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Military Technical College, Kobry Elkobbah, Cairo, Egypt
Tel: +(202) 24025292 –24036138, Fax: +(202) 22621908

Friction Induced Vibrations of Moving Bodies
Part I (Mass on Horizontal Plane)

Rabab H. Mohamed*, A. El-Butch†, M. I. Ghobrial‡ and E. Elhalawany§

Abstract: Friction is one of the most important problems in engineering systems while vibration is the dynamic motion of objects and the forces associated with it. Modeling of friction-induced vibrations in mechanical and moving bodies requires an accurate description of friction. Unfortunately, there is no universally accepted friction model or theory to cover general friction; on the other hand, the resultant vibrations often exhibit various nonlinear, transient, and unsteady features.

Small changes in interfacial parameters could have significant effect on the resultant vibrations and dynamic forces associated with those vibrations. In the present work a mathematical model of friction induced vibrations between a moving mass on a fixed horizontal plane has been proposed and the effect of the different operating conditions has been studied.

Keywords: Friction induced vibration, stick-slip

Nomenclature

k	spring constant
v	driving velocity
x	displacement of the slider
$\dot{x}$	acceleration of the slider
$\ddot{x}$	acceleration of the slider
F_{fr}	friction force between the mass and the base F_s spring force
F_{sc}	critical spring force
μ_{static}	static friction coefficient
t_{stick}	stick time
ω_n	natural frequency of the system
m	mass of the slider
g	gravitational acceleration (9.81 m/s^2)
$\mu_{dynamic}$	dynamic friction coefficient

* Research student, Helwan Co. for Engineering Industries, Egypt, rabaqbhosney@yahoo.com
† Prof., Chairman of mechanical design Dept, Faculty of engineering, Mataria, Helwan University, Egypt, elbutch@aucegypt.edu
‡ Prof., Chairman of mechanical engineering, Dept, Faculty of engineering, Helwan University, Egypt, Morcousibrahim@yahoo.com
§ Director, National Organization for Military Production, Egypt, emad-elhalawany@gmail.com

Introduction

When the surfaces of two objects are placed in contact and allowed to slide, there is a resistance to the motion. This resistance is the friction that is experienced whenever one solid body moves over another. Friction induced vibrations occur in many systems in our daily life, examples for friction contacts performing stick-slip vibrations are squeaking door frame joints, and many machine parts moving against each other. In all cases, it is important to control the stick-slip behavior in order to reduce its adverse effects. Lubricant such as oil or graphite can be to prevent their undesired oscillations.

The phenomenon known as 'stick-slip' in systems with contacts has been observed and studied for many years. Even today it manifests itself in various machines and systems, such as automotive and aircraft brakes, [1,2], robotic joints,[3,4] and machine tools, [5,6]. Stick-slip is the phenomenon of intermittent motion induced by changing friction force. It appears in applications with low relative sliding velocity. In machining, it can occur, for example, in machine tool slide ways, or during extrusion, wire drawing and deep drawing processes. It can result in low accuracy in machining, bad finish of machined surfaces and oscillating stresses that can lead to fatigue failure of machine tools.

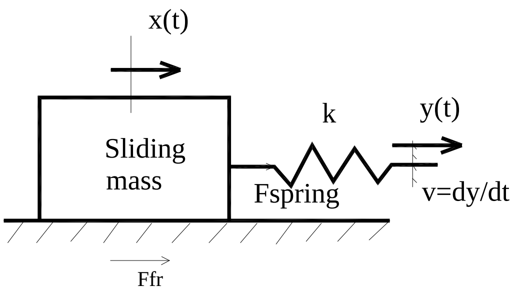
Many engineering systems with sliding friction have noticeable system vibrations associated with the interface friction process, in both normal and sliding directions, due to finite sliding mass and limited elastic properties. Many papers on the stick slip motion have been presented, [7:12]. Stick-slip friction also exists in machine elements involving relative motion, such as gears, pulleys, bearings and DC motors.

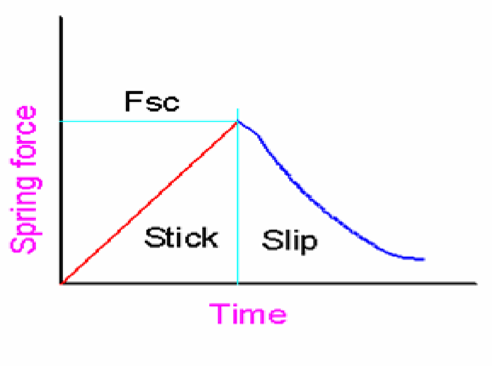
In order to clarify the relation between the stick-slip motion and the frictional characteristics of the machine tool slide way from a practical point of view, more investigations are desired. The stick–slip is characterized by a sawtooth pattern in the friction vs time curve, it could take place in a random, chaotic fashion, or repetitively. The stick–slip is usually associated with vibration that could take place over a wide range of oscillation frequencies.

In the present work, it is intended to present and study the mathematical model of a mass moving on a fixed base.

Analytical Model

The dynamic contact model is shown in Fig. 1 (a). The effect of different design and operation parameters on the motion characteristics during the slip period is investigated.





(a) **(b)**

Fig. 1 System model and stick slip motion

The dynamic contact model consists of a rigid body of mass (m) rests on a flat rough surface, the friction force between the mass and the base is F_{fr} .

The body is moving with constant velocity (v = dy/dt), and it is driven in x direction by a spring through its free end. For small pull velocities, the system exhibits stick slip motion which could be analyzed by considering two successive periods of stick and slip.

During Stick period mass is stacked to surface as static friction is higher than dynamic friction. The spring is stretched at a constant rate (v = dy/dt) until a critical spring force (F_{sc})is reached, Fig. 1 (b).

During the stick time the spring acts on the mass with a force (F_s). This force is defined by the following equation (Eq.1), [13].

$$F_s(t) = k(y - x) = k(vt - x) \tag{1}$$

where x is displacement of the slider and v is driving velocity

The motion starts at which the critical spring force is reached as given in Eq.2.

$$F_s = F_{sc} = mg\ \mu_{static} \tag{2}$$

where; F_s is spring force and μ_{static} is static friction coefficient.

The stick time (t_{stick})can be calculated from Eq.3

$$t_{stick} = \frac{mg\ \mu_{static}}{kv} = \frac{g\mu_{static}}{v\omega_n^{\ 2}} \tag{3}$$

where; $\omega_n = \sqrt{\dfrac{k}{m}}$ is the natural frequency of the system, [14].

where; k is spring constant or stiffness
m is mass of the slider

When the frictional resisting force drops from static to dynamic, the mass starts to move with an instantaneous acceleration, at $t = t_{stick} + \Delta t$ with $\Delta t \to 0$.

The equilibrium equation that represents the above can be written as:

$$(\mu_{static} - \mu_{dynamic})mg = m\ddot{x}\ \Big|_{t_{static}} = 0 \tag{4}$$

where; $\mu_{dynamic}$ is dynamic friction coefficient

From $t = t_{static}$ and during the slip period, the spring pulling force is less than Fsc until the start of the next stick period where the spring starts to be stretched again to Fsc value.

During the slip period, the problem is treated as an initial value problem (IVP) with the following initial conditions:

$$x = 0, \quad \dot{x} = \frac{dx}{dt} = 0, \quad and \quad \ddot{x} = (\mu_{static} - \mu_{dynamic})g \tag{5}$$

The differential equation of motion is:

$$m\ddot{x} = F_S - F_{fr} = k(y - x) - mg\mu_{static}\, e^{-\dot{x}} \tag{6}$$

$$\ddot{x} = \frac{k}{m}(\nu t - x) - g\mu_{static} e^{-\dot{x}} \tag{7}$$

where; $\ddot{x}$ is acceleration of the slider.

This equation represents the case of boundary lubricated contacts, (exponential form).

Equation (7) is a second order differential equation. It is solved using MATLAB software to investigate the effects of the different design and operational parameters on the motion characteristics during the slip period.

Results and Discussion

Figure 2 shows the effect of static friction coefficient during the slip period on the displacement, velocity and acceleration. The increase in static coefficient of friction with respect to dynamic one provides higher resistance at the start of motion corresponding to a higher spring force (higher extension before slipping), which tends to pull the mass a relatively a longer distances as shown in the first relation of figure 2.

Figure 3 shows the effect of the drive speed during the slip period. The increase in the drive speeds tends to increase the slip time and consequently the displacement and maximum sliding speed.

Figure 4 shows the motion characteristics during the slip period for different springs. A stiff spring, (high value of K), shortens the displacement and lowers the maximum sliding velocity. The starting value of acceleration depends mainly on (μ_{static} - $\mu_{dynamic}$). So it can be concluded that, the stiffer springs provide a better stick-slip free motion.

Figure 5 shows the motion characteristics during the slip period for different sliding masses. For big sliding mass, the slip time is relatively high and both the velocity and the acceleration of the sliding mass increase.

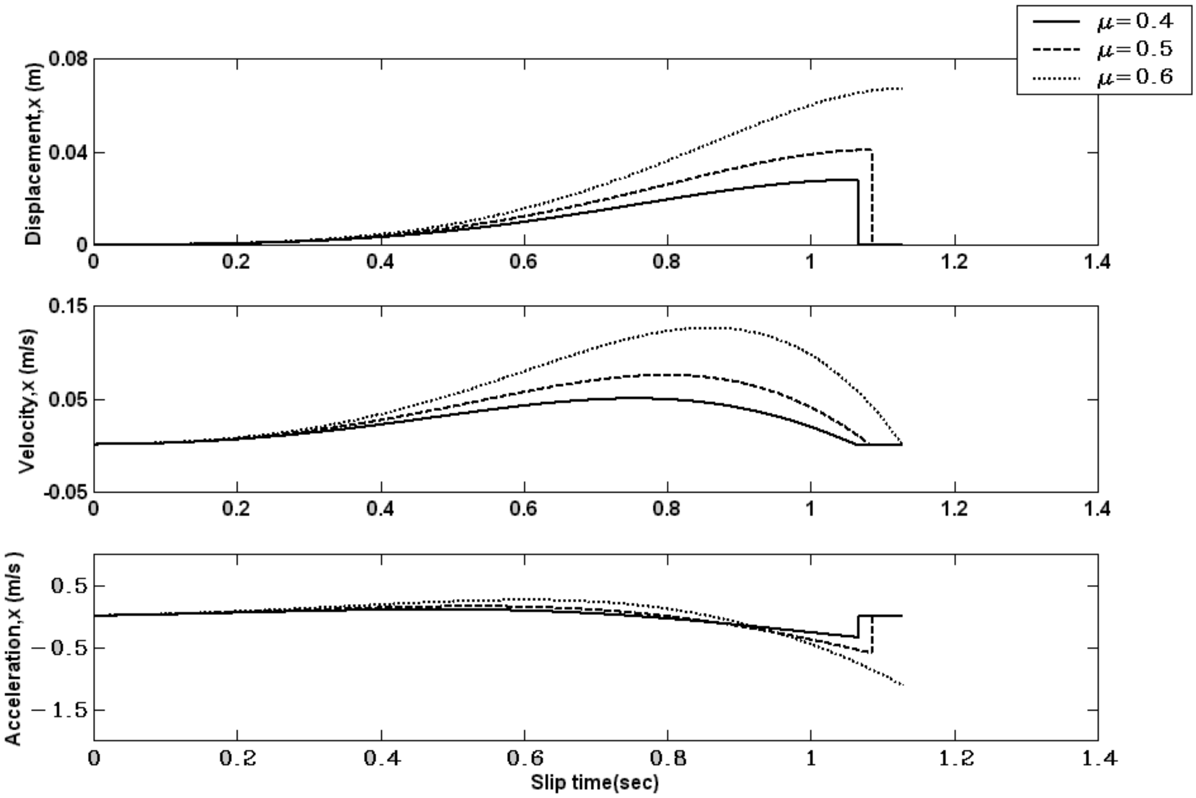


Fig.2 Effect of static friction coefficient (μ_{static}) on motion characteristics during slip period (v=0.01m/s, m=50kg, k=1000N/m, and μ_{dynamic} =0.25)

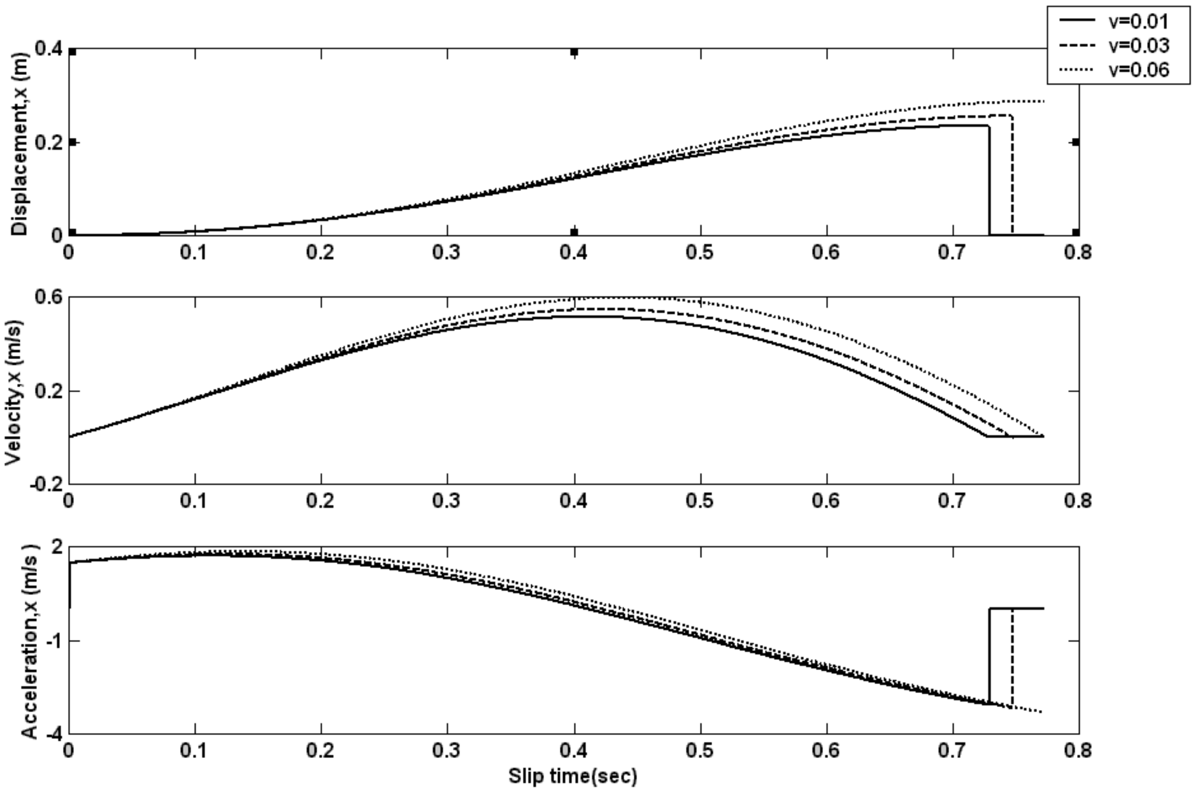


Fig.3 Effect of drive constant speed on motion characteristics during slip period (m=50kg, k=1000N/m, μ_{static} =0.4, and μ_{dynamic} =0.25)

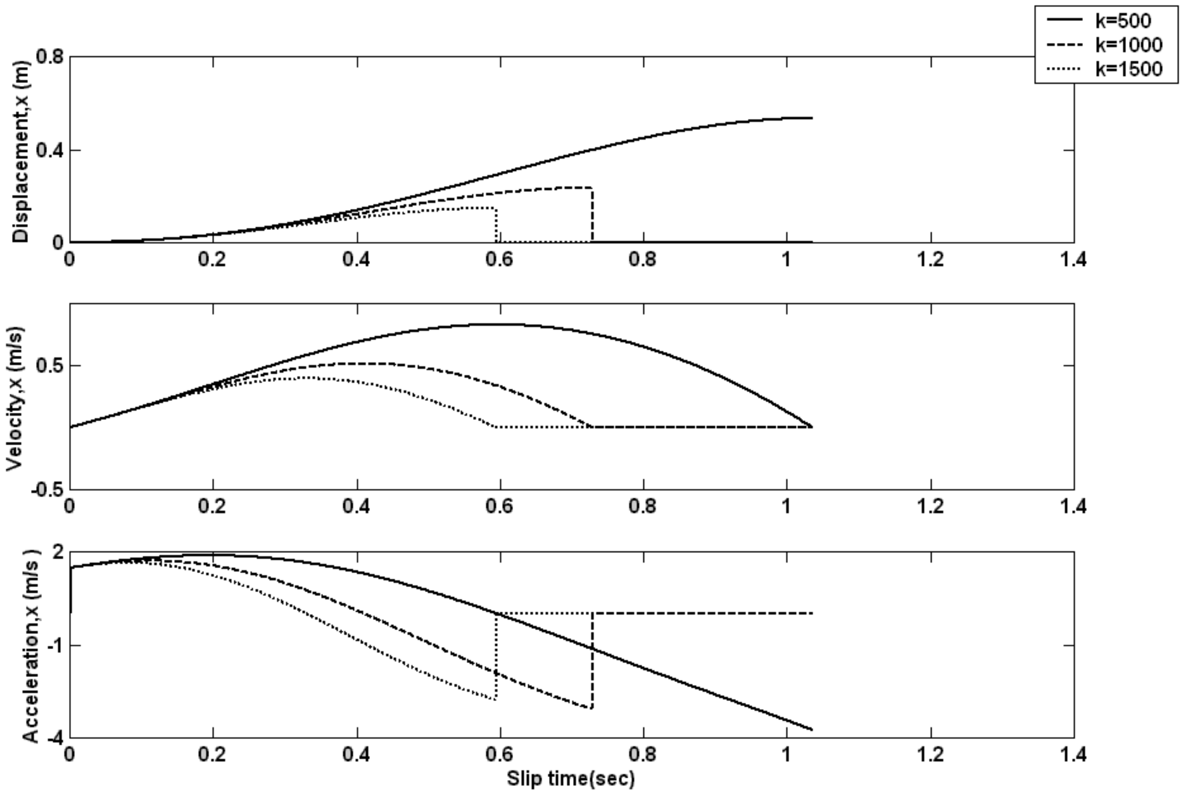


Fig.4 Motion characteristics during slip period for different springs
(m=50kg, v=0.01m/s, μ_{static} =0.4, and μ_{dynamic} =0.25)

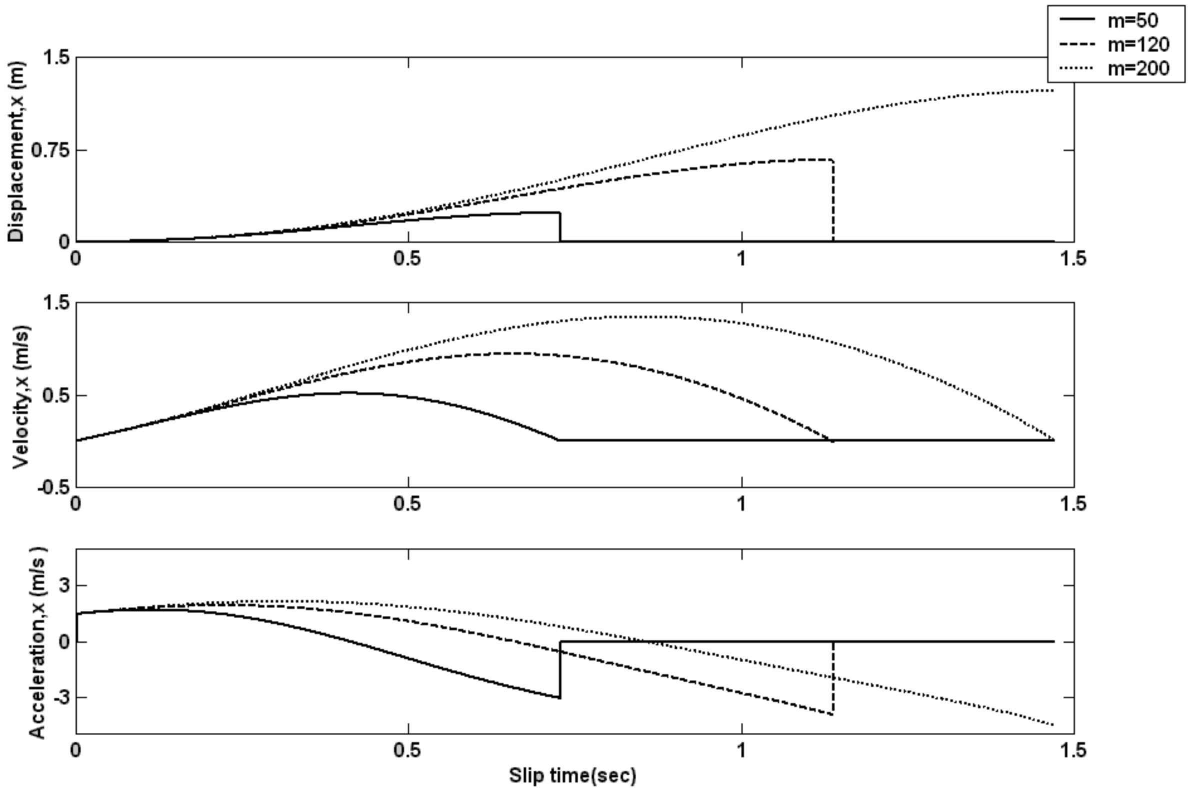


Fig. 5 Motion characteristics during slip period for different sliding masses
(k=1000N/m, v=0.01m/s, μ_{static} =0.4, and μ_{dynamic} =0.25)

Figures 6 and 7 show the effect of the spring constant (k) on the slip distance for different static coefficients of friction (μ_{static}) and drive speeds (v). It is clearly shown that stiffer springs (high value of k) provide a better stick-slip free motion. The increase in static coefficient of friction and drive speed provides a relatively longer mass displacement.

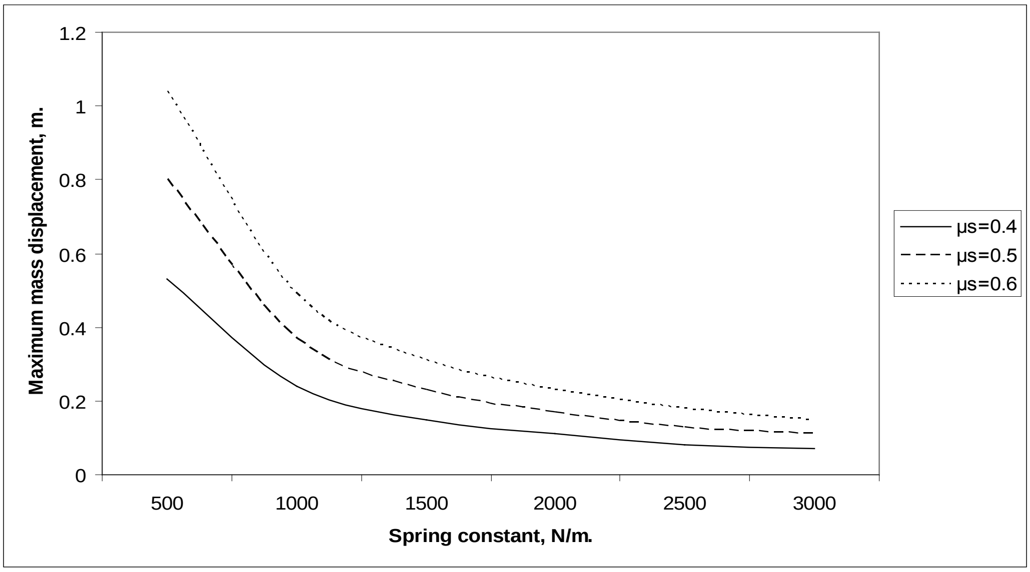


Fig. 6 Relation between spring constant and maximum slip displacement for different static coefficients of friction
(v=0.01, m=50 Kg, μ_{dynamic} =0.25)

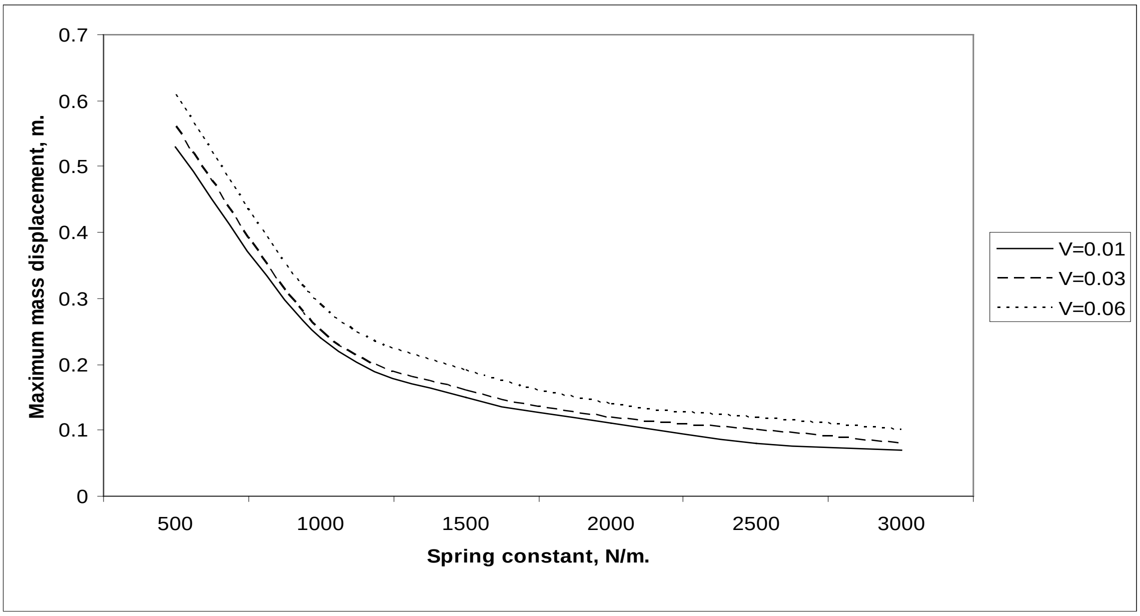


Fig.7 Relation between spring constant and maximum slip displacement for different drive speeds
(m=50 Kg, μ_{dynamic} μ_d=0.25, μ_{static} =0.4)

Figures 8 and 9 summarize the effects of the spring constant on the slip velocity for different static coefficients of friction and drive speeds. For softer springs (low value of k) and high values of static coefficient of friction and drive speed severe jerks are obtained. This may cause severe rubbing between the contacting surfaces leading to higher rates of wear and consequently, shorter service life for the contacting surfaces are expected.

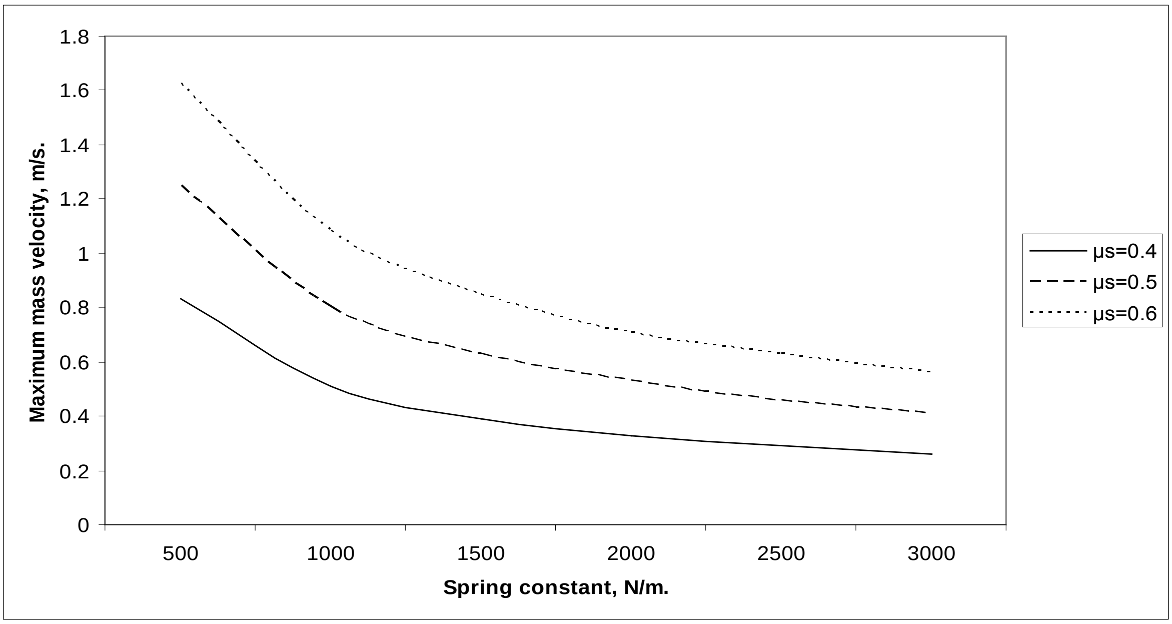


Fig.8 Relation between spring constant and maximum slip velocity for different static friction coefficients (v=0.01, m=50 Kg, μ_{dynamic} =0.25)

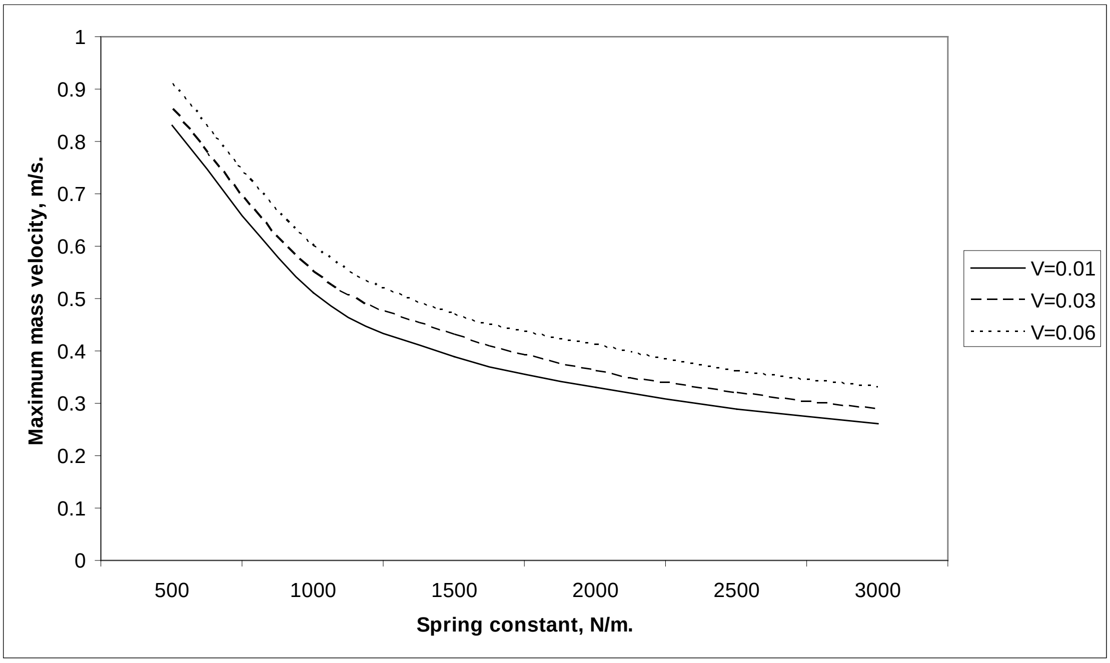


Fig.9 Relation between spring constant and maximum slip velocity for different drive speeds (m=50 Kg, μ_{dynamic} =0.25, μ_{static} =0.4)

Figures 10 and 11, plots for maximum slip acceleration with different static coefficients of friction and the drive speeds are shown. A stiff connection leads to decrease in the mass acceleration.

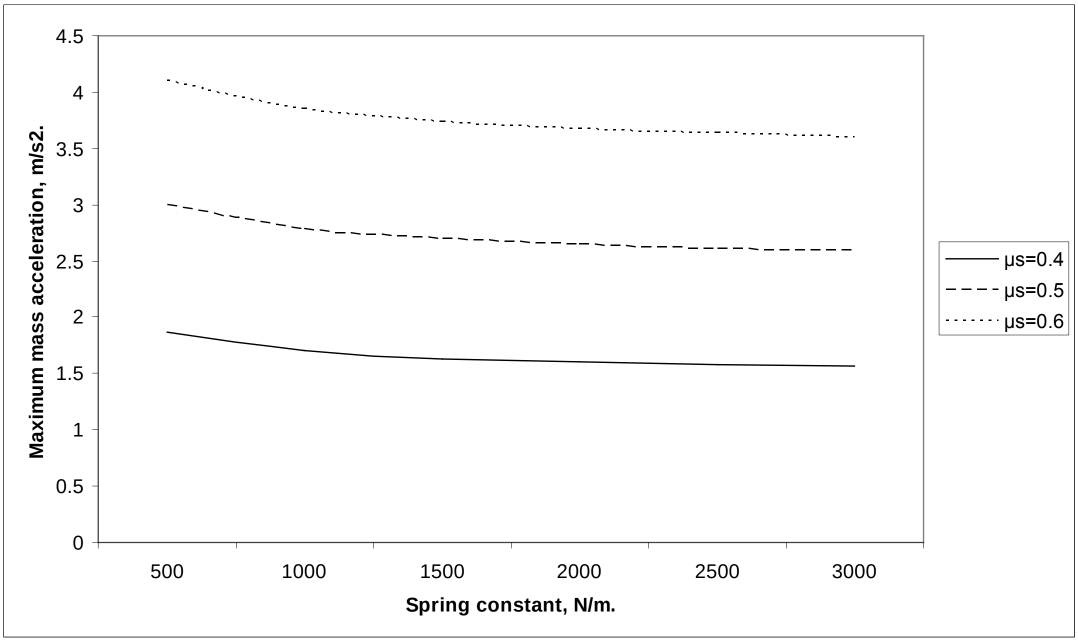


Fig.10 Relation between spring constant and maximum slip acceleration for different static friction coefficients
(v=0.01, k=1000 N/m, m=50 Kg, μ_{dynamic} =0.25)

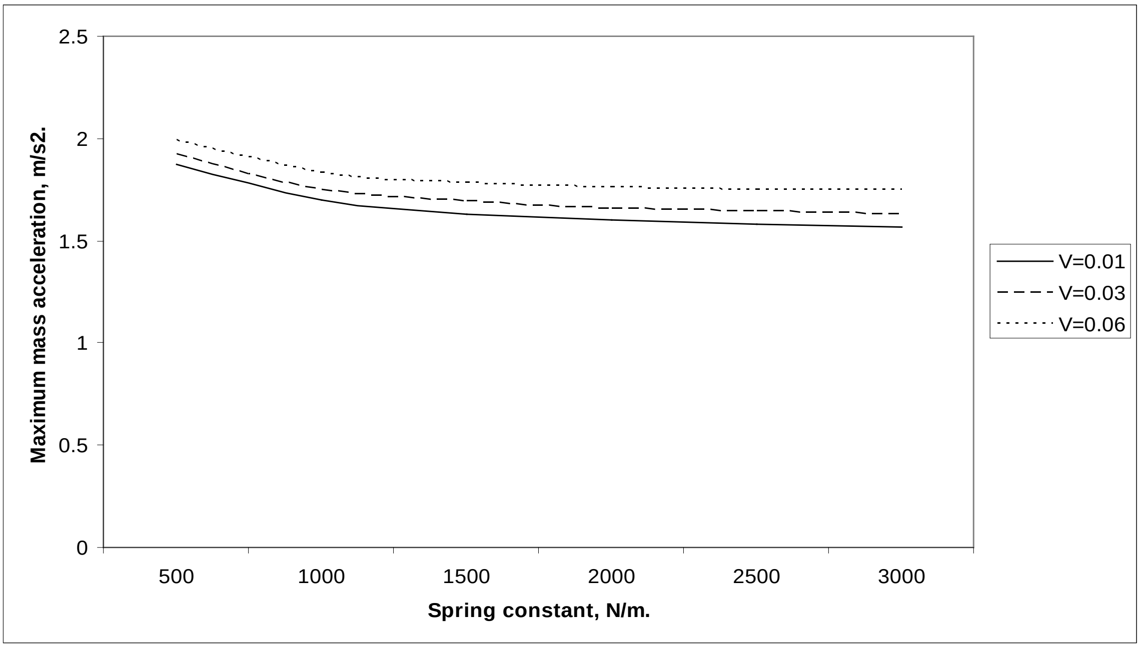


Fig.11 Relation between spring constant and maximum slip acceleration for different drive speeds
(m=50 Kg, μ_{dynamic} =0.25, μ_{static} =0.4)

Figures 12 and 13 show the effects of the sliding mass (m) on the slip distance for different static coefficients of friction and drive speeds. It is shown that low value of sliding mass m provides a better stick- slip free motion. The increase in static coefficient of friction and drive speeds provides a relatively longer displacement.

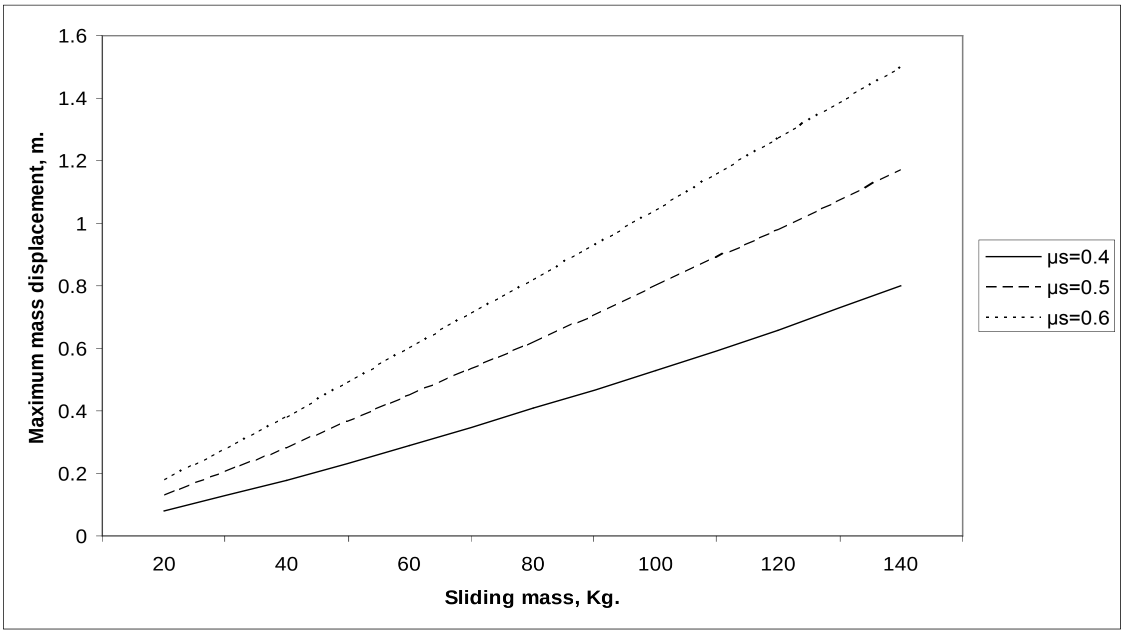


Fig.12 Relation between sliding mass and maximum slip displacement for different static friction coefficients
(v=0.01, k=1000 N/m, μ_{dynamic}=0.25)

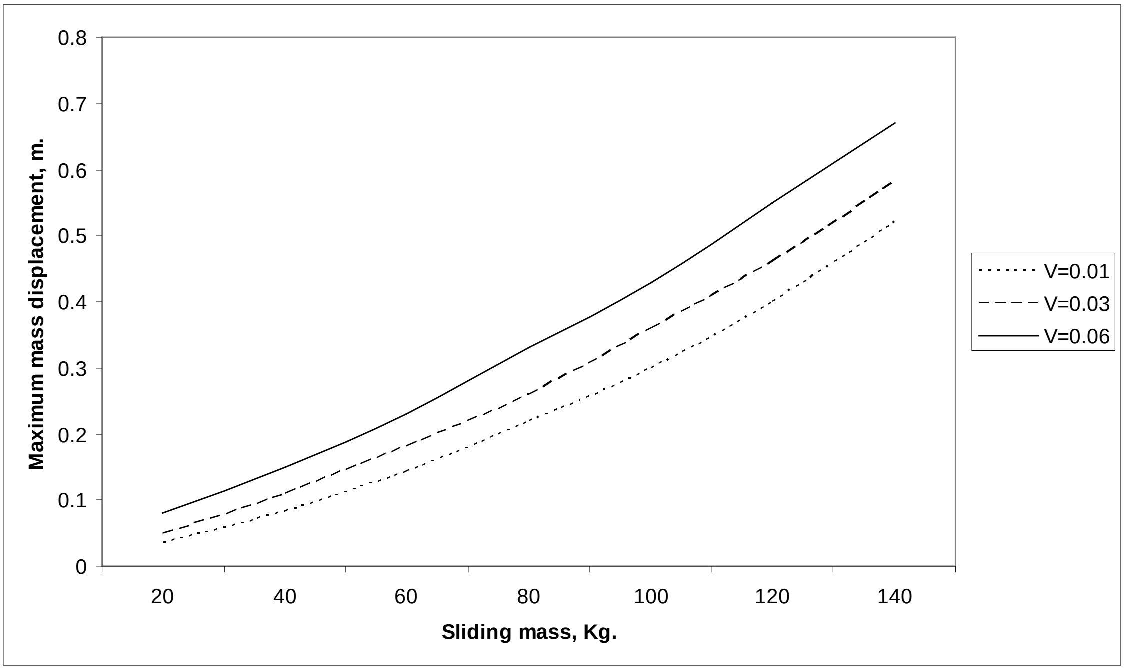


Fig.13 Relation between sliding and maximum slip displacement for different drive speeds (k=1000 N/m, μ_{dynamic} =0.25, μ_{static} =0.4)

Figures 14 and 15 show the relation between sliding mass and maximum slip velocity for different static coefficients of friction and drive speeds. For high value of sliding mass and high value of static coefficient of friction and drive speed, high values of maximum mass velocity are reached which cause sever rubbing action between the sliding masses leading to higher rates of wear and consequently, shorter service life for the contacting sliding surfaces are expected.

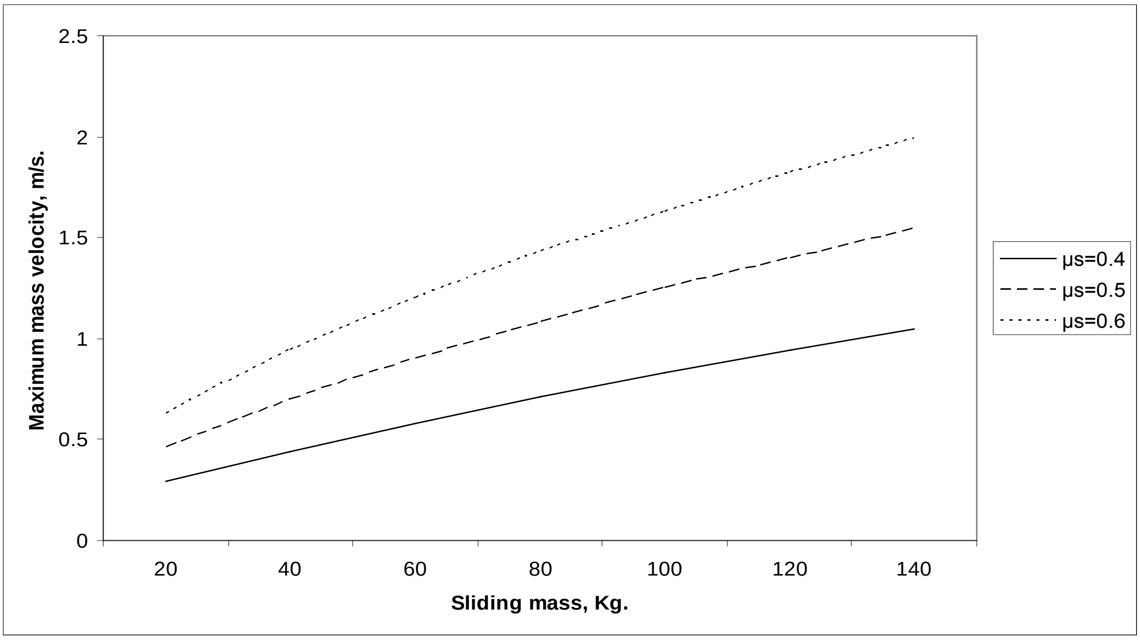


Fig.14 Relation between sliding mass and maximum slip velocity for different static friction coefficients
(v=0.01, k=1000 N/m, μ_{dynamic} =0.25)

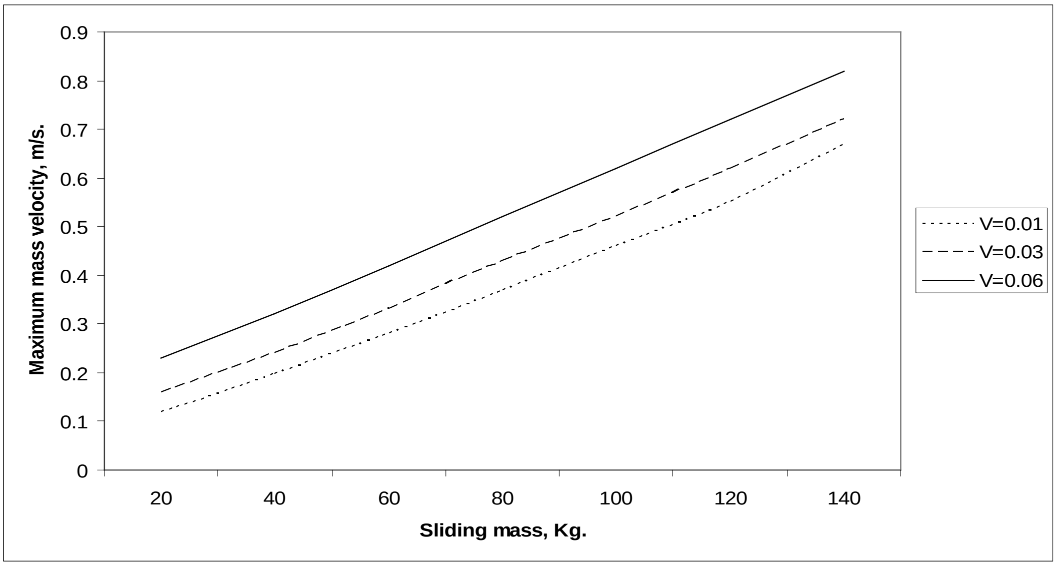


Fig.15 Relation between sliding mass and maximum slip velocity for different drive speeds
(k=1000 N/m, μ_{dynamic} =0.25, μ_{static} =0.4)

Figures 16 and 17 summarize the effects of the sliding mass on the slip acceleration for different static coefficients of friction and drive speeds. High sliding mass leads to an increase in the mass acceleration at high value of static coefficient of friction and drive speed.

The present work has shown good agreement with the previously published work, [13] considering dry friction. It gives the same trend concerning the effect of the operating parameter on the mass displacement, velocity and acceleration.

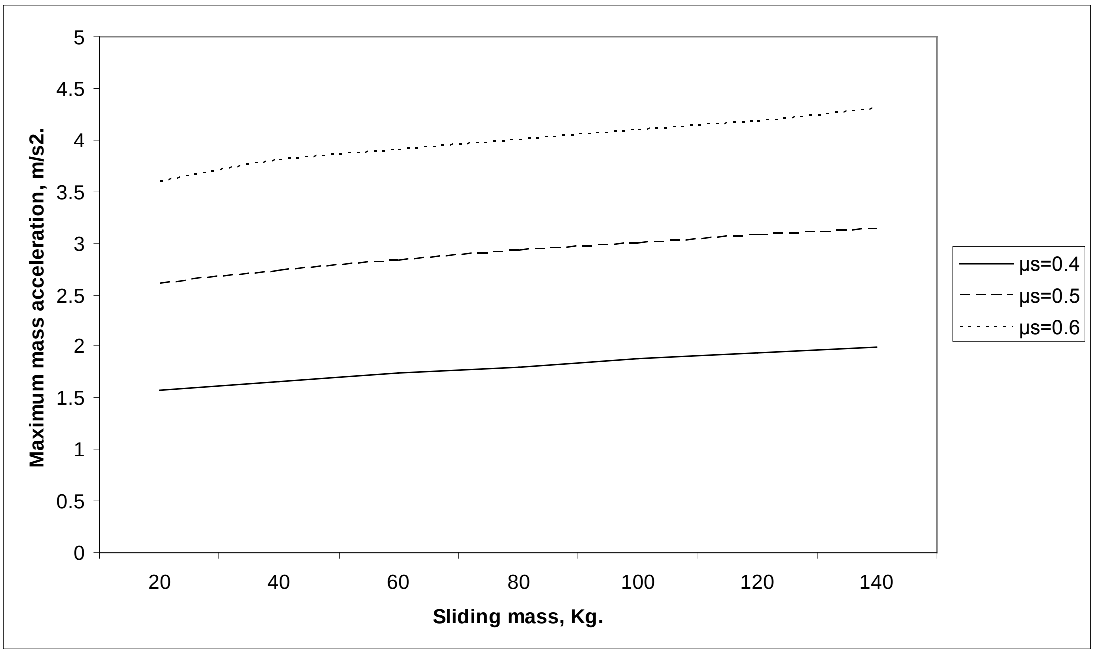


Fig.16 Relation between sliding mass and maximum slip acceleration for different static friction coefficients
(v=0.01, k=1000 N/m, μ_{dynamic} =0.25)

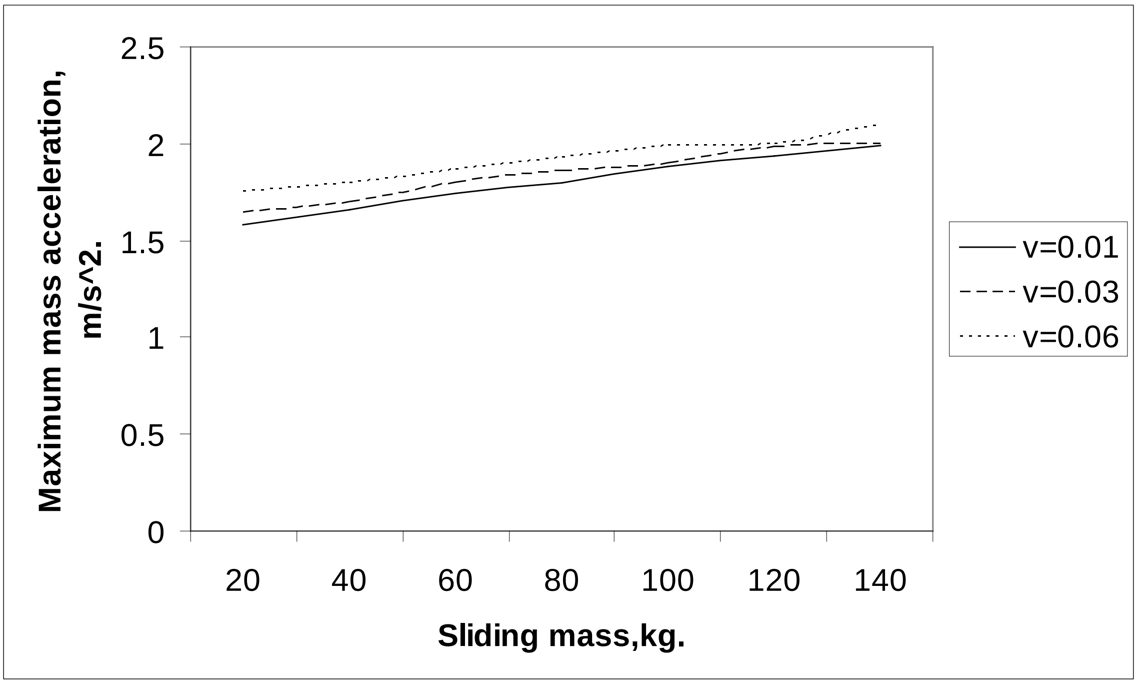


Fig.17 Relation between sliding mass and maximum slip acceleration for different drive speeds
(k=1000 N/m, μ_{dynamic} =0.25, μ_{static} =0.4)

Conclusion

It has been shown that stick-slip motion between sliding mass on horizontal plane can occur as a result of difference static and kinetic coefficients of friction. The dependence of friction vibration-induced stick-slip on the operating and design parameters has been defined explicitly through analysis.

The following concluding points can be drawn from the present study:

1- The increase in the drive speeds tends to increase the slip time and consequently the displacement and maximum sliding speed.
2- A stiff spring shortens the displacement and provides a better stick-slip free motion.
3- The starting value of the acceleration depends mainly on (μ_s-μ_d).
4- The increase in static coefficient of friction and drive speed provides a relatively longer mass displacement
5- Softer springs and high values of static coefficient of friction and drive speeds cause severe rubbing between the contacting surfaces leading to higher rates of wear.

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