_{3\times3} & \mathbf{I}_3 & \mathbf{O}_{3\times1} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} \\ \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} & 1 & \mathbf{O}_{1\times3} & \mathbf{O}_{1\times3} \\ \mathbf{j}_{41} & \mathbf{j}_{42} & \mathbf{j}_{43} & \mathbf{O}_{3\times3} & -\mathbf{j}_{42} \\ \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} & \mathbf{O}_{3\times1} & \mathbf{O}_{3\times3} & \mathbf{I}_3 \end{bmatrix} \tag{19}$$

Where, $\dot{\mathbf{w}}_{\mathbf{v}}$ is the change in wind velocity vector with respect to the projectile position; $\mathbf{j}_{42}$ and $\mathbf{j}_{43}$ are the first order projectile angle of repose derivative w.r.t. projectile velocity and spin rate respectively [21].

Finally, the elements of the transfer matrix $\mathbf{\Phi}$, which maps the state error $\Delta\mathbf{S}^{t_i}$ into the state error $\Delta\mathbf{S}^{t_{i+1}}$ is given by

$$\mathbf{\Phi}^{t_i} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{I}_3\Delta t & \mathbf{O}_{3\times1} \\ \mathbf{\Delta}_{21} & \mathbf{\Delta}_{22} & \mathbf{\Delta}_{23} \\ \mathbf{\Delta}_{31} & \mathbf{\Delta}_{32} & \mathbf{\Delta}_{33} \end{bmatrix} \tag{20}$$

Where,

$$\mathbf{\Delta}_{21} = \left[\mathbf{j}_{21} + \mathbf{j}_{24}.\mathbf{j}_{41} + \left[\mathbf{j}_{25} - \mathbf{j}_{24}.\mathbf{j}_{42} \right] w_\mathbf{X} \right] \Delta t \tag{21}$$

$$\mathbf{\Delta}_{22} = \mathbf{I}_3 + \left[\mathbf{j}_{22} + \mathbf{j}_{24}.\mathbf{j}_{42} \right] \Delta t + \left[\mathbf{j}_{25} - \mathbf{j}_{24}.\mathbf{j}_{42} \right] \dot{w}_\nu \Delta t^2 / 2 \tag{22}$$

$$\mathbf{\Delta}_{23} = \left[\dot{v}_{\dot{\gamma}} + \mathbf{j}_{24}.\mathbf{j}_{43} \right] \Delta t \tag{23}$$

$$\mathbf{\Delta}_{31} = \left[\mathbf{j}_{31} - \mathbf{j}_{32} w_\mathbf{X} \right] \Delta t \tag{24}$$

$$\mathbf{\Delta}_{32} = \mathbf{j}_{32}\Delta t - \mathbf{j}_{32}\dot{w}_\nu \Delta t^2 / 2 \tag{25}$$

$$\mathbf{\Delta}_{33} = 1 + \ddot{\gamma}_{\dot{\gamma}}\Delta t \tag{26}$$

The proposed algorithm is dealing with the correction procedure as two one-dimensional problems, in which quadrant elevation angle QE and azimuth angle AZ are obtained separately as illustrated below;

3.1. Quadrant elevation angle determination

During projectile launch angle correction process, the first angle to be estimated is QE = θ_c, where the projectile maximum range is achieved by ($\theta \approx 45°$). For a certain projectile muzzle velocity, there are two different QE give the same range with different flight time and firing QE angle (low and high firing angles). During searching, the low firing angle is used ($\theta \leq 45°$), where the flight time is decreased always and hence, reduce the wind effect.

Traditionally, the QE can be determined and corrected using firing tables lookup search methods. But building precise firing tables need a lot of firing test at to cover different firing conditions. Therefore, an iterative Newton-Raphson method will be used in this research as illustrated below

$$\theta_{j+1} = \begin{cases} \theta_j - \dfrac{X_\mathrm{P} - Range}{\left(f_1' + f_2' \right)/2} & X_\mathrm{P}\big|_{-\Delta\theta} \leq Range \leq X_\mathrm{P}\big|_{+\Delta\theta} \\[2ex] \left(\theta_j + \Delta\theta \right) - \dfrac{X_\mathrm{P}\big|_{+\Delta\theta} - Range}{2f_1' - f_2'} & Range > X_\mathrm{P}\big|_{+\Delta\theta} \\[2ex] \left(\theta_j - \Delta\theta \right) - \dfrac{X_\mathrm{P}\big|_{-\Delta\theta} - Range}{2f_2' - f_1'} & Range < X_\mathrm{P}\big|_{-\Delta\theta} \end{cases} \tag{27}$$

Where, j is the iteration number, and the corresponding range derivative w.r.t the projectile QE is given by

$$f_1' = \frac{X_P\big|_{+\Delta\theta} - X_P}{\Delta\theta}, \quad f_2' = \frac{X_P - X_P\big|_{-\Delta\theta}}{\Delta\theta} \tag{28}$$

3.2. Azimuth angle determination

Finding the projectile launch azimuth angle ψ is much easier and faster than finding QE. Then azimuth angle can be adjusted by the direction error of the impact point of the final iteration as follows,

$$\psi_{j+1} = \psi_j + \Delta\psi \tag{29}$$

$$\Delta\psi = \tan^{-1}\left(\frac{Y_P - Y_T}{X_P}\right) \tag{30}$$

5. ALGORITHM VALIDATION

The validation process of the proposed algorithm is divided into two phases, (1) the validation of the estimation accuracy of the projectile range derivative w.r.t the QE ($\partial X_P/\partial\theta$), (2) the validation of the overall correction algorithm on the mean trajectory impact point w.r.t the reference trajectory.

Firstly, the projectile range derivative w.r.t the QE ($\partial X_P/\partial\theta$) is estimated in order to compute the amount of QE needed during correction process. Then, the upper and lower bound trajectories will be computed, where these bounds are $\pm\Delta\theta$ from the reference trajectory QE.

Therefore, a number of ballistic trajectories for 105mm-HE projectile is generated as shown in Fig. 2, where $\partial X_P/\partial\theta$ is always a function of QE. The projectile mass properties and aerodynamic coefficients have been illustrated in [22]. All trajectories are generated using 4-DOF model with $\theta = 10, 20\ldots 70°$ and $\psi = 0°$, where the projectile is traveling through wind profile shown in Fig. 3. The trajectory data frequency is assumed to be 0.001s and the $\Delta\theta = 1°$.
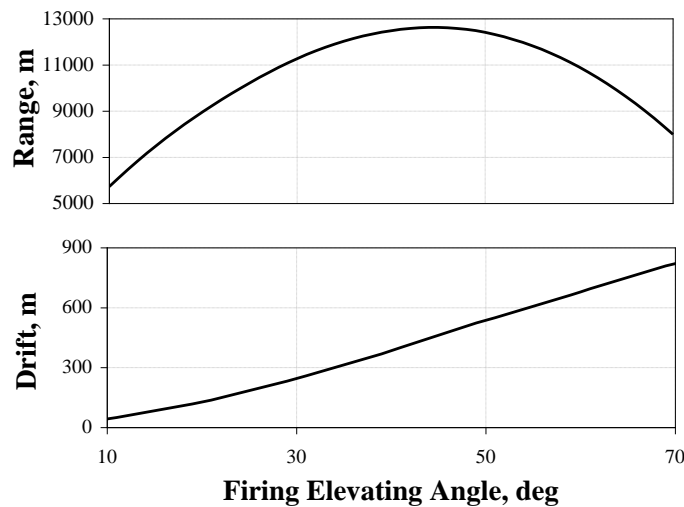

Fig. 2 The relation between the projectile QE and corresponding range and drift
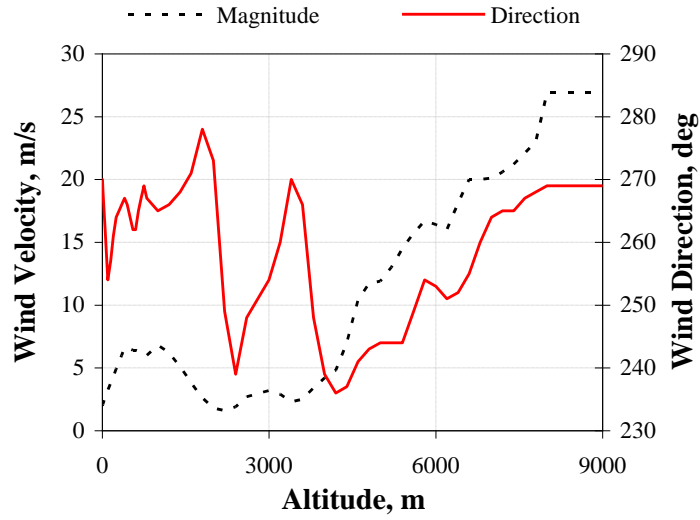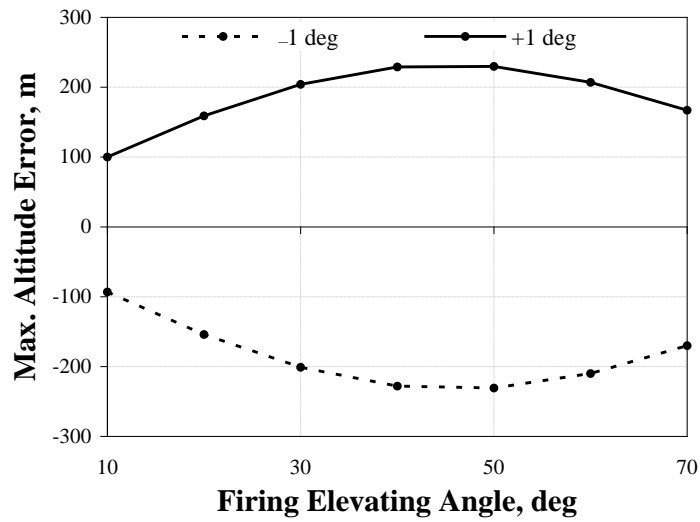
Fig. 3 Wind velocity and direction versus altitude
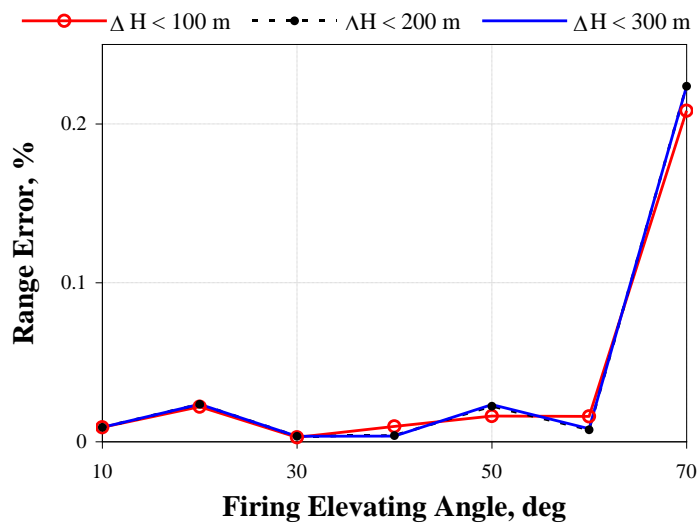

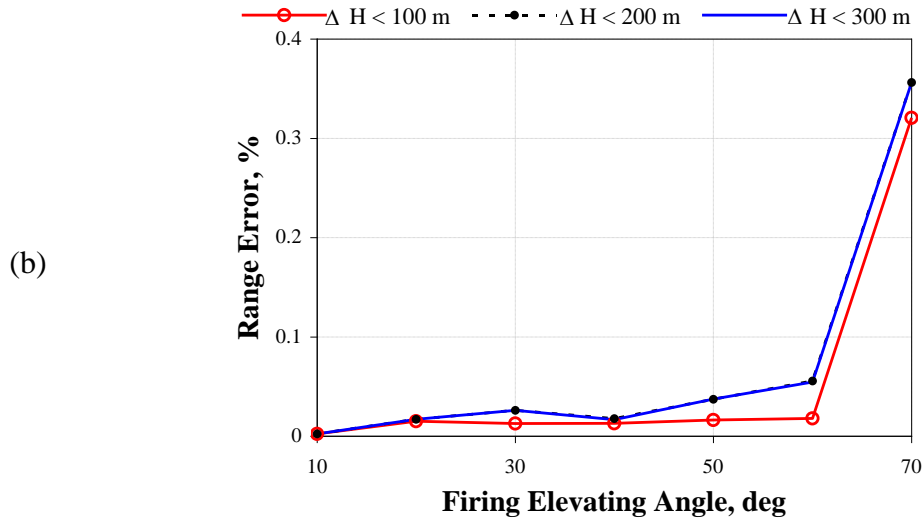Fig. 4 The maximum altitude difference between bound trajectories and reference

(a)

(b)

Fig. 5 The range error versus the projectile QE for a) $\Delta\theta = -1°$, b) $\Delta\theta = +1°$

(a)

(b)
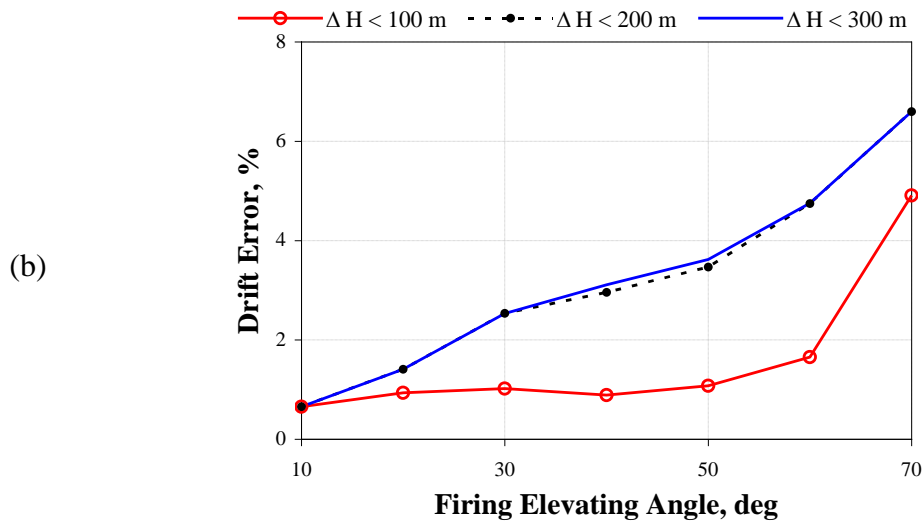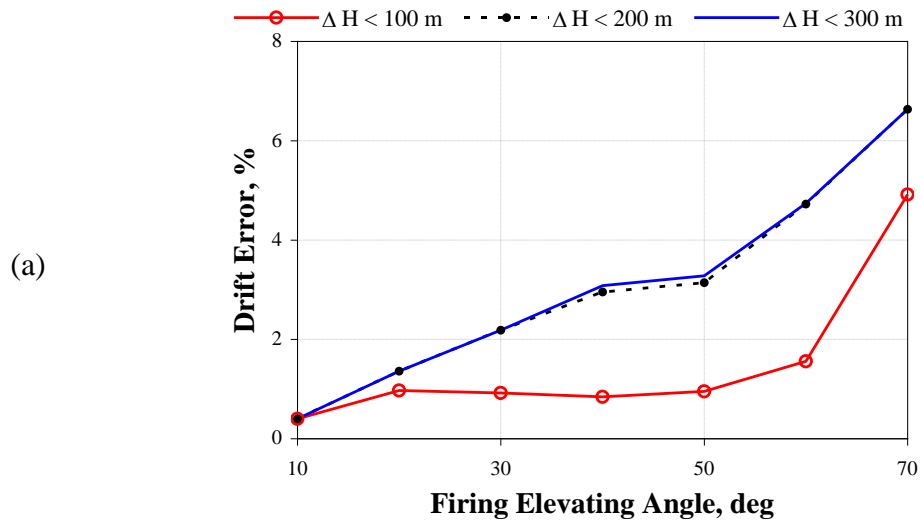

Fig. 6 The drift error versus the projectile QE for a) $\Delta\theta = -1°$, b) $\Delta\theta = +1°$

The accuracy of the state correction matrix $\mathbf{\Phi}^{t_i}$ is strongly affected by some parameters which are deviated from the reference trajectory such as the altitude and repose angle. Therefore, a comparative study has been done between different altitude bound ΔH, where if the altitude error for the estimated trajectory is greater than ΔH, Eq. (11) will be terminated and the rest of trajectory will be computed from scratch using DTTM-4DOF model.

Figures 5 and 6 show the relative range and drift accuracy respectively w.r.t the reference, where as the altitude difference (ΔH) between the reference state and the estimated state increased, the error will be increased. It can be concluded that the effect of changing ΔH has a negligible error on the projectile range error for low QE ($\theta \leq 45°$). But in case of high QE, the range and drift errors can not be neglected even at $\Delta H < 100$m due to large repose angle at high QE as shown in Fig 7. Therefore, the proposed algorithm will be used only for low QE to have best performance. As shown in Fig 8, the time consumed during the computation of the reference trajectory QE = θ and the two other trajectories QE = $\theta \pm \Delta\theta$ using 4-DOF model decreased by more than 60% using the state correction matrix $\mathbf{\Phi}^{t_i}$ for ($\Delta H < 300$m) due to decreasing number of operations performed during each time step by directly obtain two trajectories using the correction matrix multiplied by the reference trajectory instead of computing from scratch using 4-DOF model.
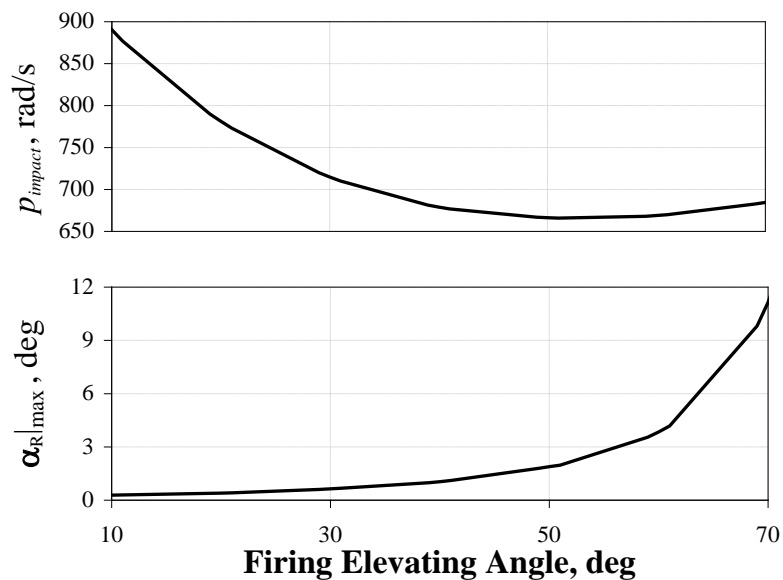

Fig. 7 The projectile impact spin rate $\dot{\gamma}$ and max. repose angle α versus QE
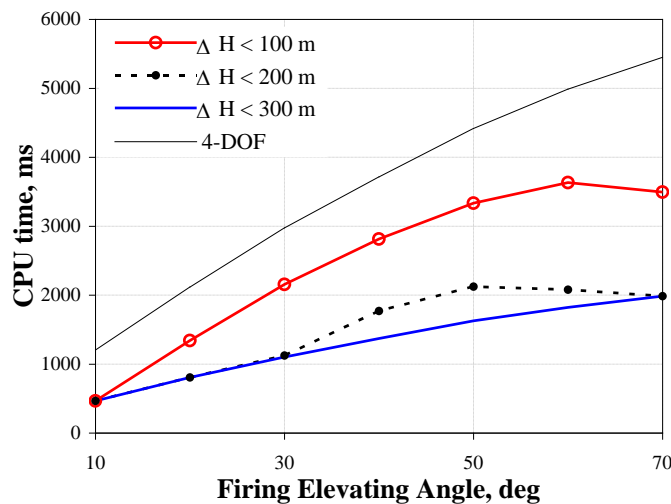

Fig. 8 The CPU time study

Table 1 Trajectory Correction

Case No.	Impact Point (real wind)		Target Point (zero wind)	
	Range, m	Drift, m	Range, m	Drift, m
1	12075.19	129.6152	12159.26	134.5334
			19047.84	494.5224
2	20047.84	594.5224	19547.84	550.5224
			20228.43	628.9069
3	21558.97	853.0912	22260.67	952.719

Secondly, using a spin-stabilized 155mm-M107 projectile [23], a complete usage of the proposed correction algorithm is done for different cases as mentioned in Table 1 to investigate the solution convergence during iterations. In case of firing at standard conditions with zero wind, the target point in case 1, 2-a, and 3 are obtained using QE = 10°, 30° and 40° respectively as listed in Table 2. But using non-standard conditions with wind profile shown in Fig. 3, the impact points are deviated from the desired targets and it's hard to obtain the corresponding firing angle to hit the target point with an appropriate accuracy.

Results of the case-1 are listed in Table 2. These results show the speed of the proposed algorithm, where the required two dimensional firing angle can be estimated just in one iteration with miss distance $\varepsilon \approx 1$, where the projectile range is strongly affected by a small variation in firing QE.

Case-2 examines the effect of miss distance on the number of iterations needed to improve the estimated firing angle accuracy.

Table 3 shows that the adjustments made in the gun azimuth ψ is always made after estimating the gun elevation θ, where the effect of range variation during iteration process has a strong impact on the corresponding azimuth ψ as mentioned in Fig 2. However, the effect of azimuth variation $\Delta\psi$ on the resultant range is very small and can be neglected. As the projectile miss-distance ε increases, the solution number of iterations is also increases.

Case-3 is used to test the proposed correction algorithm near the maximum firing elevating angle $40° \le \theta \le 45°$, where the projectile range variation relative to firing QE is very small compared to cases 1 and 2. The results for this case are listed in Table 4. These results show the robustness of the proposed algorithm. A stopping condition is applied to the proposed algorithm, where the maximum fringing elevating angle can reached is $\theta = 45°$. As expected, the solution converged and discontinued.

Table 2 Case-1 simulation results

Iteration	ε_{range}, m	ε_{drift}, m	Miss Distance, m	θ, deg	ψ, deg
0	-84.0672	-4.91824	84.21099	10	0
1	-0.71067	0.746387	1.030605	10.13755	0.013221

Table 3 Case-2 simulation results

Iteration	ε_{range}, m	ε_{drift}, m	Miss Distance, m	θ, deg	ψ, deg
0	1000	100	1004.988	30	0
1	-325.616	-12.9003	325.8712	25.30706	0
2	-3.33624	17.25996	17.57944	26.32621	0
3	0.574426	-2.14406	2.219676	26.33711	-0.05288

Table 4 Case-3 simulation results

Iteration	ε_{range}, m	ε_{drift}, m	Miss Distance, m	θ, deg	ψ, deg
0	-701.698	-99.6278	708.7352	40	0
1	-473.171	-9.719	473.2711	45	-0.200013

5. Conclusion:

A correction algorithm was developed to accurately estimate the projectile two dimensional firing angles required to impact a desired target, given the projectile launch parameters uncertainties, the current meteorological data, and launch and targeted impact locations. A limitation has been placed to the proposed algorithm, where the maximum firing elevating angle available is $\theta = 45°$. A state correction matrix was deduced using the discrete time transfer matrix for modified point mass trajectory model DTTM-4DOF. The main advantage of the proposed algorithm is decreasing the number of operations performed during each firing angle and hence; the computation time needed for estimating the projectile impact point and the corresponding impact range derivative relative to QE by approximately 60% than using traditional methods in literature. Results demonstrate that the solution of proposed correction algorithm is always converged with high accuracy.

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