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Separable warhead mathematical model of Supersonic & Hypersonic Re-entry Vehicles

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Abstract:

Separating a reentry vehicle into warhead and body is a conventional and efficient means of producing a huge decoy and increasing the kinetic energy of the warhead. This procedure causes the radar to track the body, whose radar cross section is larger, and ignore the warhead which is the most important part of the reentry vehicle. The aerodynamic Coefficients models play an essential part in the simulation and the analysis of the supersonic and hypersonic of the ballistic missiles, especially in the dynamic trajectory planning. This paper builds the aerodynamic coefficients models by the nonlinear least square method based on the separable warhead re-entry vehicle using lebedev aerodynamic calculations. The lift and drag coefficients models can be expressed with the polynomial and the exponential function. So the models fit the aero characteristic well and can be used in practical design and simulation as a reference.

1- Introduction

Ballistic missile warheads can carry nuclear, chemical or biological payloads in addition to conventional (bulk warhead or submunition warhead) ones. In fact, Tactical ballistic missiles (TBM) are regarded to be the best solution for delivering nuclear payloads. While basic types of first generation TBM do not have a separable warhead, more advanced TBM can carry one of various types of separable warhead such as a spin stabilized warhead, orientable warhead, stealthy warhead, maneuvering re-entry vehicle (RV), or multi-independent RVs[1]. These improvements have been driven by accurate targeting and ballistic missile defense penetration requirements. The re-entry phase of the ballistic missile happens after the warhead has accelerated to its maximum speed in the ballistic phase of its flight, reaching high velocity speeds of 1500 m/s to 5000+ m/s before its re-entry into the earth’s atmosphere. The tip of the missile can reach temperatures of up to 3000 c^o during flight in the re-entry atmospheric air due to air friction; this can give a spectacular warhead pattern at terminal re-entry phase. Upon re-entry, the warhead orients itself and high deceleration begins at 5g to 40+ g units as shown in figure (1). The re-entry of a ballistic object is very specific because its movements result from weight and aerodynamic effects such as lift and drag.

This paper builds the aerodynamic coefficients models of the reentry vehicle by the nonlinear least square method based on the separable warhead re-entry vehicle using lebedev aerodynamic calculations. The paper is organized as follows:

Section (2) highlights the reentry vehicle Design considerations and mode of operation. The dynamic equations that describe the reentry vehicle motion are provided in section (3). Simulation results and analysis are discussed in section (4). Section (5) shows the Aerodynamic Coefficient Model for Warhead. Paper terminated with the conclusion section.

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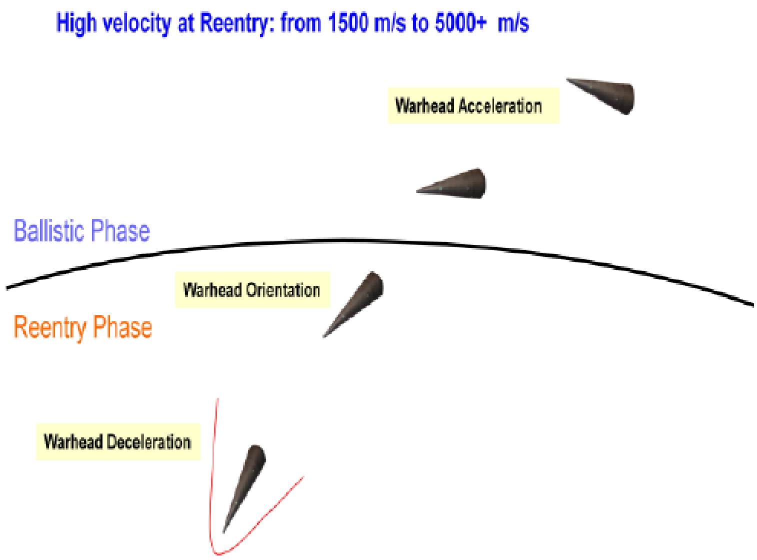

Fig. 1 Reentry vehicle trajectory profile

2- Re-entry Vehicle Design Considerations and mode of operation

There are four critical parameters considered when designing a vehicle for atmospheric entry[1]:
1) Peak heat flux. 2) Heat load. 3) Peak deceleration. 4) Peak dynamic pressure. Peak heat flux and dynamic pressure selects the thermal protection system (TPS) material. Heat load selects the thickness of the TPS material stack. Peak deceleration is of major importance for manned missions. Peak dynamic pressure can also influence the selection of the outermost TPS material if spoliation is an issue.

In BM's most of them apparently spin up the single booster stage and warhead combination starting at about 10 seconds before the termination of the powered flight at T shut-off. At this point after T shut-off of powered flight the warhead is then separated from the booster stage to fly on a re-entry trajectory that remains stable to its target. With the addition of GPS targeting or ATR using camera installed on the warhead body, the warhead accuracy is greatly enhanced [2]. There are still many in the analytical community that question, perhaps correctly, this suggested accuracy of 190 meters to over 500 m. There can be no doubt that this spin-up technology does improve the accuracy of these warheads over the previously demonstrated poor capability. Since the warheads are not tumbling it in fact enhances the interceptor sensor signature identification capability verses that of a tumbling warheads signature.

3- Dynamic Equations

Consider a vehicle in the reentry phase over a flat and non-rotating earth as illustrated in Fig. 1. Assume the RV to be a point mass with constant weight following a ballistic trajectory in which two significant forces, drag and gravity, act on the RV. Extra forces are induced by model error when assumptions are violated or the RV undertakes a maneuver. The RV trajectory model in radar coordinate (O_R, X_R, Y_R, Z_R) centered at the radar site can be written as :

$$\dot{v}_x = -g\frac{\rho v^2}{2C}cos\gamma_1 sin\gamma_2 + u_4 \quad (1)$$

$$\dot{v}_y = -\frac{\rho v^2}{2C}gcos\gamma_1 cos\gamma_2 + u_5 \quad (2)$$

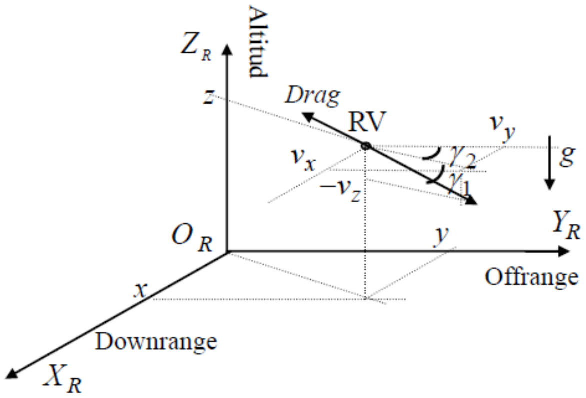

Fig. 2 Reentry vehicle flight trajectory

$$\dot{v}_z = \frac{\rho v^2}{2C}gsin\gamma_1 - g + u_6 \quad (3)$$

with position initial conditions $x(0)$, $y(0)$, $z(0)$ and velocity initial conditions $v_x(0)$, $v_y(0)$ and $v_z(0)$ in X_R, Y_R and Z_R respectively. In this model, C denotes the ballistic coefficient,

$$C = \frac{W}{SC_{D0}}$$

$$\gamma_1 = tan^{-1}\left(-\frac{v_z}{\sqrt{v_x^2 + v_y^2}}\right) \quad ,\gamma_2 = tan^{-1}\frac{v_x}{v_y}$$

v means the total velocity of the RV, v_x, v_y, and v_z express velocity components along , X_R, Y_R, and Z_R, respectively; u_4 , u_5 , and u_6 are un-modeled accelerations generated by the model errors along each axis; C_{D0}, S and W denote zero-lift drag coefficient, reference area and weight respectively. ρ stands for air density and is a function of altitude [3]. The well-known normal gravity g model is extensively used because the RV normally flies over heights of several hundred kilometers [3]. The RV separates into two objects, the warhead and the body, at a given altitude or a certain time. The warhead moves toward to a spot near the RV's destination, along a slight different trajectory from that of the original RV. The body then falls on its own rapidly after several seconds. The difference among the equations of motion for the RV, warhead, and body is only in the ballistic coefficient. Equations of motion for the warhead with the ballistic coefficient C_W after separation can be expressed as[4]:

$$\dot{v}_{wx} = -\frac{\rho v_w^2}{2C_w}gcos\gamma_{w1}sin\gamma_{w2} + u_{w4} \qquad t > t_s \quad (4)$$

$$\dot{v}_{wy} = -\frac{\rho v_w^2}{2C_w} g cos\gamma_{w1} cos\gamma_{w2} + u_{w5} \qquad t > t_s \qquad (5)$$

$$\dot{v}_{wz} = -\frac{\rho v_w^2}{2C_w} g sin\gamma_{w1} - g + u_{w6} \qquad t > t_s \qquad (6)$$

with position initial conditions $x_w(t_s), y_w(t_s), z_w(t_s)$ and velocity initial conditions $v_{wx}(t_s)$, $v_{wy}(t_s)$ and $v_{wz}(t_s)$ in X_R, Y_R, and Z_R respectively, where t_s means the time to separate, v_w denotes the total velocity of the warhead, γ_{w1} and γ_{w2} express the elevation and flight path angles, respectively; u_{w4}, u_{w5}, and u_{w6} are un-predictable input accelerations acting on the warhead. For the body with the ballistic coefficient C_b,

$$\dot{v}_{bx} = -\frac{\rho v_b^2}{2C_b} g cos\gamma_{b1} sin\gamma_{b2} + u_{b4}\ , \qquad t > t_s \qquad (7)$$

$$\dot{v}_{by} = -\frac{\rho v_b^2}{2C_b} g cos\gamma_{b1} cos\gamma_{b2} + u_{b5}\ , \qquad t > t_s \qquad (8)$$

$$\dot{v}_{bz} = -\frac{\rho v_b^2}{2C_b} g sin\gamma_{b1} - g + u_{b6}\ , \qquad t > t_s \qquad (9)$$

with position initial conditions $x_b(t_s), y_b(t_s), z_b(t_s)$ and velocity initial conditions $v_{bx}(t_s)$, $v_{by}(t_s)$ and $v_{bz}(t_s)$ in , X_R, , Y_R and , Z_R respectively. The parameters and variables are defined as in the warhead. Let the state vectors be:

$$X(t) = [\,x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_5\ \ x_6]^T \qquad (10)$$
$$= [x\ \ y\ \ z\ \ v_x\ \ v_y\ \ v_z],$$

$$X_w(t) = [\,x_{w1}\ \ x_{w2}\ \ x_{w3}\ \ x_{w4}\ \ x_{w5}\ \ x_{w6}]^T \qquad (11)$$
$$= [x_w\ \ y_w\ \ z_w\ \ v_{wx}\ \ v_{wy}\ \ v_{wz}],$$

$$X_b(t) = [\,x_{b1}\ \ x_{b2}\ \ x_{b3}\ \ x_{b4}\ \ x_{b5}\ \ x_{b6}]^T \qquad (12)$$
$$= [x_b\ \ y_b\ \ z_b\ \ v_{bx}\ \ v_{by}\ \ v_{bz}],$$

The nonlinear state equations can be written as:

$$\dot{X}(t) = F(X) + \varphi u + I_{6\times6}\xi\ , \qquad t \le t_s \qquad (13)$$

$$\dot{X}_w(t) = F(X_w) + \varphi u_w + I_{6\times6}\xi_w\ , \qquad t \le t_s \qquad (14)$$

$$\dot{X}_b(t) = F(X_b) + \varphi u_b + I_{6\times6}\xi_b\ , \qquad t \le t_s \qquad (15)$$

where , ξ , ξ_w and ξ_b stand for the process noise vectors with variance Q, Q_w and Q_b respectively, I denotes the identity matrix.

$$F(X) = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ -\frac{\rho}{2C_b}(x_4^2 + x_5^2 + x_6^2) g cos\gamma_{b1} sin\gamma_{b2} \\ -\frac{\rho}{2C_b}(x_4^2 + x_5^2 + x_6^2) g cos\gamma_{b1} cos\gamma_{b2} \\ \frac{\rho}{2C_b}(x_4^2 + x_5^2 + x_6^2) g sin\gamma_{b1} - g \end{bmatrix},$$

$$\varphi = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix},$$

$$u = [\,0 \;\; 0 \;\; 0 \;\; u_4 \;\; u_5 \;\; u_6\,]^T,$$

$$u_w = [\,0 \;\; 0 \;\; 0 \;\; u_{w4} \;\; u_{w5} \;\; u_{w6}\,]^T,$$

$$u_b = [\,0 \;\; 0 \;\; 0 \;\; u_{b4} \;\; u_{b5} \;\; u_{b6}\,]^T.$$

The precision phased array radar, which is digital radar, is used in tracking and is the only instrument in the system for detecting the RV. It predicts target's position at the next sampling period according to a set of measurement, and generates a radar beam to illuminate the predicted area to track the target. The measurement equation for the RV is then given by:

$$Z = HX + , \qquad t \leq t_s \qquad (16)$$

where denotes the measurement noise vector, which is assumed to be normally distributed with mean zero and variance R, and H is the 6×6 identity matrix. (13) and (16) are the dynamic equations for the RV during reentry. When t > t_s two sets of measurement are detected at the sampling time, one for the warhead and one for the body, that is:

$$Z_w = HX_w + , \qquad t > t_s \qquad (17)$$

$$Z_b = HX_b + , \qquad t > t_s \qquad (18)$$

(14), (17), (15), and (18) are the dynamic equations for the warhead and body, respectively, after separation. The radar is trying to estimate and predict state vectors from these two sets of measurement.

4- Simulation Analysis

The proposed algorithm performance is measured to determine the distance between the estimated and actual warhead trajectories. Let the estimation error w.r.t. the warhead signify the difference between the estimated and warhead trajectories as shown in figure (3). Similarly, the estimation error w.r.t. the body is the difference between the estimated and body trajectories as shown in figure (4). This section verifies the proposed algorithm in terms of estimation error w.r.t the warhead and body. The proposed algorithm should have a small estimation error w.r.t. the warhead to ensure the warhead is constantly tracked.

Simulation parameters

Consider an RV in reentry phase with C = 2500 kg/$m^2$and initial values of x(0) = 34km, y(0) = 36km, z(0) = .1m, v(0) = 1782.6968m/s, γ_1(0) = 42°, and γ_2 (0) = 0.35°. The RV splits into the warhead and body at t_s= 75s. The ballistic coefficients of the warhead and body are C_w = 4000kg/m2 and C_b = 8333kg/m2, respectively. equations (1)-(9) simulate the measured warhead and body trajectories

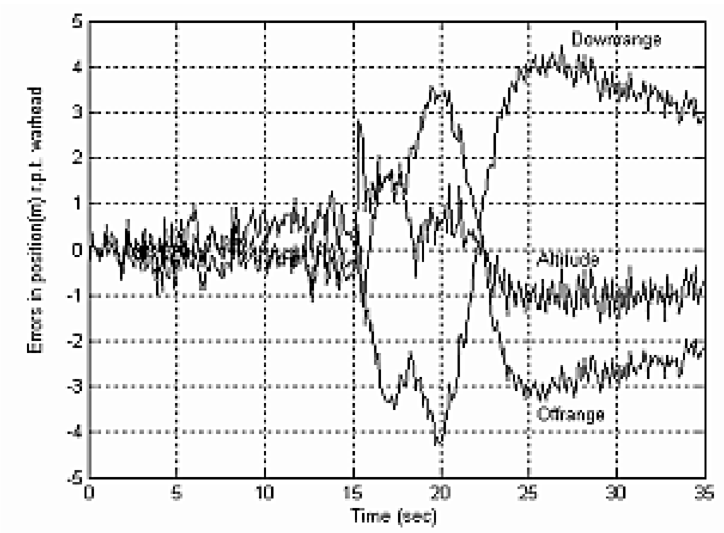

Figure.3 The estimation errors w.r.t. the warhead in position

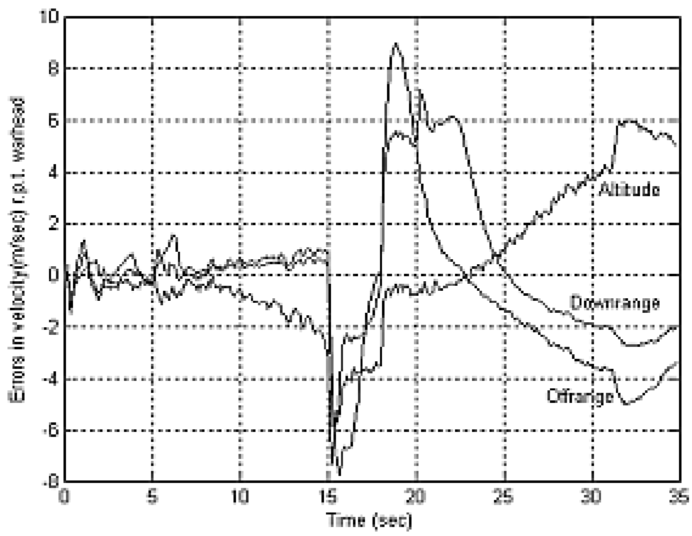

Figure.4 The estimation errors w.r.t. the warhead in Velocity

Table 1. Root mean square of the estimated error

Position [km]				**Velocity[m/s]**		
w.r.t	XR	YR	ZR	XR	YR	ZR
Warhead at impact point	2.281	1.782	.417	1186	1297	1.190
Body after ts separation	48.32	51.47	.292	1173	1268	.264

Table 2. Root mean square of the estimated error in different Ballistic Coefficient

	Position [km]			Velocity[m/s]			
C		XR	YR	ZR	XR	YR	ZR
2500	W.H Body	2.281 48.32	1.782 51.47	.417 .292	1186 1173	1297 1268	1.190 .264
3500	W.H Body	2.094 47.532	1.852 51.23	.351 .288	1166 1171.45	1259 1264	.585 .254
5000	W.H Body	1.967 46.78	1.782 50.927	.286 .263	1157 1170.02	1238 1261.38	.358 .241

5 - Aerodynamic Coefficient Model for Warhead

5.1 Aerodynamic Lift Coefficient

In order to find the relationship between the lift coefficient and the angle of attack, we depict the curve by the lift coefficient. The figure 5 shows that the lift coefficient is linear with the angle of attack very well. So we can choose the first order polynomial function to fit this relation. The relationship between the lift coefficient and the Mach
number is shown in the figure 6. The curves decrease severely in the low Mach number and become level almost in the high Mach number. So we choose the exponential function as the base function. Because there is an exponential function, so the overall function is nonlinear and we should get the parameters of the function by nonlinear least square method [5].

The lift coefficient model is shown as follows,

$$C_L = C_{L1}\alpha + C_{L2}\exp(C_{L3}v) + C_{L0} \tag{19}$$

Where $C_{L1}, C_{L2}, C_{L3}, C_{L0}$ are parameters; C_L is the lift coefficient; α is the angle of attack, degree; *v* is the velocity, Mach. The results of optimization are:

$$C_{L1} = 0.0366\,, C_{L2} = 0.2664\,, C_{L3} = -0.20\,, C_{L0} = -0.1177$$

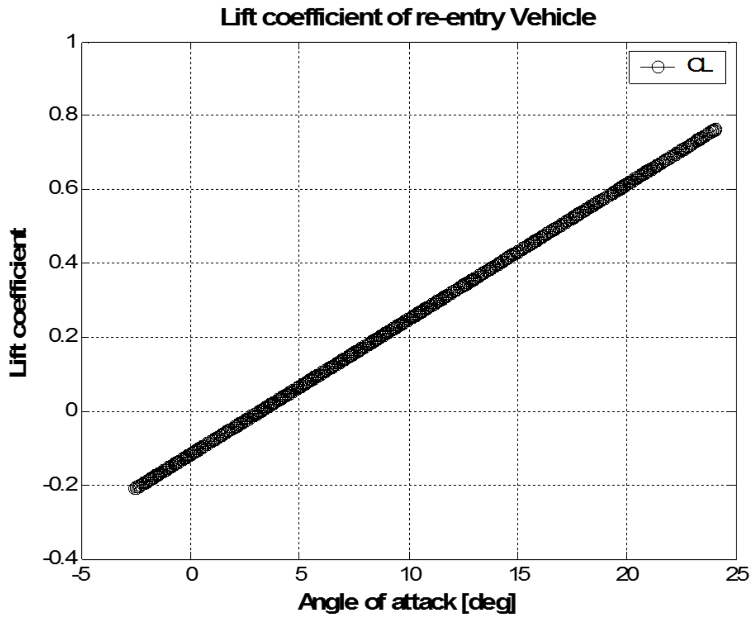

Figure.5 The Lift Coefficient w.r.t Angle of Attack

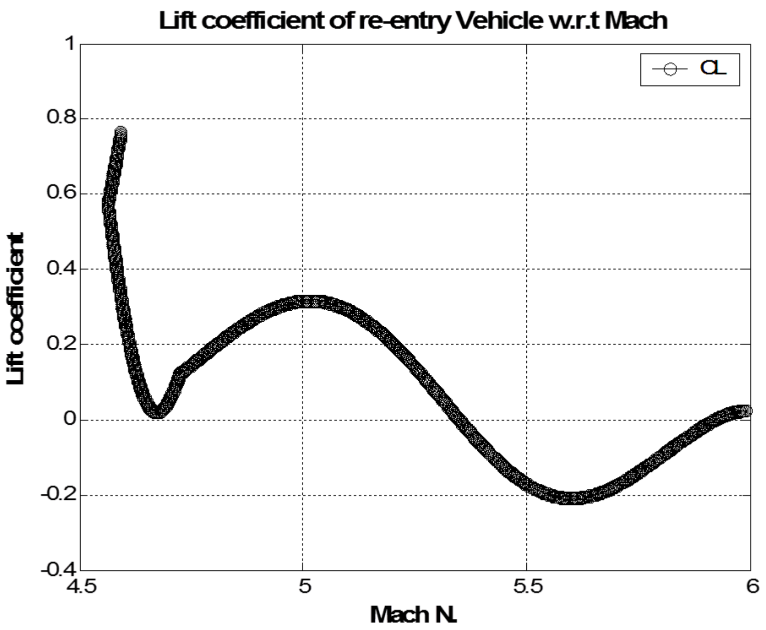

Figure.6 The Lift Coefficient w.r.t Mach Number.

5.2 Aerodynamic Drag Coefficient

As the above process, the drag coefficient model can also be built in the same way. But the function is different from the above one. Because, the drag coefficient is linear with the angle of attack square. This relationship can also be found from figure 7. From figure 7, the relationship between the drag coefficient and the angle of attack is quadratic. So the drag coefficient model is expressed as[6],

$$C_D = C_{D1}\alpha^2 + C_{D2}\exp(C_{D3}v) + C_{D0} \qquad (20)$$

Where $C_{D1}, C_{D2}, C_{D3}, C_{D0}$ are parameters; C_D is the drag coefficient The results of optimization are:

$$C_{D1} = 7.53 \times 10^{-4}\,, C_{D2} = 0.27\,, C_{D3} = -0.407, C_{D0} = 0.0442.$$

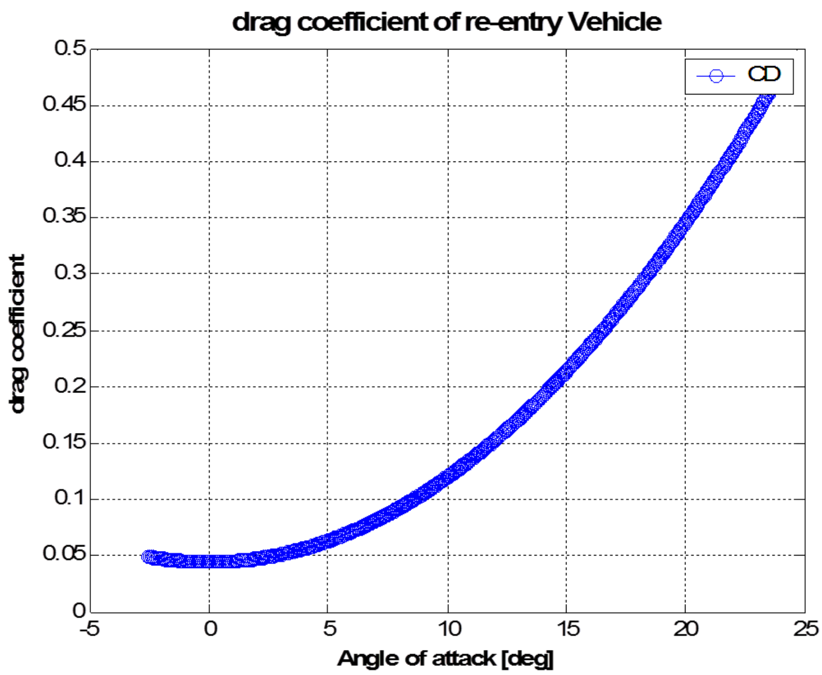

Figure.6 The Drag Coefficient w.r.t Angle of Attack

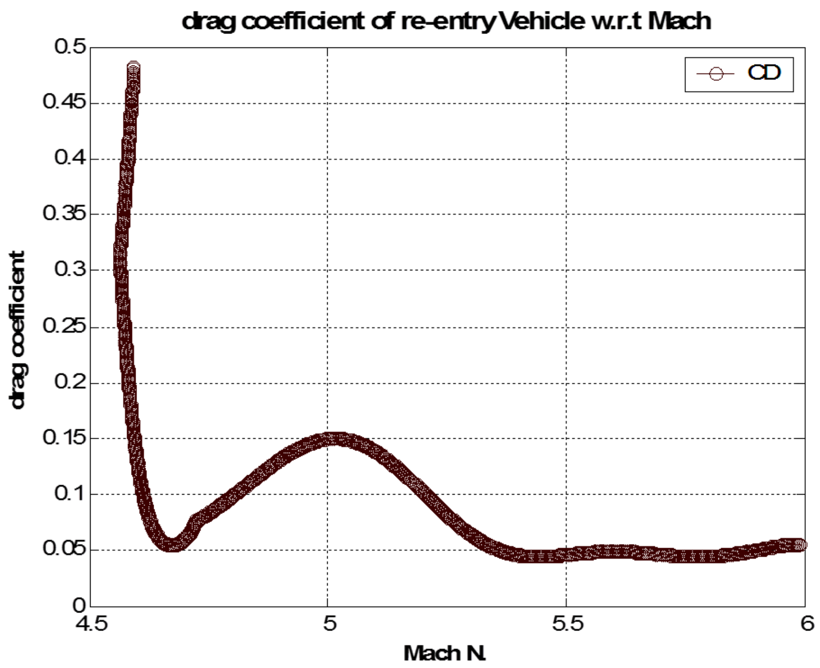

Figure.7 The Drag Coefficient w.r.t Mach Number

Conclusion

Simulation results monitor the performance of the estimation error corresponding to the warhead and body. We also focused on the aerodynamic coefficients models of the RV, which is very manful for the simulation and theoretical analysis of the hypersonic vehicle. Considering the two important factors, angle of attack and Mach number, the lift and drag Coefficients models have been built with a simple and effective function by the nonlinear least square method using Lebedev aerodynamic calculations.

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