Paper: ASAT-16-170-CT

16th International Conference on
AEROSPACE SCIENCES & AVIATION TECHNOLOGY,
ASAT - 16 – May 26 - 28, 2015, E-Mail: asat@mtc.edu.eg
Military Technical College, Kobry Elkobbah, Cairo, Egypt
Tel : +(202) 24025292 – 24036138, Fax: +(202) 22621908

Design and Analysis of Quadcopter Classical Controller

Ahmed H. Ahmed*, Ahmed N. Ouda*, Ahmed M.Kamel*, Y. Z. Elhalwagy*

Abstract:

The objective of this work is to introduce the design, simulation and control of a quadcopter, as an example of unmanned aerial vehicle (UAV).To fulfill this objective, a mathematical model of the quadcopter has been developed. The simulation of the model and controller design is developed in MATLAB/Simulink environment. Although it remains a complete nonlinear system, this paper operates with mathematical representation of the quadcopter and modeling of the intended system. A linearization of the obtained mathematical model has been achieved. In order to design the attitude controller, the transfer function of the brushless DC motors (BLDCM) which are responsible of the quadcopter motion is obtained using system identification techniques. A complete test experiment is described here to achieve this goal. The designed controller is assessed and simulation results are discussed.

Keywords:

Brushless DC motor (BLDCM), Unmanned Aerial Vehicle (UAV), Electronic Speed control (ESC).

1. Introduction

The quadcopter UAV is proven to be useful for many military and civil applications. The most important features are vertical take-off and landing (VTOL) and hover capability, so it is suited for missions such as surveillance, road traffic supervision, victims localization after natural disasters, etc. Such vehicles have also received a growing interest from academic research institutes, since they can be used as low cost test beds for robotic studies [1] , [2], [3]. To make autonomous flight of UAVs possible, control laws must be developed to replace the action of a human pilot. Linear control techniques such as PID or LQR applied for years to solve this problem [4]. Although the system is naturally nonlinear, input–output linearization must be applied to design the desired controllers. A good and robust controller is the one that can cope uncertainties and unmodelled dynamics resulted from linearization process.

Another well-known solution lies in the application of back stepping techniques, by considering the model used for control design as a chain of integrators. Back stepping has been widely applied to different UAV vehicles such as conventional helicopters [5], [6], coaxial bi-rotor helicopters [7] or four-rotor vehicles [4] In order to understand and study the behavior of systems, mathematical models are needed. These models are equations which describe the relationship between the inputs and outputs of the system. They can be used to enable forecasts to be made of the behavior of the system under specific conditions and to simulate the system under study in order to examine the proposed controller. The introduced mathematical model is based on deriving the equation of motion for a rigid air craft.

Linear models such as the one introduced in equations (1), (2) have been used extensively in the past and the control theory for linear systems is quite mature.

$$\dot{X} = AX + BU \quad (1)$$

$$\dot{Y} = C\dot{X} + DU \quad (2)$$

Where:

X (n×1) the state vector

U (m×1) the control vector

Y (p×1) the output vector, A, B, C, D matrices of appropriate dimension

In this case, the system is a nonlinear one, where the equations of motion are expressed in the state-space form as [8]:

$$\dot{X} = F(X, U)$$

(3)

Where:

X (n×1) the state vector

U (m×1) the control vector

F (…,…) Vector valued nonlinear function of the individual states and controls.

The state of a system is defined as an indication of the stored energy in that system (i.e., potential and kinetic energy) and its distribution which is completely defined by the state variables X_i.Also the output equation is required in the general form:

$$Y = G(X, U)$$

(4)

Where:

Y (P×1) output vector.

G (…,…) a set of nonlinear equation similar to F.

while the output variable Yi corresponds to the provided measured quantities from the onboard ith sensor.

The equation of motion of a rigid body can be decoupled into rotational equations and translation equations when taking the origin coordinate to be at the center of mass (center of gravity) of the rigid body[8],[9].

The state model derived later will be 6-Degrees of freedom model, three of which are the rotational motion of the rigid body about the coordinate axis which are fixed to the body center of gravity (yaw, pitch and roll) and the other will be the three components of translation of the center of gravity as shown in Figure 1[8]:

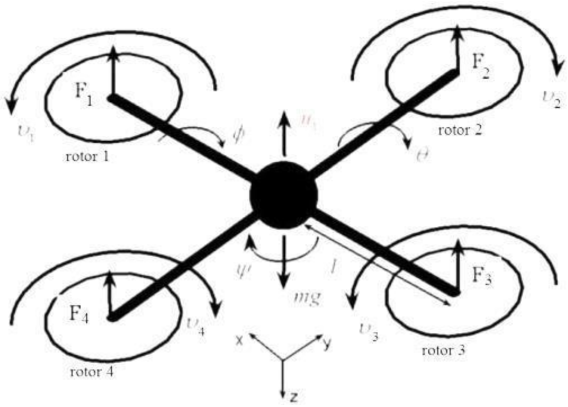

Figure 1. Body Fixed coordinate and Flat Earth coordinate

The state vector X of the proposed model consists of 12 state variables. The first set is the three components of position which specify the potential energy. Translational kinetic energy is represented by the three components of velocity; however the rotational kinetic energy is introduced by the three components of angular velocity. The last set is the attitude state, which defines the orientation relative to the gravity vector.

The main objective of this work is to design a practical and realistic controller for motion and attitude hold of a quadcopter. This controller is to be designed on a verified and close to realistic quadcopter model. To get close to reality, the transfer function of (BLDCM) is obtained using system identification techniques in an experimental work and used in the model. Through this experiment, maximum payload, time of flight and motor propeller system modeling is also estimated.

To reach this goal, this paper is organized as follows: In the next section, mathematical model formulation which can be divided into two sets, the first set is the navigation equations derived from kinematics of rigid bodies [8], and the second set is the equations of motion derived from Newton's laws [9].In section 3, the transfer function of the used (BLDCM) is obtained using system identification techniques also, motor propeller system modeling, flight time estimation and maximum payload are estimated. In section 4, a simulation program using MATLAB and Simulink has been established for modeling the underlying system[10] . In section 5, the quadcopter dynamics can be linearized to provide an easy inverse model which can be implemented in the control algorithms. In section 6, a classical control is designed to fulfill the performance requirements of time response and flight path characteristics and comparison between the designed controllers. Finally, the conclusion of this paper is obtained.

2. Mathematical Model Formulation

In the derivation of the UAV equations of motion starting from the Newton's second law of Motion, two coordinate systems are introduced. The first is the inertial coordinate O attached to the Earth frame at earth's mass center passing through north pole and stationary with respect to the fixed stars where Newton's laws of motion are valid (within the Euclidean space). The second coordinate is placed at the center of gravity of the UAV while flying and it has a fixed alignment to some convenient reference line of the UAV[11].

Assuming that the proposed system is flying near the earth surface and the air rotate with earth (the earth rotation effect is neglected), the inertial coordinate is replaced by Flat Earth coordinate E at the surface of the earth (i.e. the observer) as shown in Figure 2 with x-axis pointing to north and z-axis pointing downward[9].

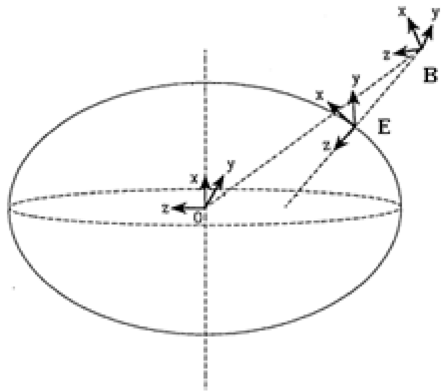

Figure 2 .The inertial, flat earth and body coordinate representation

The model equations can be divided into two sets, the first set is the navigation equations derived from kinematics of rigid bodies [8], and the second set is the equations of motion derived from Newton's laws [9].

2.1Kinematic Equations:

The rotation of the inertial frame to the body centered frame (aligned as x-forward, y-Starboard and z-down in the UAV) which describes the instantaneous attitude of the UAV is performed in the following sequence of rotations [9]:

i. Rotate about the z-axis, nose right (positive "yaw").

ii. Rotate about the new y-axis, nose up (positive θ "pitch").

iii. Rotate about the new x-axis, right arm down (positive ∅ "roll").

These angles are referred commonly as Euler angles, so the coordinate transformation is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_B = R_\emptyset R_\theta R \begin{bmatrix} x \\ y \\ z \end{bmatrix}_E$$

Let R denotes the complete transformation from the flat earth coordinate to the body Coordinate frame as follow:

$$R = R_\emptyset R_\theta R$$

Where:

$$R = \begin{bmatrix} \cos\theta\cos\emptyset & \cos\theta\sin & -\sin\theta \\ -\cos\emptyset\sin + \sin\emptyset\sin\theta\cos & \cos\emptyset\cos + \sin\emptyset\sin\theta\sin & \sin\emptyset\cos\theta \\ \sin\emptyset\sin + \cos\emptyset\sin\theta\cos & -\sin\emptyset\cos + \cos\emptyset\sin\theta\sin & \cos\emptyset\cos\theta \end{bmatrix} \quad (5)$$

The matrix R describes the attitude of the UAV as it changes with time. Its elements are functions of the Euler angles ∅,θ and.Where ∅, θ , : roll, pitch, yaw.
From the above analysis; the matrix R describes the transformation from body the coordinate to the UAV flat earth coordinate so:

$$V_E = RV_B$$

Where:

V_B … Absolute velocity vector of the UAV c.g expressed in body frame.
V_E … Absolute velocity vector of the UAV c.g expressed in inertial frame.

2.2 Dynamic Equations: From Newton's second law of linear momentum it can be written:

$$F_B + Rmg = \frac{d}{dt}(mV_B) \qquad (6)$$

Where:

F_B … The sum vector of the acting forces on the UAV (thrust and aerodynamic forces) expressed in the body coordinate.

g … The gravitational attraction vectors (expressed in the inertial coordinate) due to earth mass.

R … The matrix that rotates the g vector into the body frame.

m … mass of the UAV.

V_B … Absolute velocity vector of the UAV cg expressed in body frame (measured with respect to the inertial frame).

The angular acceleration of the UAV can be determined by applying Newton's second law to the rate of change of angular momentum of the UAV, i.e.:

$$T_B = \frac{d}{dt}(H_B) \qquad (7)$$

Where:

T_B … The net torque acting about the air craft cg.

H_B… The angular momentum vector of the rigid vehicle.

The equation of motion including thrust force, hub force, drag moment, rolling moment, pitching moment, yawing moment and forces along X, Y, Z axis can be summarized as follow:

$$\left\{\begin{array}{c} I_{xx}\ddot{\emptyset} = \dot{\theta}\dot{}(I_{yy} - I_{zz}) + J_r\dot{\theta}\Omega_r + l(-T_2 + T_4) - h\left(\sum_{i=1}^{4} H_{yi}\right) + (-1)^{i+1}\sum_{i=1}^{4} R_{mxi} \\ I_{yy}\ddot{\theta} = \dot{\emptyset}\dot{}(I_{zz} - I_{xx}) - J_r\dot{\emptyset}\Omega_r + l(T_1 - T_3) + h\left(\sum_{i=1}^{4} H_{xi}\right) + (-1)^{i+1}\sum_{i=1}^{4} myi \\ I_{zz}\ddot{\ } = \dot{\theta}\dot{\emptyset}(I_{xx} - I_{yy}) + J_r\dot{\Omega}_r + (-1)^{i}\sum_{i=1}^{4} Q_i + l(H_{x2} - H_{x4}) + l(-H_{y1} + H_{y3}) \\ m\ddot{z} = mg - (cc\emptyset)\sum_{i=}^{4} T_i \\ m\ddot{x} = (ss\emptyset + cs\theta c\emptyset)\sum_{i=1}^{4} T_i - \sum_{i=1}^{4} H_{xi} - \frac{1}{2}C_xA_c\rho\dot{x}|\dot{x}| \\ m\ddot{y} = (-cs\emptyset + ss\theta c\emptyset)\sum_{i=1}^{4} T_i - \sum_{i=1}^{4} H_{yi} - \frac{1}{2}C_yA_c\rho\dot{y}|\dot{y}| \end{array}\right\} \qquad (8)$$

Where:

I_{xx}, I_{yy}, I_{zz} body inertia moment , Ω_r: rotor speed, J_r : rotor inertia moment, ρ: air density , $\dot{\theta}\dot{}(I_{yy} - I_{zz})$:body gyro effect, $J_r\dot{\theta}\Omega_r$: propeller gyro effect, $h\left(\sum_{i=1}^{4} H_{yi}\right)$:hub moment due to forward flight, $(-1)^{i+1}\sum_{i=1}^{4} R_{mxi}$: hub moment due to sideward flight

T: motor thrust, H: hub forces, c: cosine, s: sine.

From the above equation the dynamic, kinematic and aero modules are obtained. In order to build the motor dynamic module, the transfer function of motor must be obtained using system identification techniques. To reach that goal, an experiment is setup to get the motor transfer function as can be seen in the following section.

3-Motor Identification and Flight Time Estimation

The only affecting external force that is taken into account is the thrust derived from the motor –propeller unit in the UAV, assuming indoor flying condition and traveling at low speed (by neglecting the wind effect, and aerodynamic drag). Many investigations are made on this part concerned mainly on the evaluation of a relationship between the electric input power (in other words, motor shaft rotation speed, and torque) and the output thrust from the propeller fixed at the motor shaft [11]. In order to obtain the transfer function of the used motor, the following experiment is built and analyzed.

The designed experiment comprises of the following components as seen in Figure 3, Figure 4:

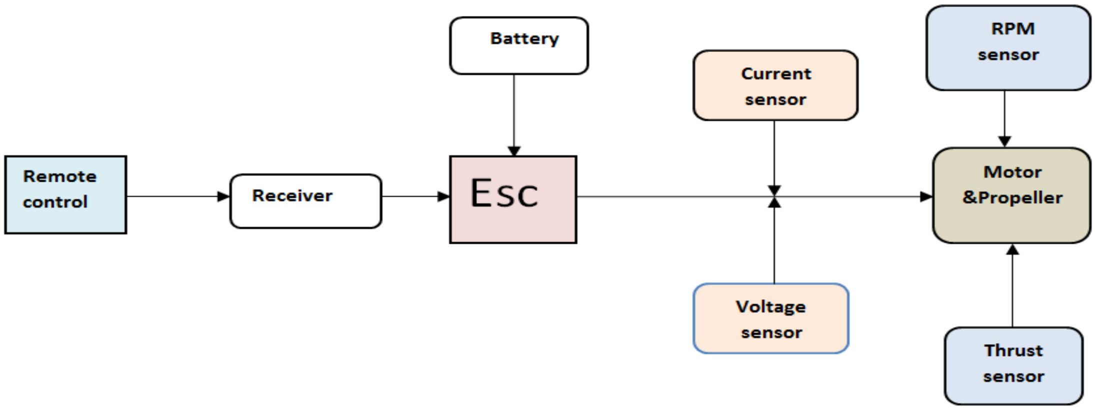

Figure 3 .block diagram of current, Thrust &RPM measuring

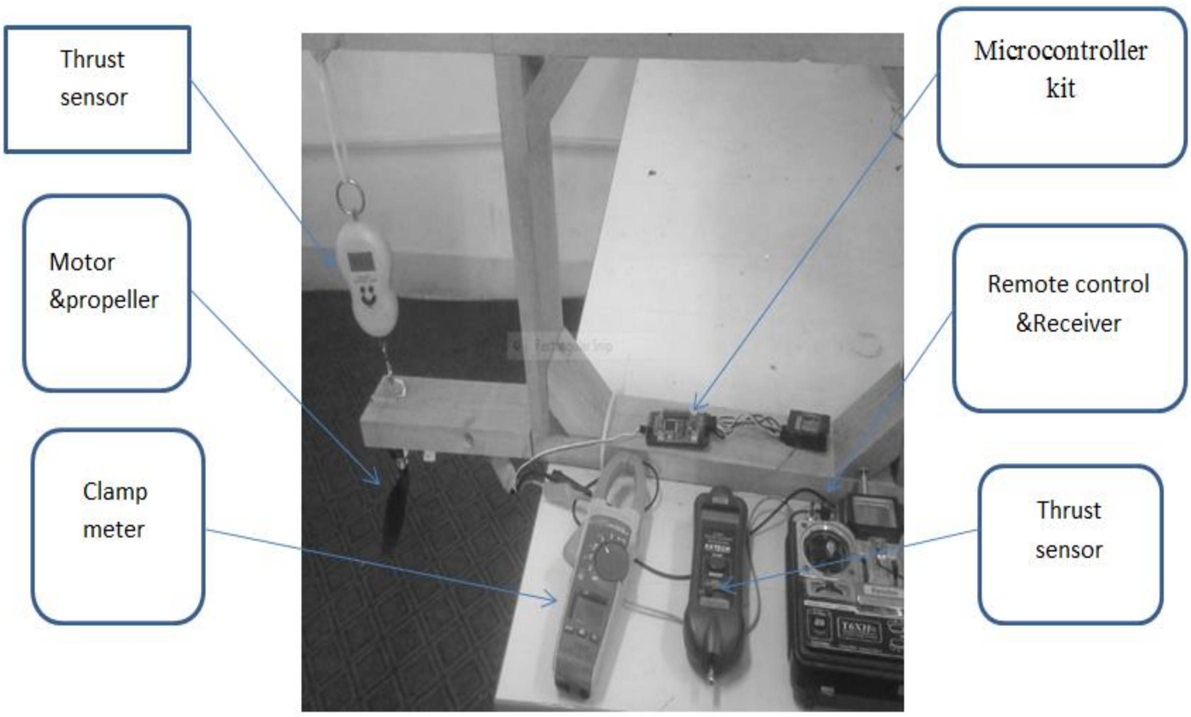

Figure 4 .experiment setup

Flight time of the quad copter is directly proportional to the used battery and can be found by dividing the battery capacity by amount of current measured in amperes being drawn from it. A laser tachometer is used to measure the motor RPM to obtain a relation between input volt and current with output RPM of the motor and consequently the output thrust. A clamp meter is used to measure the current drawn from each motor and hence the total flight time is estimated and a suitable battery is selected.

The experiment is conducted by going through the following steps:

1- Transmitting variable signals to the motor and measuring the obtained thrust by thrust sensor (KG scalar).
2- Slowly turning on the RC and increasing the thrust gradually up to certain fixed thrust bench marks.
3- 3-Recording the measured data on a table and choosing the optimum pulse width modulation (PWM) percentage that achieve the desired payload and time of flight due to drain current.

The experiment results are summarized in Table 1:

Table 1: The output data of the experimental

PWM %	Volt(V)	current(A)	RPM	weight(Kg)	Thrust
16.5	0.8	1.6	2710	0.12	1.176
33	1.6	3.5	3680	0.27	2.646
49.5	2.4	6	4620	o.44	4.312
66	3.2	9.8	5590	0.59	5.782
82.5	4	13	6520	0.7	6.86
99	4.8	16	7520	0.8	7.84

The above results reveal that the quadcopter operates at about PWM (50%), the total weight that can be carried by four motors is (4×0.44=1.76kg), the total weight without payload weight is 1.3kg, so the estimated weight of payload is about (0.43kg). The total drawn current at PWM (50%) is (4×6=24A), so the estimated time of flight with (5Ah) battery is $[(5 \times 60/24) = 12.5 \text{ min}]$. Motor propeller system can be obtained by finding the relationship between input volt and output thrust as shown in Figure 5:

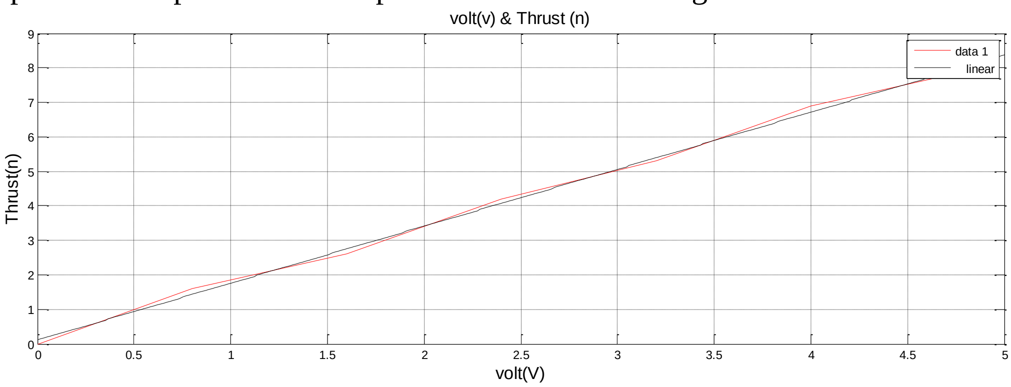

Figure 5 . The relation between input volt and output thrust

By using fitting curve, the relation between input volt and output thrust can be obtained as shown in equation (1):

$$F = 1.7 \times V + 0.01 \quad (9)$$

Where:

F… the output thrust force (N), v…the input volt (v).

The parameter (Kv) which indicates (rpm per volt) that is given in data sheet of the used motor (1200Kv) can proved by finding the slope between input volt and output rpm (slope= $(5590-4620)/(3.2-2.4)$ =1212), the obtained value is near to the given value.
Another experiment is built to obtain the motor dynamic model using system identification technique. The resulting input-output data is then used to obtain a model of the system such as a transfer function or a state-space model. By experiment that excites the system only in its linear range of behavior and collect the input/output data. Then using the data to estimate a plant model, and design a proposed controller for the obtained model [11] . A diagram of the experimental setup is shown in Figure 6:

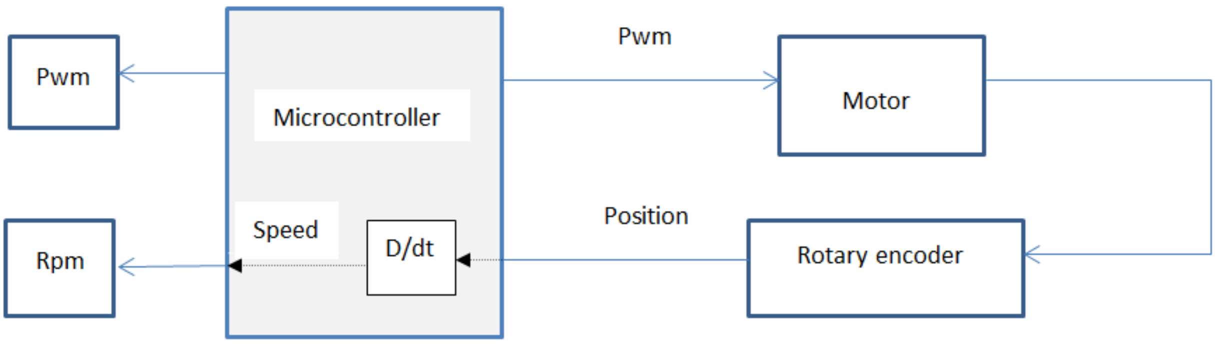

Figure 6 .The block diagram of the experimental setup

The experiment is conducted by going through the following steps:

1- A potentiometer is used to change input signal (pulse width modulation).
2- A rotary encoder to measure motor's speed corresponding to variation in input signal.
3- The measured input–output data are collected using a data acquisition ports in ATMEL ATMEGA 2560 microcontroller built in Arduino MEGA [12] developing board and interfaced with MATLAB.
4- Using system identification toolbox in MATLAB to obtain the transfer function of the used motor.

The test signals (PWM) and the corresponding speeds are shown in Figure 7. This data is divided into two parts for system estimation and results validation.

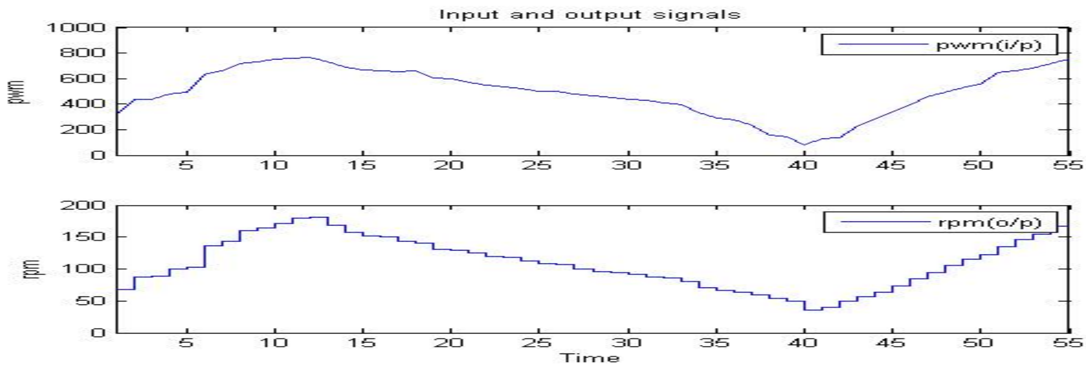

Figure 7 .Input signal (PWM) and output speed (RPM)

The fitting tests and fitting percentages of the obtained model and transfer function types are shown in the following Figure 8 and Table 2:

Figure 8 .Estimated models

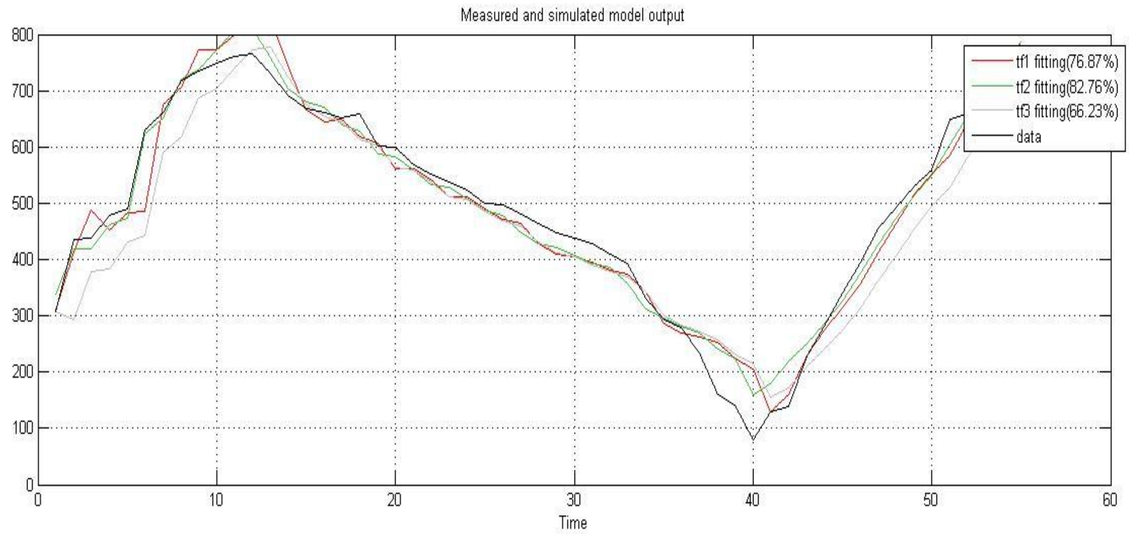

Table 2: Transfer function and fitting percentage

Transfer function	Fitting percentage (%)
$\frac{36.64}{S+8.526}$	66%
$\frac{-8.653e^{-6}s+0.0011}{s^2+0.0047+0.002}$	77%
$\frac{3.3928s^2-340.09s+39451}{s^3+74.38s^2+5589s+42107}$	83%

The transfer function corresponding to best fitting identified model is shown as follow:

$$\frac{\boldsymbol{output(rpm)}}{\boldsymbol{input(pwm)}}=\frac{3.3928s^2-340.09s+39451}{s^3+74.38s^2+5589s+42107}$$

The obtained transfer function will be used in building the MATLAB/SIMULINK model.

4. Model Design

The qudcopter model is developed in this research as mentioned earlier under MATLAB /Simulink platform and using the practically obtained motor transfer function as shown above.

The proposed model assumes the following [11] :

- The structure is supposed rigid.
- The center of gravity and the body fixed frame origin are assumed to coincide.
- The propellers are supposed rigid.
- Thrust and drag are proportional to the square of propeller's speed.

The model is designed based on the dynamic equations stated in section 2. The block diagram of this model is shown in Figure 9:

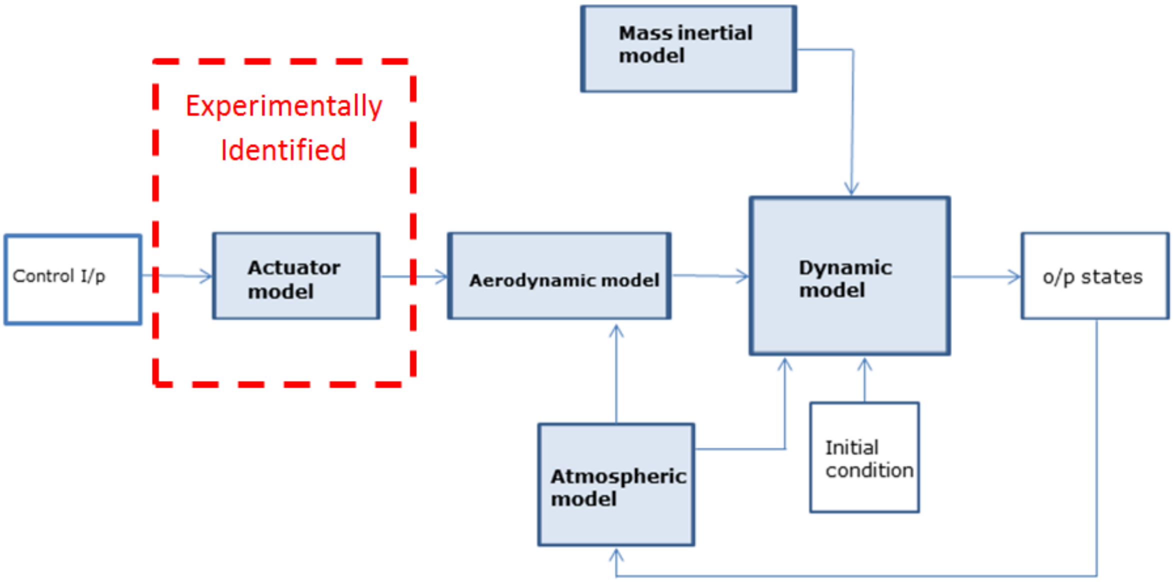

Figure 9 .Block diagram of quadcopter model

In order to achieve good control of the vehicle, a lot of feedback sensors have to be used. An inertial measurement unit (IMU) is used to provide the attitude angles (pitch (θ), roll (Ø) and yaw ()) which are necessary of vehicle stabilization and alignment. For simplicity, the model angles are taken directly as a unity feedback and no IMU dynamics are considered. For positioning, a global positioning system (GPS) receiver is assumed to continuously provide the quadcopter position. The altitude (z) is measured using barometer. Figure 10 illustrates how the feedback sensors are related to the model described above.

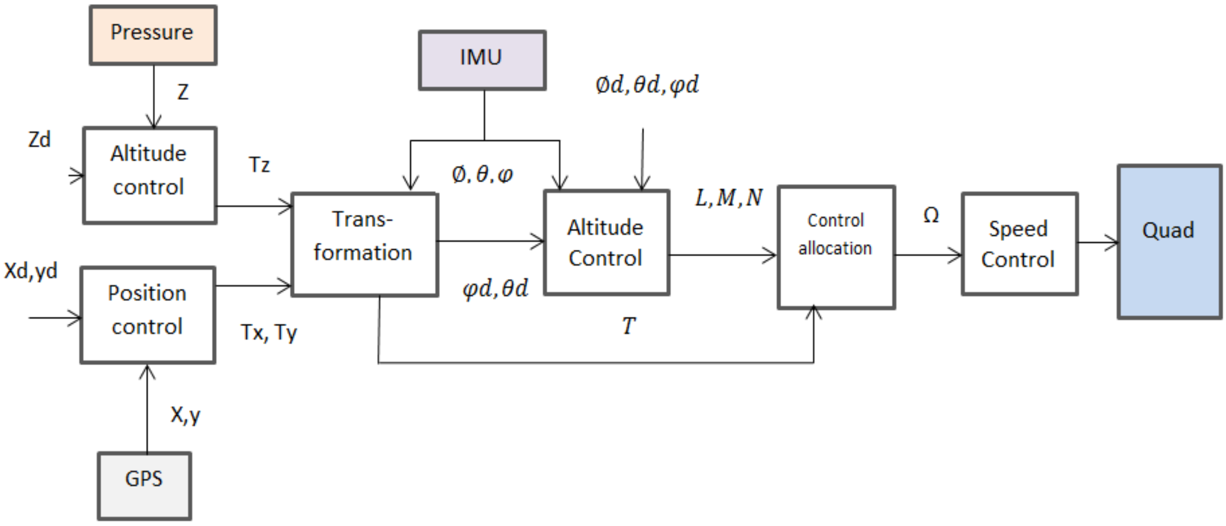

Figure 10 .full control chart of quadrotor

The obtained motor dynamic is used to show the performance of the model without controller and check model stability as shown in Figure 11:

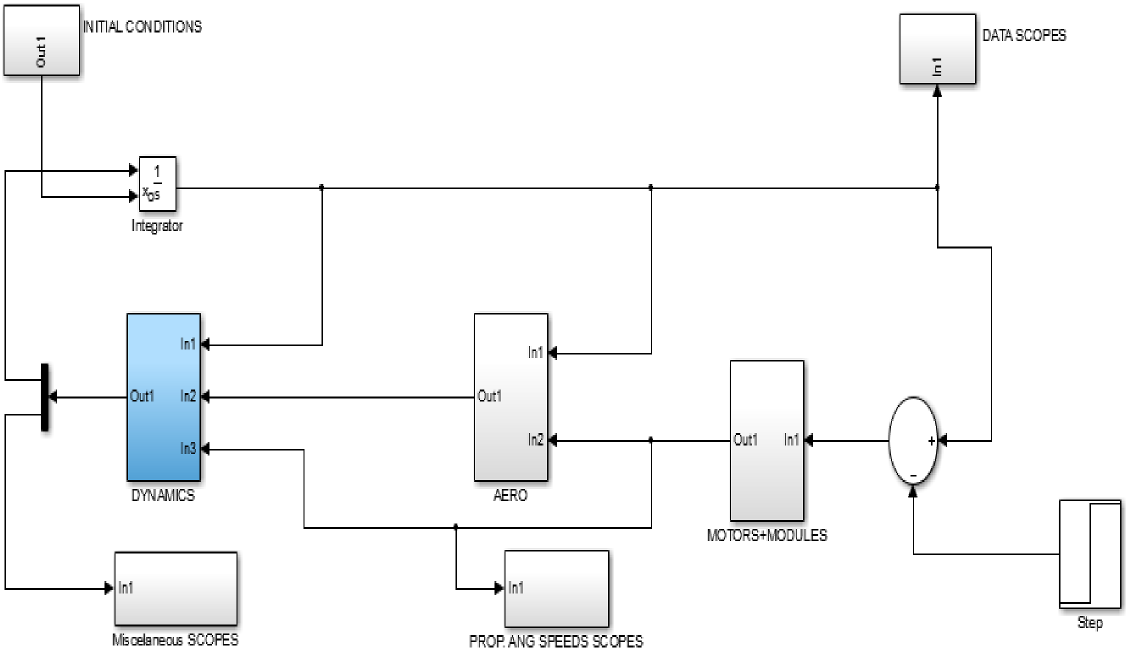

Figure 11 .Simulated model without control

The results of the obtained model for both roll, pitch, yaw and altitude channels as shown in Figure 12:

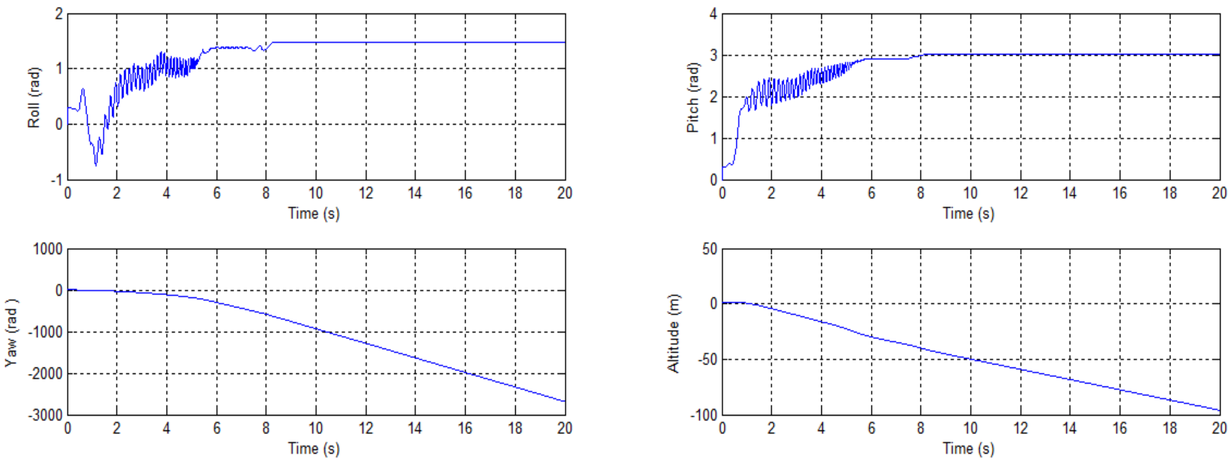

Figure 12. Model performance without control

The above figure illustrates that the model without controller has a bad performance and not stable in both control channels. So a controller will be designed to enhance the performance of the obtained model and achieve the design requirements.

5-Model linearization

The quadcopter dynamic model describes the roll, pitch and yaw rotations contains then, three terms which are the gyroscopic effect resulting from the rigid body rotation, the gyroscopic effect resulting from the propeller rotation coupled with the body rotation and finally the actuators action. The quadcopter dynamics must be linearized to provide an easy inverse model which can be implemented in the control algorithms. Since the motion of the quadcopter can be assumed close to hover condition, the two gyroscopic effects and cross coupling can be removed[11]. In this work, the controller is designed to stabilize attitude (Euler angles) and height.

According to these previous assumptions, equation (8) is reduced and simplified as follows:

$$\begin{cases} \ddot{\emptyset} = \dfrac{U_2}{I_{xx}} \\ \ddot{\theta} = \dfrac{U_3}{I_{yy}} \\ \ddot{\varphi} = \dfrac{U_4}{I_{zz}} \end{cases} \tag{10}$$

Where

$U_2 = \mathrm{l}(-\mathrm{T}_2 + \mathrm{T}_4)$ is the control thrust in roll direction.

$U_3 = \mathrm{l}(\mathrm{T}_1 - \mathrm{T}_3)$ is the control thrust in pitch direction.

$U_4 = (-1)^{\mathrm{i}} \sum_{\mathrm{i}=1}^{4} \mathrm{Q}_{\mathrm{i}}$ is the control thrust in yaw direction.

This simplified model in s domain with motor dynamic for roll channel is shown in Figure 13 :

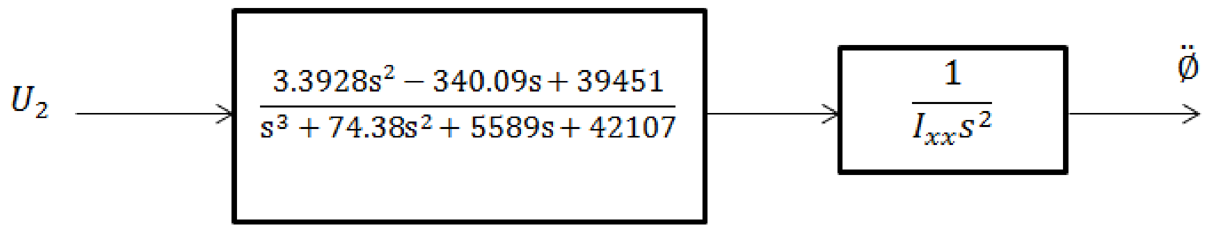

Figure 13. Simplified model in roll channel

This simplified model will be used in attitude control design with replace I_{xx} in roll channel with I_{yy} in pitch channel and I_{zz} in yaw channel.

6- Controllers design:

As a classical control – especially PID control – still play the basic and most reliable and applicable role among different control solutions. PID technique represents the basics of control, PID is often chosen because of its Simple structure, Good performance and Tuning even without a specific model of the controlled system [13]. To design PID controller, the model has been linearized around the hover situation. Hence, the gyroscopic effects haven't been taken into account in the controller design. The closed loop model has been simulated on Simulink with the full order nonlinear model. The combination of the proportional, integral, and derivative actions can be done in different ways. In the so-called ideal or non-interacting form, the PID controller is described by the following transfer function:

$$C(s) = K_p(1 + \frac{1}{T_i S} + T_d S) \quad (11)$$

$$u(s) = K_p[r(s) - y(s) + \frac{1}{sT_i}(r(s) - y(s) + \frac{sT_d}{1+\frac{sT_d}{N}}(r(s) - y(s))] \quad (12)$$

Where r(s), y(s) and u(s) are the Laplace transforms of the reference, process output and control signal respectively. K_p is the where K_p is the proportional gain, K_p / T_i is the integral gain (sometimes denoted as K_i), K_p T_d is the derivative gain (sometimes denoted as K_d), and finally N is the ratio between T_d and the time constant of an additional pole introduced to assure the properness of the controller.

PID controllers are usually tuned using hand-tuning or Ziegler-Nichols methods [13]. The general effects of control parameters are summarized in Table 3. The Control System Toolbox offers a variety of functions that allow us to examine the system's characteristics[11].

Table 3: Effect of PID gains on the response

Operation	Rise Time	Overshoot	Stability
K_p	Faster	Increases	Decreases
K_i	Slower	Decreases	Increases
K_d	Faster	Increases	Decreases

Considering a closed loop system shown in Figure 14, and using PID Tuner in MATLAB to design a SISO PID controller. The design requirements are attitude stabilization within 4 seconds and achieve about (2.8-3m) height with minimum overshoot.

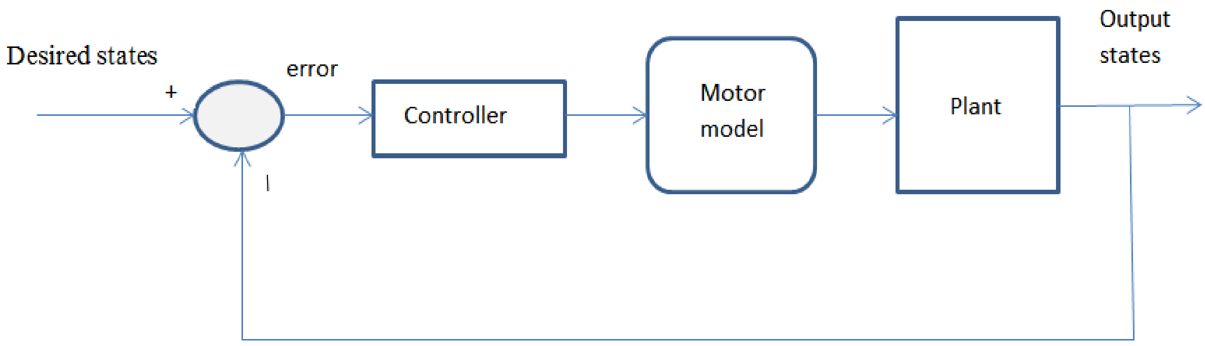

Figure 14 .Block diagram of closed loop system

The attitude control loop is responsible for stabilization of movement of the quadrotor, so that the quadrotor can also hover in the air. So the focus was mainly on attitude control as it is the heart of the control problem as shown in Figure 15:

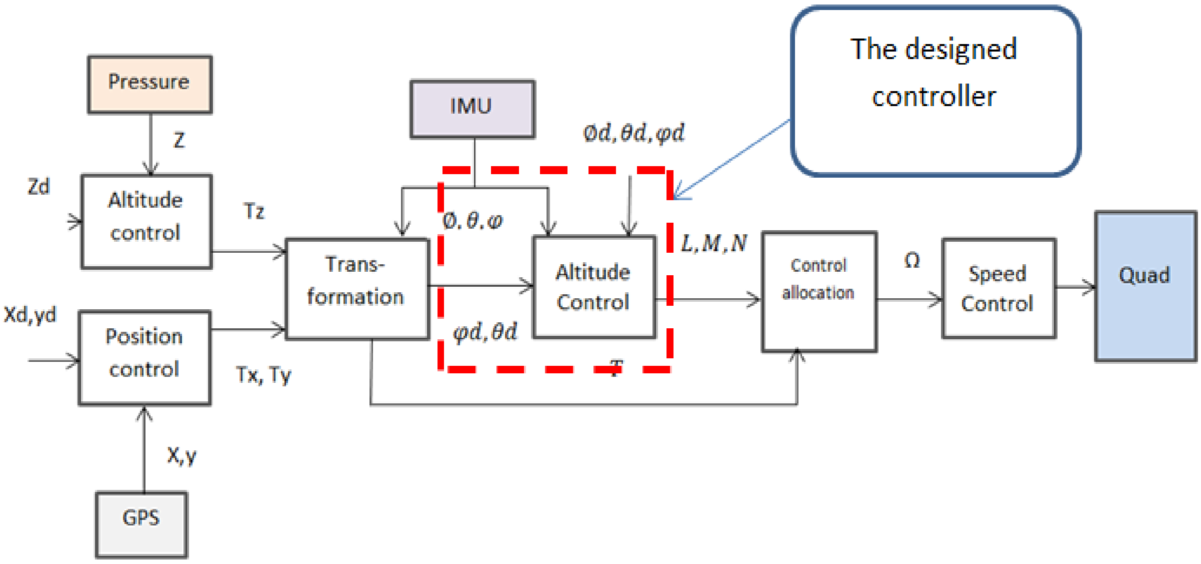

Figure 15. The model with the designed controller

The effect of designed PID controller, which based on achieving the design requirements, is shown in Figure 16:

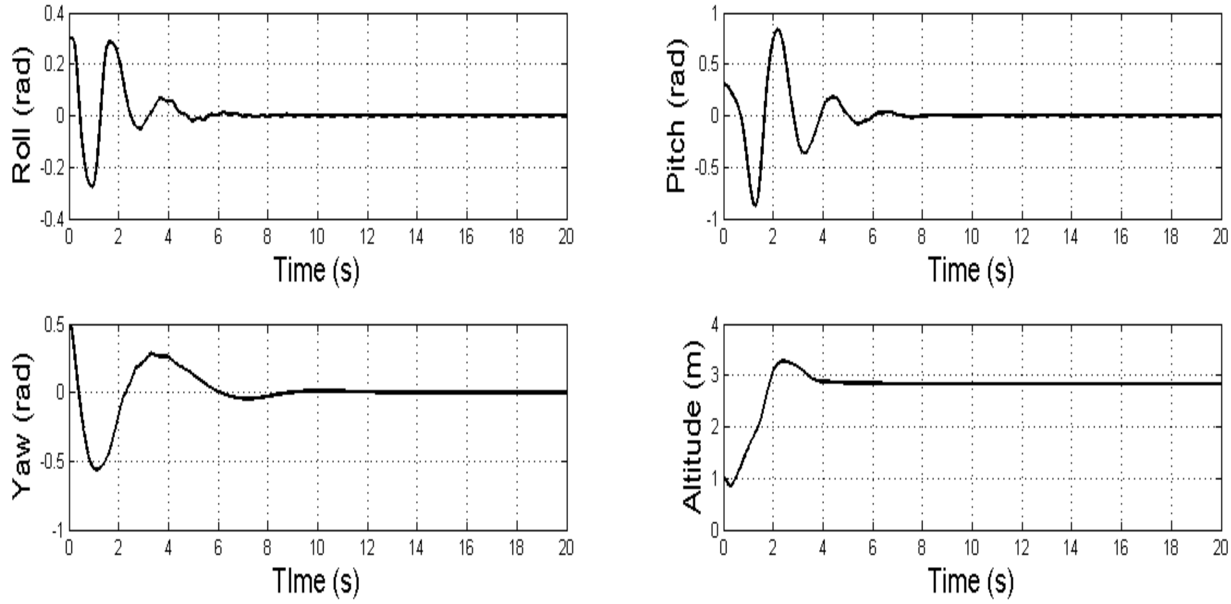

Figure 16 .The responses of PID control in roll, pitch, yaw and altitude

As shown in Figure 16 , the designed PID controller has a settling time about 4.5 seconds in both roll, pitch and altitude channels also, it has a high overshoot in each control channel.so the designed PID controller can't achieve the design requirements .Another controller is designed to enhance the performance of the obtained results .

A modified PI-D controller is designed to avoid set-point kick phenomenon so the derivative action only in the feedback path so that differentiation occurs only on the feedback signal[13].as shown in Figure 17:

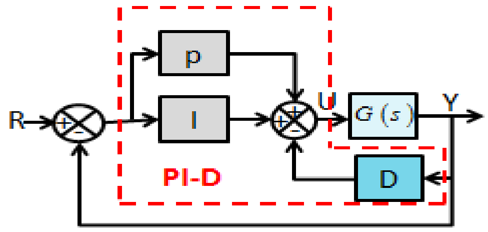

Figure 17 .Block diagram of modified PI-D controller

The responses of modified PI-D control in roll, pitch, yaw and altitude are shown in the next figure:

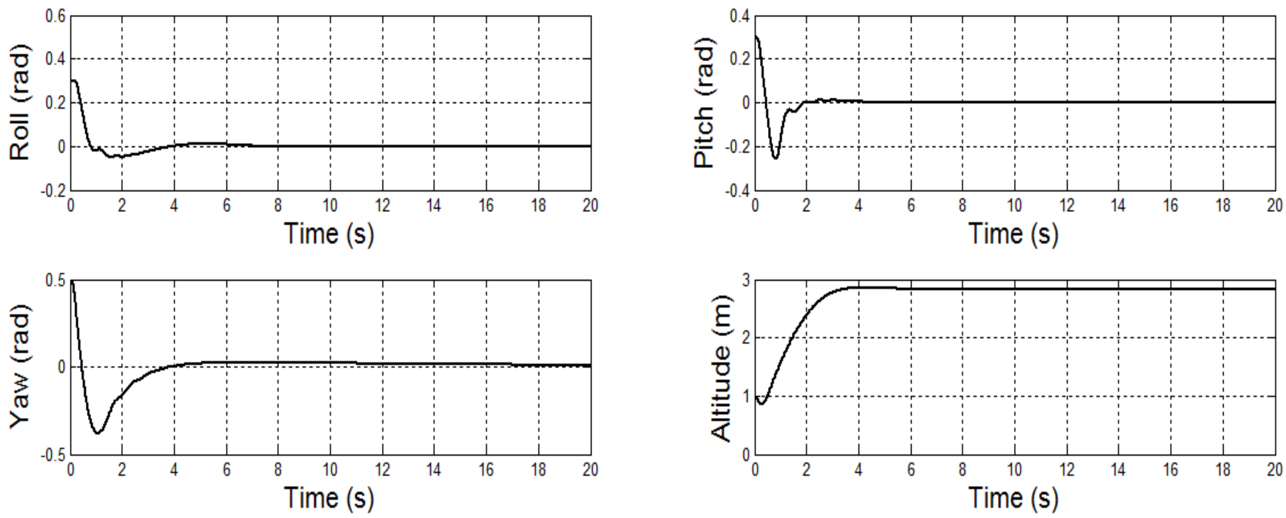

Figure 18 .The responses of modified PI-D control in roll, pitch, yaw and altitude

As shown from Figure 18, the modified PI-D controller can achieve the design requirements as it make quadcopter attitude stabilization itself within 4 seconds with minimum overshoot and minimum control effort. A comparison the between results in each channel will be shown in the next tables:

Table 4: Roll Response

	Settling time (s)	Max Overshoot (rad)	Steady state error (rad)
Designed PID	5	0.3	0
Designed PI-D	4	0.1	0

Table 5: Pitch Response

	Settling time (s)	Max Overshoot (rad)	Steady state error (rad)
Designed PID	5.5	0. 7	0
Designed PI-D	2	0. 2	0

Table 6:Yaw response

	Settling time (s)	Max Overshoot (rad)	Steady state error (rad)
Designed PID	6	0.48	0
Designed PI-D	4	0.27	0

Table 7: Altitude Response

	Settling time (s)	Rise time (s)	Max Overshoot (rad)
Designed PID	4.2	2.4	0. 3
Designed PI-D	4	2.3	0

As shown from figures and tables the designed PI-D controller gives a very stable and good performance in both attitude and altitude control than PID controller, in settling time ,steady state error, max overshoot and the response of the PI-D system is faster than the other controller.

7- Conclusion:

The paper presented the modeling of the intended system concerning the reference frames, coordinates' transformations and equations of motion. This model is built in the form of modules assigned to each process within the Quadcopter system. Then, it is programmed within MATLAB environment. The transfer function of (BLDCM) is obtained using system identification techniques via an experimental and consequently a classical controller is designed to enhance its performance. To make the simulation model near to the real case the obtained transfer function must be used in the simulation model. Different types of controllers are designed such that PID and the modified PI-D but the PI-D controller shows better in performance in both attitude and altitude than PID, in settling time, steady state error, max overshoot and the response of the PI-D system is faster than the other controller. So, the PI-D controller is more than enough for this platform. Also, maximum payload, time of flight and motor propeller system modeling can be estimated.

References:

1. Kundak, N. and B. Mettler. Experimental framework for evaluating autonomous guidance and control algorithms for agile aerial vehicles. in Proceedings of the European control conference. 2007.
2. Valenti, M., et al. Indoor multi-vehicle flight testbed for fault detection, isolation, and recovery. in Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit, Keystone, CO. 2006.
3. Waslander, S.L., et al. Multi-agent quadrotor testbed control design: Integral sliding mode vs. reinforcement learning. in Intelligent Robots and Systems, 2005.(IROS 2005). 2005 IEEE/RSJ International Conference on. 2005. IEEE.
4. Bouabdallah, S. and R. Siegwart. Backstepping and sliding-mode techniques applied to an indoor micro quadrotor. in Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on. 2005. IEEE.
5. Frazzoli, E., M.A. Dahleh, and E. Feron. Trajectory tracking control design for autonomous helicopters using a backstepping algorithm. in American Control Conference, 2000. Proceedings of the 2000. 2000. IEEE.
6. Mahony, R. and T. Hamel, Robust trajectory tracking for a scale model autonomous helicopter. International Journal of Robust and Nonlinear Control, 2004. 14(12): p. 1035-1059.
7. DZUL, A., T. HAMEL, and R. LOZANO, Modélisation et commande non linéaire pour un hélicoptère birotor coaxial. Journal européen des systèmes automatisés, 2003. 37(10): p. 1277-1295.
8. Stevens, B.L. and F.L. Lewis, Aircraft control and simulation. 2003: John Wiley & Sons.
9. Zipfel, P.H., Modeling and simulation of aerospace vehicle dynamics. 2000: Aiaa.
10. Bouabdallah, S., P. Murrieri, and R. Siegwart. Design and control of an indoor micro quadrotor. in Robotics and Automation, 2004. Proceedings. ICRA'04. 2004 IEEE International Conference on. 2004. IEEE.
11. Martinez, V., Modelling of the flight dynamics of a quadrotor helicopter. A MSc Thesis in Cranfield University, 2007.
12. Schmidt, M., Arduino. 2011: Pragmatic Bookshelf.
13. Ogata, K., Modern Control Engineering. Fifth ed. 2009.