Effect of Orbit Perturbations on the Ground Track of Low Earth Orbit Satellite

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Abstract: A low Earth orbit satellite is exposed to rapidly changing environmental conditions as it passes over various geographical features and local time zone, which significantly affect Earth's albedo heat load. The studying of the effect of this environmental changing on the satellite is associated with finding the ground track of satellite. In the present work, the governing equations of satellite motion in Keplerian low Earth orbit have been simulated within MATLAB-SIMULINK environment. The ground track of the satellite under the assumption of Keplerian orbit has been obtained. The effect of perturbation on the orbital elements have been introduced into the computer model, and the ground track of satellite under the effect of these forces has been obtained. A comparison between the two results has been performed in order to decide whether the effect of perturbation forces must be taken in consideration during the calculation of Earth's albedo based on local albedo coefficient or that the effect is of small value enough to ignore it.

Keywords: albedo, sub-satellite point, ground track of the satellite, low Earth orbit, orbit perturbations.

Nomenclature

a	=	semi-major axis of the orbit [km]
E	=	eccentric anomaly [rad]
e	=	eccentricity of the orbit
f	=	flattening factor [1/298.2947]
G	=	universal gravitational constant [6.673784×10^{-11} m^3/kg.s^2]
H	=	altitude of the satellite measured from Earth surface [km]
i	=	inclination of the orbit [rad]

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M_E	=	mass of Earth [5.9736×10^{24} kg]
n	=	mean motion of the satellite [s^{-1}]
Δn	=	variation of mean motion of the satellite [s^{-1}]
R	=	Earth equatorial radius [6378.14 km]
R_e	=	mean Earth radius [km]
r	=	distance of the satellite from the center of the Earth, i.e. magnitude of $\vec{r}$ [km]
$\vec{r}$	=	position vector of the satellite
r_i, r_j, r_k =		components of position vector $\vec{r}$ in the directions OX, OY and OZ
r_P, r_Q, r_w =		components of position vector $\vec{r}$ in the directions OP, OQ and OW
β	=	geodetic latitude [deg]
λ	=	geocentric latitude [deg]
v	=	true anomaly [rad]
ρ_a	=	radius of curvature in Meridian [km]
φ	=	geocentric and geodetic longitude [deg]
φ_{go}	=	Greenwich sidereal time or location of Greenwich at t_o [deg]
Ω	=	right ascension of the ascending node [rad]
$\dot{\Omega}$	=	nodal precession rate [rad/sec]
ω	=	argument of perigee
$\dot{\omega}$	=	argument of perigee precession rate [rad/sec]
$\omega_\oplus$	=	angular velocity of the Earth [$7.292115090 \times 10^{-5}$ rad/s]

I.   Introduction

Spacecraft thermal control is an energy process management in which environmental heating plays a major role. The principal forms of environmental heating on an orbit are the direct sunlight, the reflected sunlight from Earth (albedo), and the infrared (IR) energy emitted from Earth. During launching process or in exceptionally low orbits, there is also a free molecular heating effect caused by friction in the rarefied upper atmosphere. The overall thermal control of a satellite on an orbit is usually achieved by balancing of the energy emitted by the spacecraft as IR radiation with the energy dissipated by its internal electrical components, along with the energy absorbed from the environment; atmospheric convection is absent in space [1, 2].

The fraction of the solar radiation that is reflected from the surface and/or atmosphere of a planet is known as the planetary albedo. Its value is highly dependent on local surface, season time and atmospheric properties. For example, for the Earth, it varies from as high as 0.8 from clouds to as low as 0.05 over surface features such as water and forest [3, 4, 5, 6]. In general, an average value of Earth albedo (0.35 − 0.38) is used during calculation of heat energy reflected off Earth [7].

It is important to address the transients of the low Earth orbit (LEO) environment. Because of the low altitudes and short orbital periods, the LEO environment is dynamic and creates special difficulties for the thermal engineer. A LEO spacecraft only sees a small portion of the Earth (Spherical cap which sees the satellite) as shown in Fig. 1. As it orbits, it is exposed to rapidly changing environmental conditions as it passes over various

geographical features and local time zones, which significantly affect Earth IR and albedo heat loads.

To study the dynamic effect of Earth albedo on a satellite orbiting on LEO, we have to find a process to calculate Earth albedo based on local values of albedo coefficient which are given in atlases of albedo [3, 4, 5, 6]. The first step is the determination of the spherical cap of the Earth which affects the satellite at every moment of the satellite orbiting. As shown in Fig. 1, the determination of the Earth cap is associated with the determination of the coordinates of sub-satellite point.
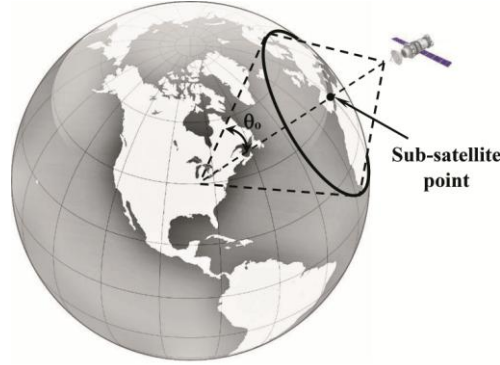

Fig. 1 Spherical cap which sees the satellite

Sub-satellite point is the point of intersection with the Earth's surface of a plumb line from the satellite to the center of the earth. The path made by the sub-satellite point when the satellite has made one complete revolution is known as satellite ground track. The motion of the sub-satellite point for an orbit shows the true relation between the satellite and the Earth's surface. If the sub-satellite paths are available for each of a variety of orbits, they will facilitate the mission planner [8]. The ground track of satellite is associated with the orbital parameters, so that, it will be affected with all perturbation forces that affect the orbital elements.

The present work is directed to decide whether the effect of perturbation forces must be taken in consideration during the calculation of Earth's albedo based on local albedo coefficient or that effect is of small value enough to ignore it.

II.   Ground track of satellite

Assuming that the motion of satellite is Keplerian, i.e. it occurs on a Keplerian orbit. In order to specify a point in Keplerian motion in space, the first step is to identify the orbit and the position of the satellite on that orbit. The six orbital elements used to completely describe the motion of a satellite within an orbit are a which defines the size of the orbit, e which defines the shape of the orbit, i which defines the orientation of the orbit with respect to the Earth's equator plane, ω which defines where the low point, perigee of the orbit, is with respect to the Earth's surface, Ω which defines the ascending and descending orbit locations with respect to the Earth's equatorial plane and v which defines where the satellite is within the orbit with respect to perigee [9 – 16]. The angles ω, i and Ω are the Euler angles characterizing the orientation of the Cartesian coordinate system $\boldsymbol{OXYZ}$, which lies in the

equatorial plane of the Earth with OX in the direction of the vernal equinox, from the inertial coordinate frame $OPQW$, which lies in the orbital plane with OP in the direction of the pericenter (periapsis).

The following equations relate the orbital elements a, e and v to the position and velocity vectors of the satellite in the inertial coordinate frame $OPQW$ [9 – 16]:

$$\vec{r} = r \cos v \, \vec{P} + r \sin v \, \vec{Q}, \tag{1}$$

$$\vec{V} = \sqrt{M_E G / a(1 - e^2)} \left(- \sin v \, \vec{P} + (e + \cos v)\vec{Q} \right), \tag{2}$$

$$r = \frac{a(1-e^2)}{(1+e \cos v)}. \tag{3}$$

Out of the six orbital elements, only v, the true anomaly, changes with time. Thus, an algorithm is needed to give the true anomaly of the satellite after a time period ΔT. For a given v at $T = 0$, the problem of finding new v_1 at other time is solved by the following equations [9 – 16]:

$$\cos E = \frac{e + \cos v}{1 + e \cos v}, \tag{4}$$

$$M = E - e \sin E, \tag{5}$$

$$n = \sqrt{\frac{\mu}{a^3}}, \tag{6}$$

$$M_1 = M + n\Delta T, \tag{7}$$

$$\cos v_1 = \frac{e - \cos E_1}{e \cos E_1 - 1}. \tag{8}$$

Once v is determined, the position and the velocity of the satellite with respect to the inertial orbital frame $OPQW$ could be determined.

To study the apparent motion of sub-satellite point on the Earth's surface, any vector defined in the orbital frame $OPQW$ should be transformed to a vector defined in the equatorial frame $OXYZ$. The following is the mathematical process for this transformation [9 – 16]:

$$\begin{bmatrix} r_i \\ r_j \\ r_k \end{bmatrix} = \tilde{R} \begin{bmatrix} r_P \\ r_Q \\ r_W \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} r_P \\ r_Q \\ r_W \end{bmatrix}, \tag{9}$$

where $R_{11} = \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i$; $R_{12} = -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i$; $R_{13} = \sin \Omega \sin i$; $R_{21} = \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i$; $R_{22} = -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i$; $R_{23} = -\cos \Omega \sin i$; $R_{31} = \sin \omega \sin i$; $R_{32} = \cos \omega \sin i$ and $R_{33} = \cos i$.

As mentioned before, sub-satellite point is the intersection point with the surface of the Earth of a straight line connecting the satellite with the center of the Earth. As seen in Fig. 2, if it is assumed that the Earth is of spherical shape with mean radius $R_e = 6371.003$ km, then

the center of Earth is point O and the sub-satellite point is S_1. The system of coordinates used to determine the position of sub-satellite point in this case is known as Geocentric coordinates. In fact, the Earth is spheroid with a polar radius $b_e = 6356.755$ km and equatorial radius $a_e = 6378.14$ km. The center of spheroid Earth is point O' and the sub-satellite point in this case is S_2. The system of coordinates used to determine the position of sub-satellite point is known as Geodetic coordinates.

The geocentric coordinates of sub-satellite point (φ, λ) can be obtained using the components of position vector of the satellite r_i, r_j and r_k by the formulae [17 – 21]

$$\lambda = \left[\tan^{-1}\left(\frac{r_k}{\sqrt{(r_i^2 + r_j^2)}} \right) \right] , \qquad (10)$$

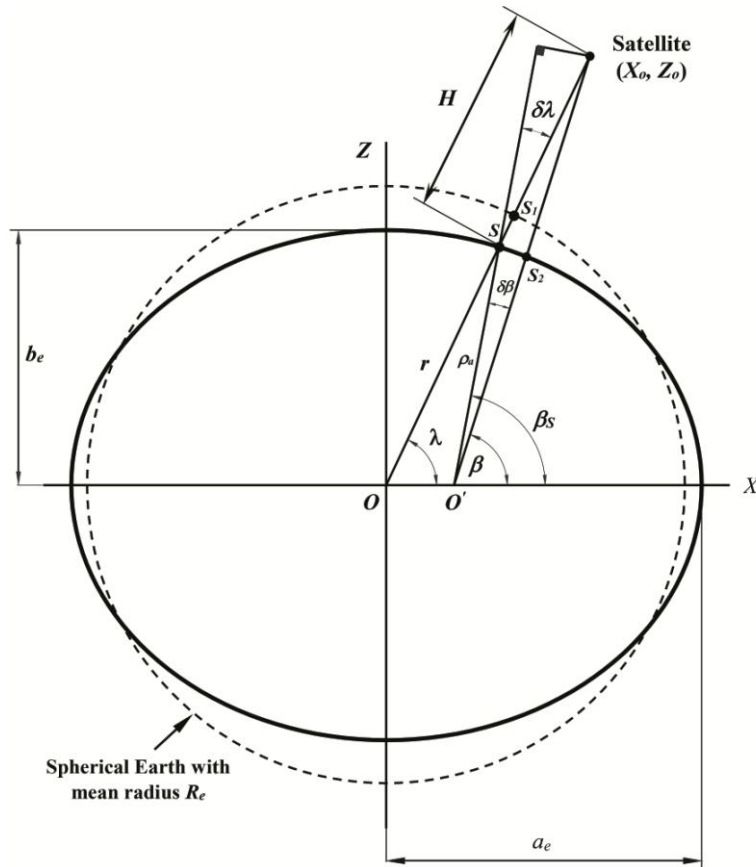$$\varphi = \tan^{-1}\frac{r_j}{r_i} - \varphi_{go} - \omega_\oplus(t - t_o) . \qquad (11)$$

Fig. 2 Geocentric versus geodetic coordinates of sub-satellite point

In order to increase the accuracy of the coordinates of sub-satellite point, we have to take in consideration the oblateness of the Earth. In works [17, 19], point S is considered to be the sub-satellite point in case of spheroid Earth and the angle βs between the line connecting the point O' with the point S and the

Equatorial plane is assumed to be geodetic latitude of sub-satellite point and is represented in the form

$$\beta_S = [\tan^{-1}(\tan(\lambda)/(1-f)^2)] . \tag{12}$$

In this case, the geodetic coordinate of sub-satellite point is $(\varphi, \ \beta_S)$. Actually, point S_2 is the accurate sub-satellite point in case of spheroid Earth and the previous assumption in works [17, 19] leads to an error of $\delta\beta$ as show in Fig. 2. To obtain more accurate result, the angle $\delta\beta$ can be calculated using the following formulae

$$\delta\beta = \tan^{-1}\frac{H\sin(\beta_S-\lambda)}{\rho_a+H\cos(\beta_S-\lambda)} , \tag{13}$$

$$H = |r| - \frac{b_e}{\cos\lambda\sqrt{(\tan\lambda)^2+(1-f)^2}} , \tag{14}$$

$$\rho_a = \frac{a_e(1-f)^2}{[1-(2f-f^2)(\sin\beta_S)^2]^{\frac{3}{2}}} . \tag{15}$$

The geodetic latitude of sub-satellite point is $\beta = \beta_S - \delta\beta$, and the geodetic coordinate of sub-satellite point is $(\varphi, \ \beta)$.

III. Perturbation of orbit

The Keplerian orbit discussed above provides an excellent reference, but other forces act on the satellite to perturb it away from the nominal orbit The difference between the true motion and the ideal Keplerian motion results from two considerations [12]:
1. The Earth is not exactly spherical and the mass distribution is not exactly spherically symmetrical;
2. The satellite feels other forces apart from the Earth's attraction: attractive forces due to other heavenly bodies and forces that can be globally categorized as frictional.

The result on the two-body equations of motion is that the orbit will precess. The effect on the orbital elements is that Ω, ω and n will change over time. The following equations describe the change in these elements over time to a degree accurate enough for our purposes.

III. 1 Perturbation because of nonspherical Earth

For a spherical Earth, the force due to gravity would only depend on the distance of the satellite from the center of the Earth. Since the earth is oblate, the force due to gravity will vary depending on where the satellite is. Thus, the orbit will have a torque resulting from the offset center of gravity. This torque results in variation of the previously mentioned orbital elements with time as following [12]:

$$\frac{\dot{\omega}}{n} = J_2 \left(\frac{R}{p}\right)^2 \left(3 - \frac{15}{4}(\sin i)^2\right) + J_2^2 \left(\frac{R}{p}\right)^4 \left[\left(\frac{27}{2} - \frac{15}{16}e^2 - \frac{9}{16}e^4\right) + \left(-\frac{507}{16} + \frac{171}{31}e^2 + \right.\right.$$

9964e4sin i2+118564−675128e2−135128e4sin i4+J4Rp4−38+158sin i2−10564sin i4

10+152e2+−154+16516sin i2−10516sin i41+32e2,　　　(16)

$$\frac{\dot{\Omega}}{n} =$$

$$-1.5 J_2 \left(\frac{R}{p}\right)^2 \cos i + J_2^2 \left(\frac{R}{p}\right)^4 \cos i \left[\left(-\frac{45}{8} + \frac{3}{4}e^2 + \frac{9}{32}e^4\right) + \left(\frac{57}{8} - \frac{69}{32}e^2 - \frac{27}{64}e^4\right)(\sin i)^2\right] +$$

$$J_4 \left(\frac{R}{p}\right)^4 \cos i \left(\frac{15}{4} - \frac{105}{16}(\sin i)^2\right)\left(1 + \frac{3}{2}e^2\right), \qquad (17)$$

$$\frac{\Delta n}{n} = 0.75 J_2 \left(\frac{R}{p}\right)^2 (2 - 3(\sin i)^2)\sqrt{1 - e^2} \times \left\{1 + 0.125 J_2 \left(\frac{R}{p}\right)^2 \left[10 + 5e^2 + 8\sqrt{1 - e^2} - \right.\right.$$

656−2512e2+121−e2sin i2−564

$$J_2^2 \left(\frac{R}{p}\right)^4 \sqrt{1 - e^2}(2 - e^2)(\sin i)^2 - \frac{45}{128}J_4 \left(\frac{R}{p}\right)^4 \sqrt{1 - e^2}(8 - 40(\sin i)^2 + 35(\sin i)^4)e^2.$$

(18)

where p is a parameter equals to $a(1 - e^2)$. The J terms are known as zonal coefficients, which account for the uneven distribution of mass throughout the Earth. However, the J_2 term accounts for the Earth oblateness and is responsible for a majority of the perturbations for Earth orbiting satellites. The values of first three dimensionless J terms are $J_2 = 0.00108263$, $J_3 = -0.00000254$ and $J_4 = -0.00000161$ [9, 12].

The previous equations can be simplified if we take in consideration that low earth orbits are of very small eccentricity e and we can deal with them as nearly circular orbits $e \approx 0$. The error involved in setting the eccentricity to zero are very small, as long as e is itself small. In addition, the terms J_2^2 and J_4 are of small effect relative to the term J_2. So, we can rewrite equations (16), (17) and (18) in the following forms [9, 12]:

$$\frac{\dot{\Omega}}{n} = -1.5 J_2 \left(\frac{R}{a}\right)^2 \cos i, \qquad (19)$$

$$\frac{\dot{\omega}}{n} = J_2 \left(\frac{R}{a}\right)^2 \left(3 - \frac{15}{4}(\sin i)^2\right), \qquad (20)$$

$$\frac{\Delta n}{n} = 0.75 J_2 \left(\frac{R}{a}\right)^2 (2 - 3(\sin i)^2). \qquad (21)$$

III. 2　Third body perturbations

The gravitational forces of the Sun and the Moon cause periodic variations in all of the orbital elements, but only the right ascension of the ascending node, argument of perigee, and mean anomaly experience secular variations. These secular variations arise from a gyroscopic precession of the orbit about the ecliptic pole. The secular variation in mean anomaly is much

smaller than the mean motion and has little effect on the orbit; however, the secular variations in right ascension of the ascending node and argument of perigee are important, especially for high-altitude orbits. For nearly circular orbits, the equations for the secular rates of change resulting from the Sun and Moon are:

$$\dot{\Omega}_{moon} = -0.00338 \cos i / n, \tag{22}$$

$$\dot{\Omega}_{sun} = -0.00154 \cos i / n, \tag{23}$$

$$\dot{\omega}_{moon} = 0.00169 \, [4 - 5(\sin i)^2]/n, \tag{24}$$

$$\dot{\omega}_{moon} = 0.00077 \, [4 - 5(\sin i)^2]/n. \tag{26}$$

As a function of time t, and starting from the origin t = 0, the orbital elements are thus:

$$\Omega(t) = \Omega + \left(\sum \dot{\Omega}\right)t, \qquad \omega(t) = \omega + \left(\sum \dot{\omega}\right)t, \qquad M(t) = M + nt + (\Delta n)t.$$

IV.   Simulation and Results analysis

Equations (1) – (15) have been simulated within the MATLAB-SIMULINK environment, in order to obtain satellite ground truck under the assumption of Keplerian orbit. Equations (19) – (26) which describe the effect of perturbation forces on the orbital elements have been introduced into the model in order to obtain the satellite ground truck under the assumption of perturbed orbit.

Simulink is a block diagram environment for multi-domain simulation and Model-Based Design. It supports simulation, automatic code generation, and continuous test and verification of embedded systems. Simulink provides a graphical editor, customizable block libraries, and solvers for modeling and simulating dynamic systems. It is integrated with MATLAB, enabling to incorporate MATLAB algorithms into models and export simulation results to MATLAB for further analysis [22].

To study the effect of perturbation forces, we will consider the satellite TRMM which is orbiting in near-circular orbit ($e = 9.96 \times 10^{-5} \approx 0$) at an altitude of 350 km, and with inclination angle of 35° and $\Omega = \omega = 0$.
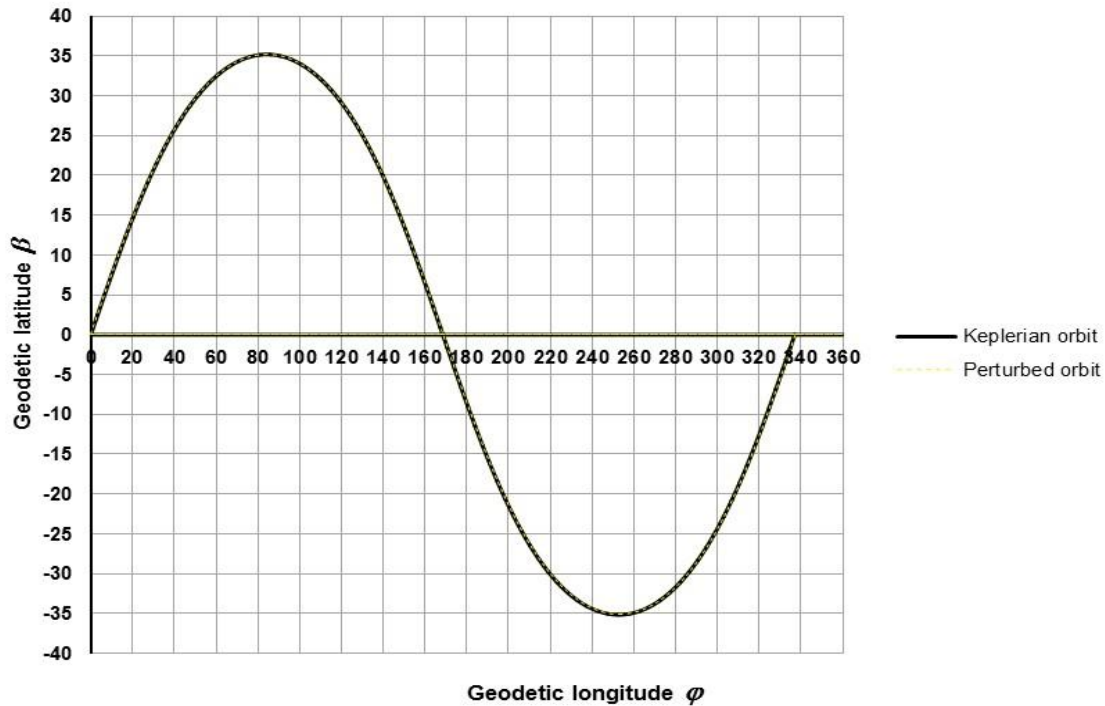
Fig. 3 Satellite TRMM ground track for the first cycle in the orbit

Fig. 3 shows the obtained ground truck of the satellite TRMM under the assumption of Keplerian orbit and perturbed orbit for the first cycle of the satellite in the orbit. The rate of change in orbital element due to perturbation forces are $\dot{\Omega} = -1.367 \times 10^{-6}$ rad/sec, $\dot{\omega} = 4.8481 \times 10^{-6}$ rad/sec and $\Delta n = 8.4561 \times 10^{-7}$ sec^{-1}. The values of orbital element variation is very small, but it will be of great effect over time. The ground track of the satellite for the first satellite cycle in the perturbed orbit is almost coincide with that in Keplerian orbit. However, there is a very small difference between the two ground tracks. The maximum difference between a sub-satellite point in perturbed orbit and the corresponding point in Keplerian orbit is 0.00294 rad = 0.1683° which is equivalent to a surface distance of 19 km.
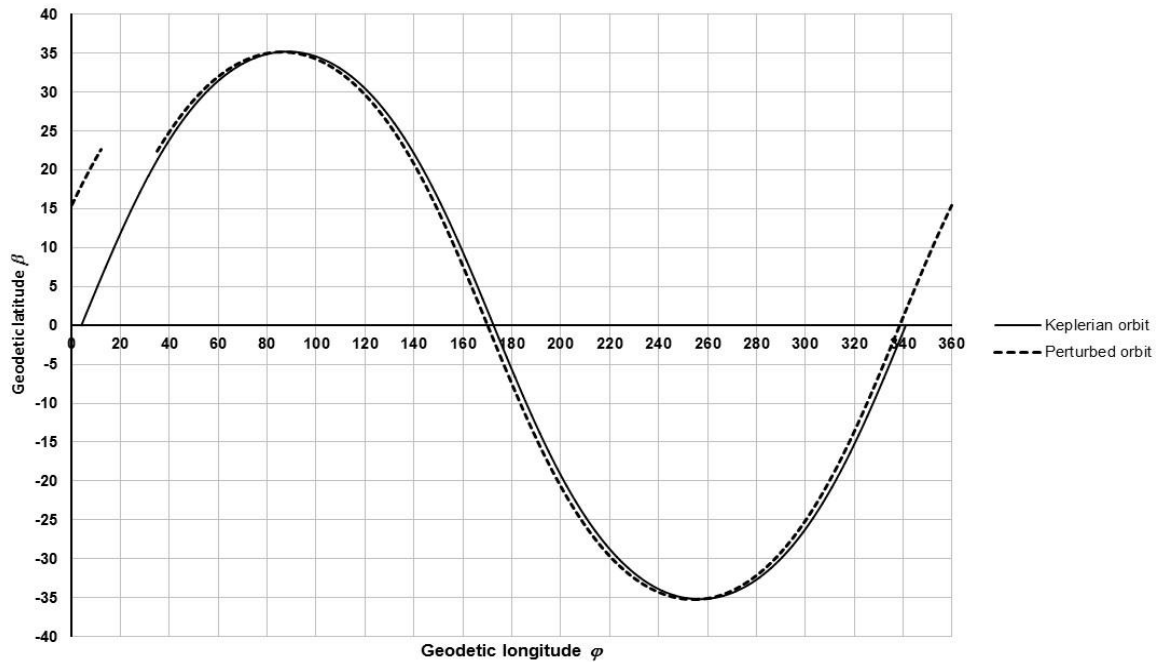
Fig. 4 Satellite TRMM ground track for the first cycle in the 10th day

Fig. 4 shows the obtained ground truck of the satellite TRMM under the assumption of Keplerian orbit and perturbed orbit for the first cycle of the satellite in the 10th day of orbiting. The difference due to perturbation forces becomes clear.

If the calculation of Earth albedo based on local albedo coefficients is performed during first satellite cycle, the 19 km difference will not affect the calculation because the maximum resolution of data given in atlases of local albedo coefficients is 5°×5° (i.e. 550 km × 550 km), but over time this distance will increase to values which must be taken in consideration.

V.   Conclusion

The equations that govern the motion of a satellite in Keplerian low Earth orbit have been simulated within MATLAB-SIMULINK environment. The ground track of satellite under the assumption of Keplerian orbit has been obtained. The governing equations of the perturbation forces which affect the orbital elements have been introduced into the computer model and the ground track of satellite under the effect of these forces has been obtained. A comparison between the two results has been performed. The comparison shows that the effect of orbit perturbations on the calculation of Earth albedo applied to a satellite in low Earth orbit can be ignored if the calculation is performed in the beginning of the satellite motion in the orbit. But for long term calculations this effect becomes of great values which must be taken in consideration during calculation.

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